

# Angle Recoding 2. Wu

## 3. MVR

20180929 Sat

Copyright (c) 2015 - 2018 Young W. Lim.

Permission is granted to copy, distribute and/or modify this document under the terms of the GNU Free Documentation License, Version 1.2 or any later version published by the Free Software Foundation; with no Invariant Sections, no Front-Cover Texts, and no Back-Cover Texts. A copy of the license is included in the section entitled "GNU Free Documentation License".

## ② MVR (Modified Vector Rotational)

two modifications

① **repetition** of elementary angles

each micro-rotation of elementary angle  
can be performed repeatedly

- more possible combinations
- smaller  $\xi_m$

② **confinement** of total micro-rotation number

confine the iteration number

in the micro-rotation phase  
to  $R_m$  ( $R_m \ll W$ )

The role of  $R_m$  is quite similar  
to the **number of non-zero digit**  
 $N_D$  in CSD recoding scheme

MV2

\* repetition of elementary angles

$$\theta = a(2) * 2 = \tan^{-1}(2^{-2}) * 2$$

AR

$$+ a(1) + a(5) - a(8) - a(10) - a(14)$$

k= 0 theta= 0.4899573 ik= 1 uik=+1 a[ 1]= 0.4636476, new theta= 0.0263097  
 k= 1 theta= 0.0263097 ik= 5 uik=+1 a[ 5]= 0.0312398, new theta=-0.0049301  
 k= 2 theta=-0.0049301 ik= 8 uik=-1 a[ 8]= 0.0039062, new theta=-0.0010239  
 k= 3 theta=-0.0010239 ik=10 uik=-1 a[10]= 0.0009766, new theta=-0.0000473  
 k= 4 theta=-0.0000473 ik=14 uik=-1 a[14]= 0.0000610, new theta= 0.0000137

Conventional  
CORDIC

$$\begin{aligned}
 &+ a(0) - a(1) + a(2) - a(3) + a(4) - a(5) + a(6) + a(7) \\
 &- a(8) - a(9) - a(10) - a(11) + a(12) + a(13) - a(14) - a(15)
 \end{aligned}$$

k= 0 theta= 0.4899573 u[ 0]=+1 a[ 0]= 0.7853982, new theta=-0.2954408  
 k= 1 theta=-0.2954408 u[ 1]=-1 a[ 1]= 0.4636476, new theta= 0.1682068  
 k= 2 theta= 0.1682068 u[ 2]=+1 a[ 2]= 0.2449787, new theta=-0.0767719  
 k= 3 theta=-0.0767719 u[ 3]=-1 a[ 3]= 0.1243550, new theta= 0.0475831  
 k= 4 theta= 0.0475831 u[ 4]=+1 a[ 4]= 0.0624188, new theta=-0.0148357  
 k= 5 theta=-0.0148357 u[ 5]=-1 a[ 5]= 0.0312398, new theta= 0.0164041  
 k= 6 theta= 0.0164041 u[ 6]=+1 a[ 6]= 0.0156237, new theta= 0.0007804  
 k= 7 theta= 0.0007804 u[ 7]=+1 a[ 7]= 0.0078123, new theta=-0.0070319  
 k= 8 theta=-0.0070319 u[ 8]=-1 a[ 8]= 0.0039062, new theta=-0.0031257  
 k= 9 theta=-0.0031257 u[ 9]=-1 a[ 9]= 0.0019531, new theta=-0.0011726  
 k=10 theta=-0.0011726 u[10]=-1 a[10]= 0.0009766, new theta=-0.0001960  
 k=11 theta=-0.0001960 u[11]=-1 a[11]= 0.0004883, new theta= 0.0002923  
 k=12 theta= 0.0002923 u[12]=+1 a[12]= 0.0002441, new theta= 0.0000481  
 k=13 theta= 0.0000481 u[13]=+1 a[13]= 0.0001221, new theta=-0.0000740  
 k=14 theta=-0.0000740 u[14]=-1 a[14]= 0.0000610, new theta=-0.0000129  
 k=15 theta=-0.0000129 u[15]=-1 a[15]= 0.0000305, new theta= 0.0000176

data :  $W$ -bit word length

the iteration number :  $N$   $N \leq W$

the restricted iteration number :  $R_m$   $R_m \ll W$

$$i \in [0, R_m - 1]$$

$$R_m < W$$

the angle quantization error

$$\xi_{m, \text{MVR}} \triangleq \theta - \sum_{i=0}^{R_m-1} \alpha(i) a(s(i))$$

the rotational sequence

$$s(i) \in \{0, 1, \dots, W-1\}$$

\* repetition allowed

the micro-rotation angle

in the  $i$ -th iteration

the directional sequence

$$\alpha(i) \in \{-1, 0, +1\}$$

the direction of the  $i$ -th

micro-rotation of  $a(s(i))$

$$\alpha(i) a(s(i)) = \tilde{\theta}(i)$$

the rotational sequence

$$s(i) \in \{0, 1, \dots, W-1\} \quad [0, 3, 6, 7]$$

the directional sequence

$$\alpha(i) \in \{-1, 0, +1\} \quad [1, -1, -1, 1]$$

$$\text{atan}(2^0) - \text{atan}(2^{-3}) - \text{atan}(2^{-6}) + \text{atan}(2^{-7})$$

