Applications of Arrays (1A)

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- Viewing an **array** as a **pointer**

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int **a**[4] ; — an array **a**

generalization

int (*a) — view **a** as a pointer

virtual pointer
- no real memory location
- constraints :
  value(&**a**) = value(**a**)

- Viewing a **pointer** as an **array**

int (*a) ; — a pointer **a**

a specific instance

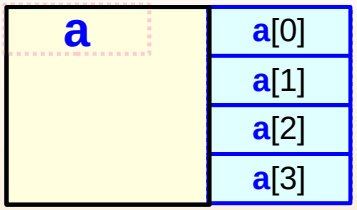
int **a**[N] — view **a** as an array

N is not fixed

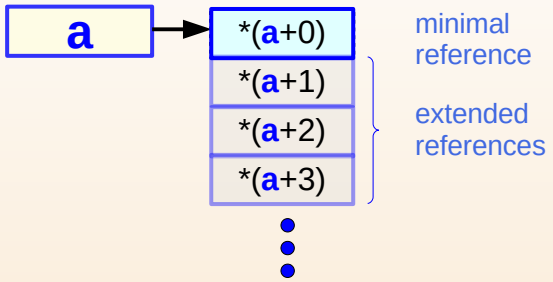
sizeof(**a**) is
not the size of the array
but of a pointer variable

Array **a** and pointer **a**

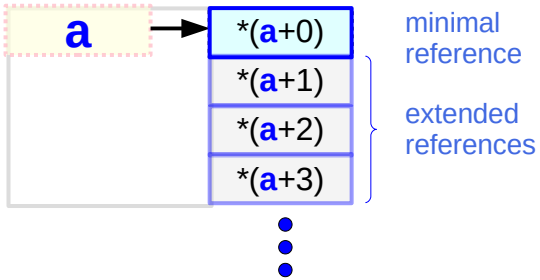
int **a**[4] ; an array **a**



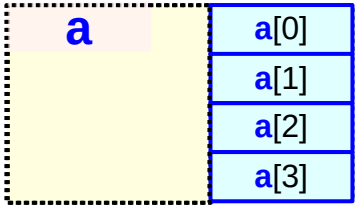
int (*a) ; a pointer **a**



int (*a) **a** as a pointer



int **a**[N] **a** as an array



Array **a** and pointer **a**

`int a[4] ;` an array **a**

- type(**a**) = int [4]
- sizeof(**a**) = an array size (16 bytes)
- value(&**a**) = value(**a**)
- fixed number of elements

`int (*a)` **a** as a pointer

a is not a real pointer

- sizeof(**a**) = an array size
- value(&**a**) = value(**a**)

`int (*a) ;` a pointer **a**

- type(**a**) = int (*)
- sizeof(**a**) = a pointer size (4 bytes)
- value(&**a**) ≠ value(**a**)
- variable number of elements

`int a[N]` **a** as an array

a is not a real array

- sizeof(**a**) ≠ an array size
  = a pointer size
- value(&**a**) ≠ value(**a**)
  = assigned address

Relationship between array and array pointer types

int **b**[4][2] ; declare a **2-d** array b

generalization

int (***b**) [2] **b** as a **1-d** array pointer

int **a**[4] ; declare a **1-d** array **a**

generalization

int (***a**) **a** as a **0-d** array pointer

int (***b**)[2] ; declare a **1-d** array pointer **b**

a specific instance

int **b**[N][2] **b** as a **2-d** array

int (***a**) ; declare a **0-d** array pointer **a**

a specific instance

int **a**[N] **a** as a **1-d** array

Array **b** and array pointer **b**

int **b**[4][2] ; **2-d** array **b**

- type(**b**) = int [4]
- sizeof(**b**) = an array size (32 bytes)
- value(&**b**) = value(**b**)
- fixed number of elements

int (*) [2] **b** as a **1-d** array pointer

b is not a real pointer

- sizeof(**b**) = an array size
- value(&**b**) = value(**b**)

int (***b**) [2] ; **1-d** array pointer **b**

- type(**b**) = int (*)
- sizeof(**b**) = a pointer size (4 bytes)
- value(&**b**) ≠ value(**b**)
- variable number of elements

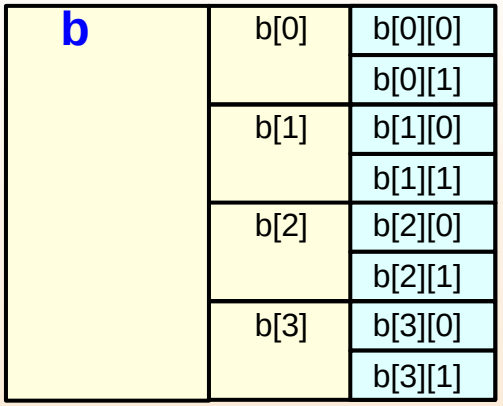
int [N][2] **b** as a **2-d** array

b is not a real array

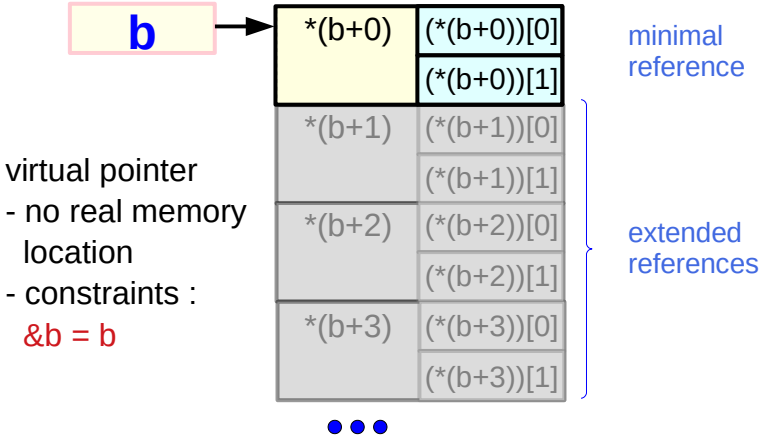
- sizeof(**b**) ≠ an array size
  = a pointer size
- value(&**b**) ≠ value(**b**)
  = assigned address

Array **b** and array pointer **b**

int **b**[4][2] ; **2-d** array **b**

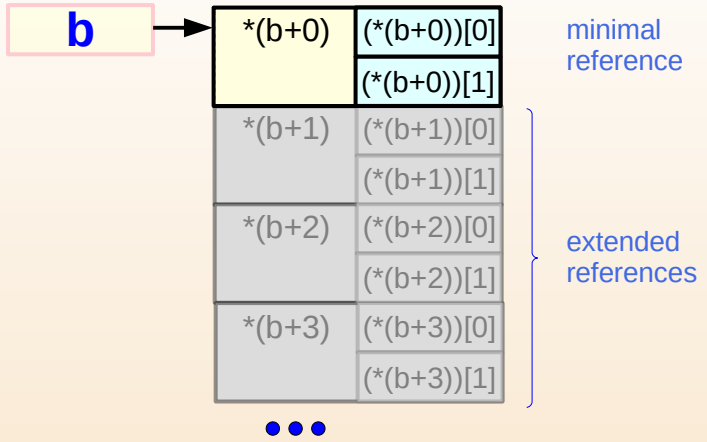


int (*) [2] **b** as a **1-d** array pointer

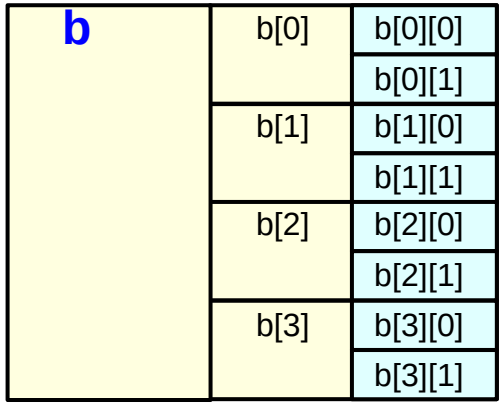


virtual pointer
- no real memory location
- constraints :
  &b = b

int (***b**) [2] ; **1-d** array pointer **b**



int [N][2] **b** as a **2-d** array



N is not fixed to 4

sizeof(b) is not the size of the array but the size of a pointer variable

Dual type - relaxing the 1st dimension of an array

int [4][2] **2-d** array

more constrained type

relaxing the 1st dimension

generalization

a specific instance

int (*)[2] **1-d** array pointer

more general type

int [4] **1-d** array

more constrained type

relaxing the 1st dimension

generalization

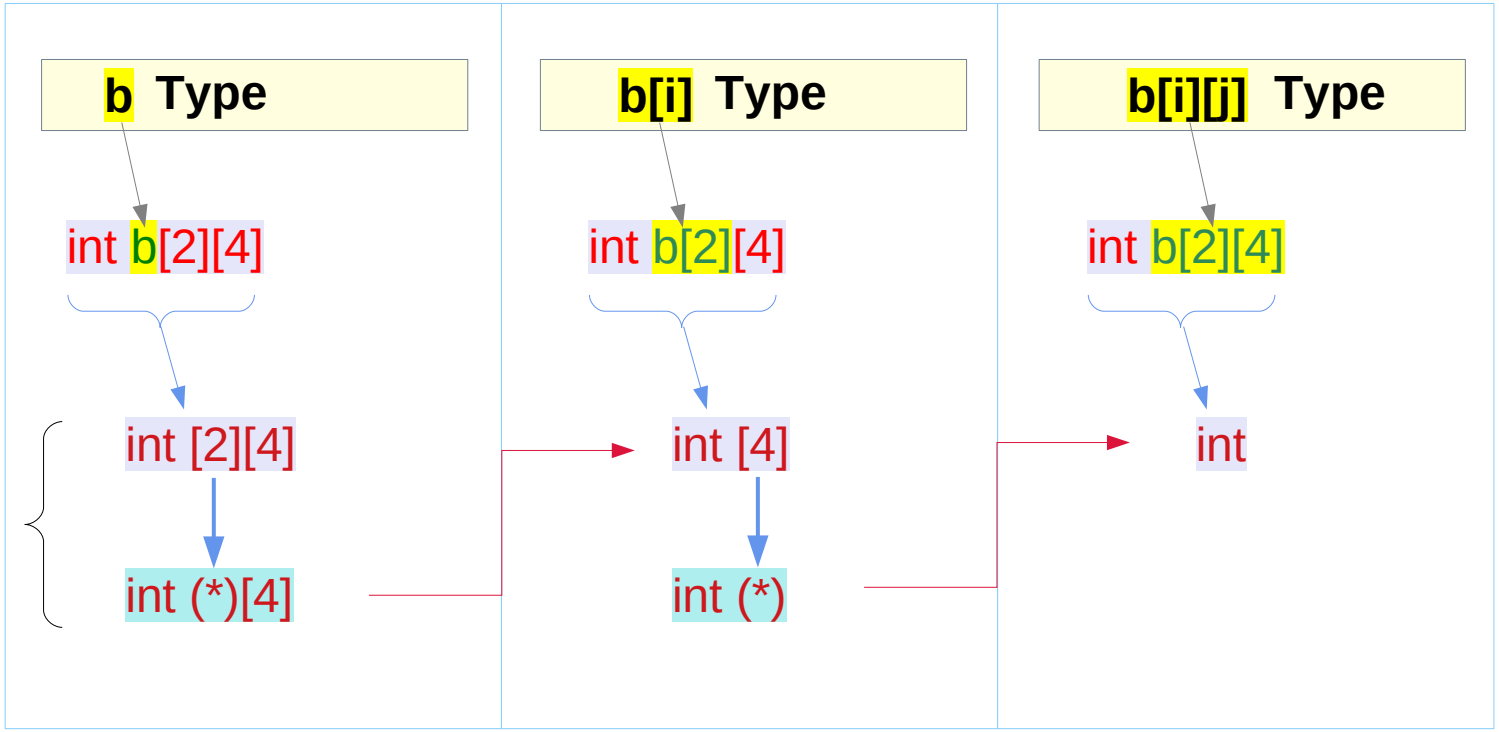
a specific instance

int (*) **0-d** array pointer

more general type

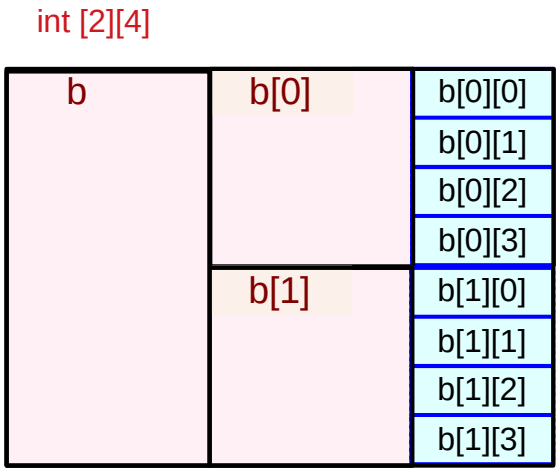
Subarray types in a **2-d** array

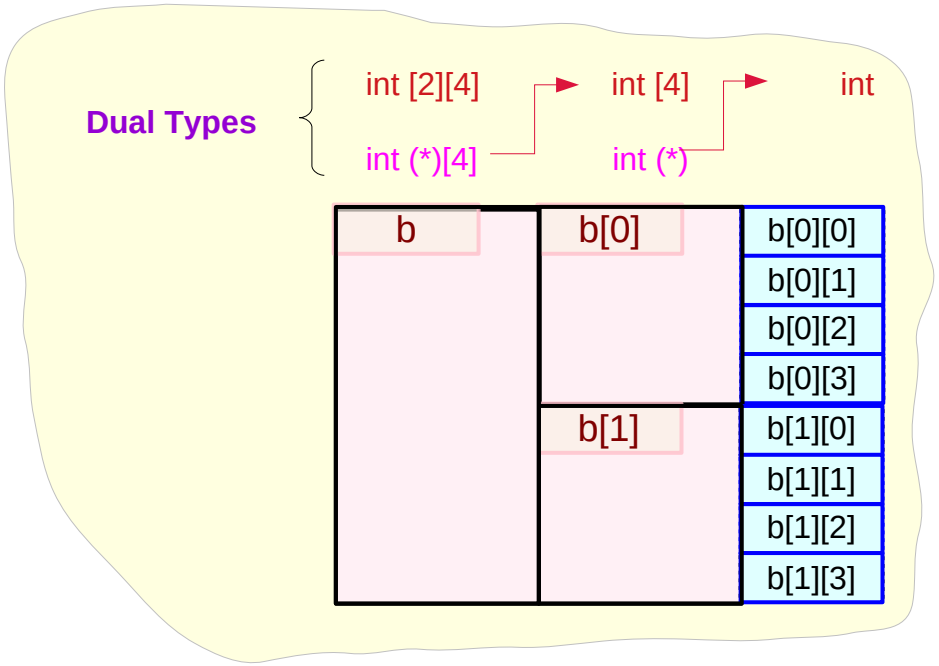
int b[2][4]; **2-d** array b

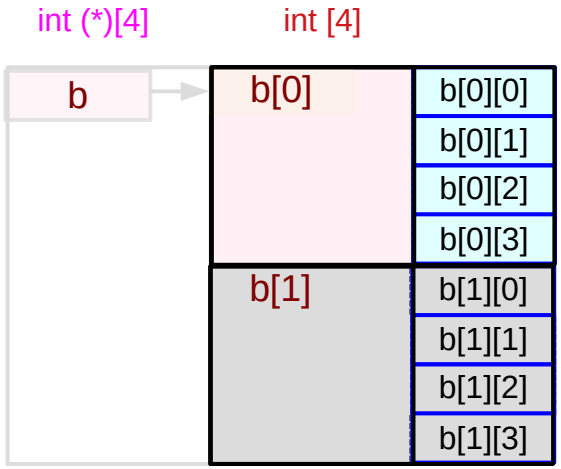


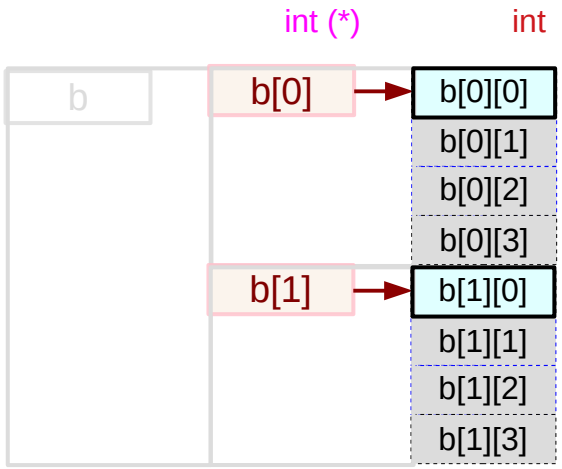
Dual types in a **2-d** array

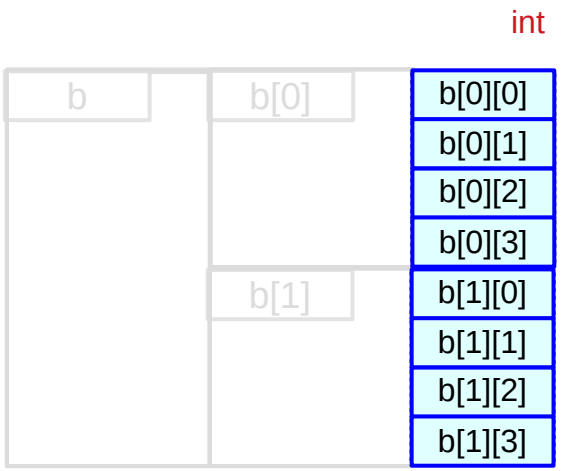
int b[2][4]; **2-d** array b











Subarray type examples

```
int a[4];
```

			relaxed type	virtual
a	int [4]	**1-d** array type	int (*)	**0-d** array pointer type
a[i]	int	**0-d** array type		

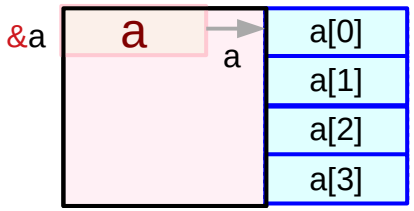
```
int b[2][4];
```

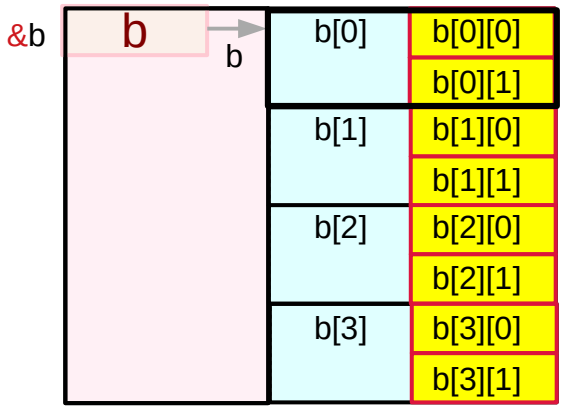
			relaxed type	virtual
b	int [2][4]	**2-d** array type	int (*)[4]	**1-d** array pointer type
b[i]	int [4]	**1-d** array type	int (*)	**0-d** array pointer type
b[i][j]	int	**0-d** array type		

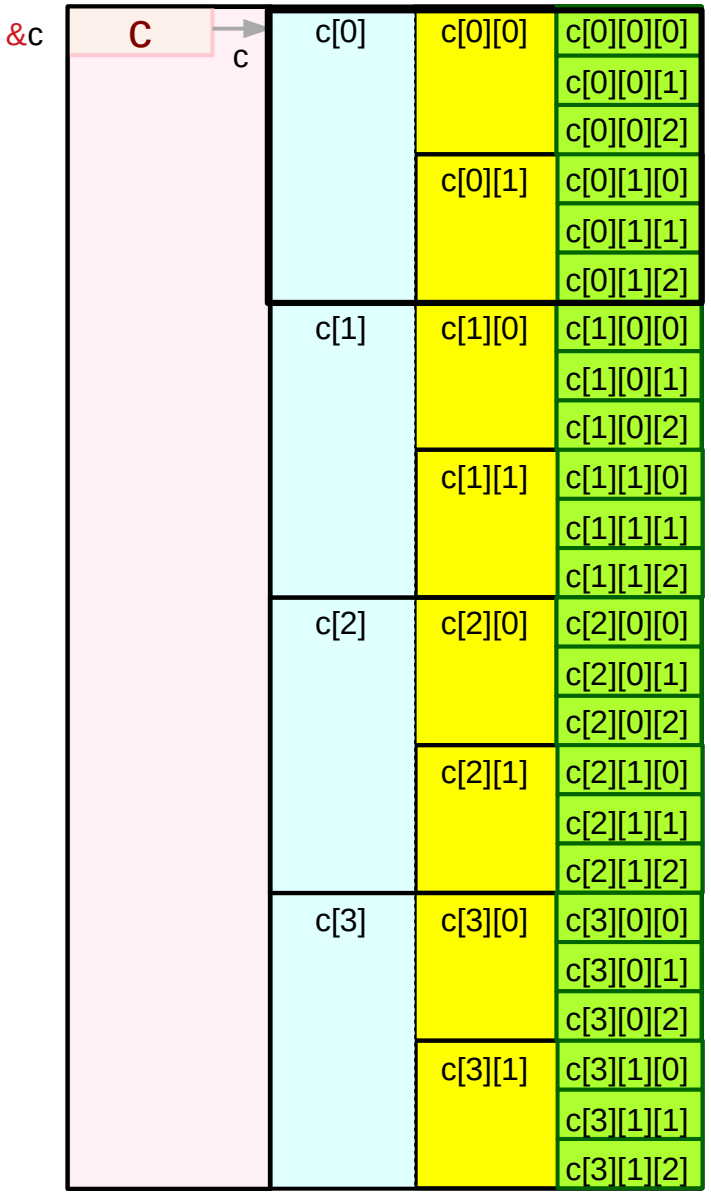
```
int c[4][2][3];
```

			relaxed type	virtual
c	int [4][2][3]	**3-d** array type	int (*)[2][3]	**2-d** array pointer type
c[i]	int [4][2]	**2-d** array type	int (*)[2]	**1-d** array pointer type
c[i][j]	int [4]	**1-d** array type	int (*)	**0-d** array pointer type
c[i][j][k]	int	**0-d** array type		

Types of **a**, **b**, **c** arrays







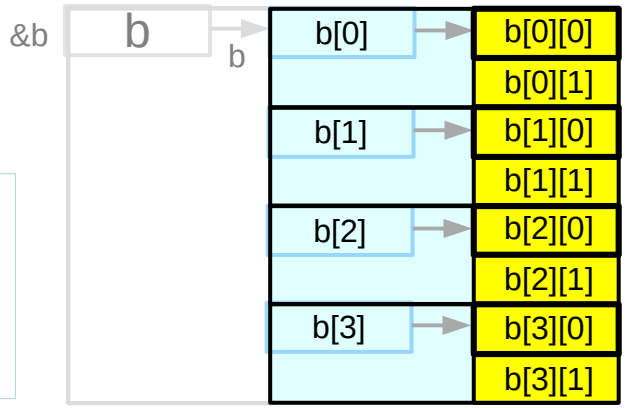

```
int a[4];
int b[2][4];
int c[4][2][3];
```

