Sum of Random Variables

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Based on
Probability, Random Variables and Random Signal Principles,
P.Z. Peebles,Jr. and B. Shi

Outline

Sum of two random variables

Definition

Let W be a random variable equal to

the sum of two independent random variables X and Y

$W = X + Y$

the probability distribution function is defined by

$F_W(w) = P\{W \leq w\} = P\{X + Y \leq w\}$

$x + y \leq w$

$F_W(w) = \int_{-\infty}^{\infty} \int_{x=-\infty}^{w-y} f_{X,Y}(x,y)dxdy$

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Leibniz Rule

Definition

the density function of the sum of two ramdom variables
that are statistically independent, is the convolution of their density functions

$$\frac{d}{dx}\left(\int_{a(x)}^{b(x)} f(x,t)dt\right) =$$
$$f(x,b(x))\frac{d}{dx}b(x) - f(x,a(x))\frac{d}{dx}a(x) + \int_{a(x)}^{b(x)} \frac{\partial}{\partial x} f(x,t)dt$$

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$$f_W(w) = \int_{-\infty}^{\infty} f_Y(y)\frac{d}{dw}\left(\int_{x=-\infty}^{w-y} f_X(w-y)dx\right) dy$$

$$f_W(w) = \int_{-\infty}^{\infty} f_Y(y) f_X(w-y)dy = (f*g)(w)$$