$$\alpha(i) \alpha(s(i)) = \tilde{\theta}(j)$$

MVR-CORDIC Algorithm with $R_n = 4$	Greedy Algorithm	3	$\bar{\alpha} = [1 \ -1 \ -1 \ -1]$ $\bar{s} = [0 \ 3 \ 6 \ 7]$	$5.2891 \cdot 10^{-4}$
	Semi-greedy Algorithm ( $D = 2$ )	4	$\bar{\alpha} = [1 \ -1 \ -1 \ 1]$ $\bar{s} = [0 \ 3 \ 5 \ 7]$	$5.2033 \cdot 10^{-4}$
	TBS Algorithm	5	$\bar{\alpha} = [1 \ 1 \ -1 \ -1]$ $\bar{s} = [1 \ 2 \ 4 \ 7]$	$2.5911 \cdot 10^{-4}$

```
>> s = [0, 3, 6, 7]
>> alpha = [1, -1, -1, -1]
>> sum(atan(2.^(-s) .* alpha))
ans = 0.63761
>>
```

```
>> s = [0, 3, 5, 7]
>> alpha = [1, -1, -1, 1]
>> sum(atan(2.^(-s) .* alpha))
ans = 0.63762
```

```
>> alpha = [1, 1, -1, -1]
>> s = [1, 2, 4, 7]
>> sum(atan(2.^(-s) .* alpha))
ans = 0.63840
```

```
0 7.85398163397448e-01
1 4.63647609000806e-01
2 2.44978663126864e-01
3 1.24354994546761e-01
4 6.24188099959574e-02
5 3.12398334302683e-02
6 1.56237286204768e-02
7 7.81234106010111e-03
8 3.90623013196697e-03
9 1.95312251647882e-03
10 9.76562189559319e-04
11 4.88281211194898e-04
12 2.44140620149362e-04
13 1.22070311893670e-04
14 6.10351561742088e-05
15 3.05175781155261e-05
```

$w=16$

0	7.85398163397448e-01	s(0)
1	4.63647609000806e-01	.
2	2.44978663126864e-01	.
3	1.24354994546761e-01	s(1)
4	6.24188099959574e-02	.
5	3.12398334302683e-02	.
6	1.56237286204768e-02	s(2)
7	7.81234106010111e-03	s(3)
8	3.90623013196697e-03	.
9	1.95312251647882e-03	.
10	9.76562189559319e-04	.
11	4.88281211194898e-04	.
12	2.44140620149362e-04	.
13	1.22070311893670e-04	.
14	6.10351561742088e-05	.
15	3.05175781155261e-05	.

0	7.85398163397448e-01	s(0)
3	1.24354994546761e-01	s(1)
6	1.56237286204768e-02	s(2)
7	7.81234106010111e-03	s(3)

$R_m = 4$

s(0)=0  
s(1)=3  
s(2)=6  
s(3)=7

0	7.85398163397448e-01
1	4.63647609000806e-01
2	2.44978663126864e-01
3	1.24354994546761e-01
4	6.24188099959574e-02
5	3.12398334302683e-02
6	1.56237286204768e-02
7	7.81234106010111e-03
8	3.90623013196697e-03
9	1.95312251647882e-03
10	9.76562189559319e-04
11	4.88281211194898e-04
12	2.44140620149362e-04
13	1.22070311893670e-04
14	6.10351561742088e-05
15	3.05175781155261e-05



# AQ & MVR CORDIC

$$\xi_{m, MVR} \triangleq \theta - \left[ \sum_{j=0}^{R_m-1} \alpha(j) a(s(j)) \right]$$

the rotational sequence  $s(j)$

$$j = 0, 1, 2, \dots, R_m-1$$



$$s(j) \in \{0, 1, \dots, W-1\} \quad \text{rotational sequence}$$

determines the micro-rotation angle  $a(s(j))$   
in the  $j$ -th iteration

the directional sequence  $\alpha(j)$

$$\alpha(j) \in \{-1, 0, +1\}$$

controls the direction of the  $j$ -th  
micro-rotation of  $a(s(j))$

$$\alpha(j) a(s(j)) = \tilde{\theta}(j)$$

$$\begin{array}{l} i = 0, 1, 2, 3, \dots, W-1 \\ s(j) = 0, 1, 2, 3, \dots, W-1 \quad \text{rotational sequence} \\ \alpha(j) = -1, 0, 0, +1, \dots, -1 \quad \text{directional sequence} \\ j = 0, -, -, 1, \dots, R_m-1 \quad \text{effective iteration number} \\ R_m \ll W \end{array}$$

