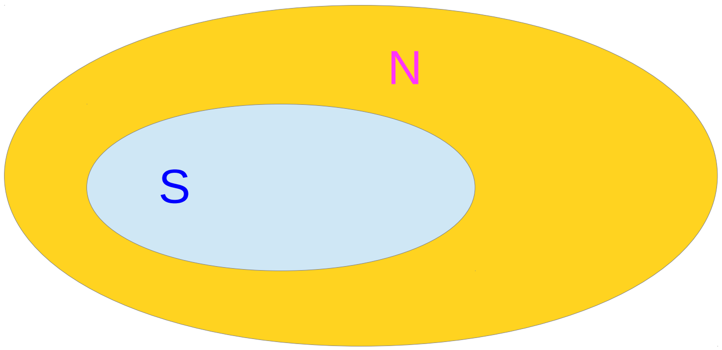
Implication (2A)

Please send corrections (or suggestions) to youngwlim@hotmail.com.

This document was produced by using OpenOffice and Octave.

Implication

$$S \longrightarrow N$$



If **S**, then N.

S implies N.

N whenever **S**.

S is sufficient for N.

S only if **N**.

not S if not **N**.

not S without **N**.

N is necessary S.

http://en.wikipedia.org/wiki/

Necessity and Sufficiency

S $\longrightarrow$ **N**

condition that guarantees N

sufficiency for N

S satisfies at least N

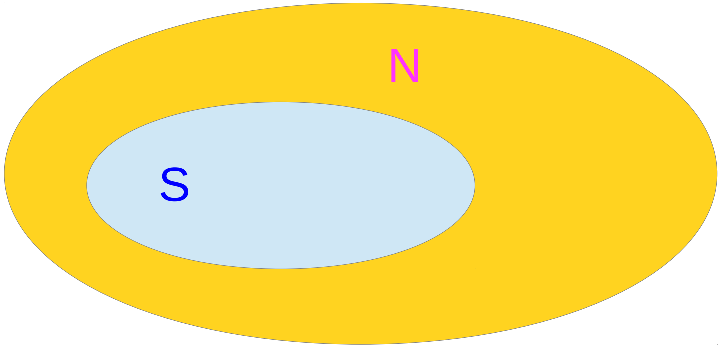
N if **S**

condition that must be satisfied for S

necessity for S

without **N**, it can't be S

S only if **N**



Other Necessary Conditions

S **N**

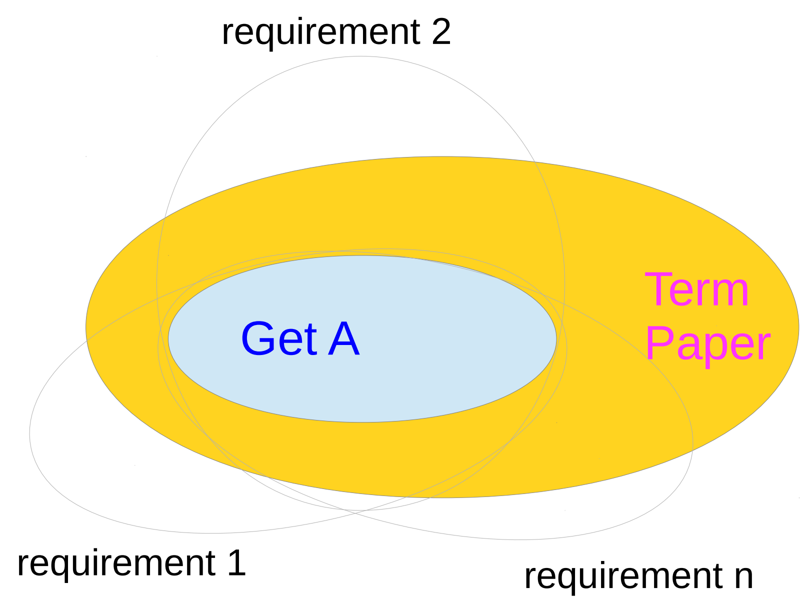
Get A → Term Paper submitted

Midterm taken

Final test taken

Rank < 15%

necessary
conditions
of S

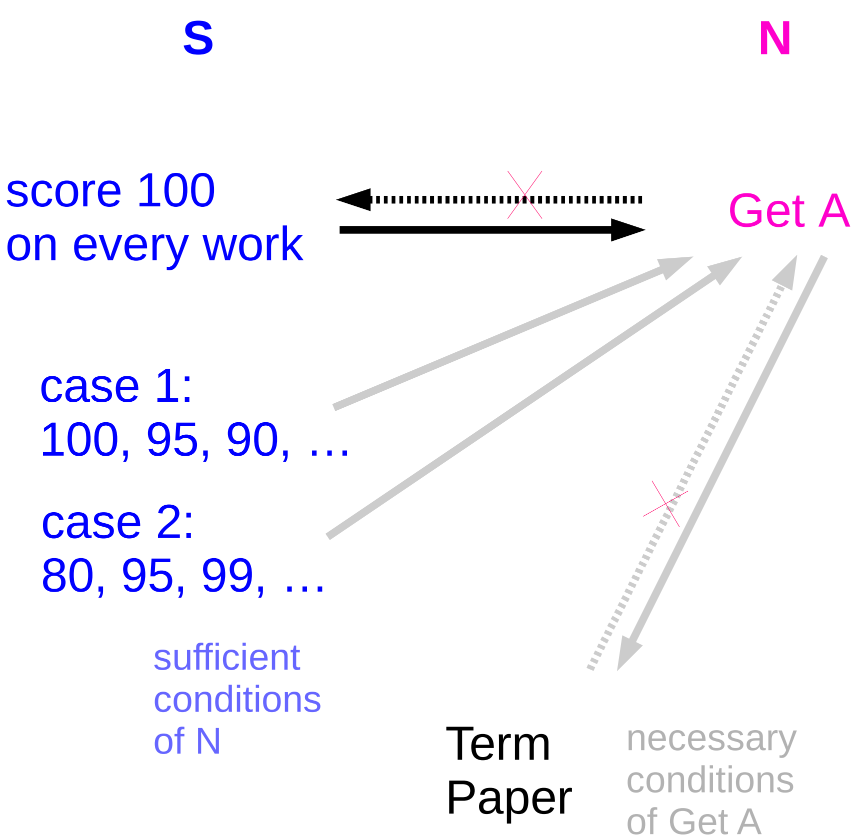


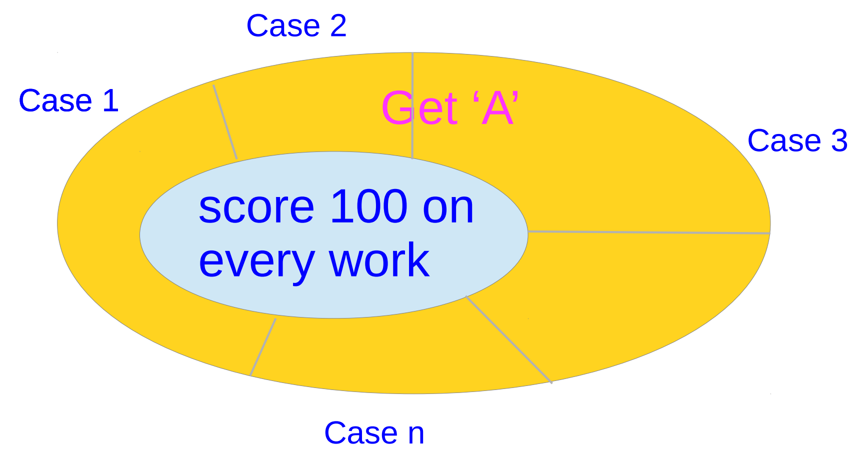
S only if N

not S if not N

http://philosophy.wisc.edu/hausman/341/Skill/nec-suf.htm`

Other Sufficient Conditions





N if S

N whenever S

http://philosophy.wisc.edu/hausman/341/Skill/nec-suf.htm`

Semantics of Implication Rule

(1) Suppose A → B is true
If A is true then B must be true

(2) Suppose A → B is false
If A is true then B must be false

(3) Suppose A → B is true
If A is false then we **do not know** whether B is true or false

	A	B	A→B
(1)	T	T	T
(2)	T	F	F
(3)	F	T	T
	F	F	T

Material Implication & Logical Implication

Given two propositions **A** and **B**,

If **A** ⇒ **B** is a tautology ← in every interpretation

It is said that **A** **logically implies** **B** (**A** ⇛ **B**)

