

Sum of Random Variables

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Based on
Probability, Random Variables and Random Signal Principles,
P.Z. Peebles,Jr. and B. Shi

Outline

1 Statistical Independence

Sum of two random variables

Definition

Let W be a random variable equal to the sum of two independent random variables X and Y

$$W = X + Y$$

the probability distribution function is defined by

$$F_W(w) = P\{W \leq w\} = P\{X + Y \leq w\}$$

$$x + y \leq w$$

$$F_W(w) = \int_{-\infty}^{\infty} \int_{x=-\infty}^{w-y} f_{X,Y}(x,y) dx dy$$

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Leibniz Rule

Definition

the density function of the sum of two random variables that are statistically independent, is the convolution of their density functions

$$\frac{d}{dx} \left(\int_{a(x)}^{b(x)} f(x, t) dt \right) = f(x, b(x)) \frac{d}{dx} b(x) - f(x, a(x)) \frac{d}{dx} a(x) + \int_{a(x)}^{b(x)} \frac{\partial}{\partial x} f(x, t) dt$$

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$$f_W(w) = \int_{-\infty}^{\infty} f_Y(y) \frac{d}{dw} \left(\int_{x=-\infty}^{w-y} f_X(w-y) dx \right) dy$$

$$f_W(w) = \int_{-\infty}^{\infty} f_Y(y) f_X(w-y) dy = (f * g)(w)$$