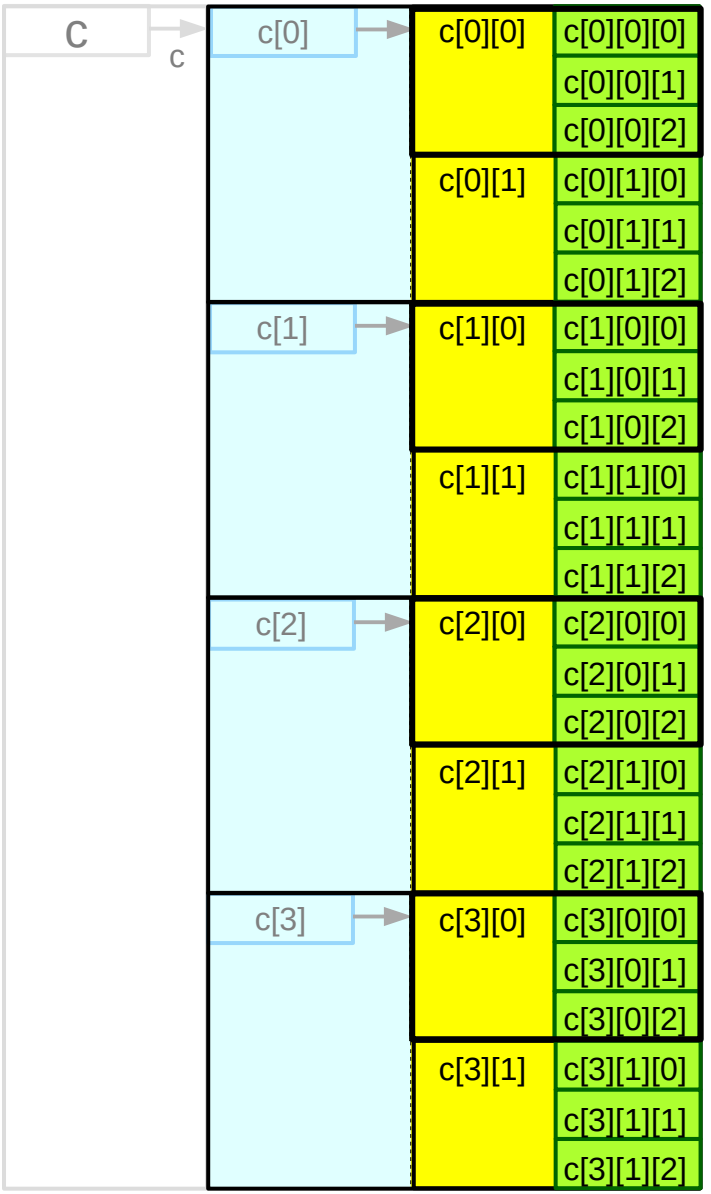
dual types

int [4] int (*)	**1-d** array **a** **0-d** array pointer **a** (virtual)	**a[i]** ***(a+i)**
int [4][2]; int (*)[2];	**2-d** array **b** **1-d** array pointer **b** (virtual)	**b[i]** ***(b+i)**
int [4][2][3]; int (*)[2][3];	**3-d** array **c** **2-d** array pointer **c** (virtual)	**c[i]** ***(c+i)**

Types of **b[i]**, **c[i]** subarrays

```
int a[4];
int b[2][4];
int c[4][2][3];
```





dual types

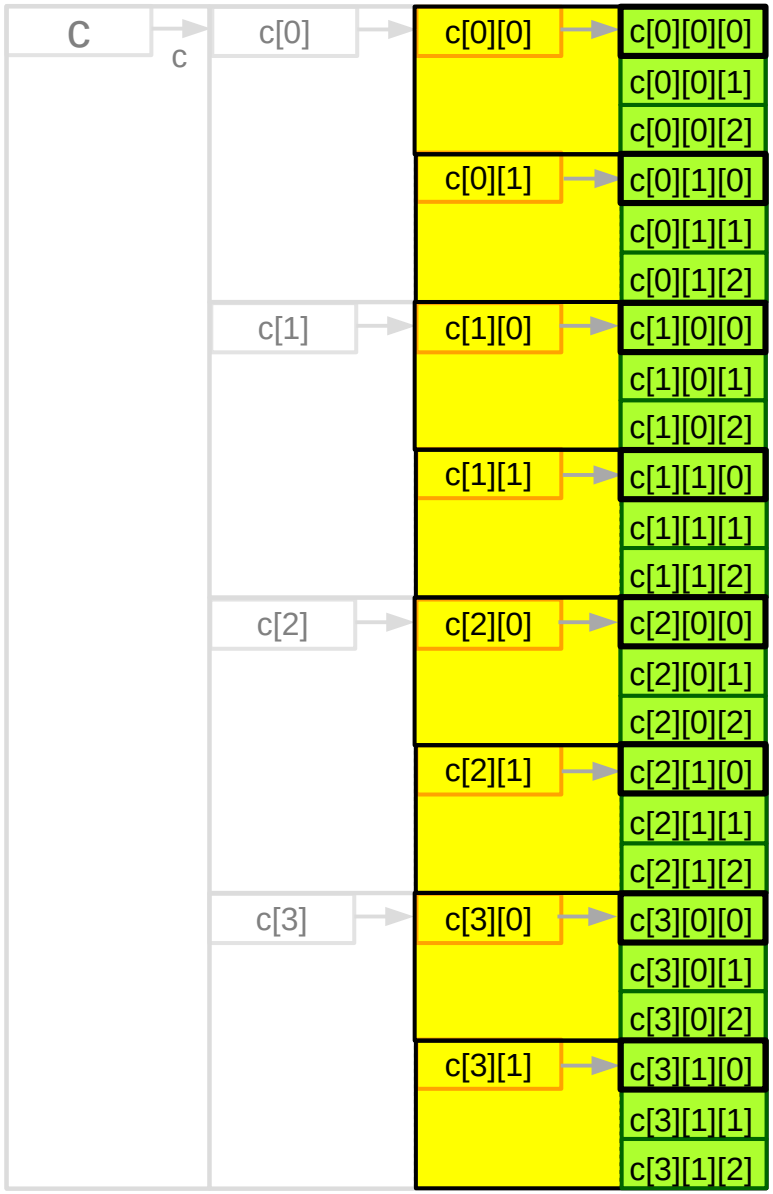
int [2]	**1-d** array **b[i]**	**b[i][j]**
int (*)	**0-d** array pointer **b[i]** (virtual)	***(b[i]+j)**
int [2][3];	**2-d** array **c[i]**	**c[i][j]**
int (*)[3];	**1-d** array pointer **c[i]** (virtual)	***(c[i]+j)**

Types of **c[i][j]** subarrays

```
int a[4];
int b[2][4];
int c[4][2][3];
```

dual types

int [3]	**1-d** array **c[i][j]**	**c[i][j][k]**
int (*)	**0-d** array pointer **c[i][j]** (virtual)	***(c[i][j]+k)**



Types of a **4-d** array and its subarrays

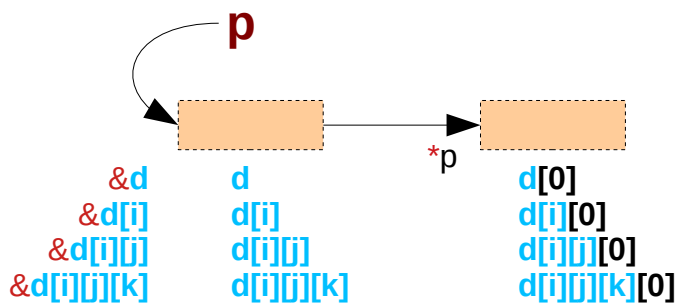
int d[4][2][3][4];

		types	
d	consider d[4][2][3][4]	int [4][2][3][4]	**4-d** array
	relax the 1st dimension	int (*)[2][3][4]	**3-d** array pointer (virtual)
d[i]	consider d[i][2][3][4]	int [2][3][4]	**3-d** array
	relax the 1st dimension	int (*)[3][4]	**2-d** array pointer (virtual)
d[i][j]	consider d[i][j][3][4]	int [3][4]	**2-d** array
	relax the 1st dimension	int (*)[4]	**1-d** array pointer (virtual)
d[i][j][k]	consider d[i][j][k][4]	int [4]	**1-d** array
	relax the 1st dimension	int (*)	**0-d** array pointer (virtual)

i,j,k are specific index values i=[0..3], j=[0..1], k=[0..2]

Initializing *n*-**d** array pointers with *n*-**d** subarrays

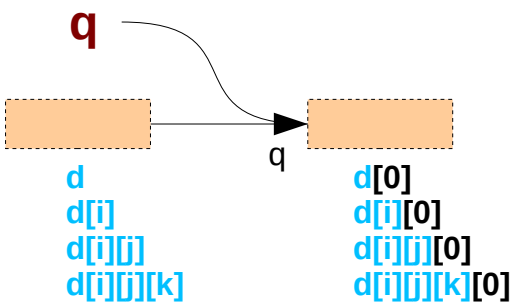
int **d**[4][2][3][4];



d	**4-d** array	**d**[4][2][3][4]	**p** = &**d**	abstract data
p	**4-d** array pointer	(***p**)[4][2][3][4]	int (***p**)[4][2][3][4] = &**d**;	
			(***p**)[i][j][k][l] ≡ **d**[i][j][k][l]	
d[i]	**3-d** array	**d[i]**[2][3][4]	**p** = &**d[i]**	abstract data
p	**3-d** array pointer	(***p**)[2][3][4]	int (***p**)[3][4] = &**d[i]**;	
			(***p**)[j][k][l] ≡ **d[i]**[j][k][l] given i	
d[i][j]	**2-d** array	**d[i][j]**[3][4]	**p** = &**d[i][j]**	abstract data
p	**2-d** array pointer	(***p**)[3][4]	int (***p**)[4] = &**d[i][j]**;	
			(***p**)[k][l] ≡ **d[i][j]**[k][l] given i, j	
d[i][j][k]	**1-d** array	**d[i][j][k]**[4]	**p** = &**d[i][j][k]**	abstract data
p	**1-d** array pointer	(***p**)[4]	int (***p**) = &**d[i][j][k]**;	
			(***p**)[l] ≡ **d[i][j][k]**[l] given i, j, k	

Initializing (*n*-1)-d array pointers with *n*-d subarrays

int d[4][2][3][4];



d	**4-d** array	d[4][2][3][4]	q = d	virtual pointer
q	**3-d** array pointer	(*q)[2][3][4]	int (*q)[2][3][4] = d;	
			q[i][j][k][l] ≡ d[i][j][k][l]	
d[i]	**3-d** array	d[i][2][3][4]	q = d[i]	virtual pointer
q	**2-d** array pointer	(*q)[3][4]	int (*q)[3][4] = d[i];	
			q[j][k][l] ≡ d[i][j][k][l]	given i
d[i][j]	**2-d** array	d[i][j][3][4]	q = d[i][j]	virtual pointer
q	**1-d** array pointer	(*q)[4]	int (*q)[4] = d[i][j];	
			q[k][l] ≡ d[i][j][k][l]	given i, j
d[i][j][k]	**1-d** array	d[i][j][k][4]	q = d[i][j][k]	virtual pointer
q	**0-d** array pointer	(*q)	int (*q) = d[i][j][k];	
			q[l] ≡ d[i][j][k][l]	given i, j, k

Aggregate Data Types
Abstract Data Types
Virtual Array Pointers

Aggregate data type

an aggregate type

consists of **N** elements

each element
- starting address
- size

int [4][4]

dual type

a virtual pointer

the address and value of
a virtual pointer
are the *same*

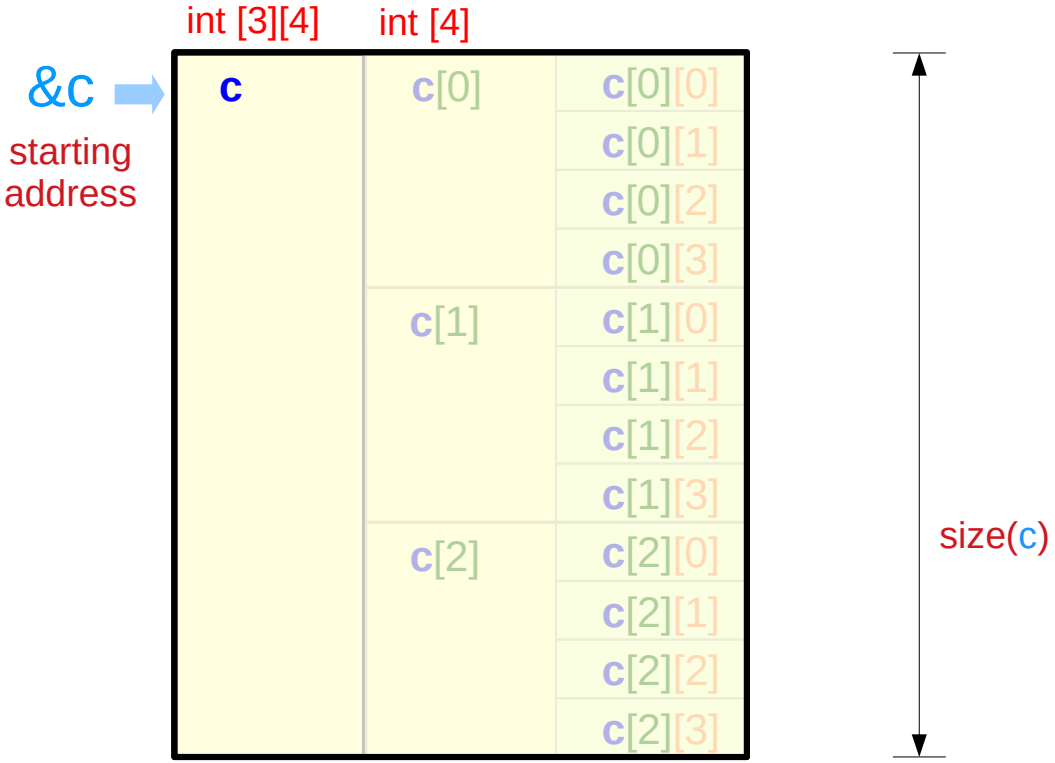
int (*)[4]

an abstract type

is considered as one unit

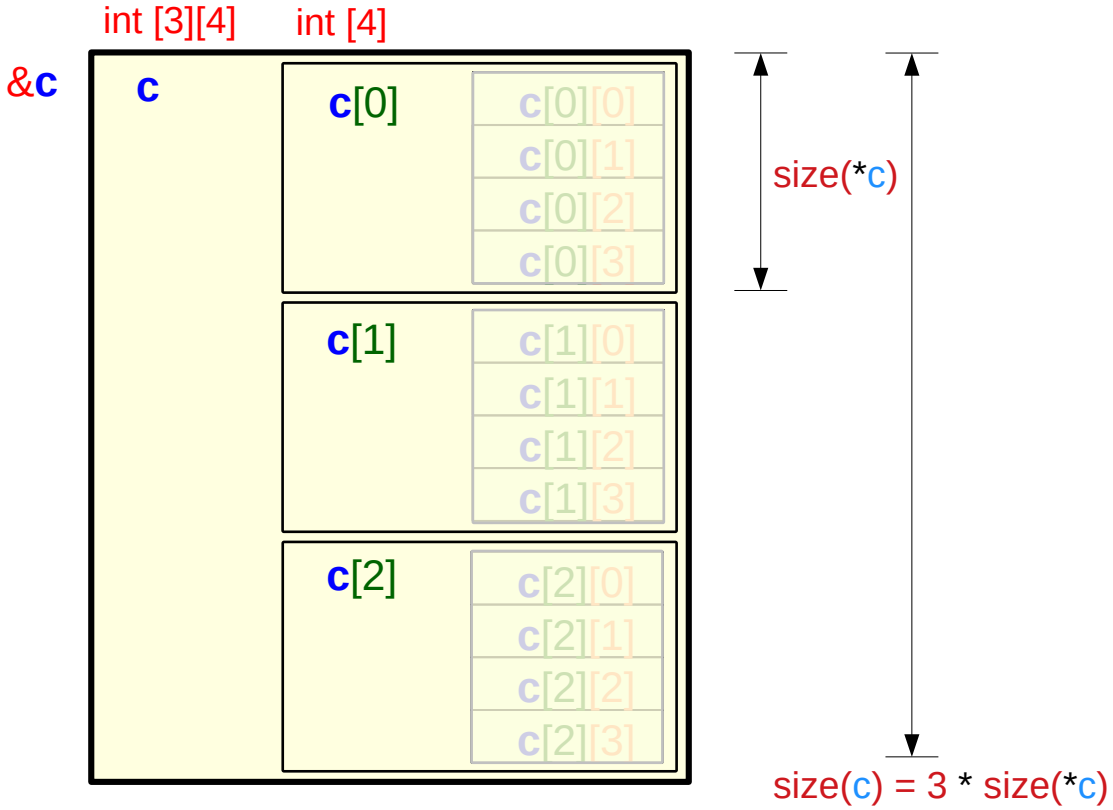
- starting address
- size

Abstract data **c**



an abstract data — c
- start address — &c
- size — sizeof(c)

Aggregate data **c**

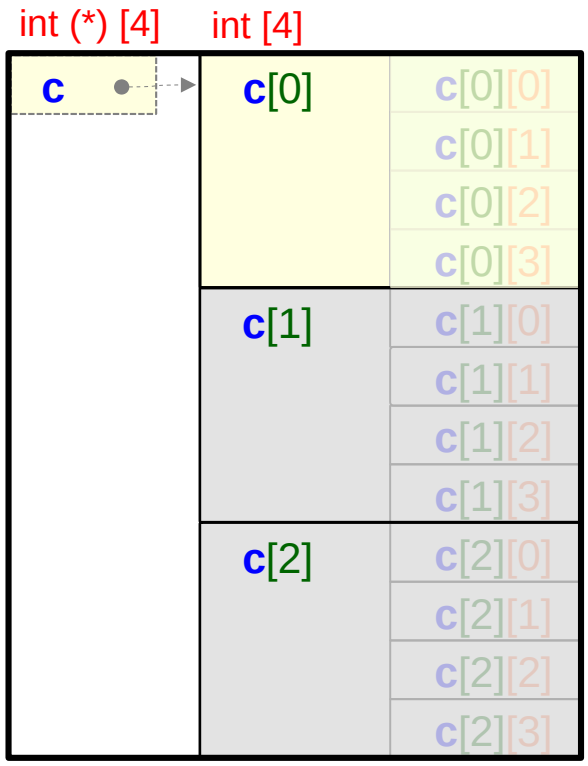


an aggregate type data **c**

- 1st element **c**[0]
- 2nd element **c**[1]
- 3rd element **c**[2]

Virtual pointer **c**

&c = c = &c[0]



size(*c)

a virtual pointer **c**
- pointer address &**c**
- pointer value **c** = &**c**[0]

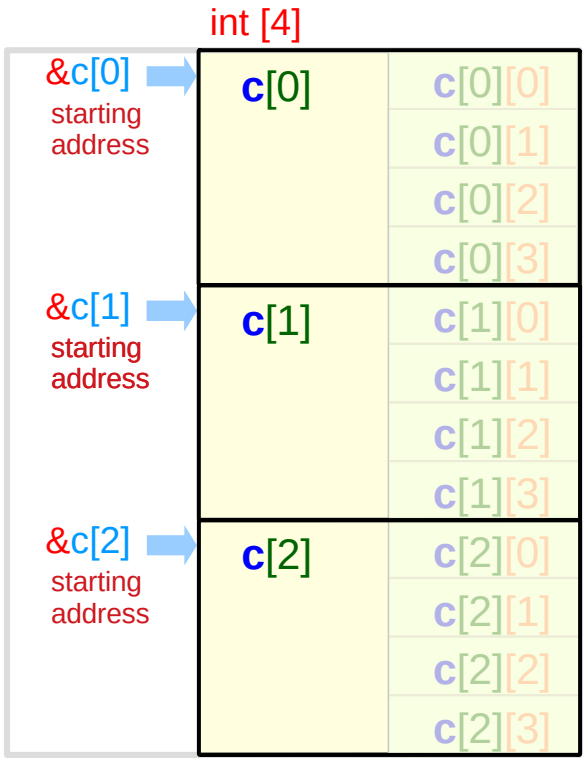
with the constraint
c = &**c**

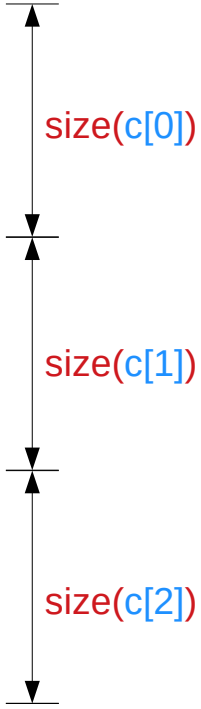
an abstract data **c**[0] = ***c**
- start address &**c**[0] = **c**
- size sizeof(**c**[0])

virtual pointer **c** points
to abstract data **c**[0]

virtual pointers
- no physical memory locations are allocated
- address and data have the same value

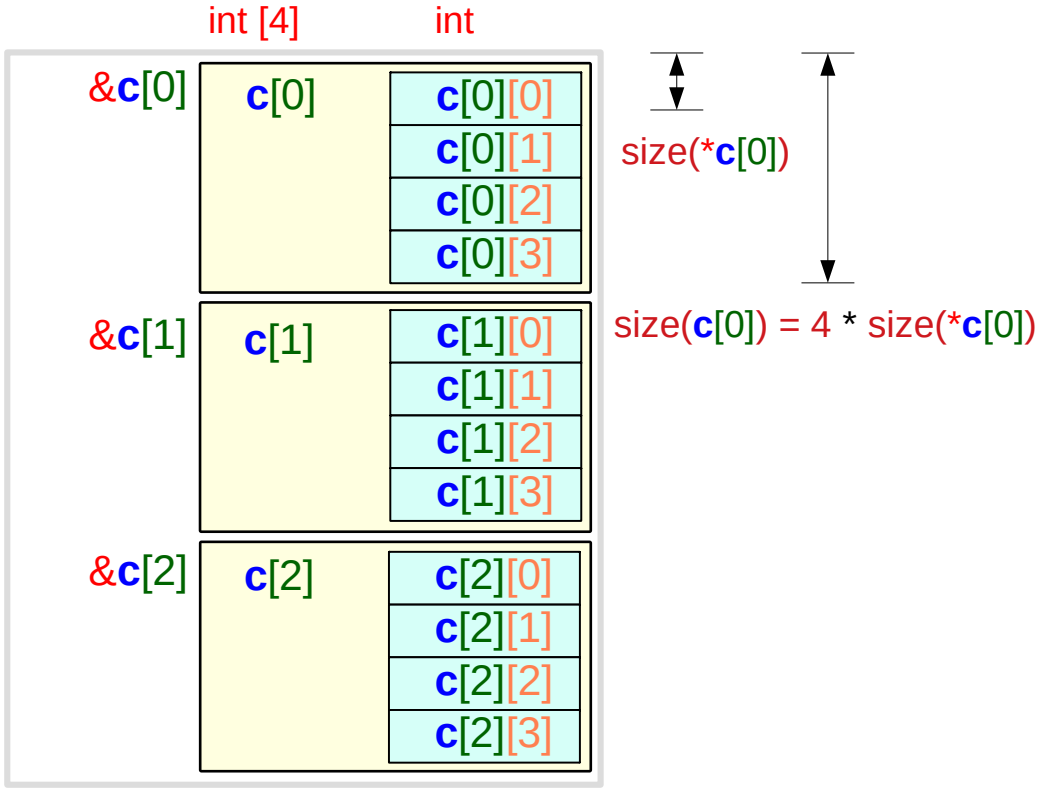
Abstract data **c[i]**





- an abstract data c[0]
  - start address &c[0]
  - size sizeof(c[0])
- an abstract data c[1]
  - start address &c[1]
  - size sizeof(c[1])
- an abstract data c[2]
  - start address &c[2]
  - size sizeof(c[2])

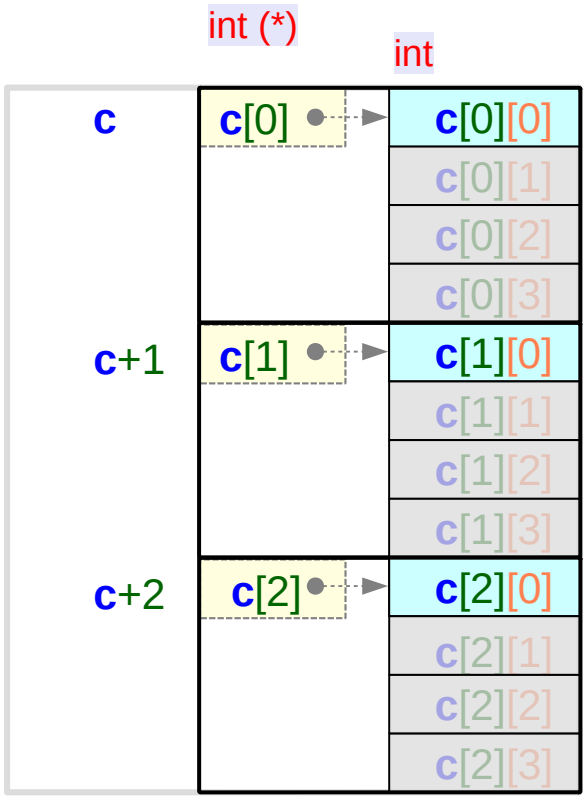
Aggregate data **c[i]**



an aggregate type data **c**[i]

- 1st element **c**[i][0]
- 2nd element **c**[i][1]
- 3rd element **c**[i][2]
- 4th element **c**[i][3]