Material Implication **A** ⇒ **B** (not a tautology) **A** → **B**

Logical Implication **A** ⇛ **B** (a tautology) **A** ⇒ **B**

A	B	A⇒B
T	T	T
T	F	F
F	T	T
F	F	T

A	B	A∧B	A∧B ⇒ A
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

tautology

A∧B ⇛ **A**

Interpretations and Models

	A	B	A⇒B	
Interpretation **I1**	T	T	T	a **Model** of A⇒B
Interpretation **I2**	T	F	F	~~a **Model** of A⇒B~~
Interpretation **I3**	F	T	T	a **Model** of A⇒B
Interpretation **I4**	F	F	T	a **Model** of A⇒B

	A	B	A∧B	A∧B ⇒ A	
Interpretation **I1**	T	T	T	T	a **Model** of A∧B
Interpretation **I2**	T	F	F	T	~~a **Model** of A∧B~~
Interpretation **I3**	F	T	F	T	~~a **Model** of A∧B~~
Interpretation **I4**	F	F	F	T	~~a **Model** of A∧B~~

Entailment **A∧B ⊨ A**, or **A∧B ⭆ A**

Entailment

if **A**→**B** holds in every model then **A** ⊨ **B**,
and conversely if **A** ⊨ **B** then **A**→**B** is true in every model

any model that makes **A∧B** true

also makes **A** true **A∧B ⊨ A**

No case : True ⇒ False

A	B	A⇒B
T	T	T
T	F	F
F	T	T
F	F	T

A	B	A∧B	A∧B ⇒ A
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

Entailment **A∧B ⊨ A**, or **A∧B ⇛ A**

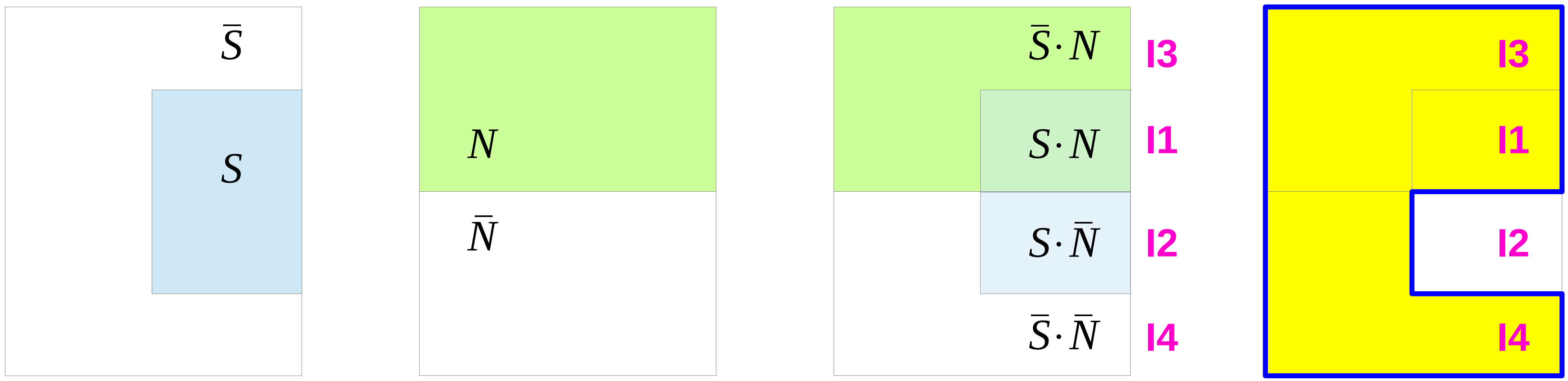
in every model

in every interpretation

Material Implication and Venn Diagram

	S	N	S⇒N
Interpretation **I1** ➡	T	T	T
Interpretation **I2** ➡	T	F	F
Interpretation **I3** ➡	F	T	T
Interpretation **I4** ➡	F	F	T

S⇒N is True



When S⇒N is a true statement

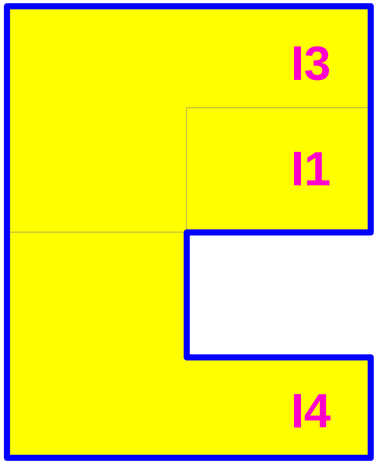
	S	N	S⇒N	
Interpretation **I1** ➡	T	T	T	(1)
Interpretation **I2** ➡	T	F	F	
Interpretation **I3** ➡	F	T	T	
Interpretation **I4** ➡	F	F	T	(2)