$i$        $j$        $S(j)$   
①      0       $S(0) = 0$

1

2

3

④      1, 4       $S(1) = 4, S(4) = 4$

repetition allowed

⑤      2       $S(2) = 5$

6

7

⑧      3       $S(3) = 8$

9

rotational  
sequence

10

effective  
iteration  
number

11

12

13

14

$W-1 = 15$

$i$	Conventional	$j$	$S(j)$
0	$S(0) = 0$	0	$S(0) = 0$
1	$S(1) = 1$		
2	$S(2) = 2$		
3	$S(3) = 3$		
4	$S(4) = 4$	1	$S(1) = 4$
5	$S(5) = 5$	2	$S(2) = 5$
6	$S(6) = 6$		
7	$S(7) = 7$		
8	$S(8) = 8$	3	$S(3) = 8$
9	$S(9) = 9$		
10	$S(10) = 10$		
11	$S(11) = 11$		
12	$S(12) = 12$		
13	$S(13) = 13$		
14	$S(14) = 14$		
15	$S(15) = 15$		

effective iteration number

rotational sequence

$W-1 =$

sub-angle  $(\alpha(j) a(s(j))) \sim \tilde{\theta}(j)$

$$\xi_{m,AR} = \theta - \left[ \sum_{j=0}^{N'-1} \tilde{\theta}(j) \right], \quad \tilde{\theta}(j) = \tan^{-1}(\alpha(j) \cdot 2^{-s(j)})$$
$$= \theta - \left[ \sum_{j=0}^{N'-1} \tan^{-1}(\alpha(j) \cdot 2^{-s(j)}) \right]$$

$$N' \triangleq \sum_{j=0}^{N-1} |\mu(j)| \quad \text{the effective iteration number}$$

EAS formed by MVR-CORDIC  
is the same as AR  
also performs AQ

the EAS consists of all possible values of  $\tilde{\theta}(j)$

the EAS  $S_1$  in AR

$$S_1 = \{ \tan^{-1}(\alpha^* \cdot 2^{-s^*}) : \alpha^* \in \{-1, 0, +1\}, s^* \in \{0, 1, \dots, N-1\} \}$$

The major difference

1) the total number of sub-angles  $N_A$

the total iteration number

in the micro-rotation phase

is kept fixed to a pre-defined value of  $R_m$

$$N_A = R_m$$

2) the sub-angle  $\theta_i$  corresponds to  $\alpha^{(j)} a(s^{(j)})$

$$\theta_j = \alpha^{(j)} a(s^{(j)}) = \tilde{\theta}_j$$

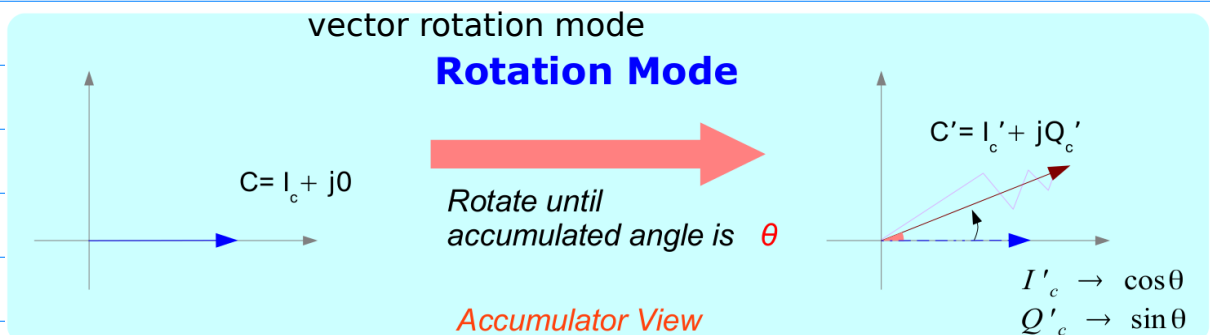
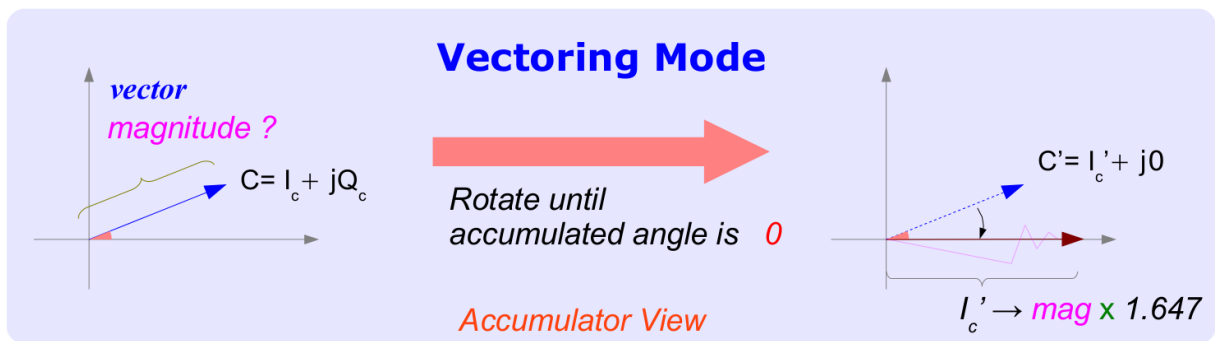
# MVR (Modified Vector Rotation)