Virtual pointer **c[i]**



a virtual pointer **c[i]**
- pointer address **&c[i]**
- pointer value **c+i** = **&c[i]**

with the constraint
c[i] = **&c[i]**

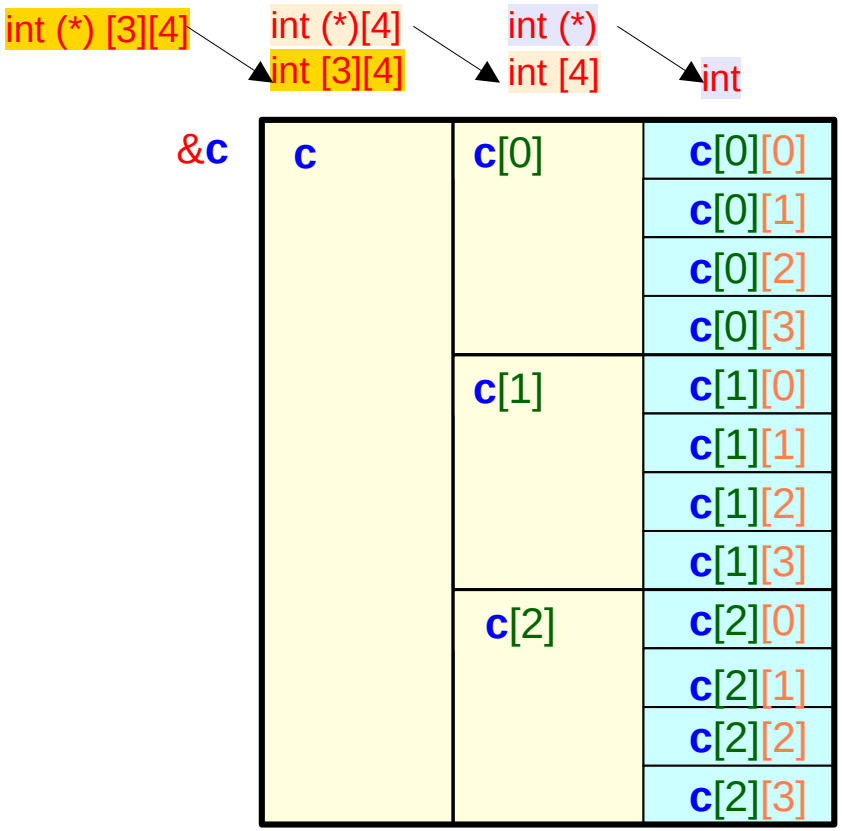
an primitive data **c[i][0]** = ***c[i]**
- start address **&c[i][0]** = **c[i]**
- size sizeof(**c[i][0]**)

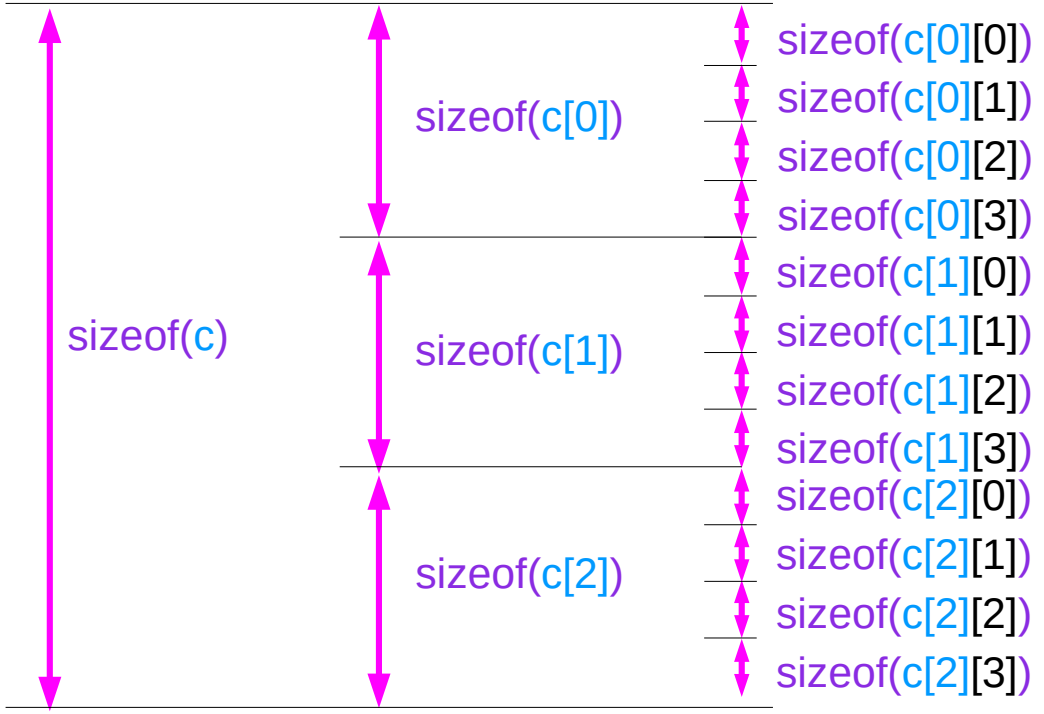
virtual pointer **c[i]**
points to primitive data **c[i][0]**

virtual pointers
- no physical memory locations are allocated
- address and data have the same value

A **2-d** array and its **1-d** sub-arrays – a size view

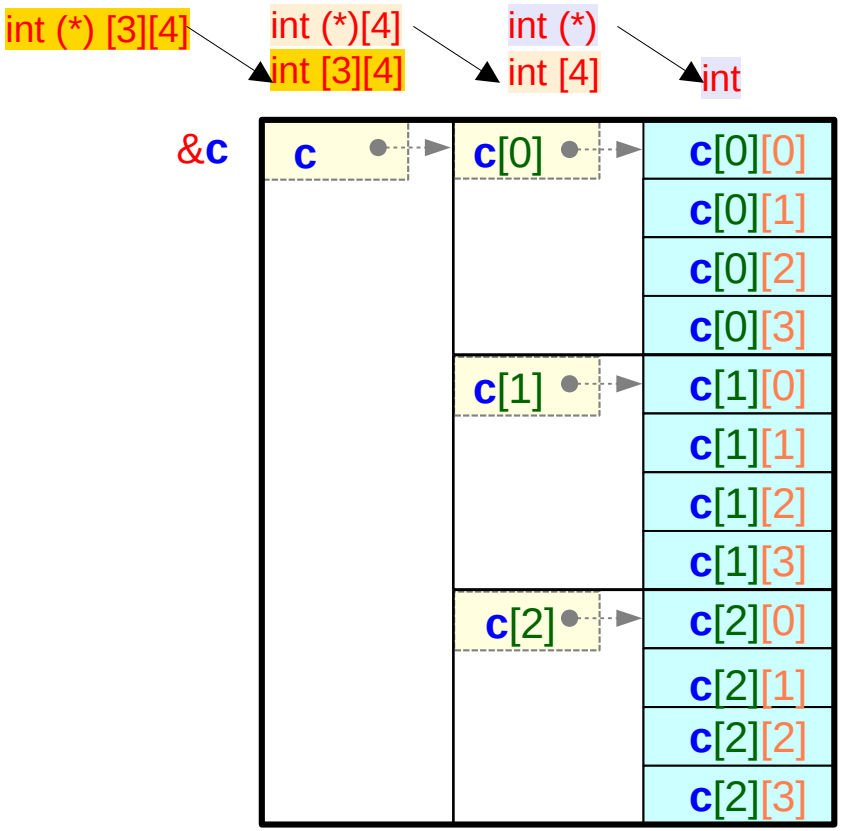
```
int c[3][4];
```





A **2-d** array and its **1-d** sub-arrays – a virtual pointer view

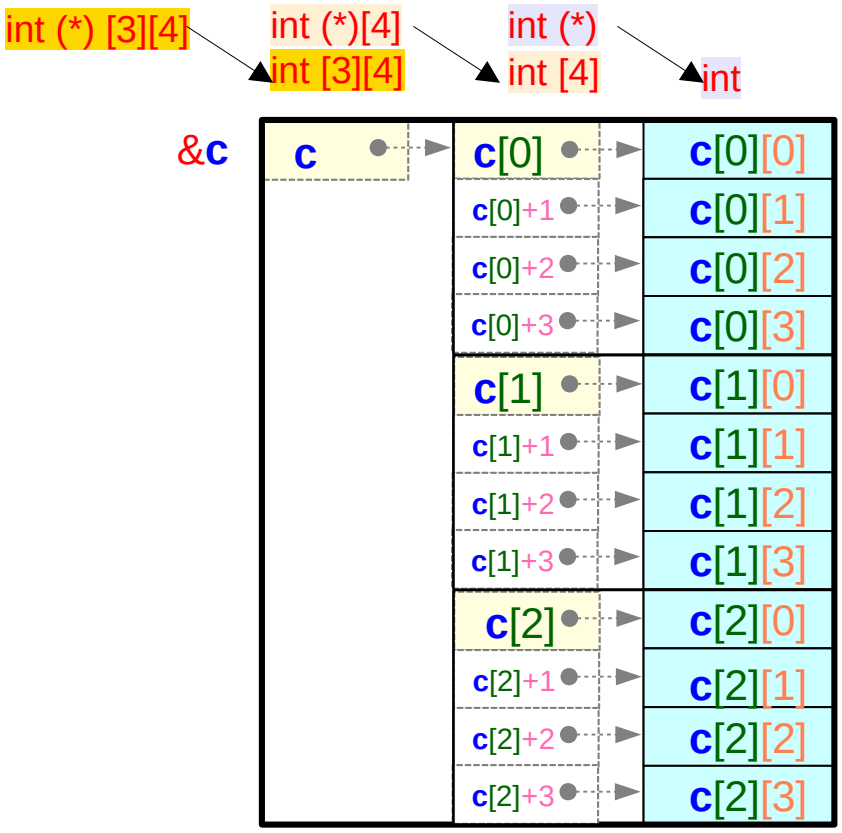
```
int c[3][4];
```




```
value(  c) = value(  c[0]) = value(&c[0][0])
value(&c)  = value(&c[0])  = value(&c[0][0])

           value(  c[1])   = value(&c[1][0])
           value(&c[1])    = value(&c[1][0])

           value( c[2])    = value(&c[2][0])
           value(&c[2])    = value(&c[2][0])


address(c) = address(c[0])  = address(c[0][0])
           address(c[1])    = address(c[1][0])
           address(c[2])    = address(c[2][0])
```

A **2-d** array and its **1-d** sub-arrays – size relation

```
int c[3][4];
```



sizeof(c) = sizeof(c[0]) * 3 … *leading element*
sizeof(c+1) = pointer size (4/8 bytes)
sizeof(c+2) = pointer size (4/8 bytes)

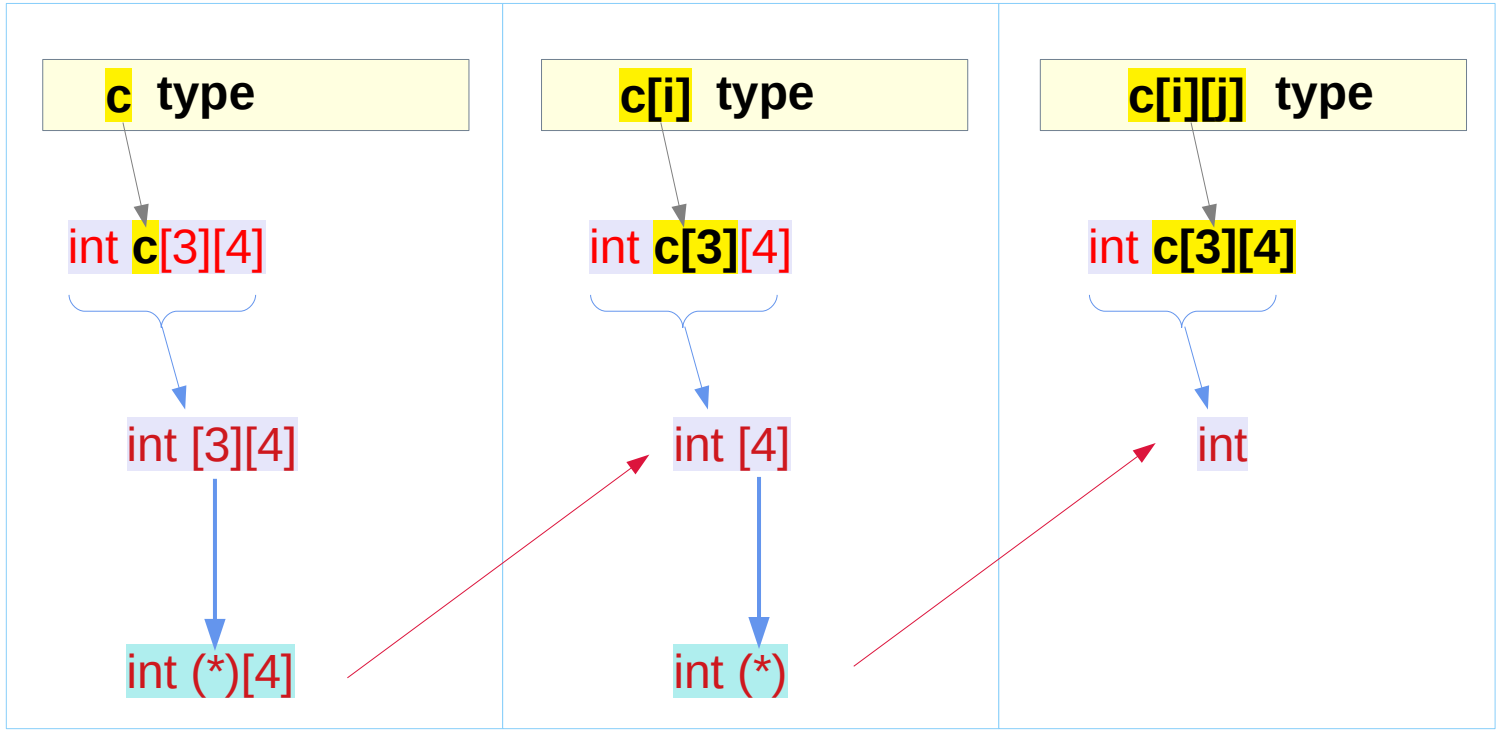
sizeof(c[0]) = sizeof(c[0][0]) * 4 … *leading element*
sizeof(c[0]+1) = pointer size (4/8 bytes)
sizeof(c[0]+2) = pointer size (4/8 bytes)
sizeof(c[0]+3) = pointer size (4/8 bytes)

sizeof(c[1]) = sizeof(c[1][0]) * 4 … *leading element*
sizeof(c[1]+1) = pointer size (4/8 bytes)
sizeof(c[1]+2) = pointer size (4/8 bytes)
sizeof(c[1]+3) = pointer size (4/8 bytes)

sizeof(c[2]) = sizeof(c[2][0]) * 4 … *leading element*
sizeof(c[2]+1) = pointer size (4/8 bytes)
sizeof(c[2]+2) = pointer size (4/8 bytes)
sizeof(c[2]+3) = pointer size (4/8 bytes)

Sub-array types in a **2-d** array

int c[3][4]; **2-d** array c



Dual Types

- **Identifying nested arrays**

  in a 2-d array declaration

Nested arrays in a 2-d array declaration

int c[3] [4] ;

int **c[3]** [4] ;

c : a 3 element array
c[i] : each element

int c[3] [4] ;

c[i]'s type 1 : int [4]
an array of 4 integers

c[i]'s type 2: int (*)
a pointer to an integer

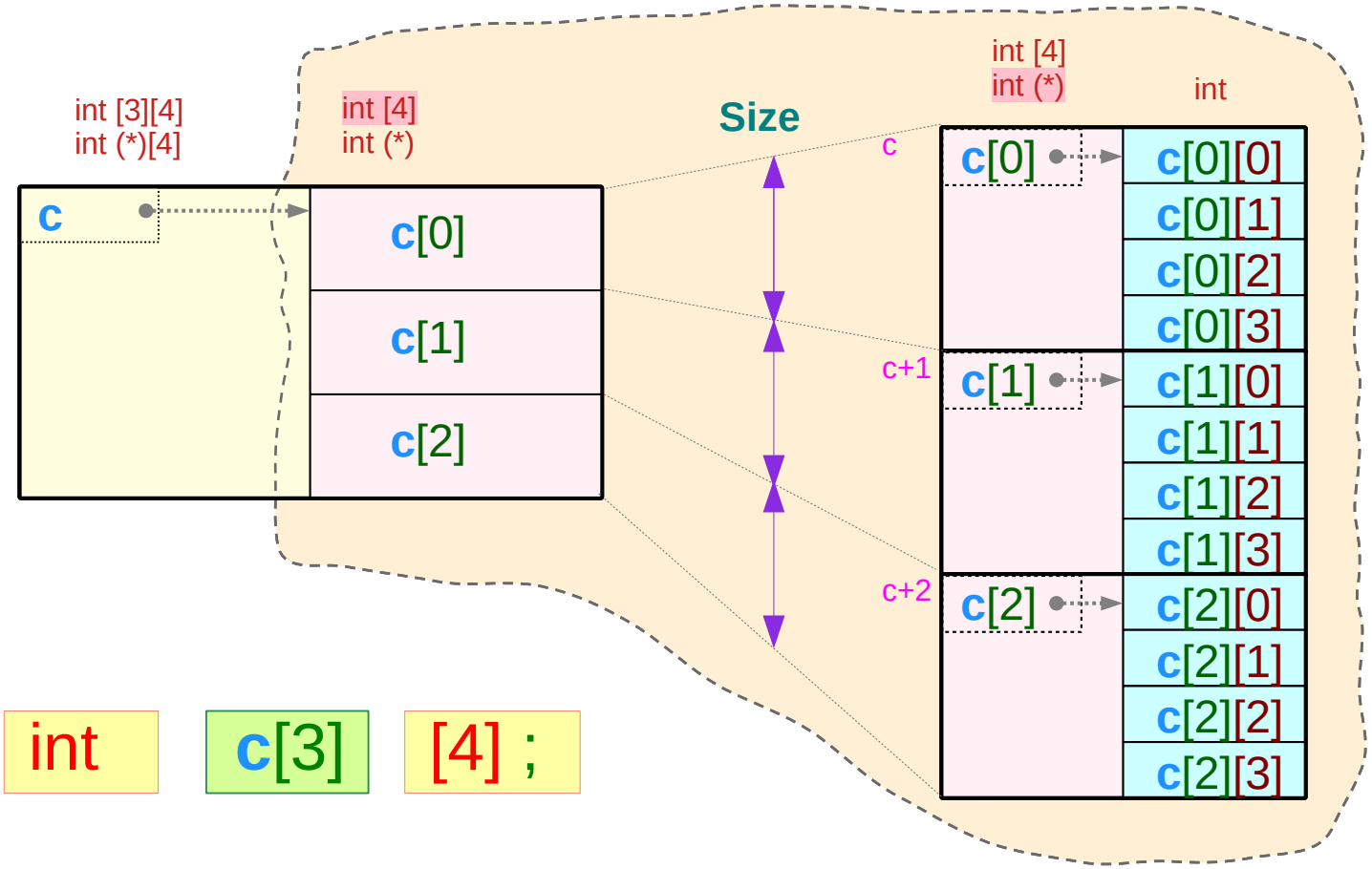
Nested arrays

c[3]

c : a 3 element array
c[i] : each element

int [4] ;

c[i]'s type 1 : int [4]
c[i]'s type 2 : int (*)



Address

&c[0][0] ➡c[0] ➡c

&c[1][0] ➡c[1]

&c[2][0] ➡c[2]

int

c[3]

[4] ;

c : 3-element array

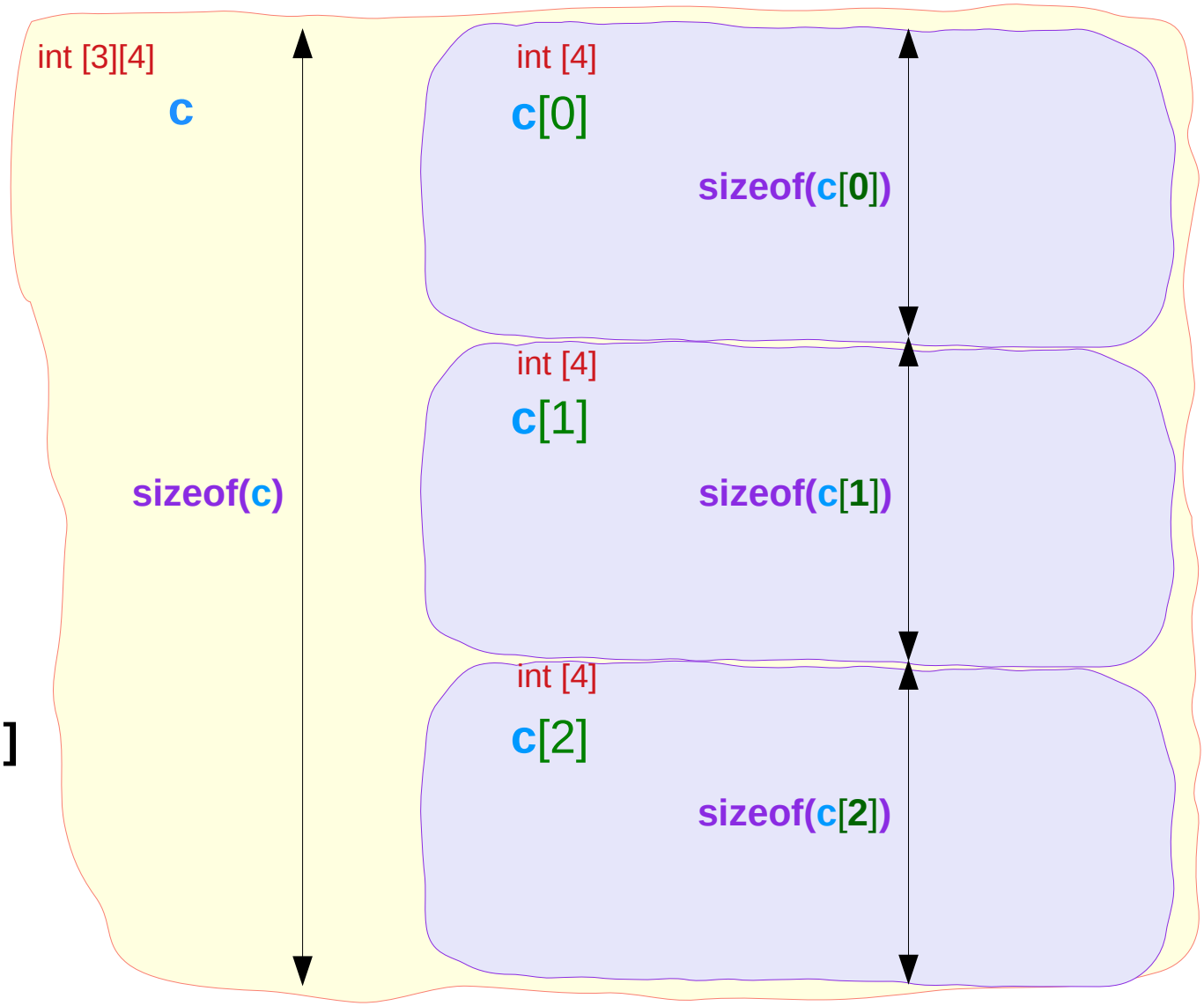
c	**2-d** array	int [3][4]
c[i]	**1-d** array	int [4]

```
int  c  [3]  [4] ;
```