if the conditional statement (**S⇒N**) is a **true** statement,

(1) then the consequent **N** must be **true** **if** **S** is **true**

(2) the antecedent **S** can not be **true** **without** **N** being **true**

S⇒N is True

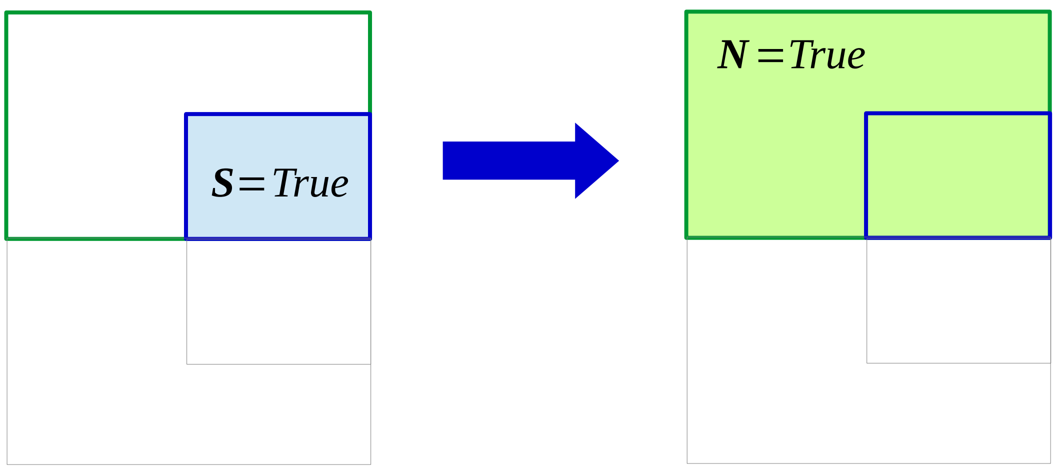


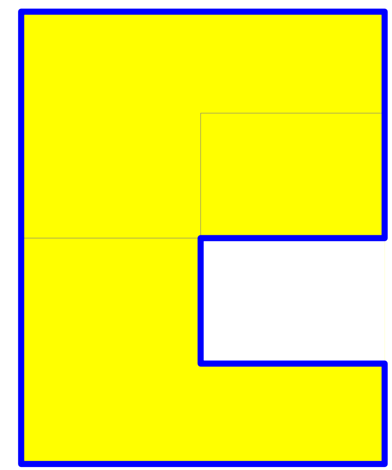
S ⊆ N

if the conditional statement **(S⇒N)** is a **true** statement,

then the consequent **N** must be **true** **if** **S** is **true**

S⇒N





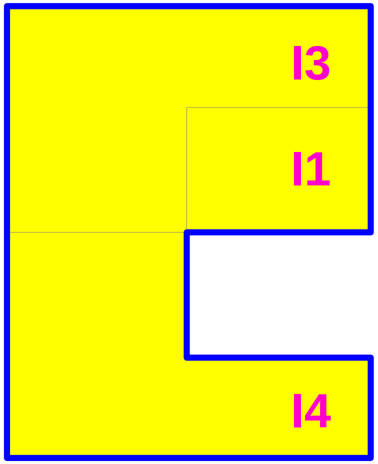
x∈**S**   x∈**N**

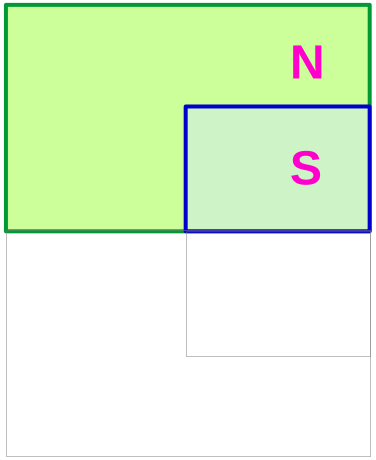
S⊆N

If **S** is **true** when S⇒N is a true statement

	S	N	S⇒N
Interpretation **I1**	T	T	T
Interpretation **I2**	T	F	F
Interpretation **I3**	F	T	T
Interpretation **I4**	F	F	T

S⇒N is True





if the conditional statement (**S⇒N**) is a **true** statement,

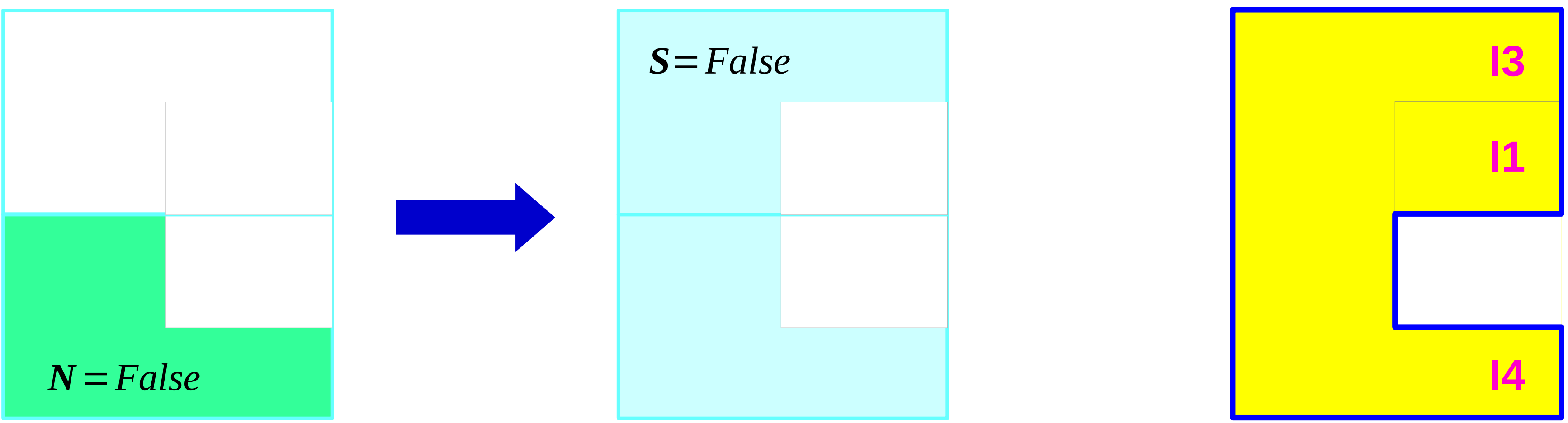
(1) then the consequent **N** must be **true** **if** **S** is **true**

~N ⊆ ~S

if the conditional statement (S⇒N) is a **true** statement,

the antecedent **S** can <u>not</u> be **true** <u>without</u> **N** being **true**

¬N⇒¬S

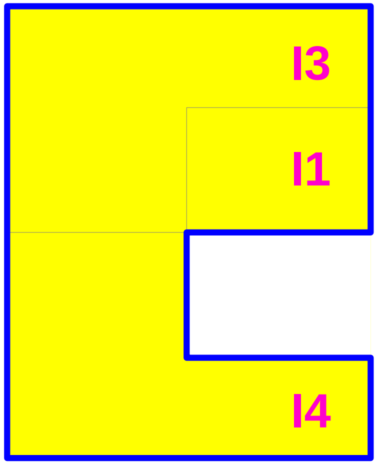


x∈¬**N** x∈¬**S** **¬N⊆¬S**

If **N** is not **true** when S⇒N is a true statement

	S	N	S⇒N
Interpretation **I1**	T	T	T
Interpretation **I2**	T	F	F
Interpretation **I3**	F	T	T
Interpretation **I4**	F ←	$\boxed{\text{F}}$	T