1) Repeat of Elementary Angles  $\theta_i, \theta_i$

2) fixed total micro-rotation Number  $R_m$

\* Vector Rotation Mode

\* and the rotation angles are known in advance



# Modified Vector Rotational MVR CORDIC

- reduce the iteration number
- maintaining the SQNR performance
- modifying the basic microrotation procedure

## Three Searching Algorithm

- ① the selective prerotation
- ② the selective scaling
- ③ iteration-tradeoff scheme

# Optimization Problem

EAS point of view

Given  $\theta$ , find the combination of  $R_m$  elementary angles from EAS  $S_i$ , such that the angle quantization error  $|\xi_{m, \text{MUR}}|$  is minimized.

semi-greedy algorithm  
trade offs between computational complexities  
and performance



key issue in the MVR-CORDIC  
is to find the best sequences of  
 $s(i)$  and  $\alpha(i)$  to minimize  $|\xi_m|$   
subject to the constraint that  
the total iteration number is confined to  $R_m$

- 1) Greedy Algorithm
- 2) Exhaustive Algorithm
- 3) Semigreedy Algorithm

data :  $W$ -bit word length

the iteration number :  $N$   $N \leq W$

the restricted iteration number :  $R_m$   $R_m \ll W$

# Hu's greedy algorithm

$$\theta^{(0)} = \theta, \quad \{\mu^{(i)} = 0, \quad 0 \leq i \leq N-1\}, \quad k=0$$

repeat until  $|\theta^{(k)}| < \alpha(N-1)$  Do

choose  $i_k, \quad 0 \leq i_k \leq N-1$

$$| |\theta^{(k)}| - a(i_k) | = \underset{0 \leq i \leq N-1}{\text{Min}} | |\theta^{(k)}| - a(i_k) |$$

$$\theta^{(k+1)} = \theta^{(k)} - \mu(i_k) a(i_k)$$

$$\mu(i_k) = \text{Sign}(\theta^{(k)})$$

$$J(i) = | \theta^{(i)} - \alpha(i) a(i) | \text{ is minimized}$$

# 1) Greedy Algorithm

given  $\theta$ ,  $W$ ,  $R_m$

try to approach the target rotation angle,  $\theta$ , step by step  
in each step, decisions are made on  $\alpha(i)$  and  $s(i)$   
by choosing the best combination of  $\alpha(i)$  and  $s(i)$   
so as to minimize  $|\xi_m|$

$\alpha(i)$  and  $s(i)$  are determined such that

the error function  $J(i) = |\theta(i) - \alpha(i) a(s(i))|$  is minimized

$\theta(i)$ : the residue angle in the  $i$ -th step

$$\theta(i) = \theta - \sum_{m=0}^{i-1} \alpha(m) a(s(m))$$

the searching is terminated

$\left\{ \begin{array}{l} \text{if no further improvements can be found} \\ J(i) \geq J(i-1) \\ \text{if the iteration } (i) \text{ reaches } R_m - 1 \end{array} \right.$

$\alpha(R_m - 1)$  and  $s(R_m - 1)$  are determined

at the end of the searching

the greedy algorithm terminates

only when the residue angle error  
cannot be further reduced.