3-element array c

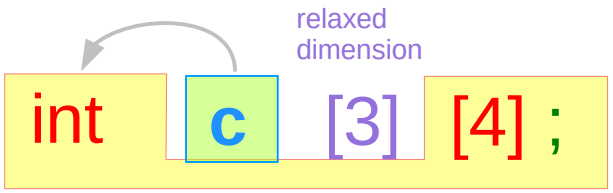
abstract data element **c[i]**

each element **c[i]** has the
1-d array type int [4]



c : pointer to a 4-element array

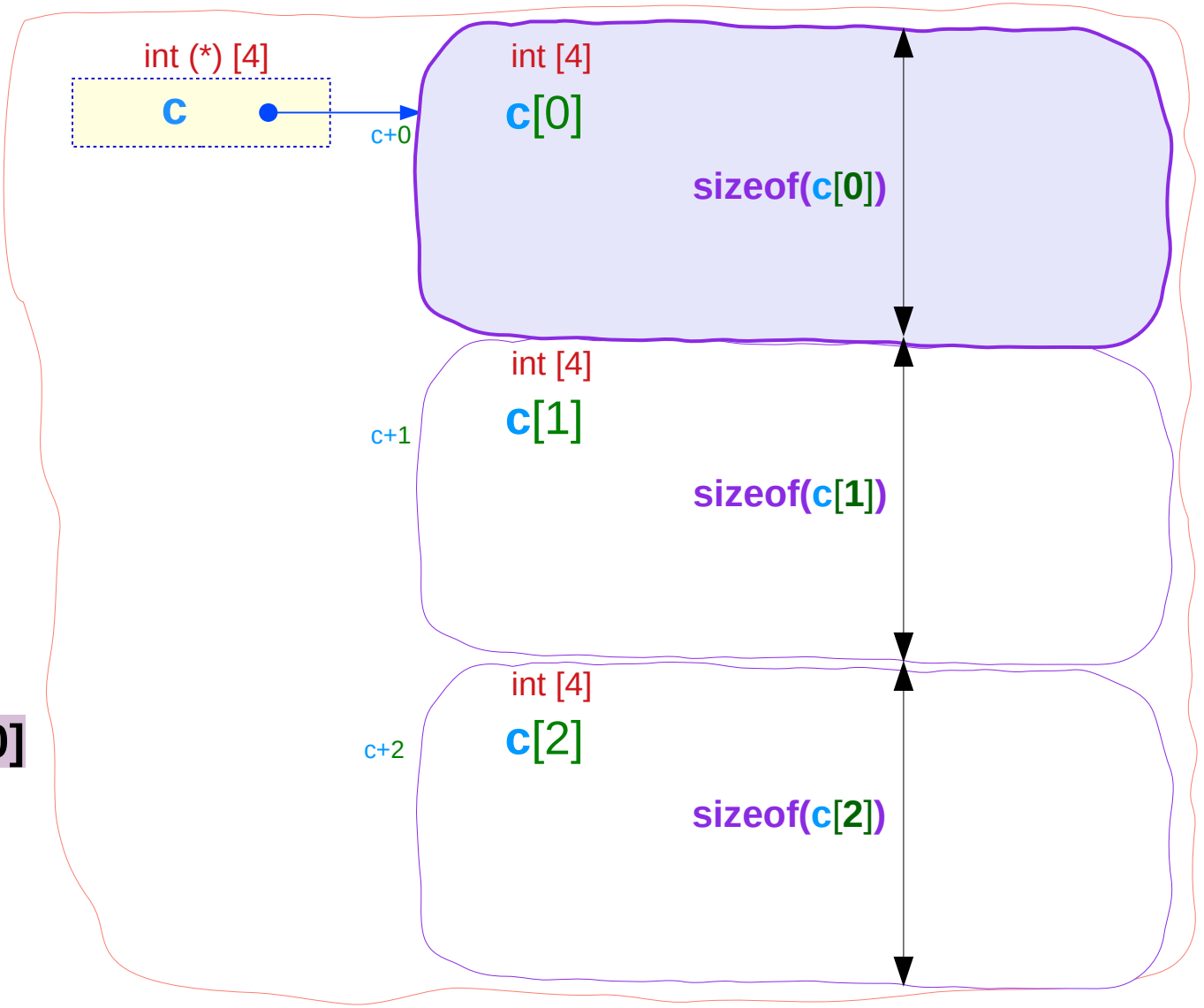
c	**1-d** array pointer	int (*)[4]
c[i]	**1-d** array	int [4]



pointer c

abstract data element **c[0]**

each element **c[i]** has the
1-d array type int [4]



c[i] : 4-element array

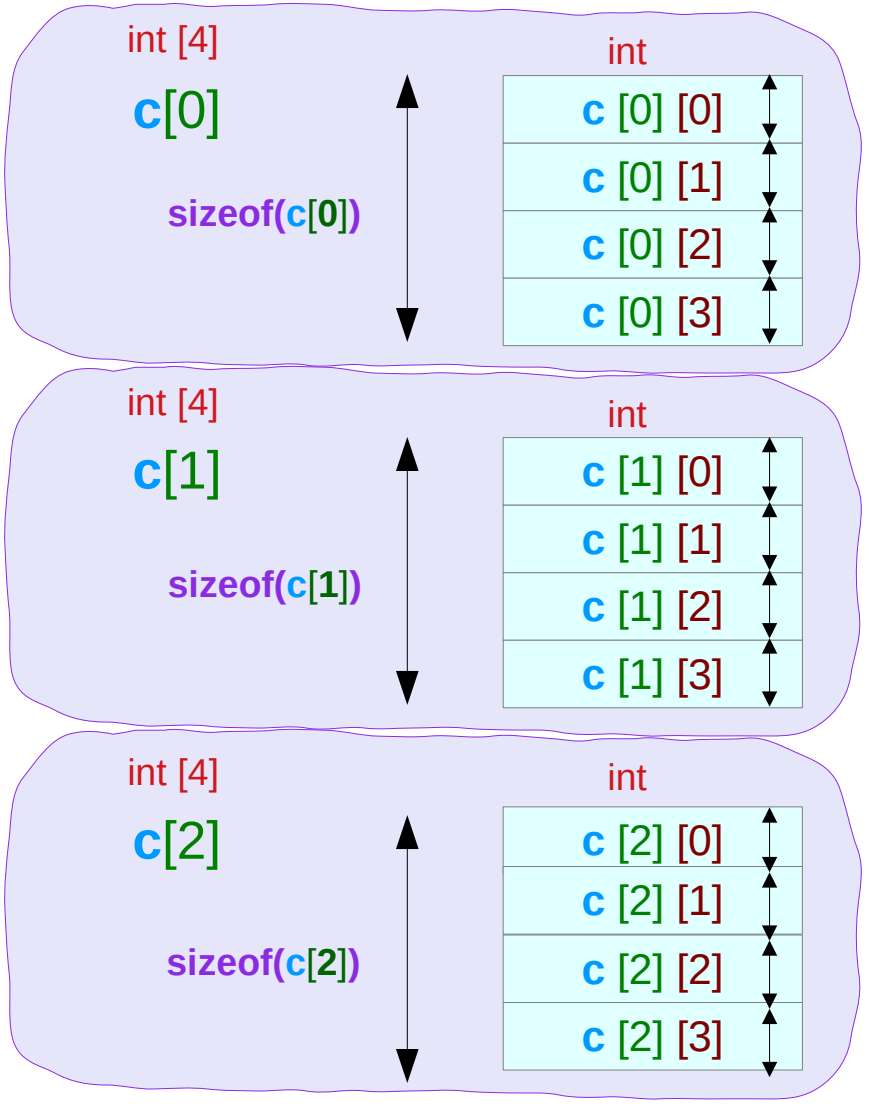
c[i]	**1-d** array	int [4]
c[i][j]	**0-d** array	int

int c [3] [4] ;

4-element array c[i]

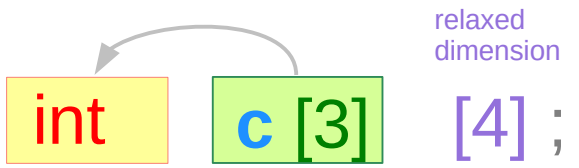
primitive data element **c[i][j]**

each element **c[i][j]** has
the primitive type int



c[i] : pointer to a primitive data

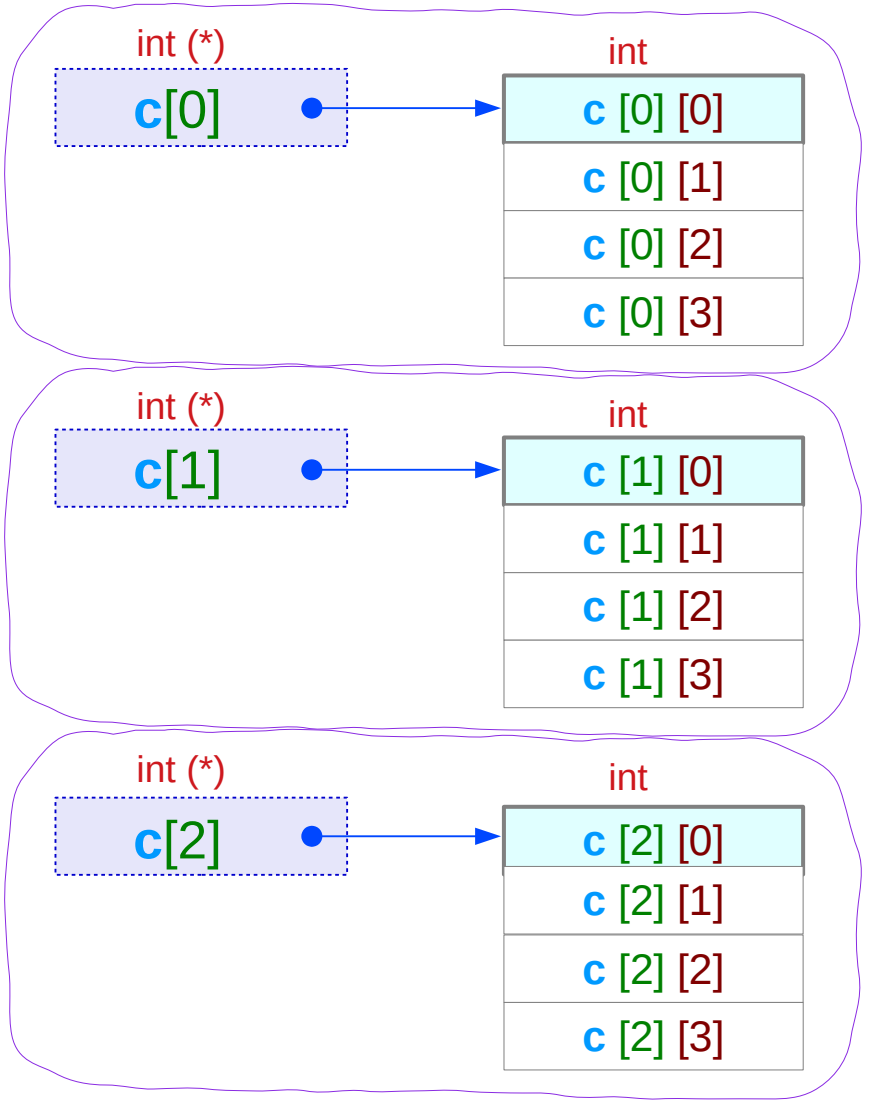
c[i]	**0-d** array pointer	int (*)
c[i][j]	**0-d** array	int



pointer c[i]

primitive data element **c[i][0]**

each element **c[i][j]** has
the primitive type int



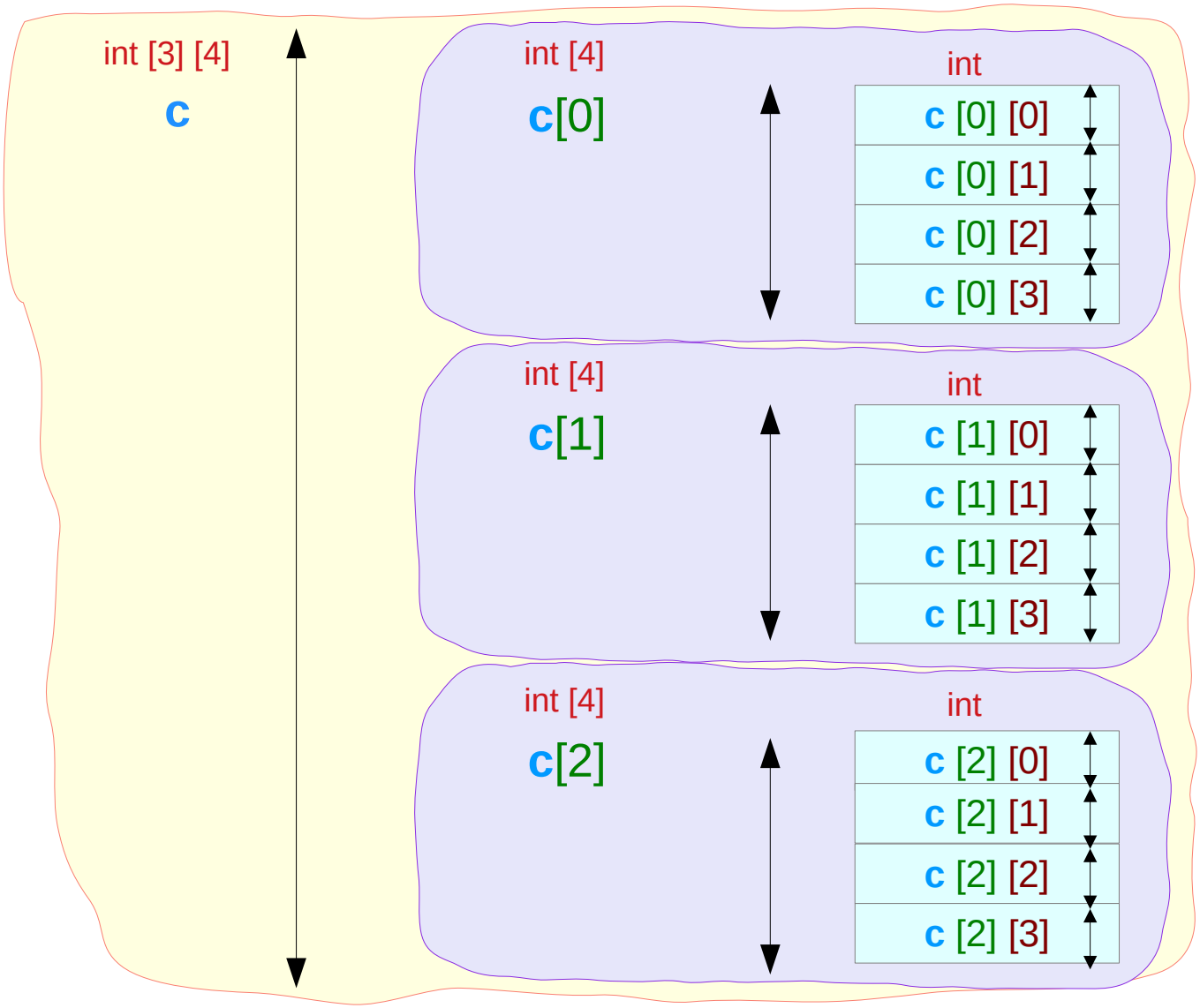
Recursive data view

c	**2-d** array	int [3][4]
c	**1-d** array pointer	int (*)[4]
c[i]	**1-d** array	int [4]
c[i]	**0-d** array pointer	int (*)
c[i][j]	**0-d** array	int

int c[3] [4] ;

3-element array c

4-element array c[i]



Pointer view

c	**2-d** array	int [3][4]
c	**1-d** array pointer	int (*)[4]
c[i]	**1-d** array	int [4]
c[i]	**0-d** array pointer	int (*)
c[i][j]	**0-d** array	int

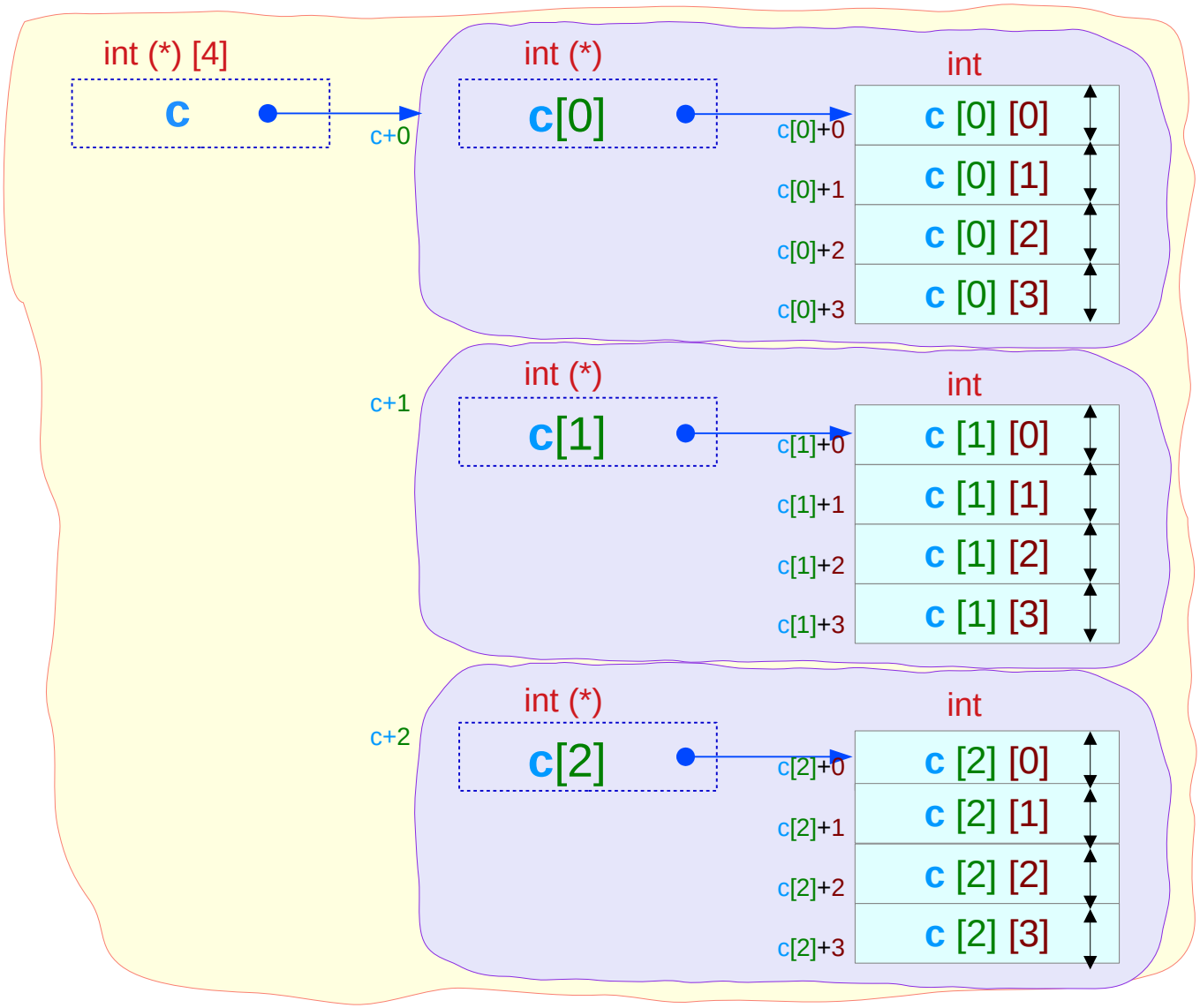
int c[3] [4] ;

v(c) = v(c[0]) = v(&c[0][0])

v(c[1]) = v(&c[1][0])

v(c[2]) = v(&c[2][0])

v ≡ value



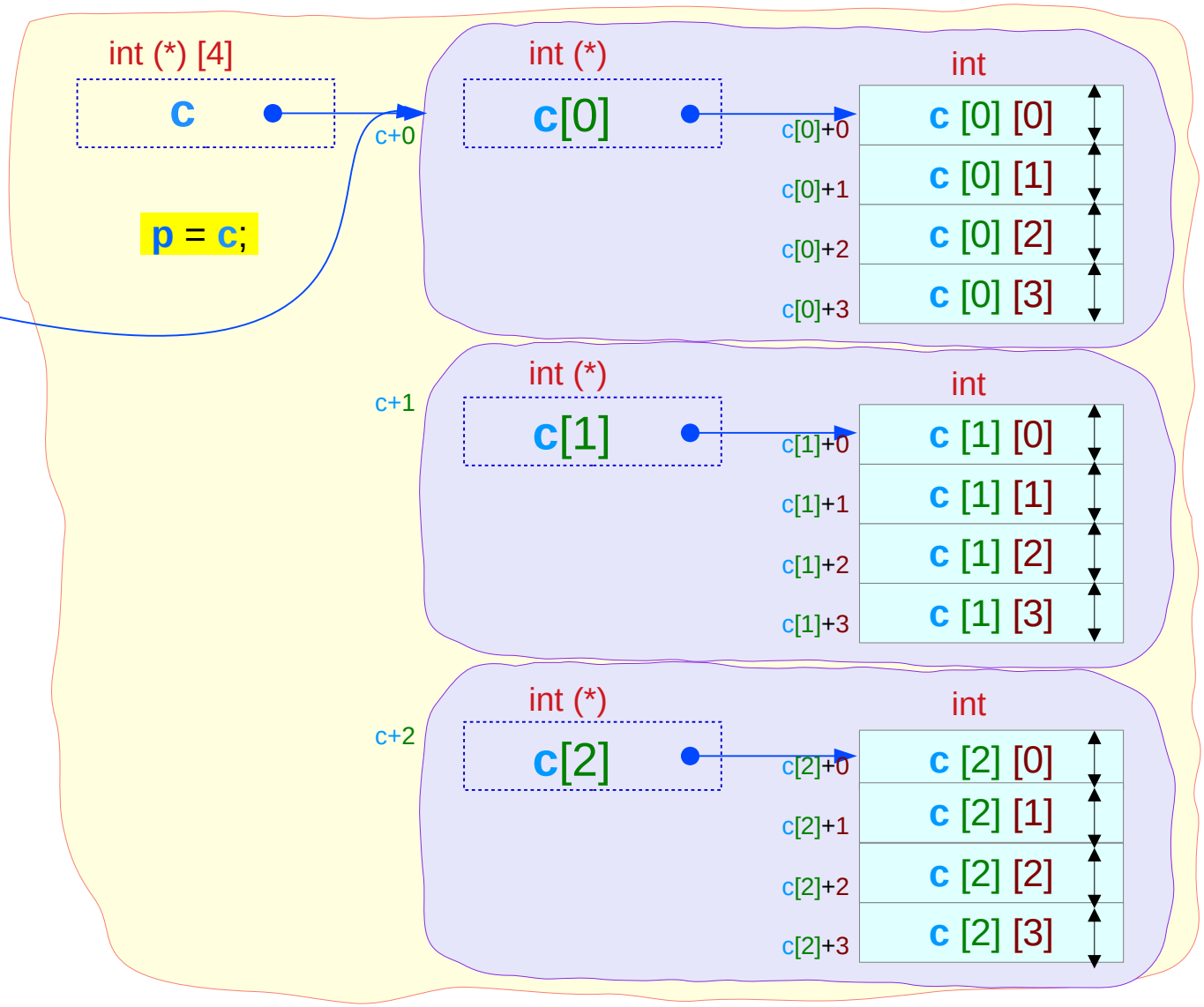
1-d array pointer

```
int     (*p)    [4] ;
```

```
int     c[3]    [4] ;
```

v(c) = v(c[0]) = v(&c[0][0])
v(c[1]) = v(&c[1][0])
v(c[2]) = v(&c[2][0])

