# Characteristics of Multiple Random Variables

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Based on
Probability, Random Variables and Random Signal Principles,
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# Outline

1. Simulation of Multiple Random Variables

# Estimate of mean
N Gaussian random variables

### Definition

$$\hat{\bar{x}}_N = \frac{1}{N} \sum_{n=1}^{N} x_n$$

$$\hat{\bar{X}}_N = \frac{1}{N} \sum_{n=1}^{N} X_n$$

# Mean of the estimate of mean
$N$ Gaussian random variables

### Definition

$$E\left[\hat{\overline{X}}_N\right] = E\left[\frac{1}{N}\sum_{n=1}^{N} X_n\right] = \frac{1}{N}\sum_{n=1}^{N} E[X_n] = \overline{X}$$

# Variance of the estimate of mean (1)
$N$ Gaussian random variables

### Definition

$$E\left[\left(\hat{\overline{X}}_N - \overline{X}\right)^2\right] = \sigma_{X_N}^2 = E\left[\hat{\overline{X}}_N^2 - 2\overline{X}\hat{\overline{X}}_N + \overline{X}^2\right]$$

$$= E\left[\hat{\overline{X}}_N^2\right] - \overline{X}^2 = -\overline{X}^2 + E\left[\frac{1}{N}\sum_{m=1}^{N} X_m \frac{1}{N}\sum_{n=1}^{N} X_n\right]$$

$$= -\overline{X}^2 + \frac{1}{N^2}\sum_{m=1}^{N}\sum_{n=1}^{N} E\left[X_m X_n\right]$$

# Variance of the estimate of mean (2)
$N$ Gaussian random variables

### Definition

$$E[X_m X_n] = E[X^2] \qquad (n = m)$$

$$E[X_m X_n] = \overline{X}^2 \qquad (n \neq m)$$

$$\sigma_{X_N}^2 = -\overline{X}^2 + \frac{1}{N^2}\left[NE[X^2] + (N^2 - N)\overline{X}^2\right]$$

$$= \frac{1}{N}\left[E[X^2] - \overline{X}^2\right] = \sigma_X^2/N$$

# Variance of the estimate of mean (3)
*N* Gaussian random variables

### Definition

$$P\left\{\left|\hat{\overline{X}}_N - \overline{X}\right| < \varepsilon\right\} \geq 1 - (\sigma_{X_N}^2/\varepsilon^2) = 1 - \frac{\sigma_X^2}{N\varepsilon^2}$$

$$\widehat{X_N^2} = \frac{1}{N}\sum_{n=1}^{N} X_n^2$$

$$\widehat{\sigma_N^2} = \frac{1}{N-1}\sum_{n=1}^{N}(X_n - \hat{\overline{X}}_N)^2$$