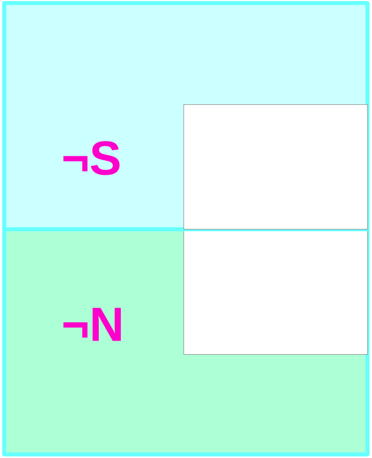
S⇒N is True



if the conditional statement (**S⇒N**) is a **true** statement,

(2) the antecedent **S** can not be **true** without **N** being **true**

¬N⇒¬S



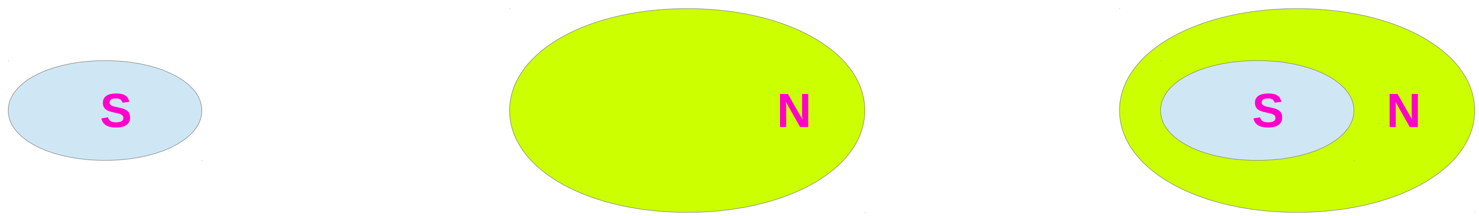
Summary

if the conditional statement **(S⇒N)** is a **true** statement,

then the consequent **N** must be **true** **if** **S** is **true**

S⇒N

S⊆N

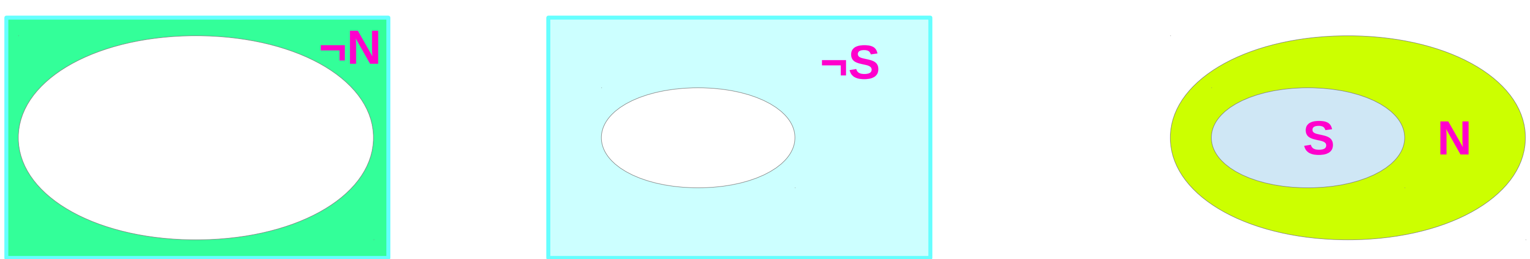


if the conditional statement **(S⇒N)** is a **true** statement,

the antecedent **S** can <u>not</u> be **true** <u>without</u> **N** being **true**

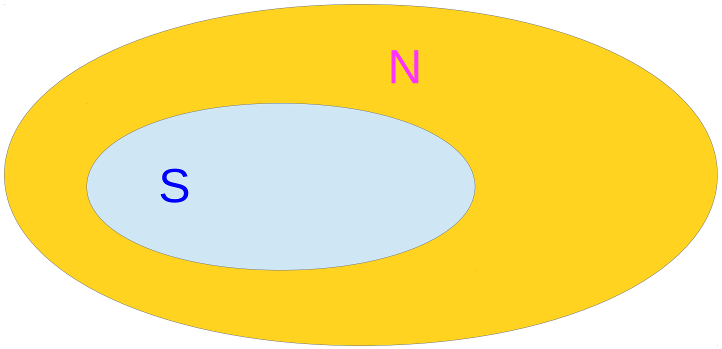
¬N⇒¬S

¬N⊆¬S



Implication

S → N



If **S**, then N.

S implies N.

N whenever **S**.

S is sufficient for N.

S only if **N**.

not S if not **N**.

not S without **N**.

N is necessary S.