v ≡ value

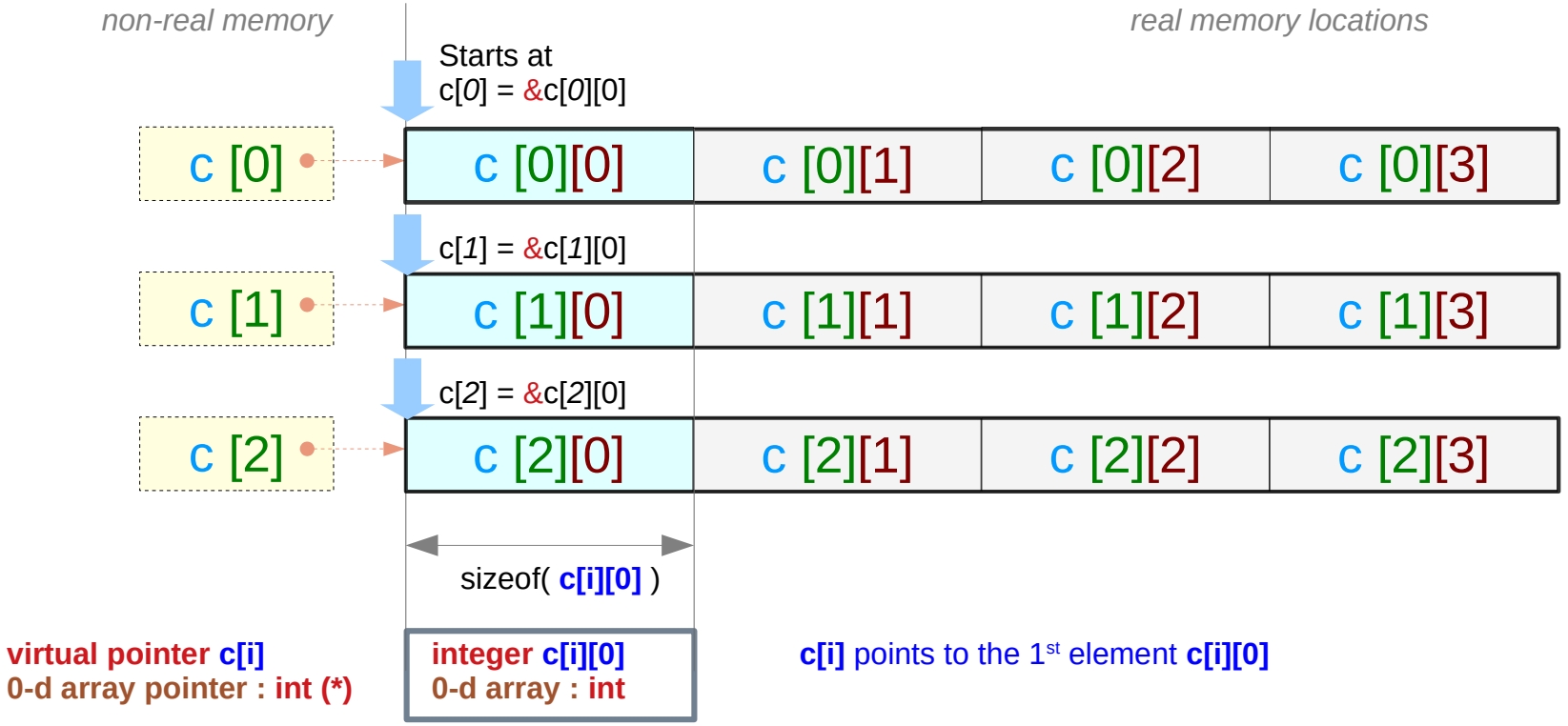


Pointer **c[i]** and integer **c[i][0]**

```
int c[3] [4];
```

non-real pointer **c[i]** : value(**c[i]**)= &**c[i][0]**

0-d array pointer

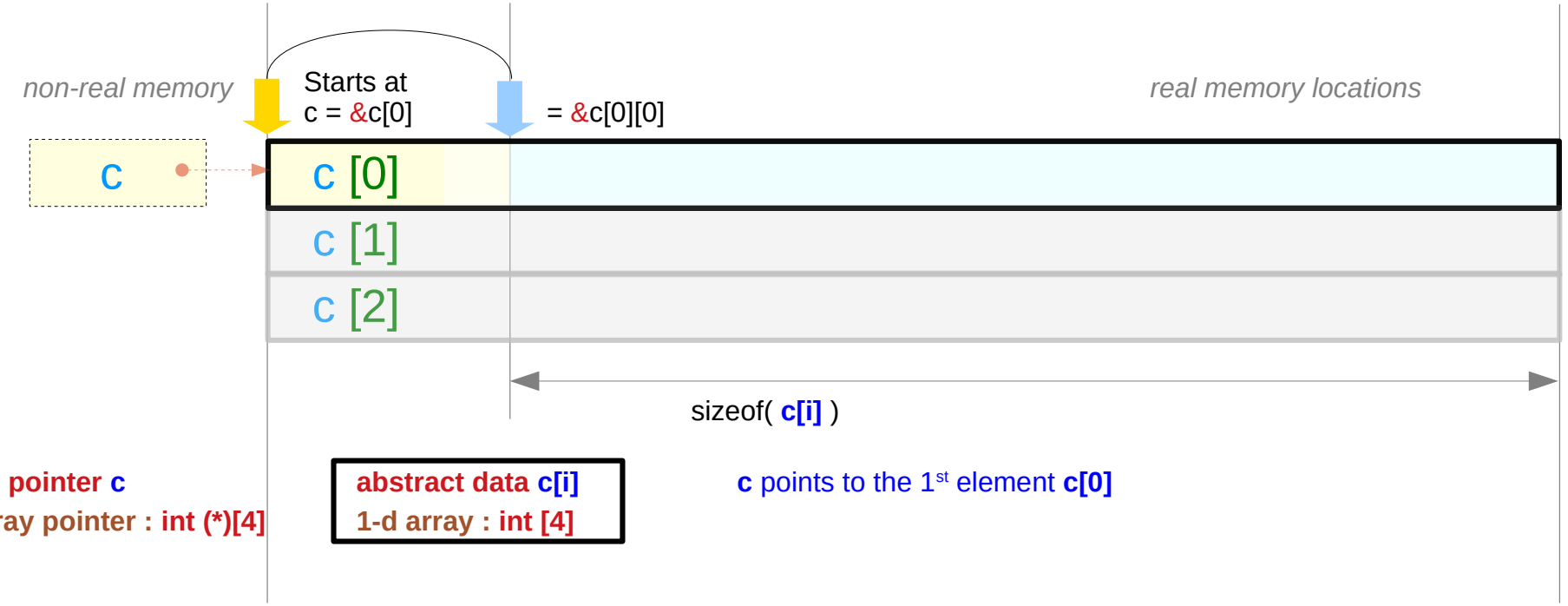


Pointer **c** and abstract data **c[i]**

```
int c [3] [4];
```

non-real pointer **c** : value(**c**) = &**c[0]** = &**c[0][0]**
abstract data **c[i]** : sizeof(**c[i]**) = 4 * sizeof(**int**)

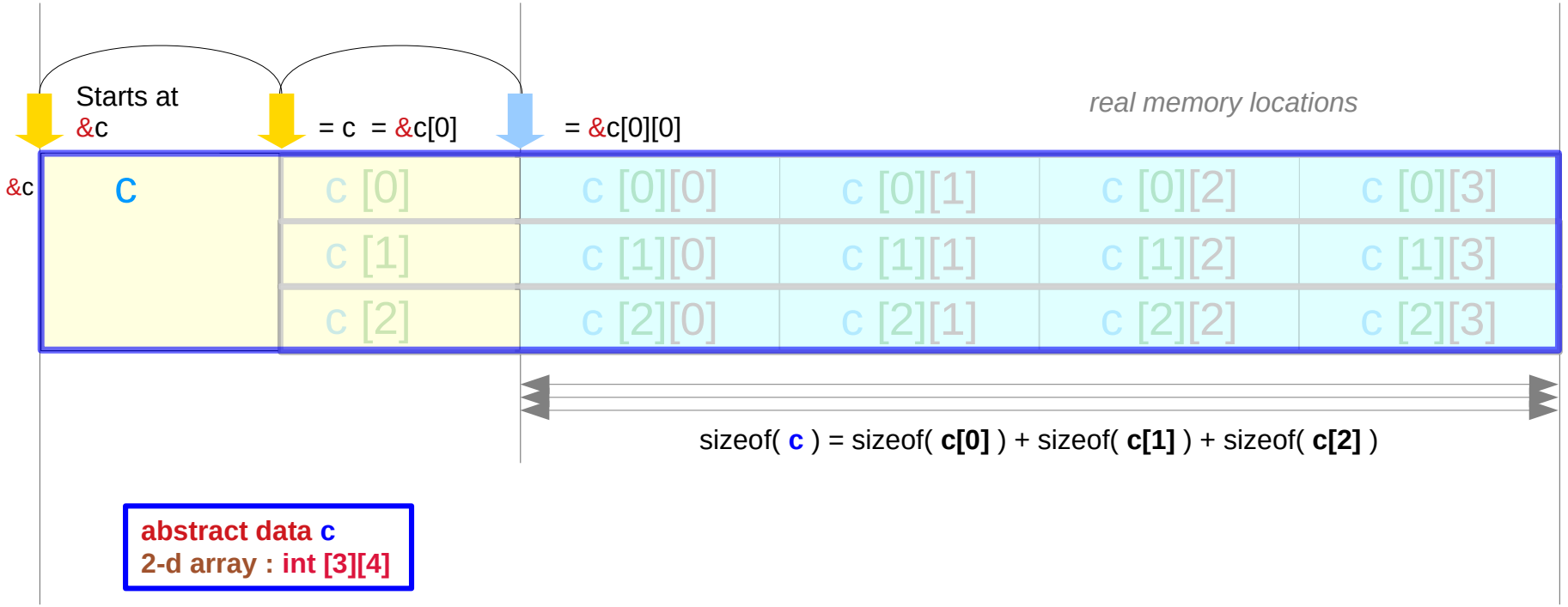
1-d array pointer
1-d array



Abstract data **c**

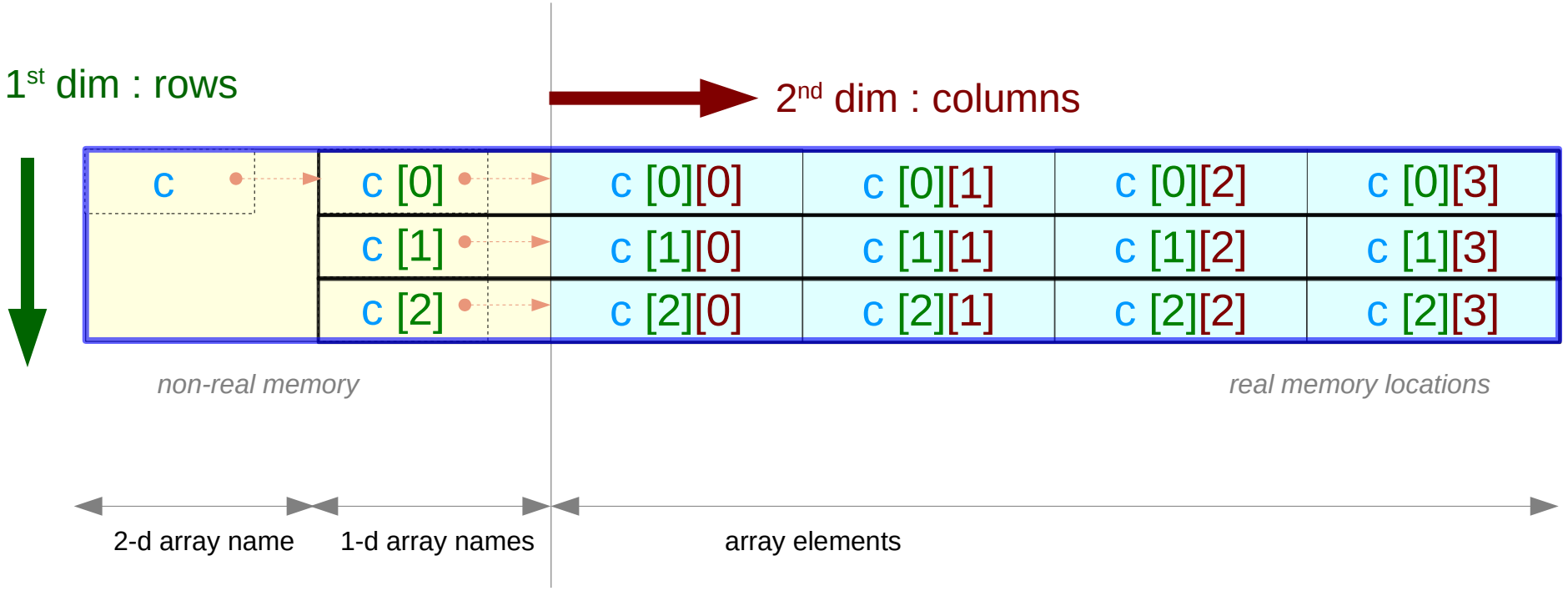
```
int c [3] [4];
```

abstract data **c:** sizeof(**c**) = 3 * sizeof(**c[i]**) **2-d** array



Rows and columns of a **2-d** array **c**

```
int c[3] [4];
```



The name of a 2-d array

```
int         a [4];

int      c [4] [4];
```

1. the name of the nested array (recursive definition)

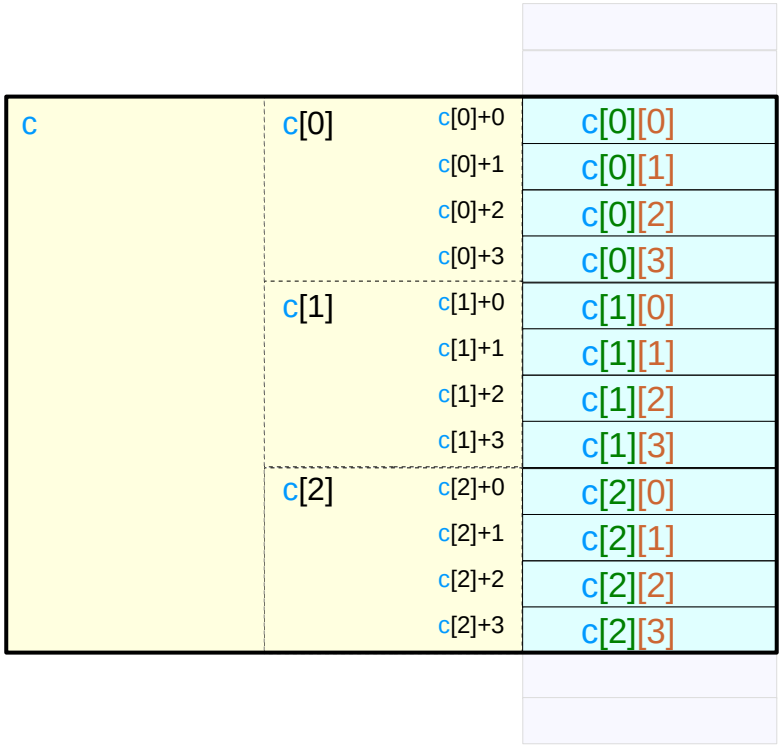
2. a double pointer

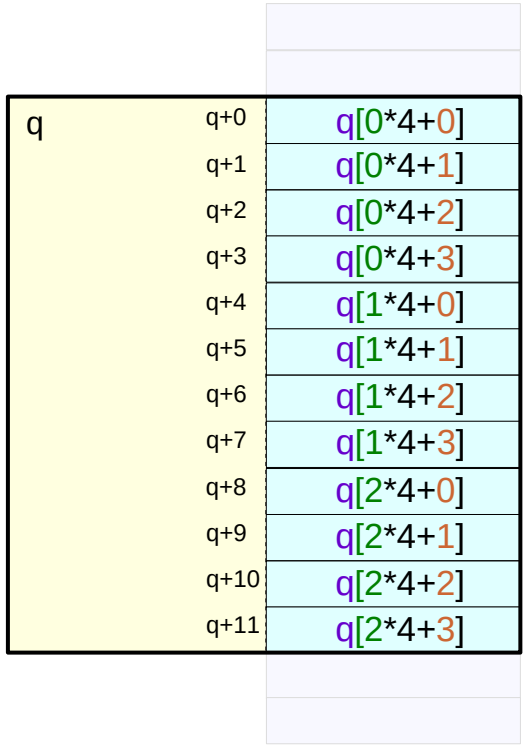
3. a pointer to an array

2-d array c and 1-d array q

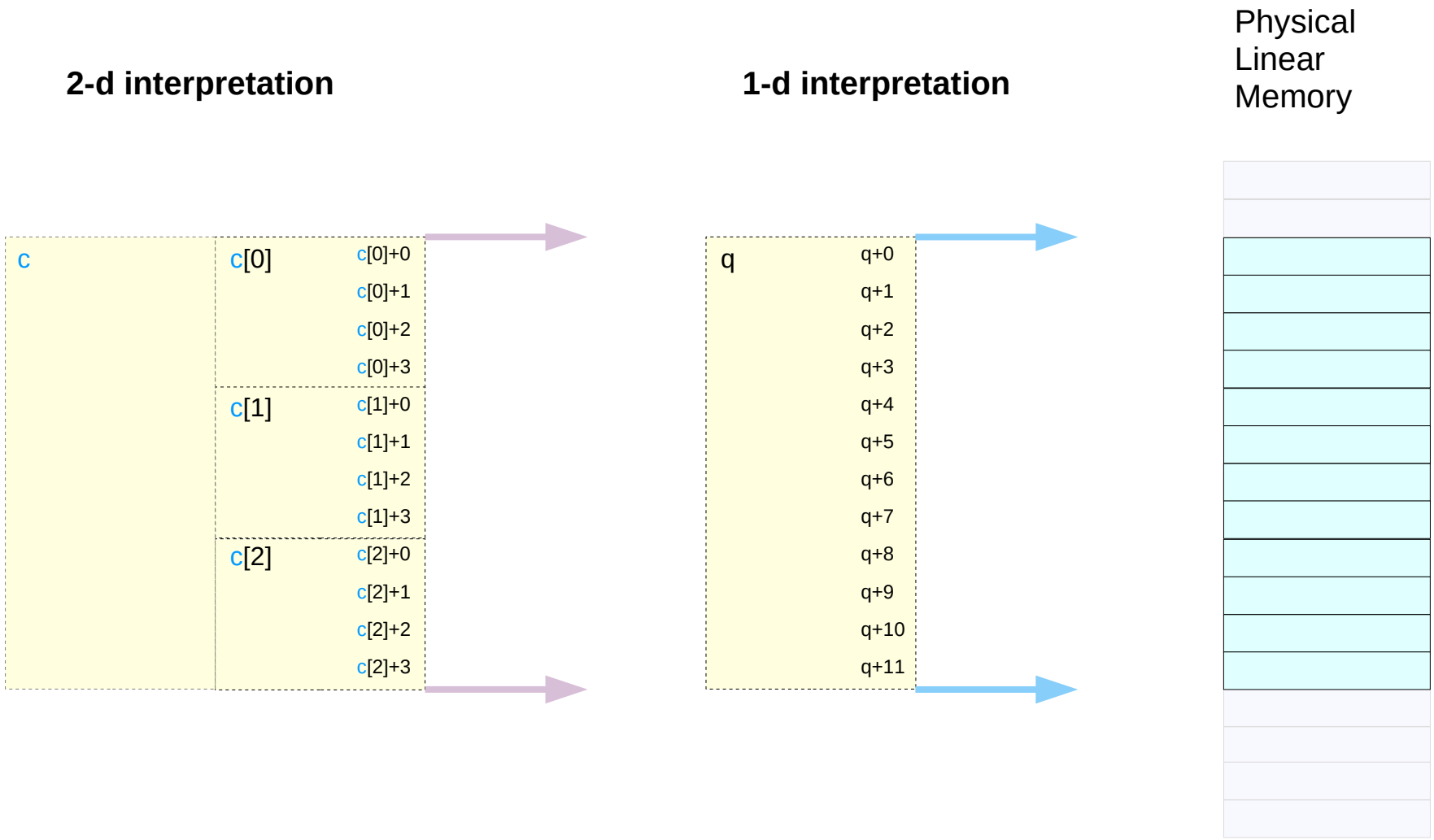
```
int c [3] [4];
```

```
int q [3*4];
```





2-d and 1-d interpretations of linear memories



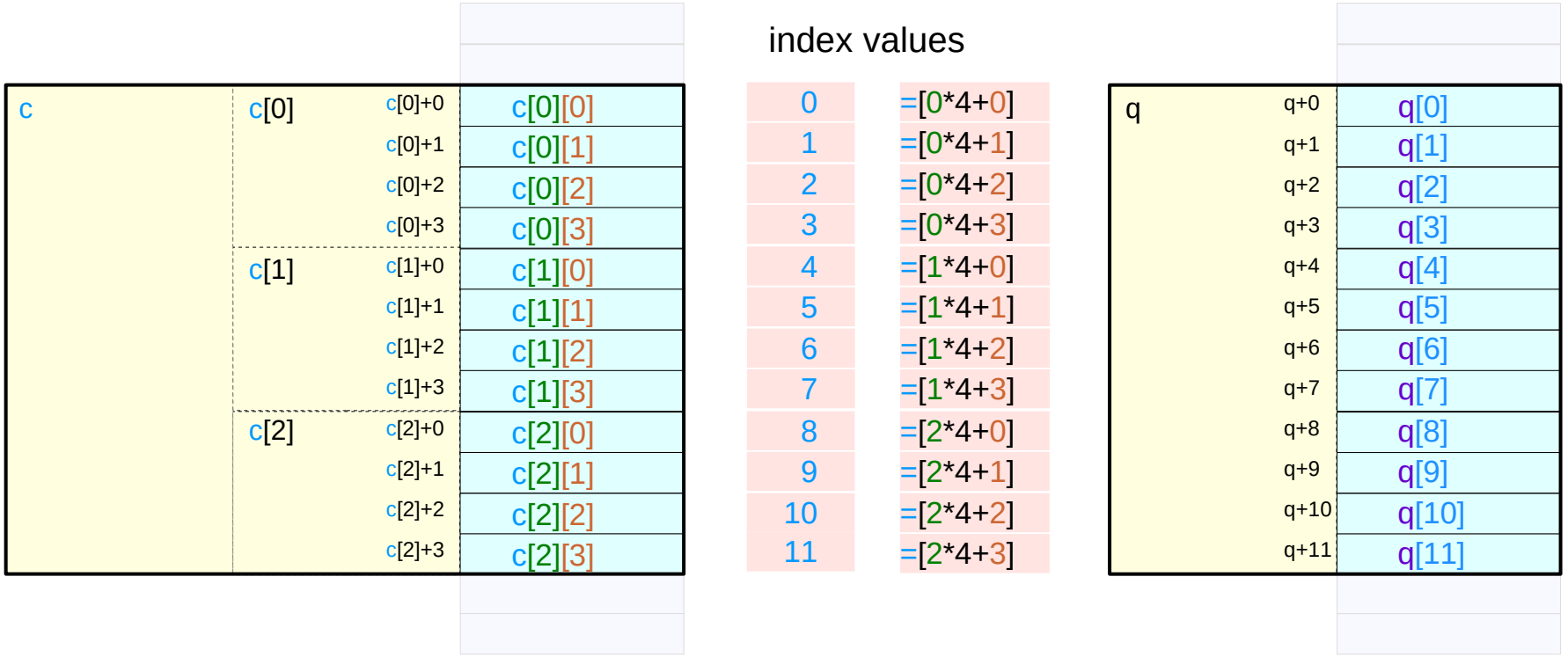
A **2-d** array stored as a **1-d** array (row major order)

```
int c [3] [4];
```

c[i][j]

[i*4+j]

[k]



2-d array access via a single pointer

```
int      *p = c[0] ;
```

```
int c [3][4];
```

```
p[ i*4 + j ]
```

```
c[ i ][ j ]
```

```
*(p+ i*4 + j)
```

```
*(*(c+i)+ j)
```

```
*(p +k)
```

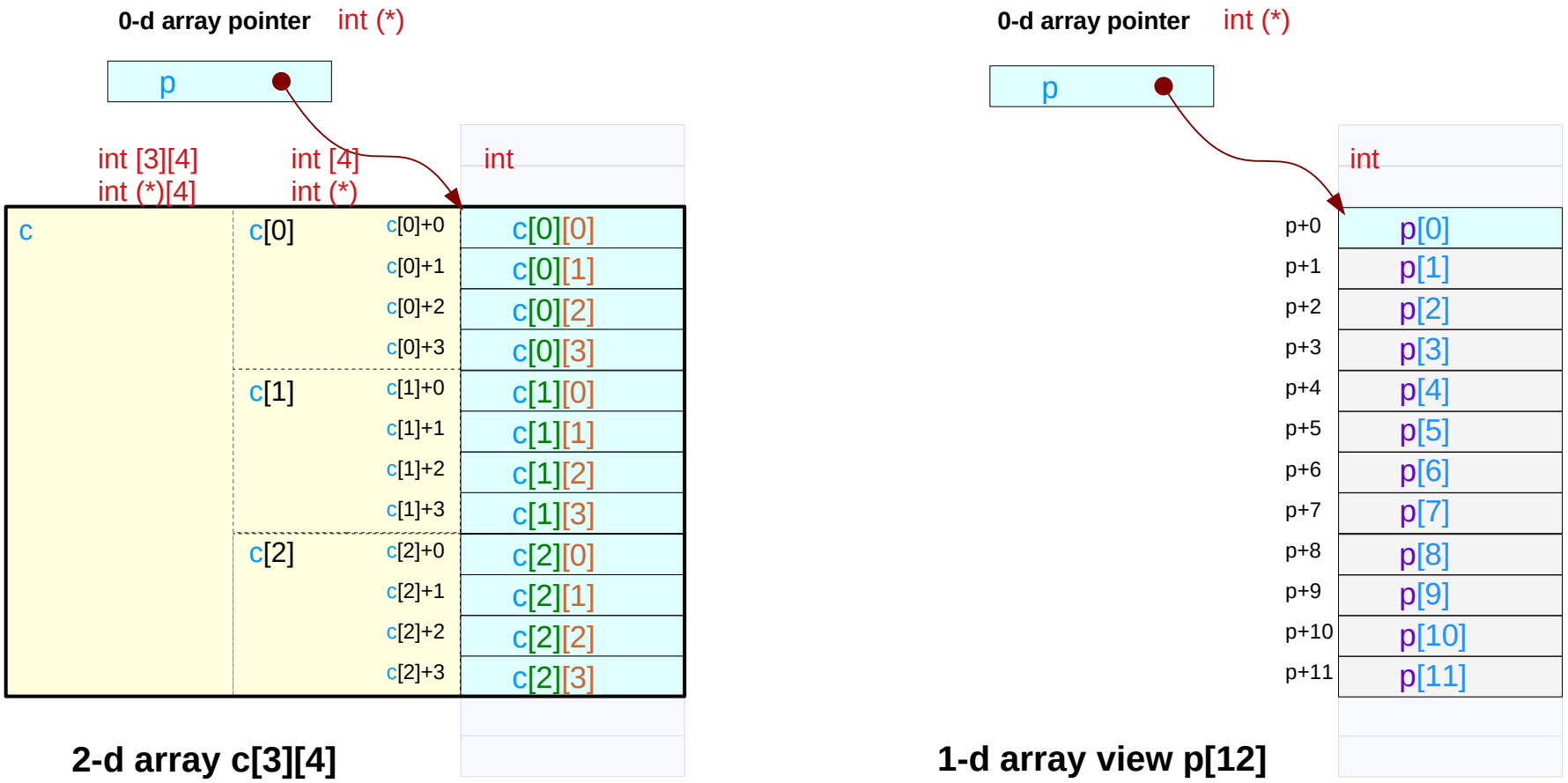
```
i = k / 4;
j = k % 4;
```

View a **2-d** array as a **1-d** array

```
int c [3][4];
```

```
int *p = c[0];
```

c, c[0], &c[0][0]



