First Order Logic – Semantics (3A)

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1/31/17

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Based on

Contemporary Artificial Intelligence,
R.E. Neapolitan & X. Jiang

Logic and Its Applications,
Burkey & Foxley

Model

First specify a **signature**

Constant Symbols

Predicate Symbols

Function Symbols

Determines the language

Given a language

A **model** is specified

A **domain** of discourse

An **interpretation**

Model

1. a nonempty set D of **entities** called a **domain of discourse**
   - this domain is a set
   - each element in the set : entity
   - each constant symbol : one entity in the domain

2. an **interpretation**
   (a) an entity in D is assigned to each of the constant symbols.
   Normally, every entity is assigned to a constant symbol.
   (b) for each **function**,
   an entity is assigned to each possible input of entities to the **function**
   (c) the predicate '**True**' is always assigned the value T
   The predicate '**False**' is always assigned the value F
   (d) for every other **predicate**,
   the value T or F is assigned
   to each possible input of entities to the **predicate**

Model

Constant assignment

(a) an entity → the constant symbols.

Function assignments

(b) an entity → each possible input of entities to the **function**

Truth value assignments

(c) the value T →the predicate '**True**'
the value F → the predicate '**False**'

(d) for every other **predicate**,
the value T or F is assigned → every other predicate
to each possible input of entities to the **predicate**

Signature Model Examples

Signature

1. constant symbols = { Mary, Fred, Sam }
2. predicate symbols = { married, young }
   married(x, y) : arity two
   young(x) : arity one

Model

1. domain of discourse D : the set of three particular *individuals*

   - this domain is a set
   - each element in the set : entity (= individuals)
   - each constant symbol : one entity in the domain (= one individual)

2. interpretation
   (a) a different individual is assigned to each of the constant symbols

   (a) an entity in D is assigned to each of the constant symbols.
   Normally, every entity is assigned to a constant symbol.

Signature Model Examples

(b) for each **function**,
an entity is assigned to each possible input of entities to the **function**

(c) the predicate '**True**' is always assigned the value T
The predicate '**False**' is always assigned the value F

(d) the truth value assignments

young(Mary) = F, young(Fred) = F, young(Sam) = T

married(Mary, Mary) = F, married(Mary, Fred) = T, married(Mary, Sam) = F
married(Fred, Mary) = T, married(Fred, Fred) = F, married(Fred, Sam) = F
married(Sam, Mary) = F, married(Sam, Fred) = F, married(Sam, Sam) = F

(d) for every other **predicate**,
the value T or F is assigned
to each possible input of entities to the **predicate**

Signature Model Examples

Signature

1. constant symbols = { Fred, Mary, Sam }
2. predicate symbols = { love } love(x, y) : arity two
3. function symbols = { mother } mother(x) : arity one

Model

1. domain of discourse D : the set of three particular individuals
2. interpretation
   (a) a different individual is assigned to each of the constant symbols
   (b) the truth value assignments
   love(Fred, Fred) = F, love(Fred, Mary) = F, love(Fred, Ann) = F
   love(Mary, Fred) = T, love(Mary, Mary) = F, love(Mary, Ann) = T
   love(Ann, Fred) = T, love(Ann, Mary) = T, love(Ann, Ann) = F
   (c) the function assignments
   mother(Fred) = Mary, mother(Mary) = Ann, mother(Ann) = - (no assignment)

Signature Model Examples

Signature

1. constant symbols = { Fred, Mary, Sam }
2. predicate symbols = { love } love(x, y) : arity two
3. function symbols = { mother } mother(x) : arity one

Model

1. domain of discourse D : the set of three particular individuals
2. interpretation
   (a) a different individual is assigned to each of the constant symbols
   (b) the truth value assignments
   love(Fred, Fred) = F, love(Fred, Mary) = F, love(Fred, Ann) = F
   love(Mary, Fred) = T, love(Mary, Mary) = F, love(Mary, Ann) = T
   love(Ann, Fred) = T, love(Ann, Mary) = T, love(Ann, Ann) = F
   (c) the function assignments
   mother(Fred) = Mary, mother(Mary) = Ann, mother(Ann) = - (no assignment)

The truth value and sentences (1)

The truth values of **all sentences** are assigned :

1. the truth values for **sentences** developed with the symbols ¬, ∧, ∨, ⇒, ⇔ are assigned as in propositional logic.