Initialization:

given  $\theta$  angle

$W$  wordlength

$R_m$  restricted iteration number

$$\theta(i) = \theta - \sum_{n=0}^{i-1} \alpha(n) a(s(n))$$

Select  $\alpha(i) \in \{-1, 0, +1\}$   
 $s(i) \in \{0, 1, 2, \dots, W-1\}$   
to minimize  $J(i) = \theta(i) - \alpha(i) a(s(i))$

$s(m)$  repetition allowed

N  
Decision:  $J(i) < J(i-1)$

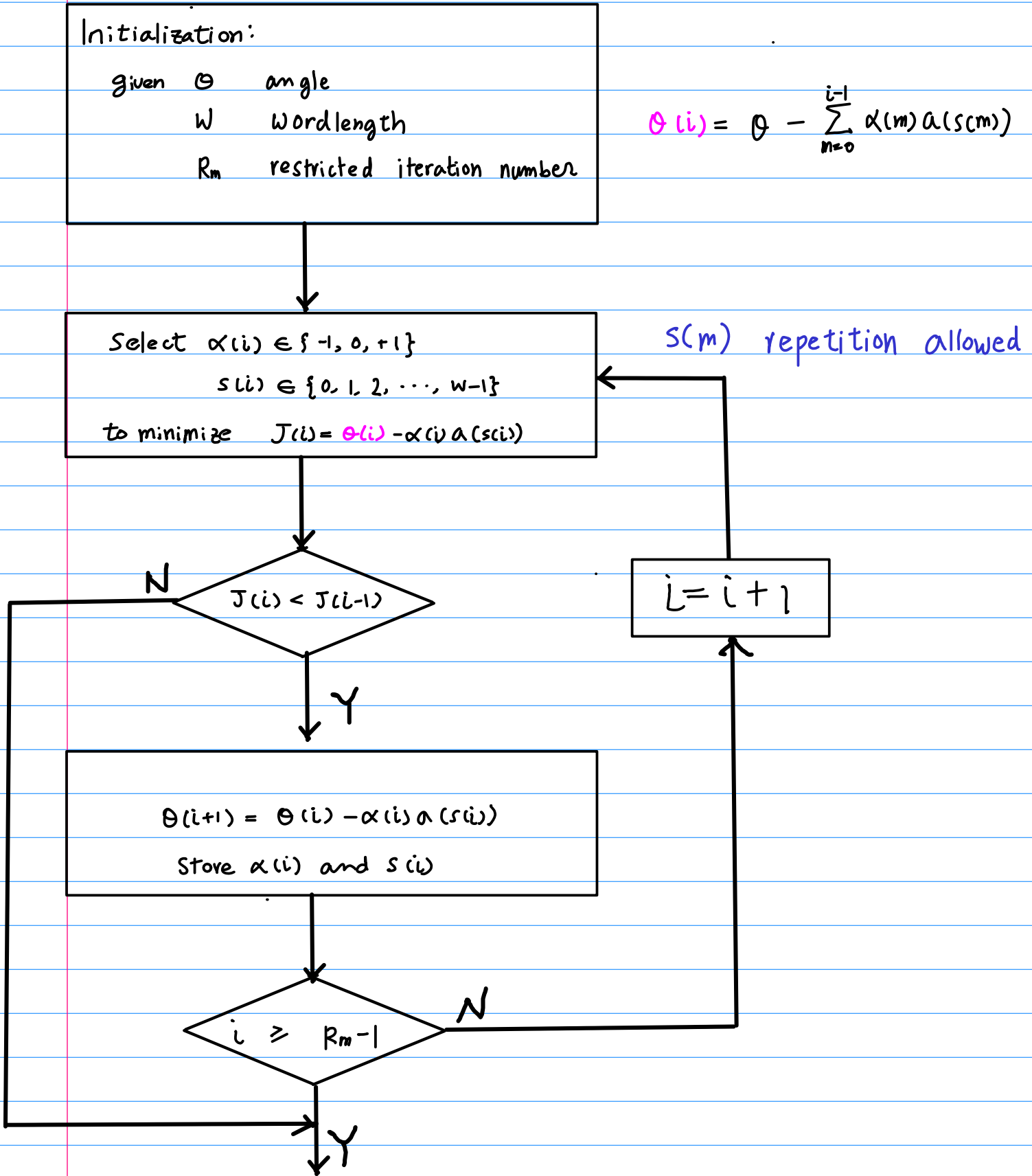
Y

$\theta(i+1) = \theta(i) - \alpha(i) a(s(i))$   
Store  $\alpha(i)$  and  $s(i)$

Decision:  $i \geq R_m - 1$   
N

Y

$i = i + 1$



## 2) Exhaustive Algorithm

search for the entire solution space

$$\begin{array}{ccc} \alpha(i) & a(s(i)) & i \\ \{-1, 0, +1\} & \{s(0), s(1), \dots, s(W-1)\} & \{0, 1, \dots, R_m-1\} \\ 3 & W & R_m \end{array} \Rightarrow (3W)^{R_m}$$

all possible combinations of

$$\sum_{i=0}^{R_m-1} \alpha(i) a(s(i))$$

in a single step

decisions for  $\alpha(i)$  and  $s(i)$ ,  $0 \leq i \leq R_m-1$   
by minimizing the error function