2-d array c[3][4]

1-d array view p[12]

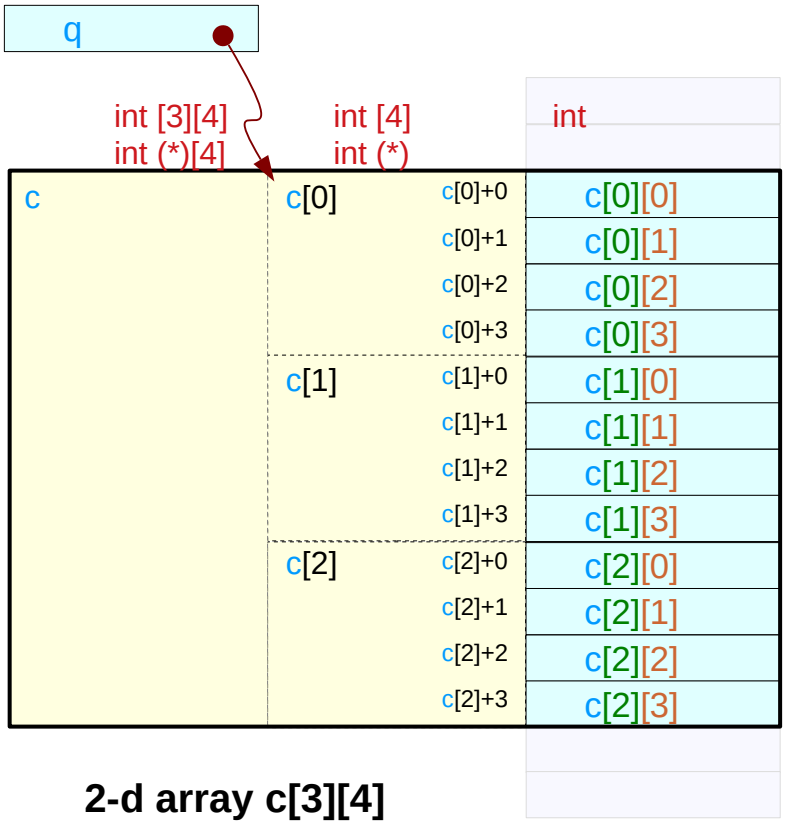
View a **2-d** array as another **2-d** array

```
int c [3][4];
```

```
int (*q) [3] = (int (*) [3]) c;
```

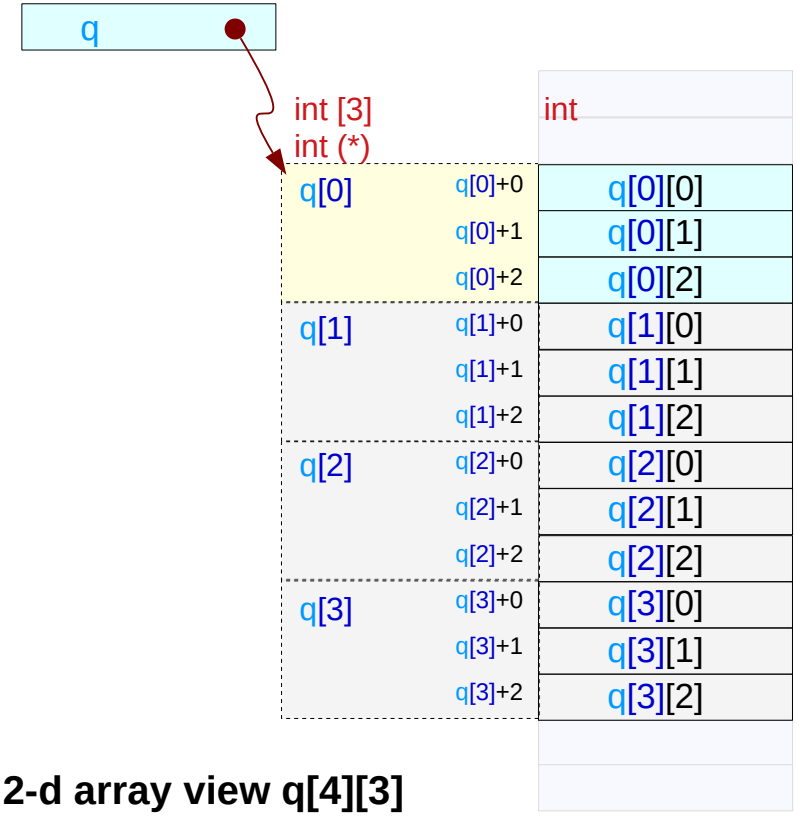
c, c[0], &c[0][0]

1-d array pointer int (*) [3]



2-d array c[3][4]

1-d array pointer int (*) [3]



2-d array view q[4][3]

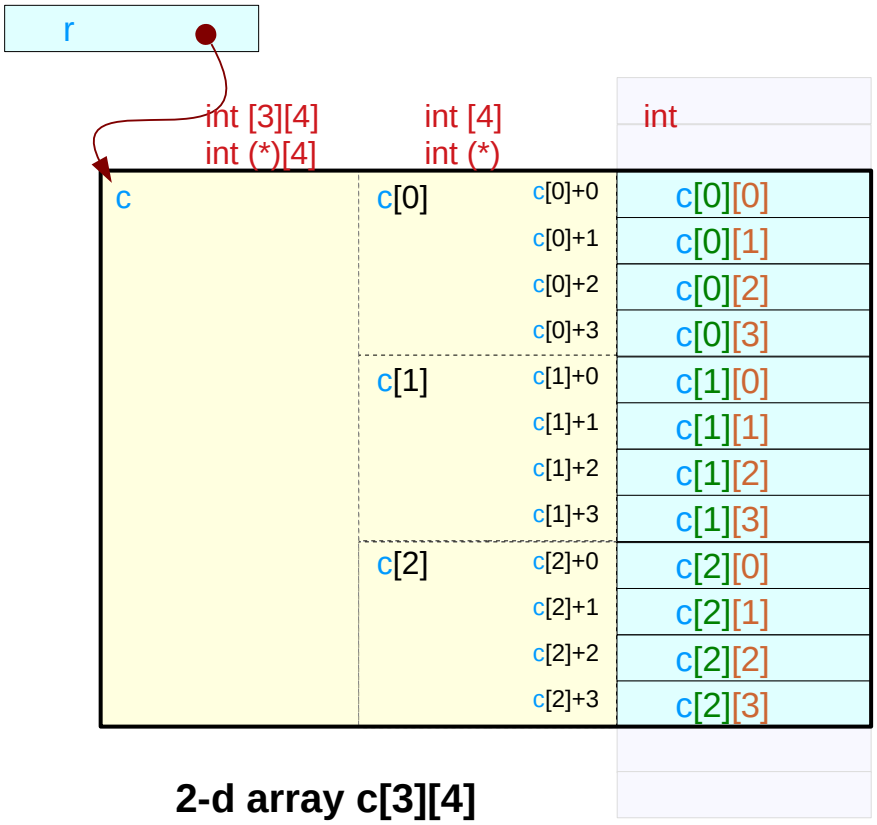
A **2-d** array stored as a **1-d** array (row major order)

```
int c [3] [4];
```

```
int (*r) [2][2] = (int (*) [2][2]) c;
```

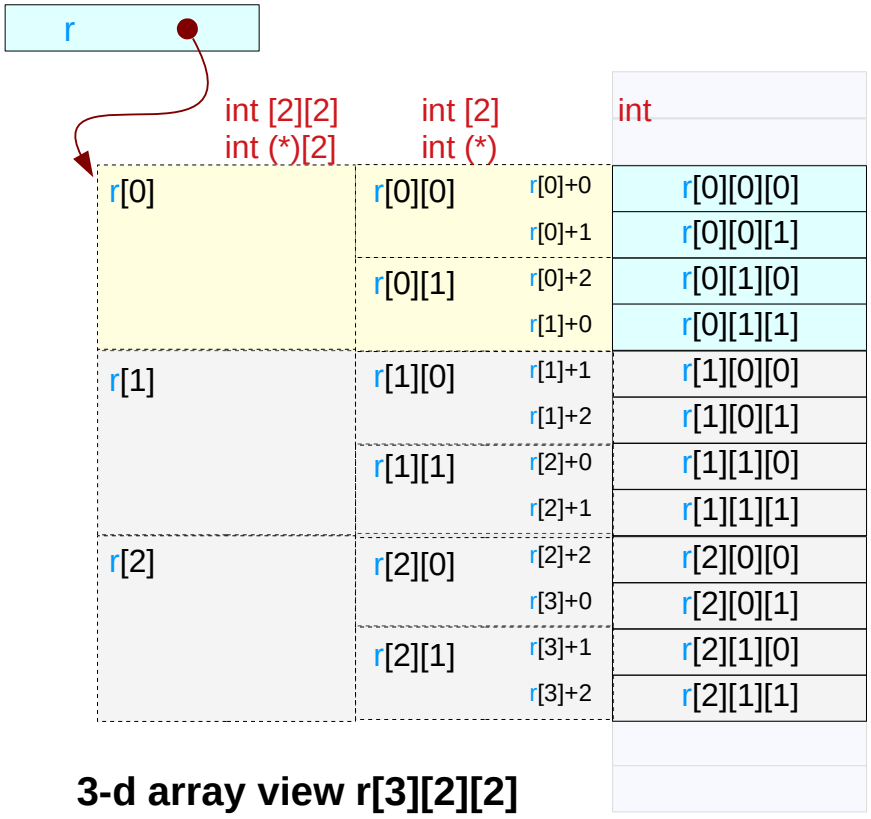
c, c[0], &c[0][0]

2-d array pointer int (*) [2][2]



2-d array c[3][4]

2-d array pointer int (*) [2][2]



3-d array view r[3][2][2]

2-d array access via pointers

```
int c [3][4];
```

```
int      *p = c[0] ;
```

1. recursive pointers

```
c [ i ][ j ]
```

```
(*(c+i))[ j ]
```

```
int (*p)[4];
```

```
*(c[ i ]+ j)
```

```
*(*(c+i)+ j)
```

```
int **q;
```

2. linear array pointers

```
p[ i*4 + j ]
```

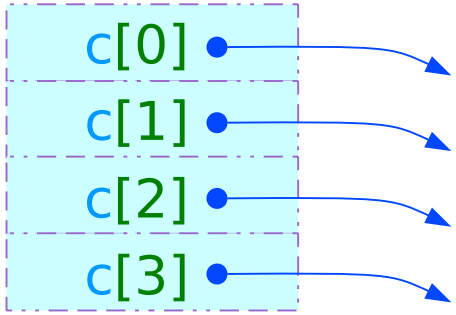
```
*(p+ i*4 + j )
```

Static Allocation of a 2-d Array

```
int A [3][4];
```

A in %eax,
i in %edx,
j in %ecx

```
sall    $2, %ecx                ;; j * 4
leal    (%edx, %edx, 2), %edx   ;; i * 3
leal    (%ecx, %edx, 4), %edx   ;; j * 4 + i * 12
movl    (%eax, %edx), %eax      ;; read M[ XA+4(3i +j) ]
```



The pointer array :
not allocated
in the memory

c[0]+0	*(c [0]+0)
c[0]+1	*(c [0]+1)
c[0]+2	*(c [0]+2)
c[0]+3	*(c [0]+3)
c[1]+0	*(c [1]+0)
c[1]+1	*(c [1]+1)
c[1]+2	*(c [1]+2)
c[1]+3	*(c [1]+3)
c[2]+0	*(c [2]+0)
c[2]+1	*(c [2]+1)
c[2]+2	*(c [2]+2)
c[2]+3	*(c [2]+3)

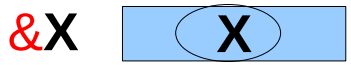
Pointers, arrays, and operator precedence

Address-of & and dereference * operators

Address-of operation

&X	=	value(&X)	*rvalue*
C Expressions		Mixed Expressions	

rvalue *lvalue*



*&**X** evaluates the address value of a variable **X***

& is a mathematical operator (the inverse operator of *)

value(&**X**) = value(value(&**X**)) = value(&**X**) = &**X**

Dereference operation

*X	=	*value(X)	*lvalue*
C Expressions		Mixed Expressions	

rvalue *lvalue*

lvalue must be evaluated to rvalue

***X** must be evaluated to an address before de-referencing*

Equivalences in address replications

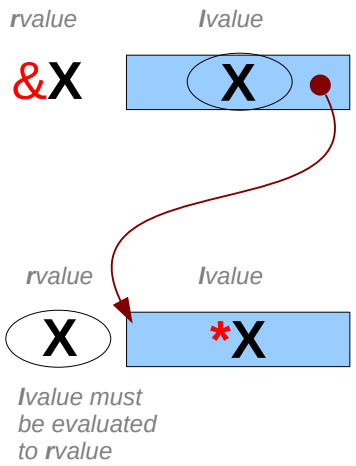
Eqivalences in Address Replications

a pointer variable ***X***

value(&**X**) ≡ value(**X**)

at the pointed address ***X***

value(**X**) ≡ ***value(**X**)

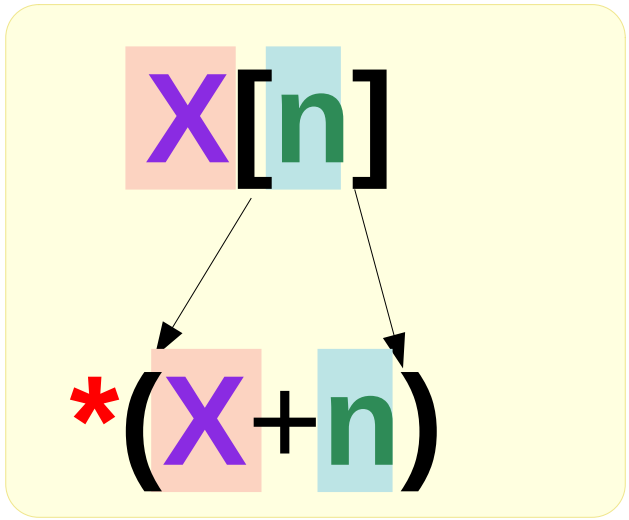


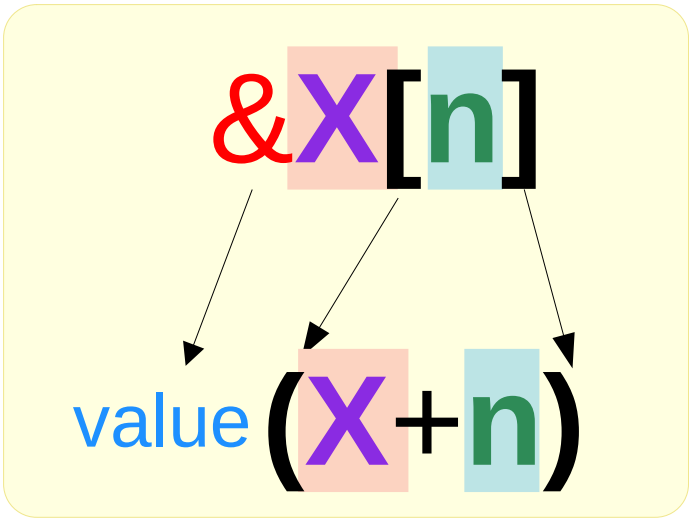
*&**X** and **X** have different types but have the same value*

***X** and ***X** have different types but have the same value*

value(&**X**) ≡ value(**X**) ≡ ***value(**X**)

Equivalences in array notations

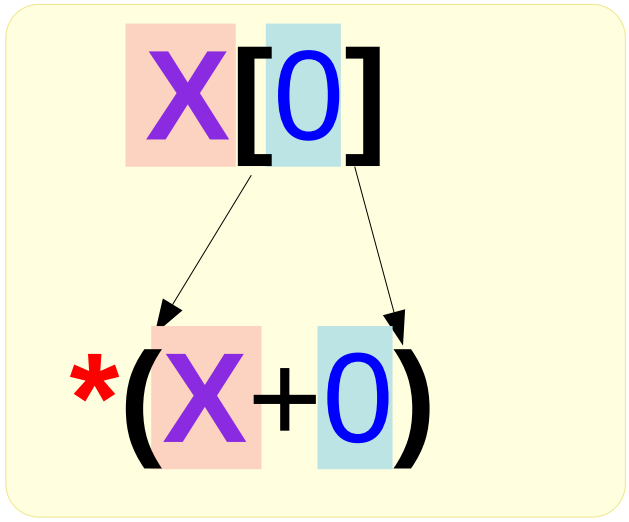


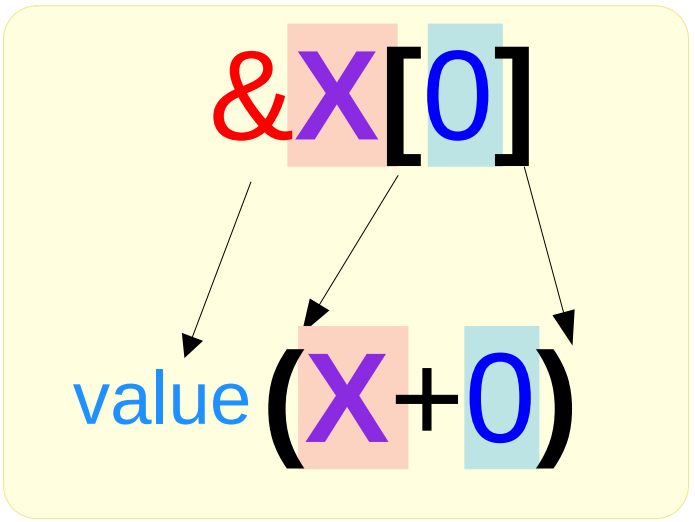


C operator &
≠ inverse of *

math operator &
= inverse of *

value(&X[n])
= value(&*(X+n))
= value(X+n)





C operator &
≠ inverse of *

math operator &
= inverse of *

value(&X[0])
= value(&*(X+0))
= value(X+0)

Pointer Arithmetic

• increment / decrement	**++X, --X, X++, X--**
• addition of an integer	**X + i**
• subtraction of an integer	**X – i**
• subtracting two pointers of the same type	**X – Y**
• comparison of pointers	**==, !=, >, >=, <, <=**
• adding two pointers are not allowed	~~**X + Y**~~

pointer variables: **X**, **Y**

integer compatible variables : **i**

(**int**, **short**, **char**, ...)

Pointer Addition / Subtraction

pointer variables: **X**, **Y**

primitive variables : **A, B**

value(**X**) is used to avoid confusion
between pointer additions
and arithmetic additions

X + A
X – A
the variable **A** must have integer compatible types, otherwise error

X + Y error!

X – Y o.k.

value(**A**) = **A** primitive variable **A**

value(**X**) ≠ **X** pointer variable **X**

value(**X** + **A**) = value(**X**) $+_a$ **A** * sizeof(***X**)

value(**X – A**) = value(**X**) $-_a$ **A** * sizeof(***X**)

value(**X + Y**) error!

value(**X – Y**) = value(**X**) $-_a$ value(**Y**)

$+_a$ is used to denote arithmetic additions

$-_a$ is used to denote arithmetic subtractions

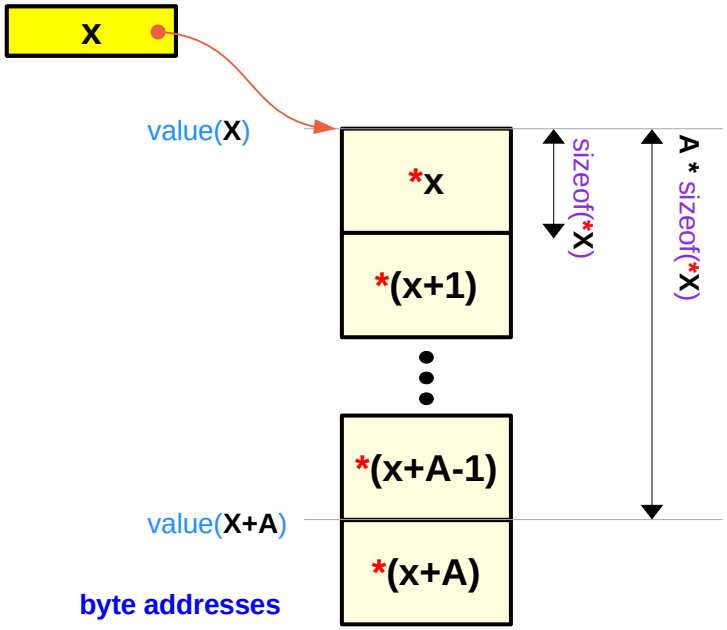
Pointer Addition / Subtraction

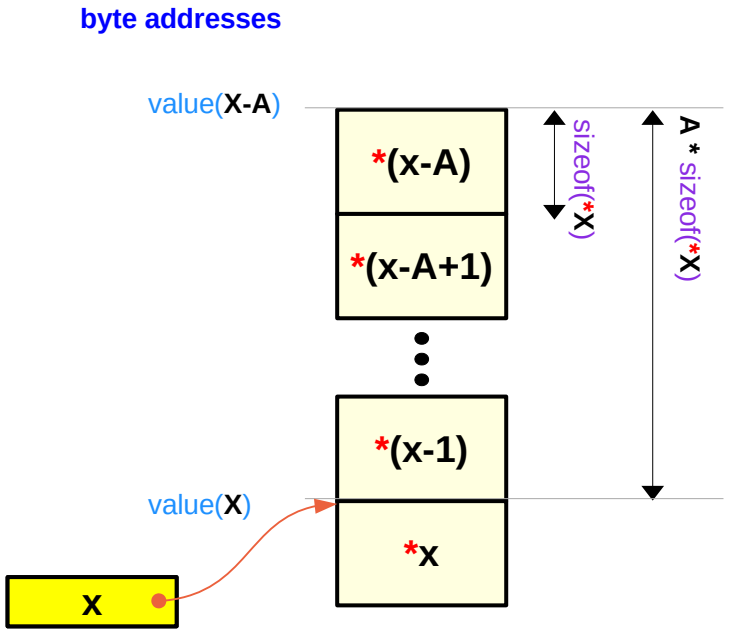
X + A : pointer addition

value(**X** + **A**) = value(**X**) $+_a$ **A***sizeof(***X**)

X – A : pointer subtraction

value(**X** – **A**) = value(**X**) $-_a$ **A***sizeof(***X**)





Pointer variables vs. primitive variables

pointer variables: **X**	value(**X**) ≠ **X**
integer compatible variables : **A**	value(**A**) = **A**