http://en.wikipedia.org/wiki/

Necessity and Sufficiency

$$\mathbf{S} \longrightarrow \mathbf{N}$$

condition that guarantees N

sufficiency for N

S satisfies at least N

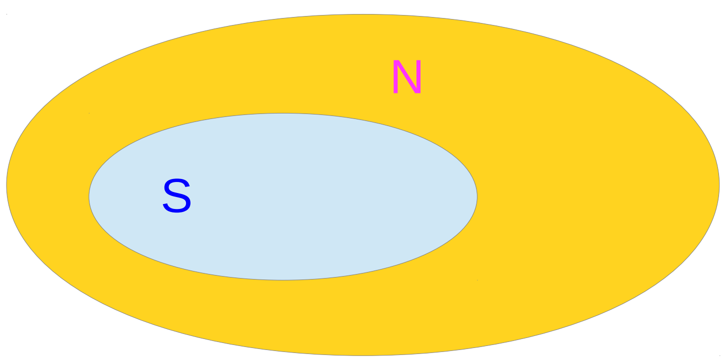
N if **S**

condition that must be satisfied for S

necessity for S

without **N**, it can't be S

S only if **N**



Other Necessary Conditions

S **N**

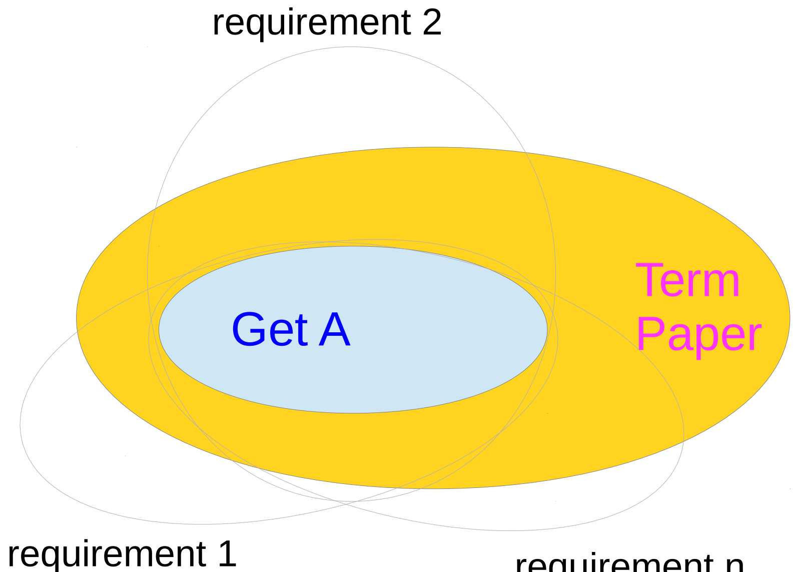
Get A → Term Paper submitted

Midterm taken

Final test taken

Rank < 15%

necessary
conditions
of S

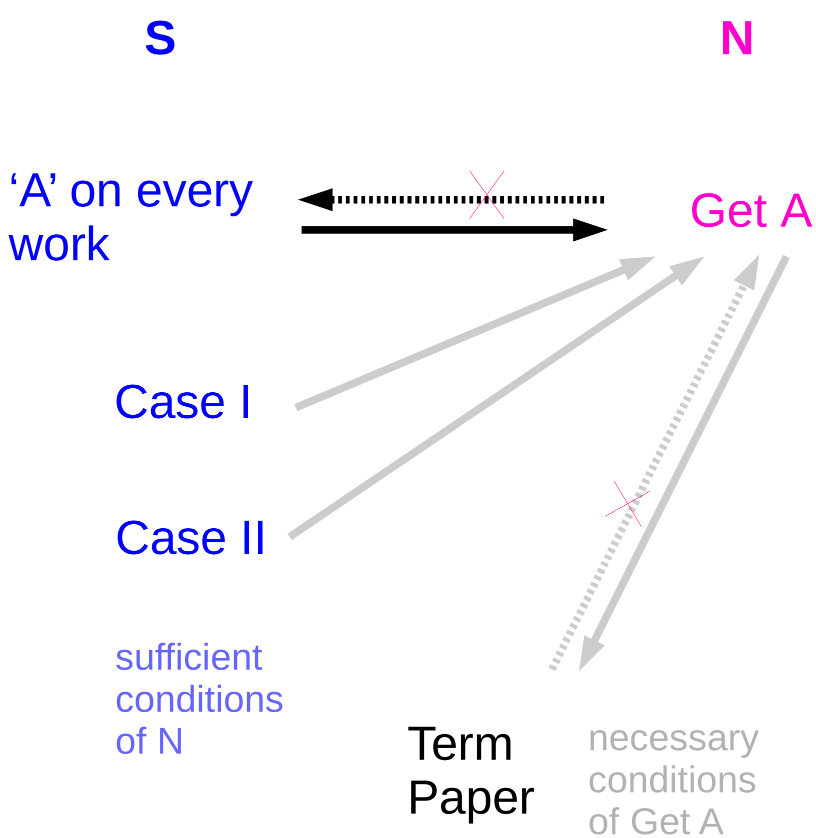


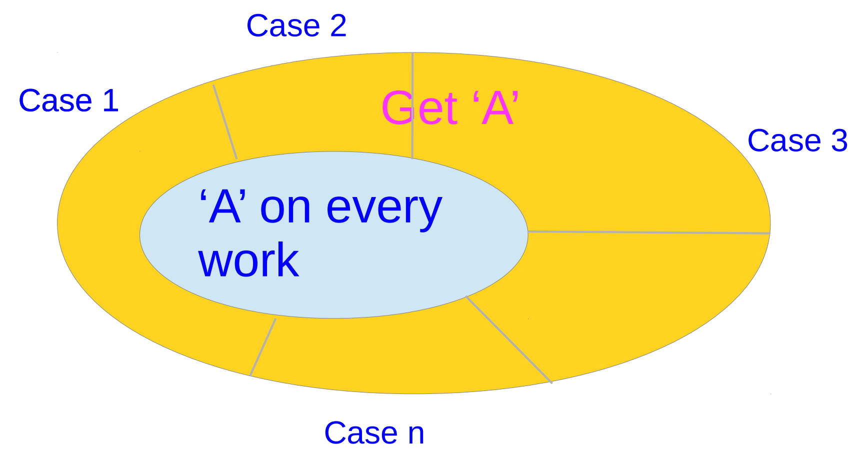
S only if N

not S if not N

http://philosophy.wisc.edu/hausman/341/Skill/nec-suf.htm`

Other Sufficient Conditions





N if S

N whenever S

http://philosophy.wisc.edu/hausman/341/Skill/nec-suf.htm`

Necessity Definition

Definition: A **necessary condition** for some state of affairs **S** is a condition that must be satisfied in order for **S** to obtain.

a necessary condition for getting an A in 341 is that a student hand in a term paper.

This means that if a student does not hand in a term paper, then a student will not get an A,

or, equivalently, if a student gets an A, then a student hands in a term paper.

http://philosophy.wisc.edu/hausman/341/Skill/nec-suf.htm

Sufficiency Definition

Definition: A **sufficient condition** for some state of affairs **N** is a condition that, if satisfied, guarantees that **N** obtains.

a sufficient condition for getting an A in 341 is
getting an A on every piece of graded work in the course.

This means that if a student gets an A on every piece of graded work in the course, then the student gets an A.

Handing in a term paper is not a sufficient condition for getting an A in the course.

It is possible to hand in a term paper and not to get an A in the course.

Getting an A on every piece of graded work is not a necessary condition for getting an A in the course.

It is possible to get an A in the course even though one fails to get an A on some piece of graded work.

http://philosophy.wisc.edu/hausman/341/Skill/nec-suf.htm`

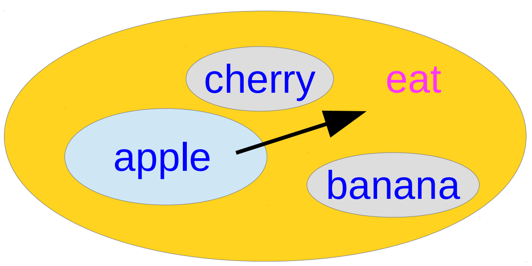
IF / Only IF

"Madison will eat the fruit if it is an apple."

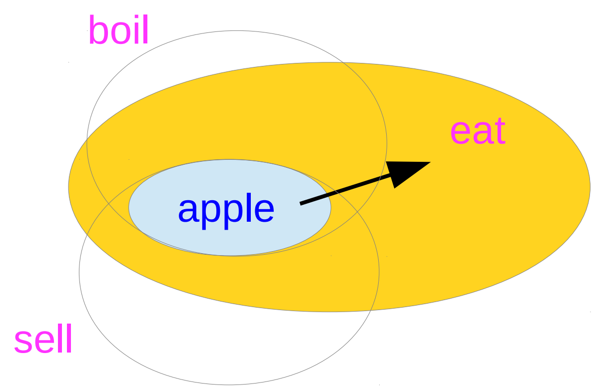
"Only if Madison will eat the fruit, is it an apple;"

"Madison will eat the fruit ← fruit is an apple"

- This states simply that Madison will eat fruits that are apples.
- It does not, however, *exclude* the possibility that Madison might also eat bananas or other types of fruit.
- All that is known for certain is that she will eat any and all apples that she happens upon.
- That the fruit is an apple is **a sufficient condition** for Madison to eat the fruit.