$$J = \left| 0 - \sum_{i=0}^{R_m-1} \alpha(i) a(s(i)) \right|$$

global optimal solution

Initialization:

given  $\Theta, W, R_m$

let  $\theta(0) = \Theta,$

$i = 0$

$J(-1) = \infty$

Select  $\alpha(i) \in \{-1, 0, +1\}$

$s(i) \in \{0, 1, 2, \dots, W-1\}$

for  $0 \leq i \leq R_m - 1$

to minimize  $J(i) = \theta - \sum_{l=0}^{R_m-1} \alpha(l) a(s(i))$

$$(3 \cdot W) \cdot (3 \cdot W) \dots (3 \cdot W) \\ = 3^{R_m} \cdot W^{R_m}$$

Store  $\alpha(i)$  and  $s(i)$

for  $0 \leq i \leq R_m - 1$

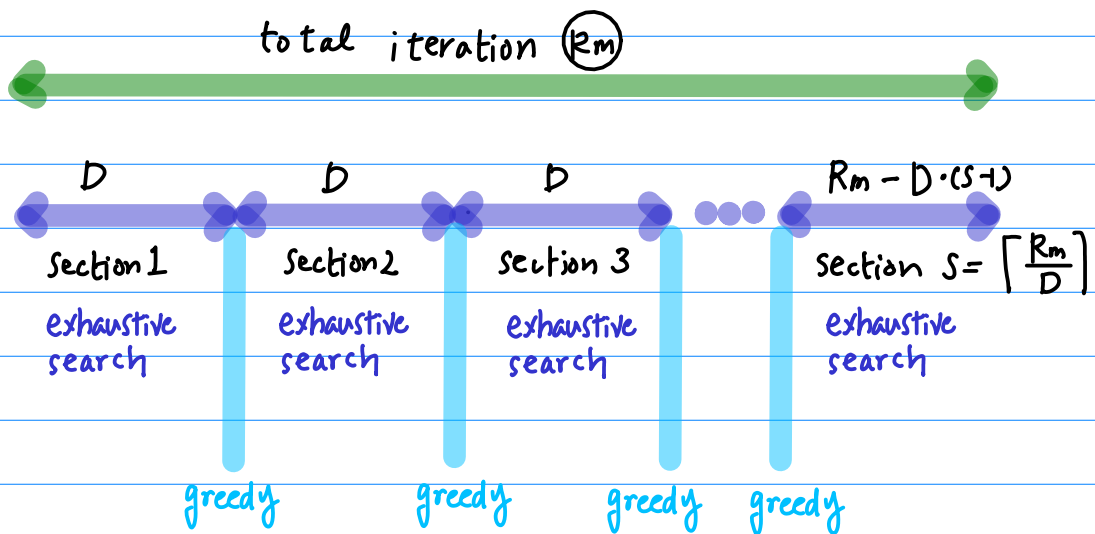
### 3) Semi-greedy Algorithm

a combination of greedy and exhaustive algorithm

the search space of  $\alpha(i)$  and  $s(i)$  for  $0 \leq i \leq R_m - 1$  are divided into several sections

with  $D$  iterations as a segment  
↓                      ↓  
block length        block

the segmentation scheme



in the  $i$ -th block

decision of  $\alpha(k)$  and  $s(k)$  for  $iD \leq k \leq (i+1)D-1$

$$\text{minimizes } J = \left| \theta(i) - \sum_{k=iD}^{(i+1)D-1} \alpha(k) a(s(k)) \right|$$

$$\text{where } \theta(i) = \theta - \sum_{m=0}^{i-1} \left[ \sum_{k=mD}^{(m+1)D-1} \alpha(k) a(s(k)) \right]$$

the residue angle in the  $i$ -th step

$$s = \left\lceil \frac{R_m}{D} \right\rceil$$

$$\theta(i) = \theta - \left[ \sum_{k=0D}^{1D-1} \alpha(k) a(s(k)) + \sum_{k=1D}^{2D-1} \alpha(k) a(s(k)) + \dots + \sum_{k=(i-1)D}^{iD-1} \alpha(k) a(s(k)) \right]$$