X	C expression	+ ➡ pointer additions – ➡ pointer subtractions	value(**X** + **A**) = value(**X**) $+_a$ **A***sizeof(***X**) value(**X** – **A**) = value(**X**) $-_a$ **A***sizeof(***X**)
A	C expression	+ ➡ pointer additions – ➡ pointer subtractions *for a pointer type operand*	value(**X** + **A**) = value(**X**) $+_a$ **A***sizeof(***X**) value(**X** – **A**) = value(**X**) $-_a$ **A***sizeof(***X**)
		+ ➡ arithmetic additions $+_a$ – ➡ arithmetic subtractions $-_a$ *for a non pointer type operand*	
value(**X**)	Math expression	+ ➡ arithmetic additions $+_a$ – ➡ arithmetic subtractions $-_a$	
value(**A**)	Math expression	+ ➡ arithmetic additions $+_a$ – ➡ arithmetic subtractions $-_a$	

Recursive application of value()

value(value(**A**)) = value(**A**) = **A**

value(value(**X**)) = value(**X**) ≠ **X**

value(value(**X** + **i**))

= value(value(**X**) $+_a$ **i** * sizeof(***X**))

= value(value(**X**)) $+_a$ value(**i** * sizeof(***X**))

= value(**X**) $+_a$ **i** * sizeof(***X**)

= value(**X**) + **i** * sizeof(***X**) in math expression

Operator Precedence of * and [ ]

*x[m]	≡	*(x[m])
x[m][n]	≡	(x[m])[n]
**x	≡	*(*x)

[ ] has a higher priority than *

[ ] has left-to-right associativity

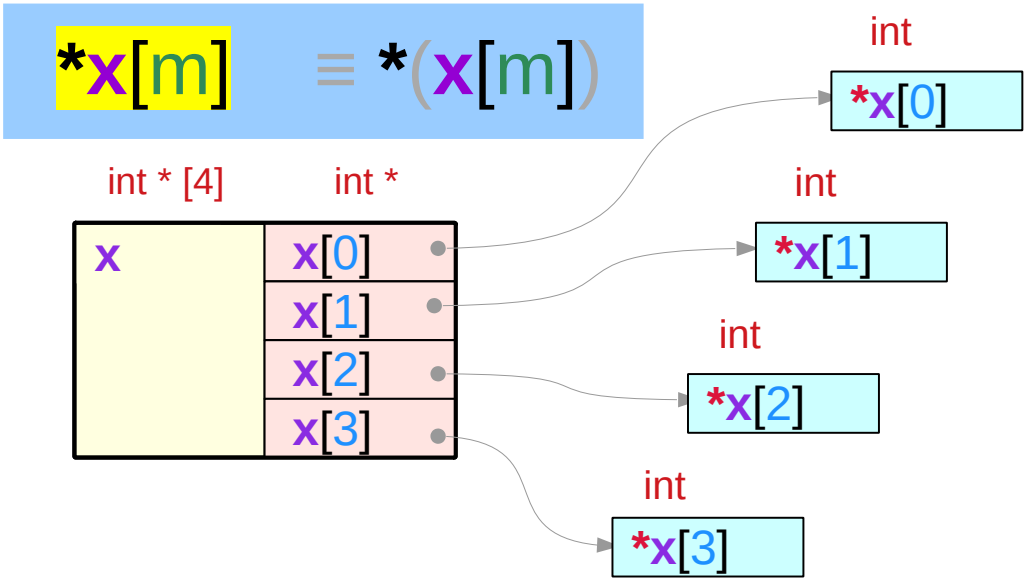
* has right-to-left associativity

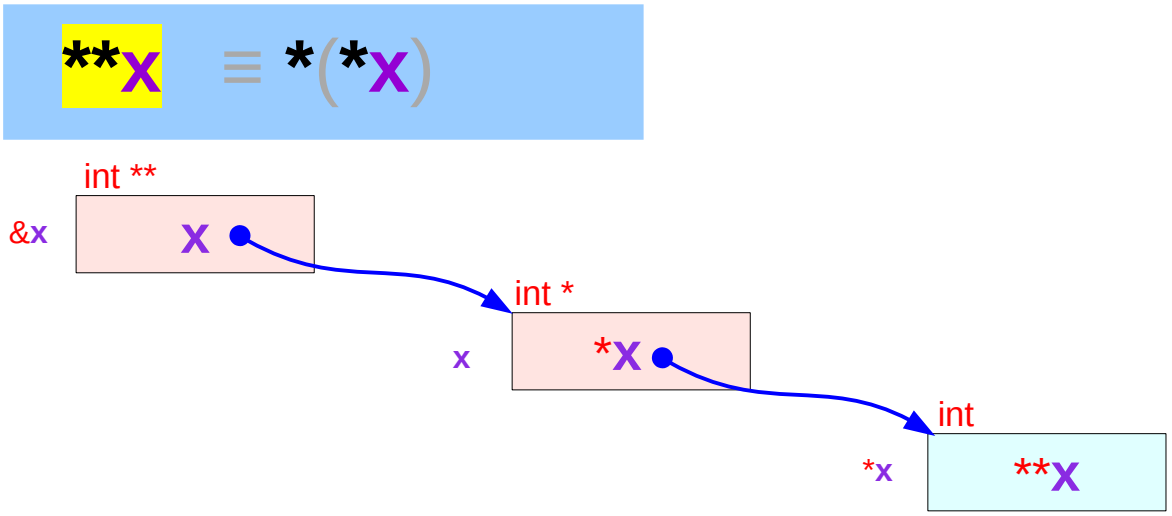
(*x)[m][n]	⟺	((*x)[m])[n]
(*x[m])[n]	⟺	(*(x[m]))[n]

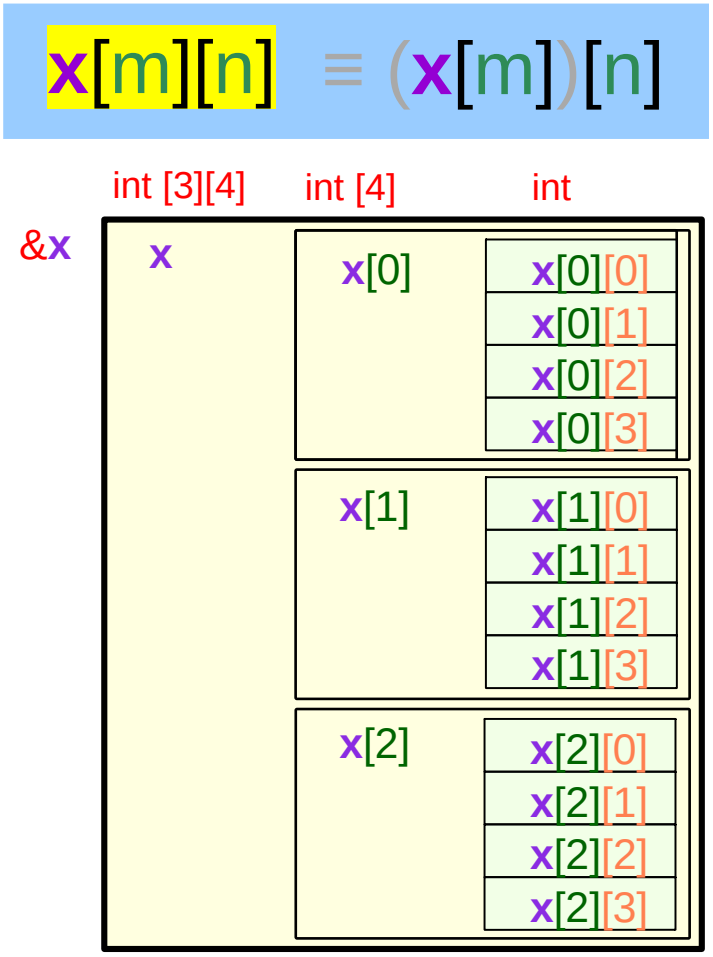
red parentheses ( ) must not be removed

gray parentheses ( ) can be removed

Operator Precedence of * and [ ]

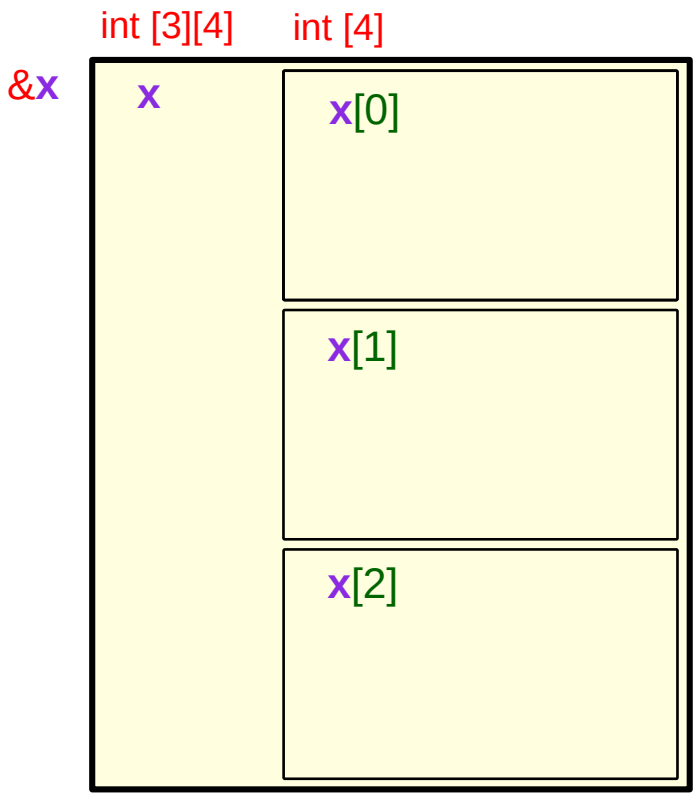






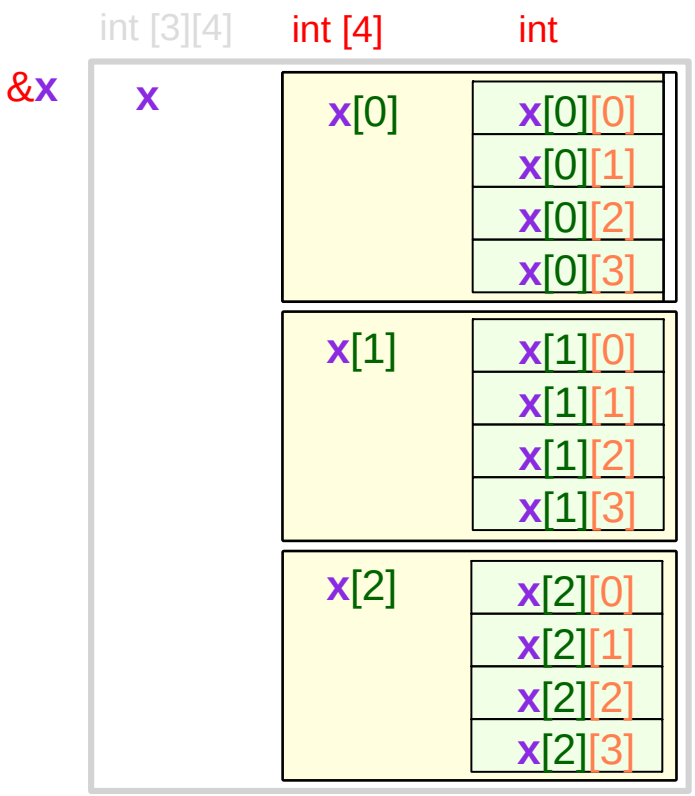
Abstract Data **x** and **x[i]**

x[3] **x** has 3 elements



array element **x[i]**

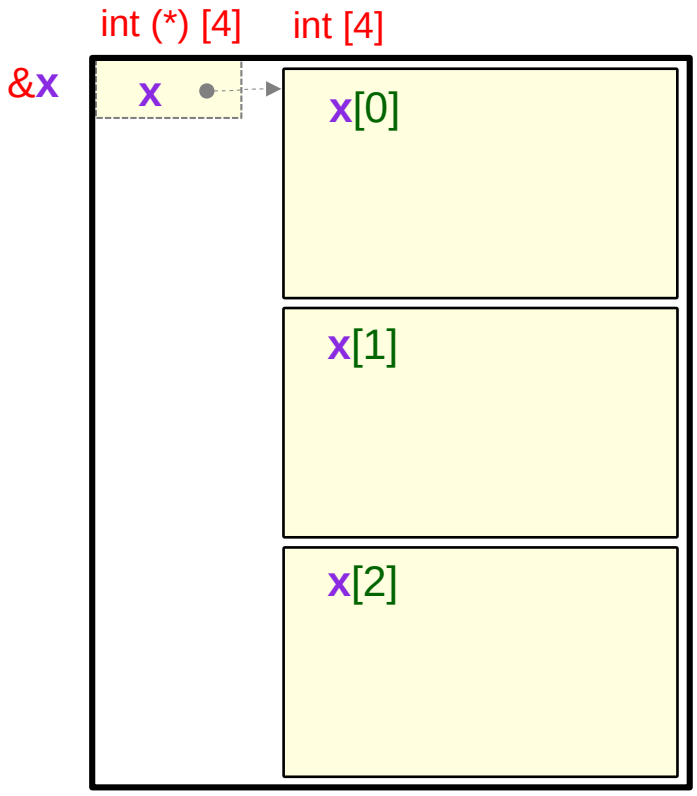
(**x**[3])[4] each **x[i]** has 4 elements



array name **x[i][j]**

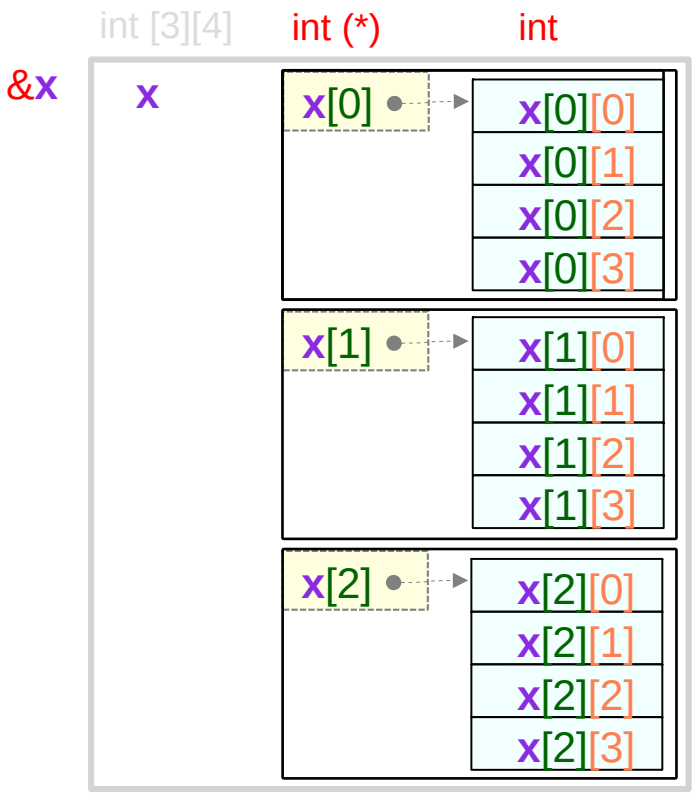
Virtual Pointers **x** and **x[i]**

x[3]



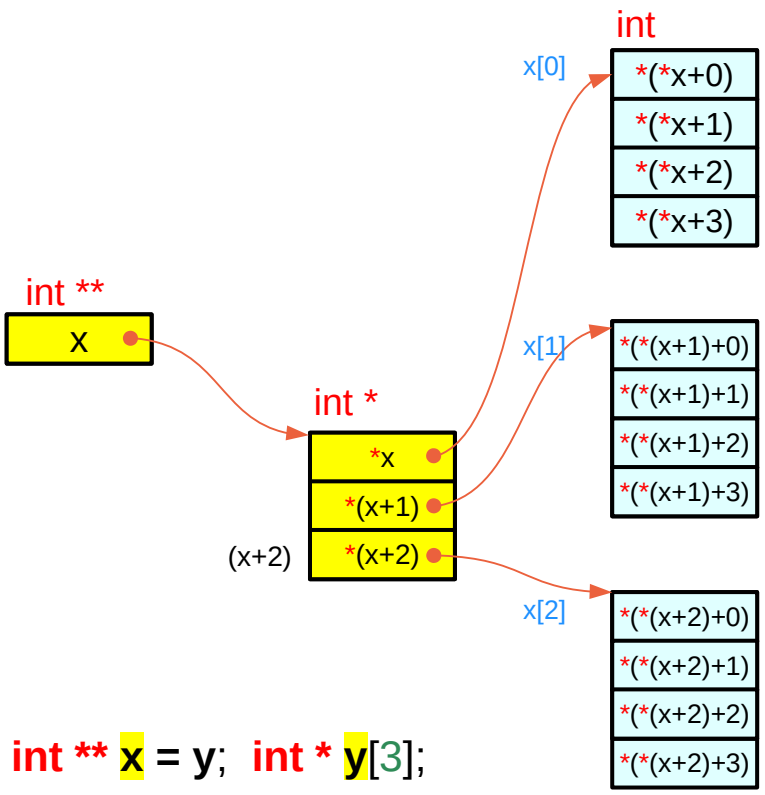
array name **x** ……..virtual pointer
array element **x**[**i**] …… abstract data

(**x**[3])[4]



array name **x**[**i**] ……. virtual pointer
array element **x**[**i**][**j**] …. primitive data

Virtual Pointers **x** and **x[i]**

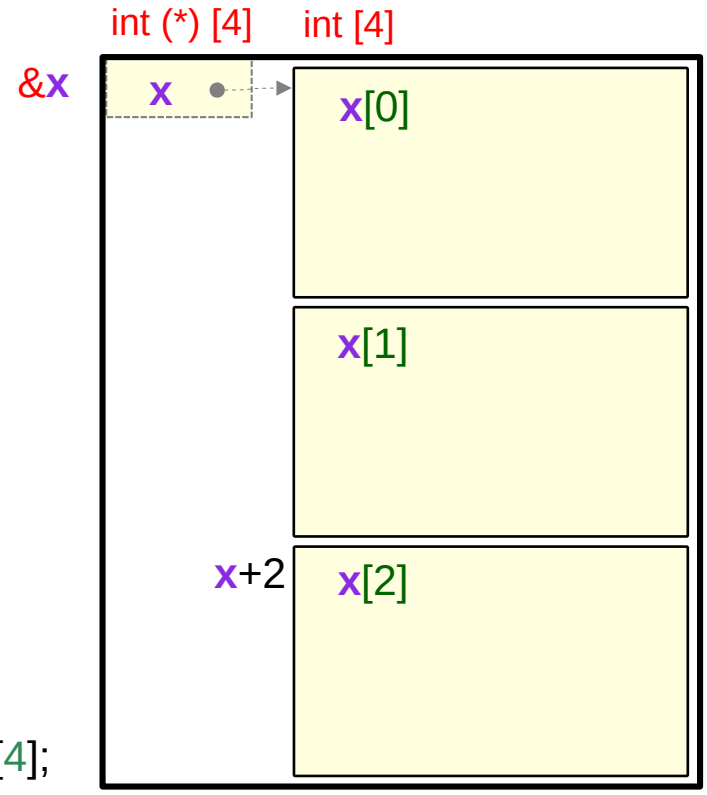


int ** x = y; int * y[3];

Pointer Array Approach

value(**x** + **i**)

= value(**x**) $+_a$ **i** * sizeof(***x**)



int x[3][4];

Array Pointer Approach

value(**x** + **i**)

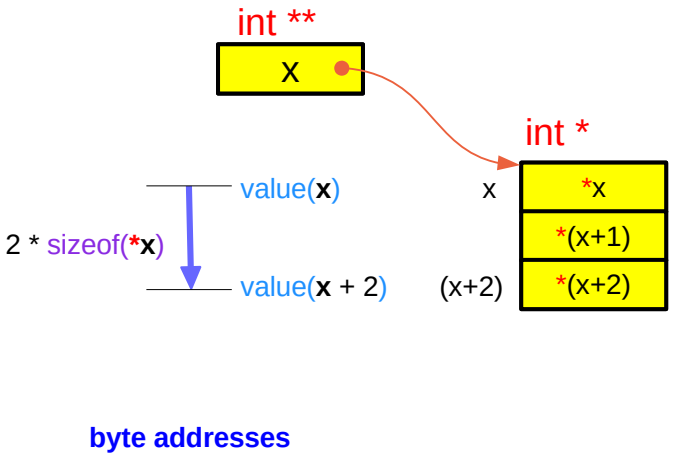
= value(**x**) $+_a$ **i** * sizeof(**x**[0])

Base and offset in byte addresses

pointer variable X

value(**x** + **i**) pointer addition +

= value(**x**) $+_a$ **i** * sizeof(***x**) arithmetic addition $+_a$

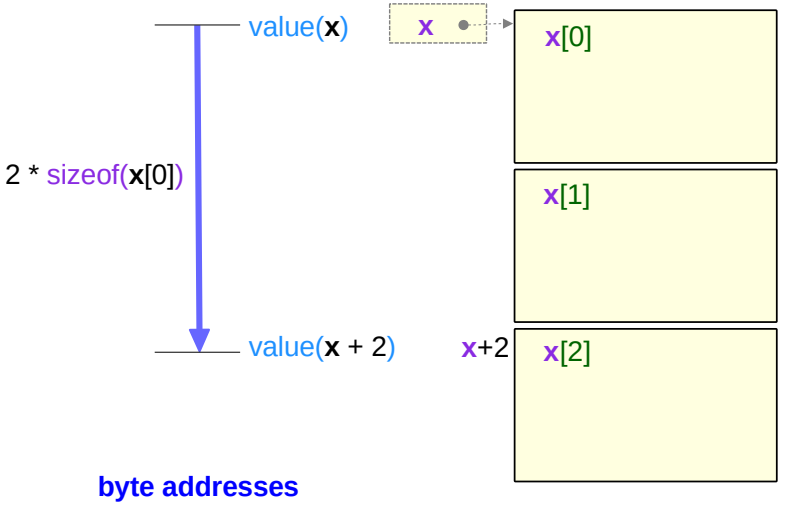


int ** x = y; int * y[3];

array variable X

value(**x** + **i**) pointer addition +

= value(**x**) $+_a$ **i** * sizeof(**x**[0]) arithmetic addition $+_a$



int x[3][4];

Base and offset in byte address (1)

pointer variable **X**	array variable **X**
value(**x** + **i**) pointer addition +	value(&**x**[**i**]) pointer addition +
= value(**x**) $+_a$ **i** * sizeof(***x**) arithmetic addition $+_a$	= value(**x**) $+_a$ **i** * sizeof(**x**[0]) arithmetic addition $+_a$
= value(**x**) + **i** * sizeof(***x**) in math expression	= value(**x**) + **i** * sizeof(**x**[0]) in math expression

value(**X**) may be involved
in only arithmetic additions / subtractions,
therefore the arithmetic addition $+_a$
can be replaced with the ordinary +

= value(**x**) $+_a$ **i** * sz, sz = sizeof(***x**)	= value(**x**) $+_a$ **i** * sz, sz = sizeof(**x**[0])
= value(**x** + **i**)$_{sz}$	= value(**x** + **i**)$_{sz}$

int ** x = y; int * y[3];

int x[3][4];

Base and offset in byte address (2)

pointer variable X

value(*(x + i) + j) value(value(x)) = value(x)

= value(*(x + i)) $+_a$ j * sizeof(**x)

= *value(x + i) $+_a$ j * sizeof(**x)

= *(value(x) $+_a$ i * sizeof(*x)) $+_a$ j * sizeof(**x)

= *(value(x) + i * sizeof(*x)) + j * sizeof(**x)

in math expressions

= *(value(x) + i * s1) + j * s2

= (*(x + i)$_{s1}$ + j)$_{s2}$

s1 = sizeof(*x), s2 = sizeof(**x)

```
int ** x = y;  int * y[3];
```

array variable X

value(x[i] + j) address replications

= value(x[i]) $+_a$ j *sizeof(x[i][j])

= *(value(x) $+_a$ i *sizeof(x[0])) $+_a$ j*sizeof(x[0][0])

= (value(x) $+_a$ i *sizeof(x[0])) $+_a$ j *sizeof(x[0][0])