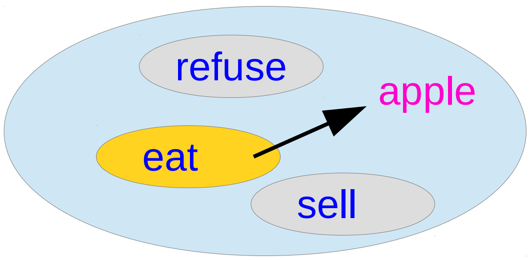
sufficient conditions



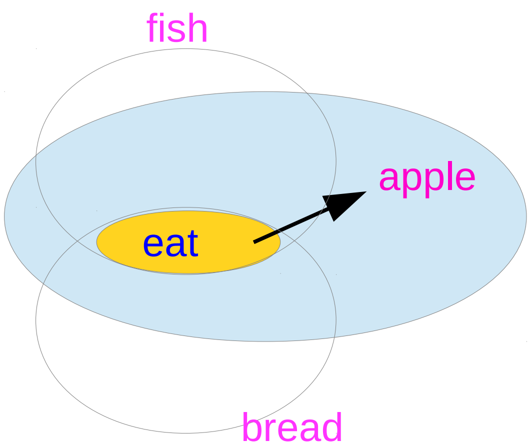
necessary conditions

http://en.wikipedia.org/wiki/Derivative

IF / Only IF



apple : a necessary condition



eat : a sufficient condition

"Madison will eat the fruit only if it is an apple."

"If Madison will eat the fruit, then it is an apple"

"Madison will eat the fruit → fruit is an apple"

- This states that the only fruit Madison will eat is an apple.
- It does not, however, *exclude* the possibility that Madison will refuse an apple if it is made available
- in contrast with (1), which requires Madison to eat any available apple.
- In this case, that a given fruit is an apple is **a necessary condition** for Madison eating it.
- It is not a sufficient condition since Madison might not eat all the apples she is given.

http://en.wikipedia.org/wiki/Derivative

IF / Only IF

"Madison will eat the fruit if it is an apple."

"Only if Madison will eat the fruit, is it an apple;"

"Madison will eat the fruit ← fruit is an apple"

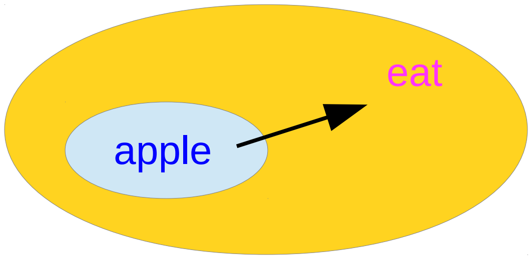
- This states simply that Madison will eat fruits that are apples.
- It does not, however, *exclude* the possibility that Madison might also eat bananas or other types of fruit.
- All that is known for certain is that she will eat any and all apples that she happens upon.
- That the fruit is an apple is **a sufficient condition** for Madison to eat the fruit.

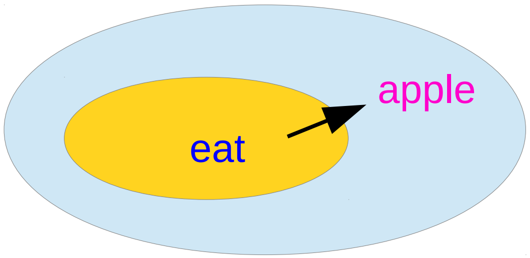
"Madison will eat the fruit only if it is an apple."

"If Madison will eat the fruit, then it is an apple"

"Madison will eat the fruit → fruit is an apple"

- This states that the only fruit Madison will eat is an apple.
- It does not, however, *exclude* the possibility that Madison will refuse an apple if it is made available
- in contrast with (1), which requires Madison to eat any available apple.
- In this case, that a given fruit is an apple is **a necessary condition** for Madison eating it.
- It is not a sufficient condition since Madison might not eat all the apples she is given.





http://en.wikipedia.org/wiki/Derivative

apple

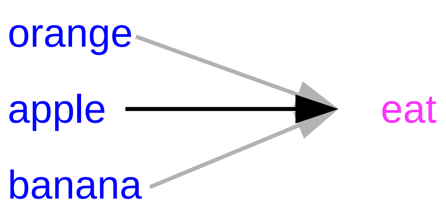
IF / Only IF

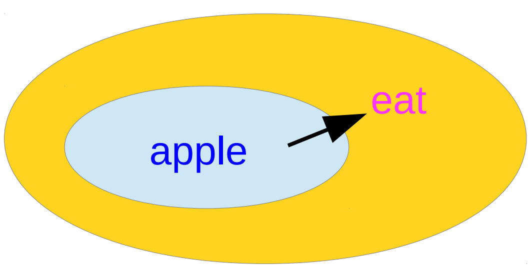
Madison will eat the fruit if it is an apple.

Only if Madison will eat the fruit, is it an apple.
If Madison will not eat the fruit, it is not an apple.

fruit is an apple → Madison will eat the fruit ←

A → E





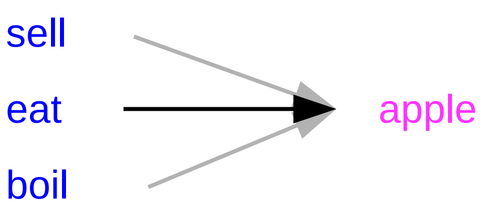
apple: a sufficient condition

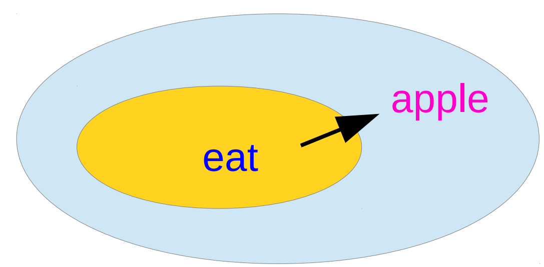
Madison will eat the fruit only if it is an apple.
Madison will not eat the fruit, if it is not an apple.

If Madison will eat the fruit, then it is an apple.

Madison will eat the fruit → fruit is an apple

E → A





apple: a necessary condition

http://en.wikipedia.org/wiki/Derivative

IFF

"Madison will eat the fruit if and only if it is an apple"

"Madison will eat the fruit ↔ fruit is an apple"

- This statement makes it clear that Madison will eat all and only those fruits that are apples.
- She will not leave any apple uneaten, and
- she will not eat any other type of fruit.
- That a given fruit is an apple is both a necessary and a sufficient condition for Madison to eat the fruit.

http://en.wikipedia.org/wiki/Derivative

Necessary Condition (1)

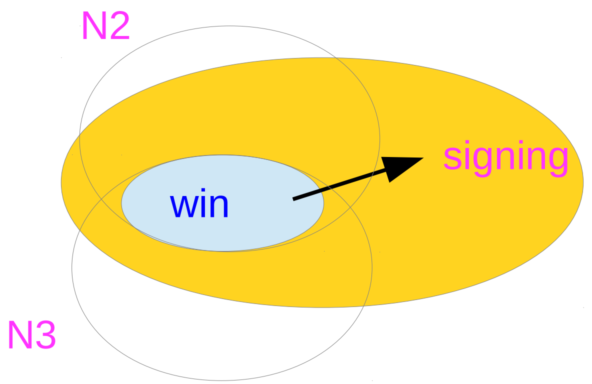
Definition: A condition that is necessary for a particular outcome to be achieved.