Initialization:

given  $\theta$ ,  $w$ ,  $R_m$

let  $\theta(0) = \theta$ ,

$i = 0$

$J(-1) = \infty$

Select  $\alpha(k) \in \{-1, 0, +1\}$

$s(k) \in \{0, 1, 2, \dots, w-1\}$

for  $iD \leq k \leq (i+1)D - 1$

to minimize  $J(i) = \theta(i) - \sum_{k=iD}^{(i+1)D-1} \alpha(k) a(s(k))$

N  
Decision:  $J(i) < J(i-1)$

$\theta(i+1) = \theta(i) - \sum_{k=iD}^{(i+1)D-1} \alpha(k) a(s(k))$

Store  $\alpha(k)$ ,  $s(k)$

Decision:  $i \geq \lceil \frac{R_m}{D} \rceil - 1$   
N

$i = i + 1$

Y

```
#include <stdio.h>
#include <math.h>

#define N 16
#define Rm 8

//-----
// conventional cordic
// input initial angle : angle[0]
// output residue angles : angle[1] .. angle[N]
//-----
void conventional_cordic(double a[], double angle[]) {
    int k = 0, uk;

    printf("* conventional cordic ... \n");

    for (k=0; k<N; ++k) {
        uk = (angle[k] >= 0) ? +1 : -1;

        printf("k=%2d angle[%2d]=%10.7f ", k, k, angle[k]);
        printf("uk=%+d ", uk);

        angle[k+1] = angle[k] - uk * a[k];

        printf("a[%2d]=%10.7f ", k, a[k]);
        printf("angle[%2d]=%+10.5f \n", k+1, angle[k+1]);
    }
}
```

```

void angle_repeat_cordic(double a[], double angle[]) {
    int k, m, mm, km, i, uk;
    double residue, minval;

    for (k=0; k<N; ++k) {

        mm = -1;
        minval = 1e+100;

        // perform conventional cordic from k-th step
        for (i=k; i<N; ++i) {
            uk = (angle[i] >= 0) ? +1 : -1;
            angle[i+1] = angle[i] - uk * a[i];
            // printf("angle[%2d]=%10f \n", i+1, angle[i+1]);
        }

        uk = (angle[k] >= 0) ? +1 : -1;

        // m: repeating depth
        for (m=1; m+k<N; ++m) {

            // expected residue angle by repeating a[k+m] (m+1) times
            residue = angle[k] - uk * (m+1)*a[k+m];

            // find the best angle repetition
            if (fabs(angle[k+m+1]) >= fabs(residue)) {
                if (minval >= fabs(residue)) {
                    printf("m=%2d residue=%f angle[%2d]=%10f \n",
                        m, residue, k+m+1, angle[k+m+1]);
                    minval = fabs(residue);
                    mm = m;
                }
            }
        }
    }
}

```

```

m = mm;

if (m > 0) {
    km = k+m;
    for (i=0; i<=m; i++) {
        printf("angle[%2d]=%10.7f ", k, angle[k]);
        printf("i=%2d uk=%+d m=%2d ", i, uk, m-i);

        angle[k+1] = angle[k] - uk * a[km];

        printf("a[%2d]=%10.7f, ", km, a[km]);
        printf("angle[%2d]=%+10.5e \n", k+1, angle[k+1]);

        k++;
    }
    k--;
} else {
    printf("angle[%2d]=%10.7f ", k, angle[k]);
    printf("i=%2d uk=%+d m=%2d ", 0, uk, m);

    angle[k+1] = angle[k] - uk * a[k];

    printf("a[%2d]=%10.7f, ", k, a[k]);
    printf("angle[%2d]=%+10.5e \n", k+1, angle[k+1]);
}

} // end of k

}

```

```

int main(void) {
    double a[N], angle[N+1];
    // double theta = 0.63761;
    // double theta = 0.27623;
    double theta; // = 4*atan(pow(2,-5));

    int i, s;

    for (i=0; i<N; ++i) {
        a[i] = atan(1./pow(2, i));
    }

    for (s=2; s<3; ++s) {
        angle[0] = theta = s * atan(pow(2, -6));
        printf("theta= %2d * atan(pow(2,-6) = %10g \n", s, theta);
        conventional_cordic(a, angle);
        angle_repeat_cordic(a, angle);
    }
}