2. the truth values for two terms connected by the = symbol is
T if both terms refer to the same entity; otherwise it is F

3. the truth values for ∀x p(x) has value
T if p(x) has value T for **every assignment** to x of an entity in the domain D; otherwise it has value F

4. the truth values for ∃x p(x) has value
T if p(x) has value T for **at least one assignment** to x of an entity in the domain D; otherwise it has value F

The truth value and sentences (1)

5. the operator **precedence** is as follows $\neg$, $=$, $\wedge$, $\vee$, $\Rightarrow$, $\Leftrightarrow$

6. the **quantifiers** have precedence over the operators

7. **parentheses** change the order of the precedence

Formulas and Sentences

An **formula**

- A **atomic formula**
- The operator ¬ followed by a **formula**
- Two formulas separated by $\wedge$, $\vee$, $\Rightarrow$, $\Leftrightarrow$
- A quantifier following by a variable followed by a formula

A **sentence**

- A **formula** with no free variables

∀x love(x,y)	: free variable y	: not a sentence
∀x tall(x)	: no free variable	: a sentence

Finding the truth value

Find the truth values of **all sentences**

1. ¬, ∧, ∨, ⇒, ⇔
2. = symbol
3. ∀x p(x)
4. ∃x p(x)
5. the **operator precedence** is as follows ¬, =, ∧, ∨, ⇒, ⇔
6. the **quantifiers** (∀, ∃) have precedence over the **operators**
7. **parentheses** change the order of the precedence

Formal Language

Signature

∀ ∃

Constant Symbols = {Socrates, Plato, Zeus, Fido}
Predicate Symbols = {human, mortal, legs} all arity one

Model
D: the set of these four particular individuals

Interpretation

(a) a different individual is assigned to each of the constant symbols
(b) the truth value assignment
human(Socrates)=T, human(Plato)=T, human(Zeus)=F, human(Fido)=F
mortal(Socrates)=T, mortal(Plato)=T, mortal(Zeus)=F, mortal(Fido)=T
legs(Socrates)=T, legs(Plato)=T, legs(Zeus)=T, legs(Fido)=T

Formal Language

Sentence 1: human(Zeus) ∧ human(Fido) ∨ human(Socrates) = T
F ∧ F ∨ T

Sentence 2: human(Zeus) ∧ (human(Fido) ∨ human(Socrates)) = F
F ∧ (F ∨ T)

Sentence 3: ∀x human(x) = F
human(Zeus)=F, human(Fido)=F

Sentence 4: ∀x mortal(x) = F
mortal(Zeus)=F

Sentence 5: ∀x legs(x) = T
legs(Socrates)=T, legs(Plato)=T, legs(Zeus)=T, legs(Fido)=T

Sentence 6: ∃x human(x) = T
human(Socrates)=T, human(Plato)=T

Sentence 7: ∀x (human(x) ⇒ mortal(x)) = T

Formal Language

Sentence 7: ∀x (human(x) ⇒ mortal(x)) = T

human(Socrates)=T, human(Plato)=T, human(Zeus)=F, human(Fido)=F
mortal(Socrates)=T, mortal(Plato)=T, mortal(Zeus)=F, mortal(Fido)=T
T ⇒ T = T T ⇒ T = T F ⇒ F = T F ⇒ T = T

References

[1] en.wikipedia.org
[2] en.wiktionary.org
[3] U. Endriss, “Lecture Notes : Introduction to Prolog Programming”
[4] http://www.learnprolognow.org/ Learn Prolog Now!
[5] http://www.csupomona.edu/~jrfisher/www/prolog_tutorial
[6] www.cse.unsw.edu.au/~billw/cs9414/notes/prolog/intro.html
[7] www.cse.unsw.edu.au/~billw/dictionaries/prolog/negation.html
[8] http://ilppp.cs.lth.se/, P. Nugues,` An Intro to Lang Processing with Perl and Prolog