The condition does **not** guarantee the outcome; but

if the condition does **not** hold, the outcome will **not** achieved

if the Cubs **win** the World Series, we can be sure that they **signed** a right-handed relief pitcher,

since, **without** such a **signing**, they would **not** have **won** the World Series.



Discrete Mathematics, Johnsonbaugh

Necessary Condition (2)

if the Cubs **win** the World Series, we can be sure that they **signed** a right-handed relief pitcher,

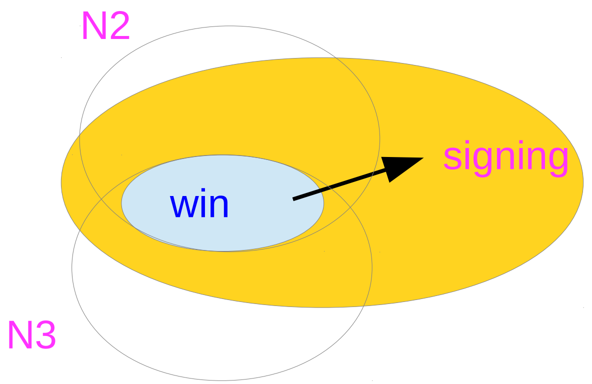
since, **without** such a **signing**, they would **not** have **won** the World Series.

The equivalent statement:

if the Cubs **win** the World Series,

then they **signed** a right-handed relief pitcher

The conclusion expresses a necessary condition



Discrete Mathematics, Johnsonbaugh

Necessary Condition (3)

if the Cubs **win** the World Series, we can be sure that they **signed** a right-handed relief pitcher,

since, **without** such a **signing**, they would **not** have **won** the World Series.

Not equivalent statement:

if the Cubs **sign** a right-handed relief pitcher,

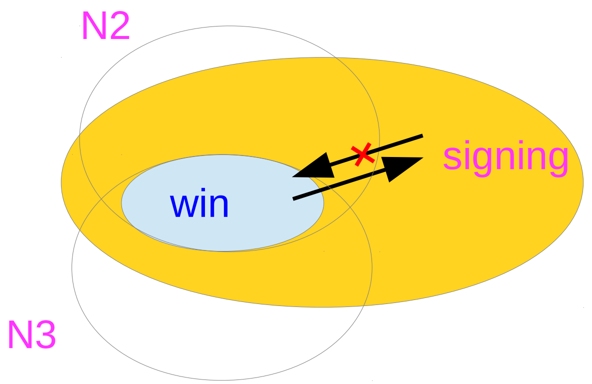
then they **win** the World Series

Signing a right-handed relief pitcher

does **not** guarantee a World Series **win**.

However, **not signing** a right-handed relief pitcher

guarantees that they will **not win** the World Series



Discrete Mathematics, Johnsonbaugh

Sufficient Condition (1)

Definition: a condition that suffices to guarantee a particular outcome.

If the condition does **not** hold,

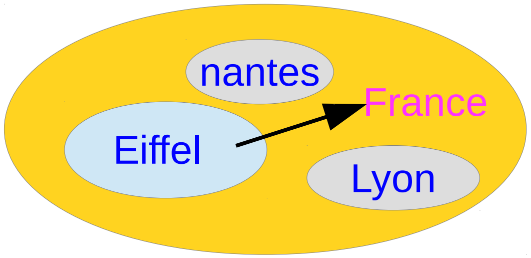
the outcome might be achieved **in other ways**

or it might **not** be achieved at all; but

If the condition does hold, the outcome guaranteed.

To be sure that Maria visits **France**, it suffices for her to go to the **Eiffel Tower**.

There are surely **other ways** to ensure that Maria visits **France**; for example, she could go to **Lyon**.



Discrete Mathematics, Johnsonbaugh

sufficient conditions

Sufficient Condition (2)

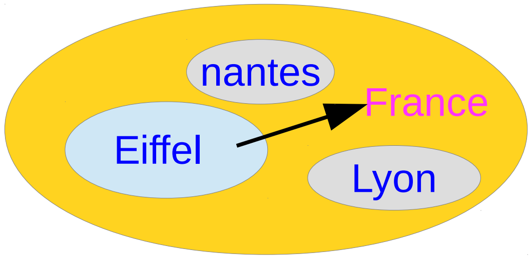
To be sure that Maria visits **France**, it suffices for her to go to the **Eiffel Tower**.

There are surely **other ways** to ensure that Maria visits **France**; for example, she could go to **Lyon**.

The equivalent statement:

If Maria goes to the **Eiffel Tower**, then she visits **France**

The hypothesis expresses a sufficient condition



Discrete Mathematics, Johnsonbaugh

sufficient conditions

Sufficient Condition (3)

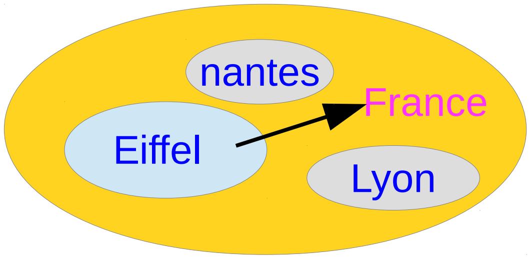
To be sure that Maria visits **France**, it suffices for her to go to the **Eiffel Tower**.

There are surely **other ways** to ensure that Maria visits **France**; for example, she could go to **Lyon**.

Not equivalent statement:

If Maria visits **France**, then she goes to the **Eiffel Tower**.

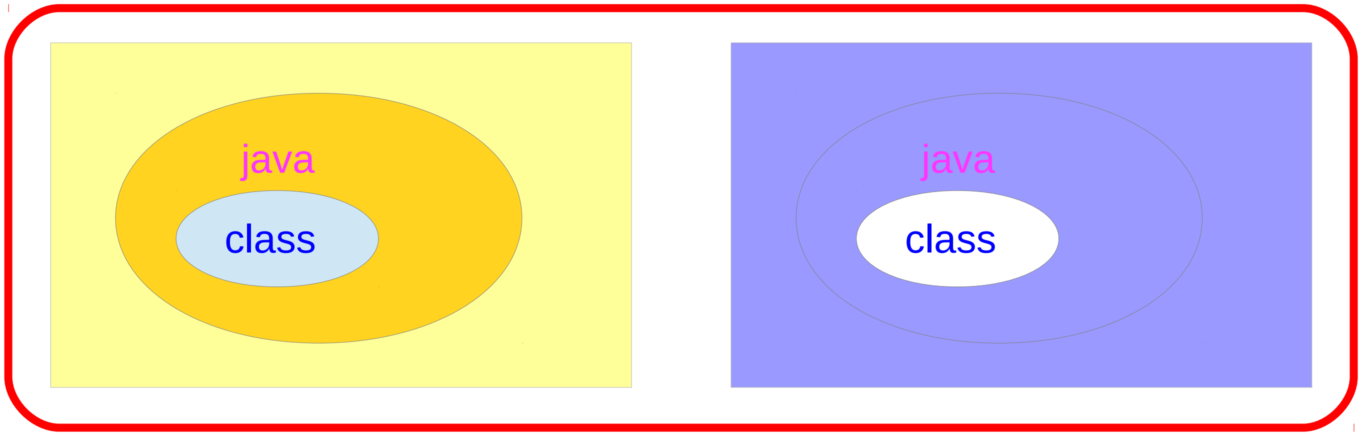
There are **ways other than** going to the **Eiffel Tower** to ensure that Maria visits **France**