```

```

theta= 2 * atan(pow(2,-6) = 0.0312475
* conventional cordic ...
k= 0 angle[ 0]= 0.0312475 uk==+1 a[ 0]= 0.7853982 angle[ 1]= -0.75415
k= 1 angle[ 1]=-0.7541507 uk=-1 a[ 1]= 0.4636476 angle[ 2]= -0.29050
k= 2 angle[ 2]=-0.2905031 uk=-1 a[ 2]= 0.2449787 angle[ 3]= -0.04552
k= 3 angle[ 3]=-0.0455244 uk=-1 a[ 3]= 0.1243550 angle[ 4]= +0.07883
k= 4 angle[ 4]= 0.0788306 uk==+1 a[ 4]= 0.0624188 angle[ 5]= +0.01641
k= 5 angle[ 5]= 0.0164118 uk==+1 a[ 5]= 0.0312398 angle[ 6]= -0.01483
k= 6 angle[ 6]=-0.0148281 uk=-1 a[ 6]= 0.0156237 angle[ 7]= +0.00080
k= 7 angle[ 7]= 0.0007956 uk==+1 a[ 7]= 0.0078123 angle[ 8]= -0.00702
k= 8 angle[ 8]=-0.0070167 uk=-1 a[ 8]= 0.0039062 angle[ 9]= -0.00311
k= 9 angle[ 9]=-0.0031105 uk=-1 a[ 9]= 0.0019531 angle[10]= -0.00116
k=10 angle[10]=-0.0011573 uk=-1 a[10]= 0.0009766 angle[11]= -0.00018
k=11 angle[11]=-0.0001808 uk=-1 a[11]= 0.0004883 angle[12]= +0.00031
k=12 angle[12]= 0.0003075 uk==+1 a[12]= 0.0002441 angle[13]= +0.00006
k=13 angle[13]= 0.0000634 uk==+1 a[13]= 0.0001221 angle[14]= -0.00006
k=14 angle[14]=-0.0000587 uk=-1 a[14]= 0.0000610 angle[15]= +0.00000
k=15 angle[15]= 0.0000023 uk==+1 a[15]= 0.0000305 angle[16]= -0.00003
angle[ 0]= 0.0312475 i= 0 uk==+1 m=-1 a[ 0]= 0.7853982, angle[ 1]=-7.54151e-01
angle[ 1]=-0.7541507 i= 0 uk=-1 m=-1 a[ 1]= 0.4636476, angle[ 2]=-2.90503e-01
m= 1 residue=-0.041793 angle[ 4]= 0.078831
angle[ 2]=-0.2905031 i= 0 uk=-1 m= 1 a[ 3]= 0.1243550, angle[ 3]=-1.66148e-01
angle[ 3]=-0.1661481 i= 1 uk=-1 m= 0 a[ 3]= 0.1243550, angle[ 4]=-4.17931e-02
angle[ 4]=-0.0417931 i= 0 uk=-1 m=-1 a[ 4]= 0.0624188, angle[ 5]=+2.06257e-02
angle[ 5]= 0.0206257 i= 0 uk==+1 m=-1 a[ 5]= 0.0312398, angle[ 6]=-1.06141e-02
angle[ 6]=-0.0106141 i= 0 uk=-1 m=-1 a[ 6]= 0.0156237, angle[ 7]=+5.00960e-03
angle[ 7]= 0.0050096 i= 0 uk==+1 m=-1 a[ 7]= 0.0078123, angle[ 8]=-2.80274e-03
angle[ 8]=-0.0028027 i= 0 uk=-1 m=-1 a[ 8]= 0.0039062, angle[ 9]=+1.10349e-03
angle[ 9]= 0.0011035 i= 0 uk==+1 m=-1 a[ 9]= 0.0019531, angle[10]=-8.49636e-04
m= 1 residue=0.000127 angle[12]= -0.000361
m= 2 residue=-0.000117 angle[13]= -0.000117
angle[10]=-0.0008496 i= 0 uk=-1 m= 2 a[12]= 0.0002441, angle[11]=-6.05496e-04
angle[11]=-0.0006055 i= 1 uk=-1 m= 1 a[12]= 0.0002441, angle[12]=-3.61355e-04
angle[12]=-0.0003614 i= 2 uk=-1 m= 0 a[12]= 0.0002441, angle[13]=-1.17214e-04
m= 1 residue=0.000005 angle[15]= -0.000056
angle[13]=-0.0001172 i= 0 uk=-1 m= 1 a[14]= 0.0000610, angle[14]=-5.61793e-05
angle[14]=-0.0000562 i= 1 uk=-1 m= 0 a[14]= 0.0000610, angle[15]=+4.85585e-06
angle[15]= 0.0000049 i= 0 uk==+1 m=-1 a[15]= 0.0000305, angle[16]=-2.56617e-05

```

```

void exhaustive_choice(double a[], double ntheta[][N], theta) {
    int u, i;

    printf("* exhaustive searching cordic... \n");

    for (u=0; u<3; u++) {
        for (i=0; i<N; ++i) {

            ntheta[u][i] = theta - (u-1) * a[i];
        }
    }
}

```

```

void exhaustive_cordic(double a[], double angle[], int uk[], int ak[]) {
    int k, u, i;

    printf("* exhaustive greedy searching cordic ... \n");

    for (k=0; k<N; k++) {
        theta = angle[k]

        minval = 1e+100;
        for (u=0; u<3; u++) {
            for (i=0; i<N; ++i) {
                ntheta = theta - (u-1) * a[i];
                if (minval > ntheta) {
                    angle[k+1] = ntheta
                    uk[k] = u;
                    ak[k] = i;
                }
            }
        }
    }
}

```