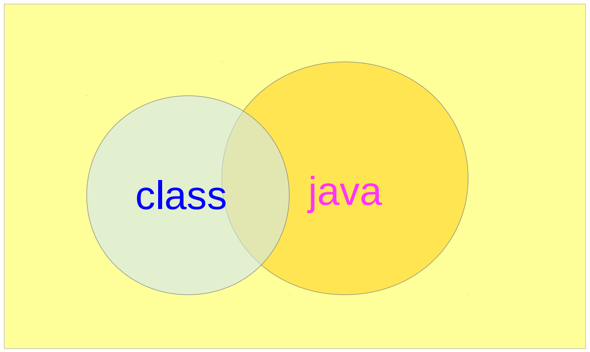
Discrete Mathematics, Johnsonbaugh

sufficient conditions

Every student in this **class** has studied **Java**.



$$\forall x\, (C(x) \rightarrow J(x))$$

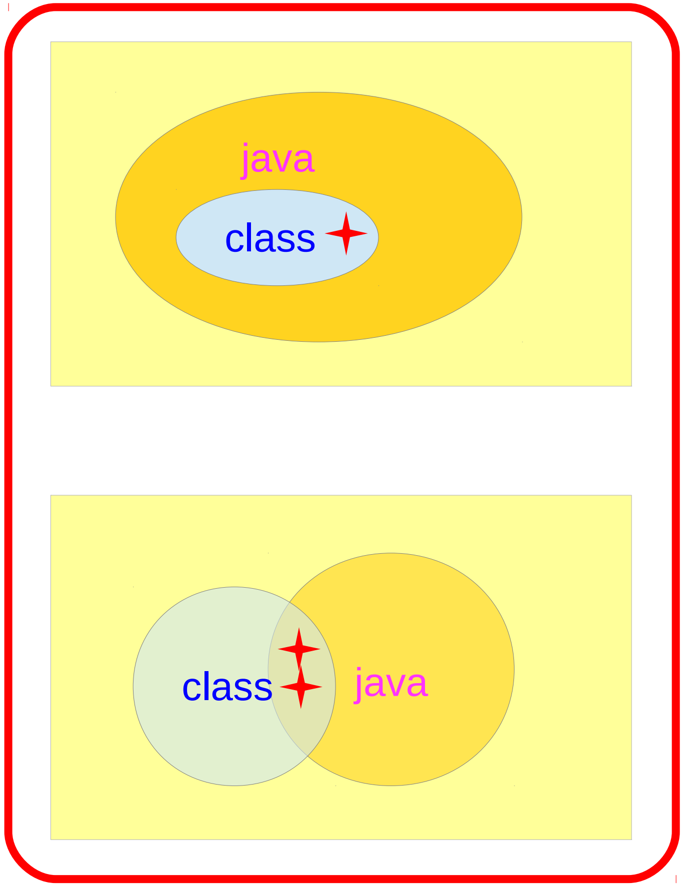


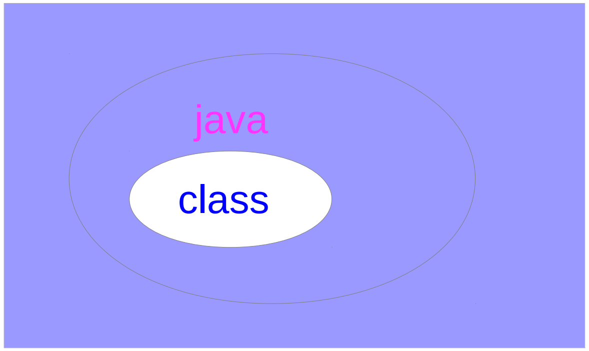
Discrete Mathematics, Johnsonbaugh

sufficient conditions

Implication in First Order Logic

Some student in this **class** has studied **Java**.





$$\exists x \left(C(x) \land J(x) \right)$$

Discrete Mathematics and its Applications, Rosen

sufficient conditions

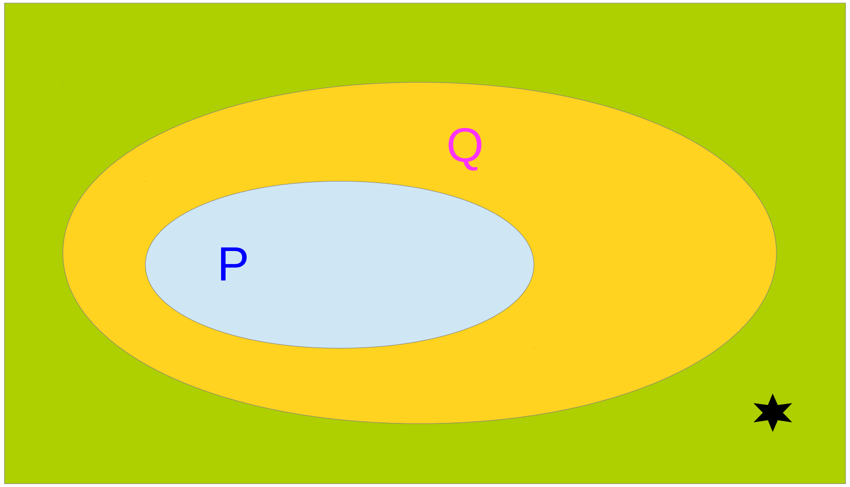
To prove implications by contradiction

P ⟶ Q

Assume this is false

$$\neg(\neg p \lor q)$$

$$p \land \neg q$$



Assume P is true and Q is false

Derive contradiction

Discrete Mathematics and its Applications, Rosen

Indirect Proof

contradiction

$\neg p$	p	q	r	$p \rightarrow q$	$p \wedge \neg q$	$r \wedge \neg r$	$(p \wedge \neg q) \rightarrow (r \wedge \neg r)$
F	T	T	T	T	F	F	T
F	T	T	F	T	F	F	T
F	T	F	T	F	T	F	F
F	T	F	F	F	T	F	F
T	F	T	T	T	F	F	T
T	F	T	F	T	F	F	T
T	F	F	T	T	F	F	T
T	F	F	F	T	F	F	T

References

[1] http://en.wikipedia.org/
[2] http://web.stanford.edu/class/archive/cs/cs103/cs103.1132/lectures/02/Small02.pdf