= value(x) + i * sizeof(x[0]) + j * sizeof(x[0][0])

in math expressions

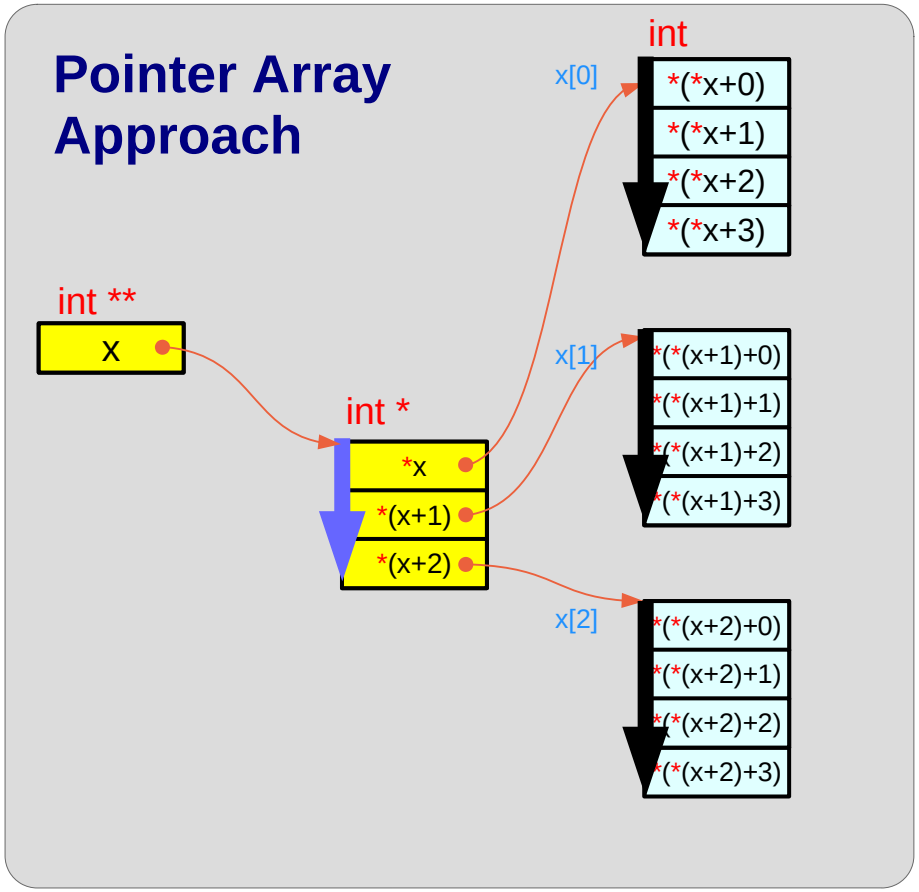
= value(x) + i * s1 + j * s2

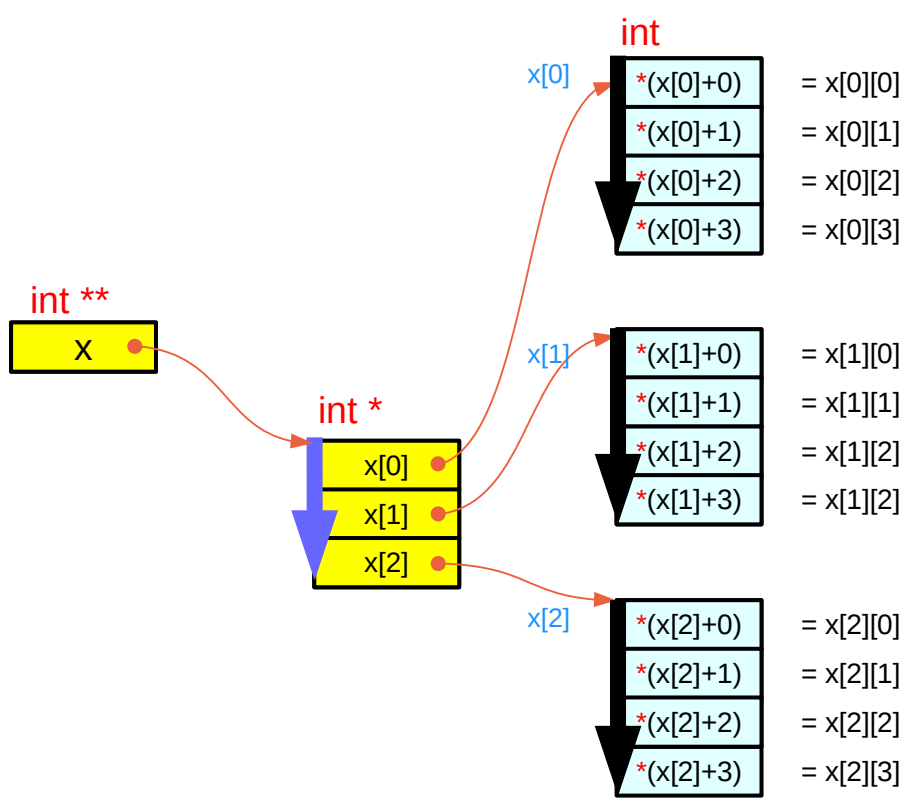
= (*(x + i)$_{s1}$ + j)$_{s2}$

s1 = sizeof(x[0]), s2 = sizeof(x[0][0])

```
int x[3][4];
```

* into [ ] notations – Pointer Array Approach

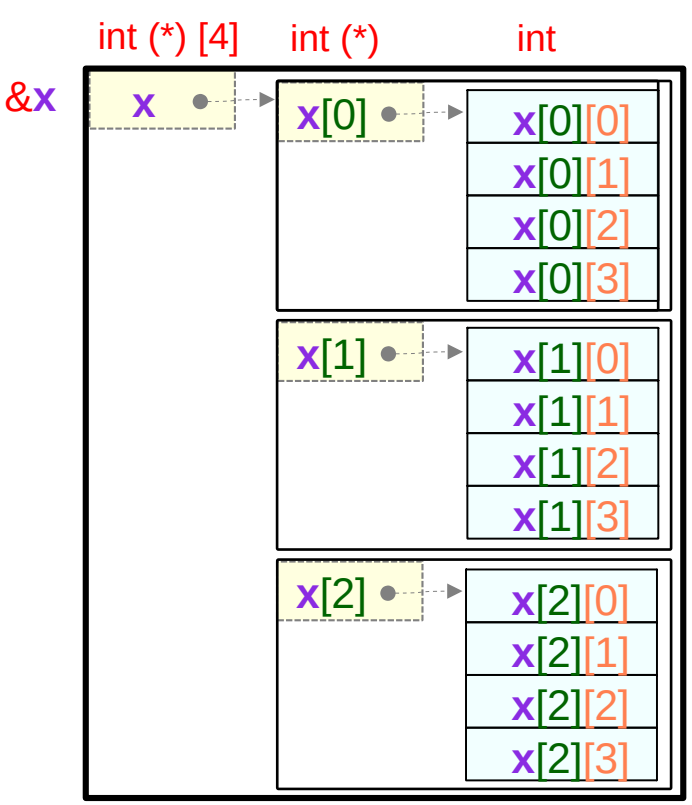


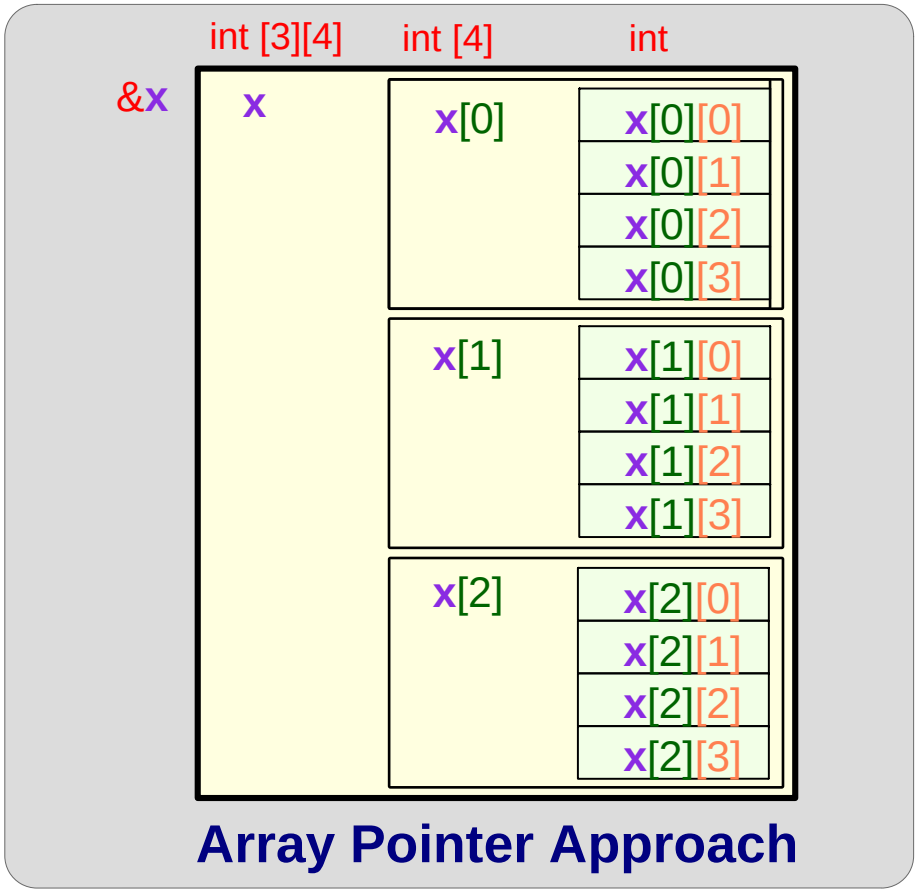


C expression $*(*(x+i)+j)$ → $x[i][j]$

Math expression $*(*(x+i)_{1\cdot4}+j)_{1\cdot4}$

* and [ ] notations – Array Pointer Approach



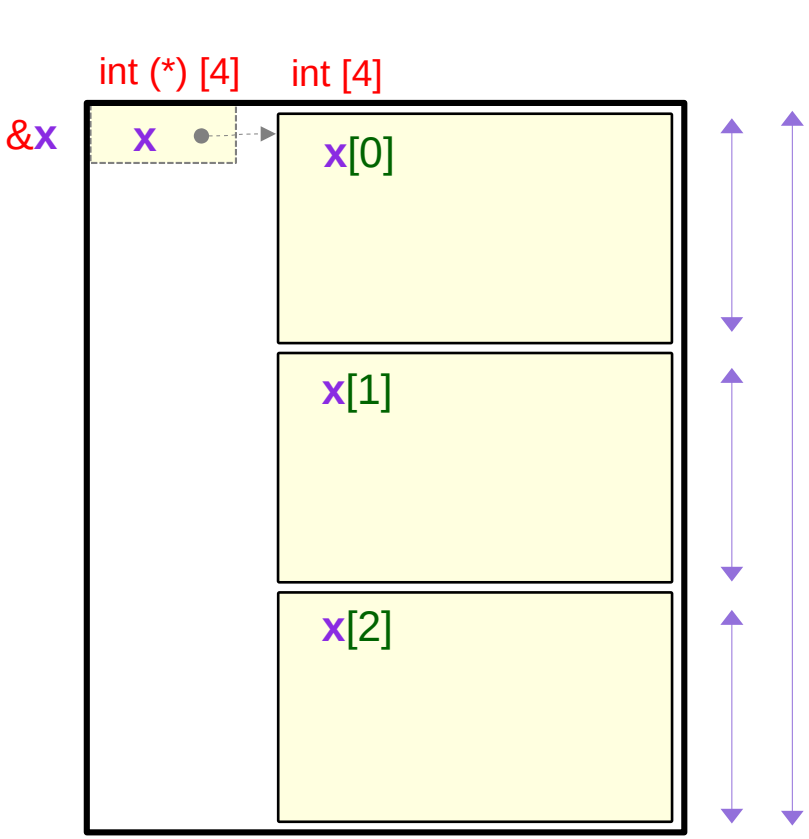


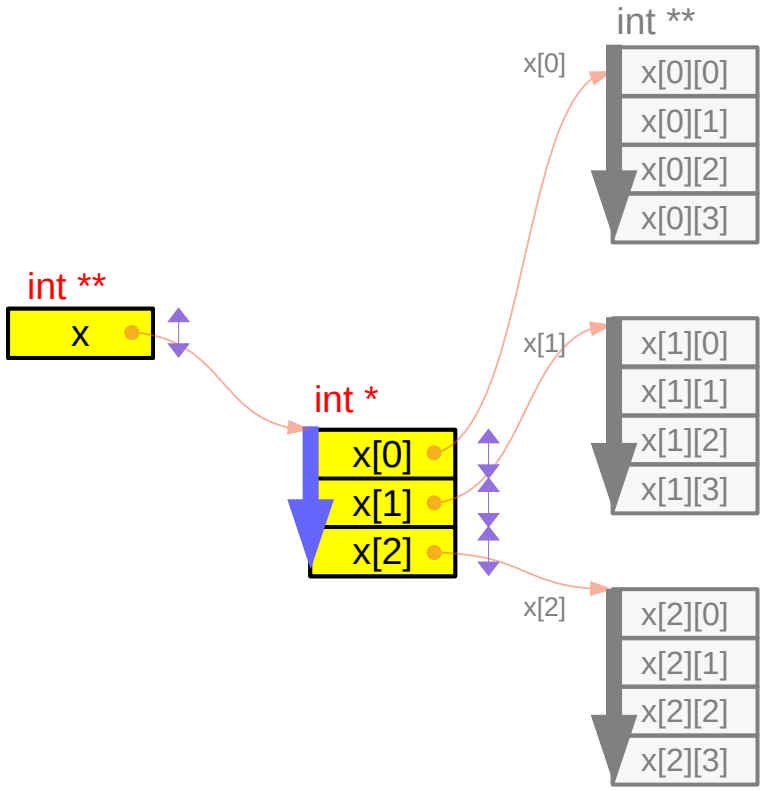
C expression $*(*(x+i)+j)$

Math expression $*(*(x+i)_{4\cdot 4}+j)_{1\cdot 4}$

$x[i][j]$

Virtual pointers vs. real pointers (1)





value(**&x**) = value(**x**) address replications

sizeof(**x**) = 3 * sizeof(***x**) abstract data size

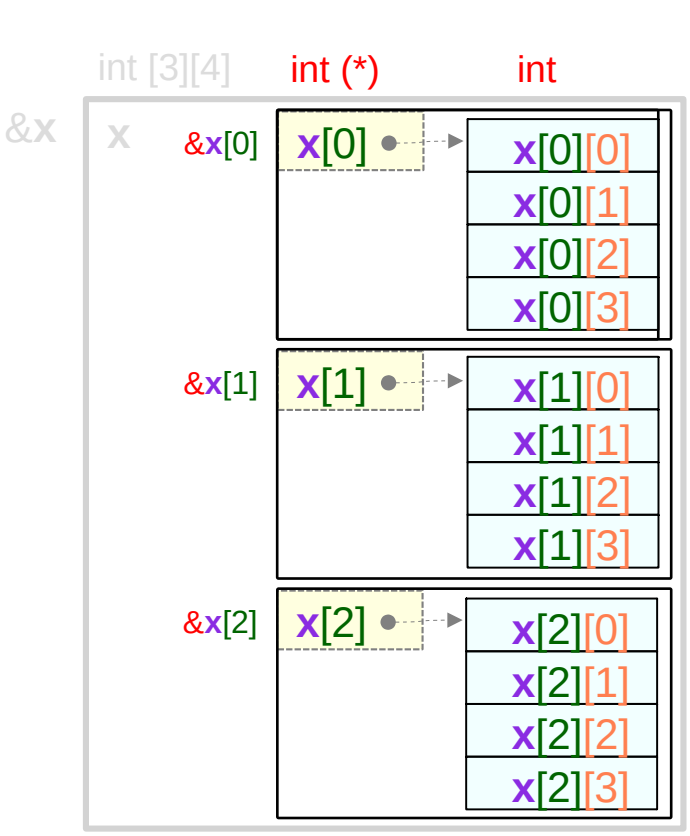
value(**x**+**i**) = value(**x**) + **i** * sizeof(***x**)
= value(**x**) + **i** * 4 *4 pointer arithmetic

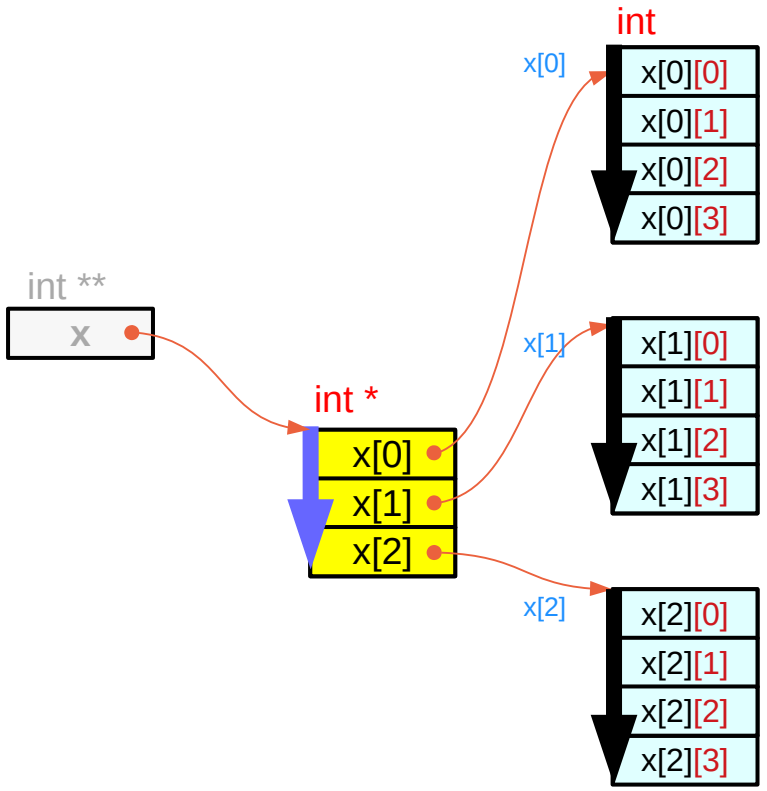
value(**&x**) ≠ value(**x**) physical memory

sizeof(**x**) = sizeof(***x**) = 4 pointer size

value(**x**+**i**) = value(**x**) + **i** * sizeof(***x**)
= value(**x**) + **i** * 4 pointer arithmetic

Virtual pointers vs. real pointers (2)





value(&**x**[**i**]) = value(**x**[**i**]) address replications

sizeof(**x**[**i**]) = 4 * sizeof(***x**[**i**]) abstract data size

value(**x**[**i**]+**j**) = value(**x**[**i**]) + **j** * sizeof(***x**[**i**])
= value(**x**[**i**]) + **j** * 4 pointer arithmetic

value(&**x**[**i**]) ≠ value(**x**[**i**]) physical memory

sizeof(**x**[**i**]) = sizeof(***x**[**i**]) = 4 pointer size

value(**x**[**i**]+**j**) = value(**x**[**i**]) + **j** * sizeof(***x**[**i**])
= value(**x**[**i**]) + **j** * 4 pointer arithmetic

Relaxing the outermost dimension

p[i]	≡	*(p+i)
p[i][j]	≡	*(p[i]+j)
p[i][j][k]	≡	*(p[i][j]+k)

&p[i]	≡	value(p+i)
&p[i][j]	≡	value(p[i]+j)
&p[i][j][k]	≡	value(p[i][j]+k)

*value(**X**) = ***X**

p[0]	≡	*p
p[i][0]	≡	*p[i]
p[i][j][0]	≡	*p[i][j]

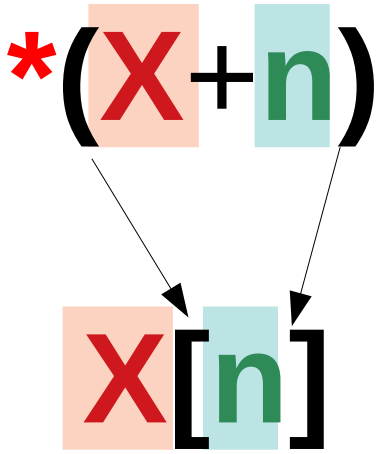
&p[0]	≡	value(p)
&p[i][0]	≡	value(p[i])
&p[i][j][0]	≡	value(p[i][j])

valid for proper **i**, **j**, **k** values

Relaxing all the dimensions

p[i]	≡	*(p+i)	→	*(p+i)
p[i][j]	≡	*(p[i]+j)	→	*(*(p+i)+j)
p[i][j][k]	≡	*(p[i][j]+k)	→	*(*(*(p+i)+j)+k)

*(X+n) ≡ X[n]



valid for proper **i**, **j**, **k** values

Left-to-right and right-to-left associative operators

p[i]	≡	p[i]	➡	*(p+i)
p[i][j]	≡	(p[i])[j]	➡	*(*(p+i)+j)
p[i][j][k]	≡	((p[i])[j])[k]	➡	*(*(*(p+i)+j)+k)

*p	≡	*(p)	➡	p[0]
**p	≡	*(*(p))	➡	(p[0])[0]
***p	≡	*(*(*(p)))	➡	((p[0])[0])[0]

Address Calculation (1) Array Pointer Approach

```
int  c [2][3][4] ;
```

c[i]	≡ *(c + i)
c[i][j]	≡ *(c[i] + j)
c[i][j][k]	≡ *(c[i][j] + k)

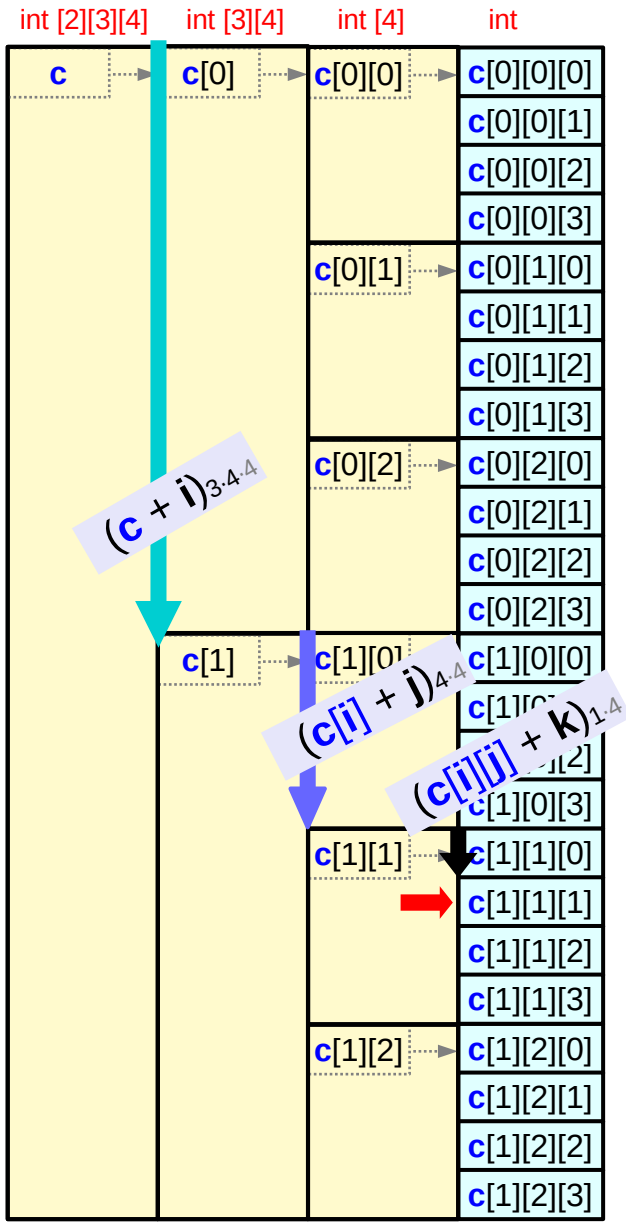
&c[i]	≡ value(c + i)
&c[i][j]	≡ value(c[i] + j)
&c[i][j][k]	≡ value(c[i][j] + k)

address replication

value(c[i][j][k]) ≠ value(&c[i][j][k]) ← primitive data & address

value(c[i][j])	= value(&c[i][j])	= value(&c[i][j][0])
value(c[i])	= value(&c[i])	= value(&c[i][0][0])
value(c)	= value(&c)	= value(&c[0][0][0])

skip **i** elements of **c** → skip **i*3*4** primitive elements of **c**
skip **j** elements of **c[i]** → skip **j*4** primitive elements of **c**
skip **k** elements of **c[i][j]** → skip **k** primitive elements of **c**



Address Calculation (2) Pointer Array Approach

```
int ** c [2] ;
int *  b [2*3] ;
int    a [2*3*4] ;
```

c[i] = &b[3*i] (= b + 3*i)

b[j] = &a[4*j] (= a + 4*j)

b[j] ≡ (a+j*4)

*(b[j]+k) = *(a+j*4+k);

b[j][k] ≡ a[j*4+k]

c[i] ≡ (b+i*3)

*(c[i]+j) = *(b+i*3+j);

c[i][j] ≡ b[i*3+j]

c[i][j] ≡ (a+(i*3+j)*4)

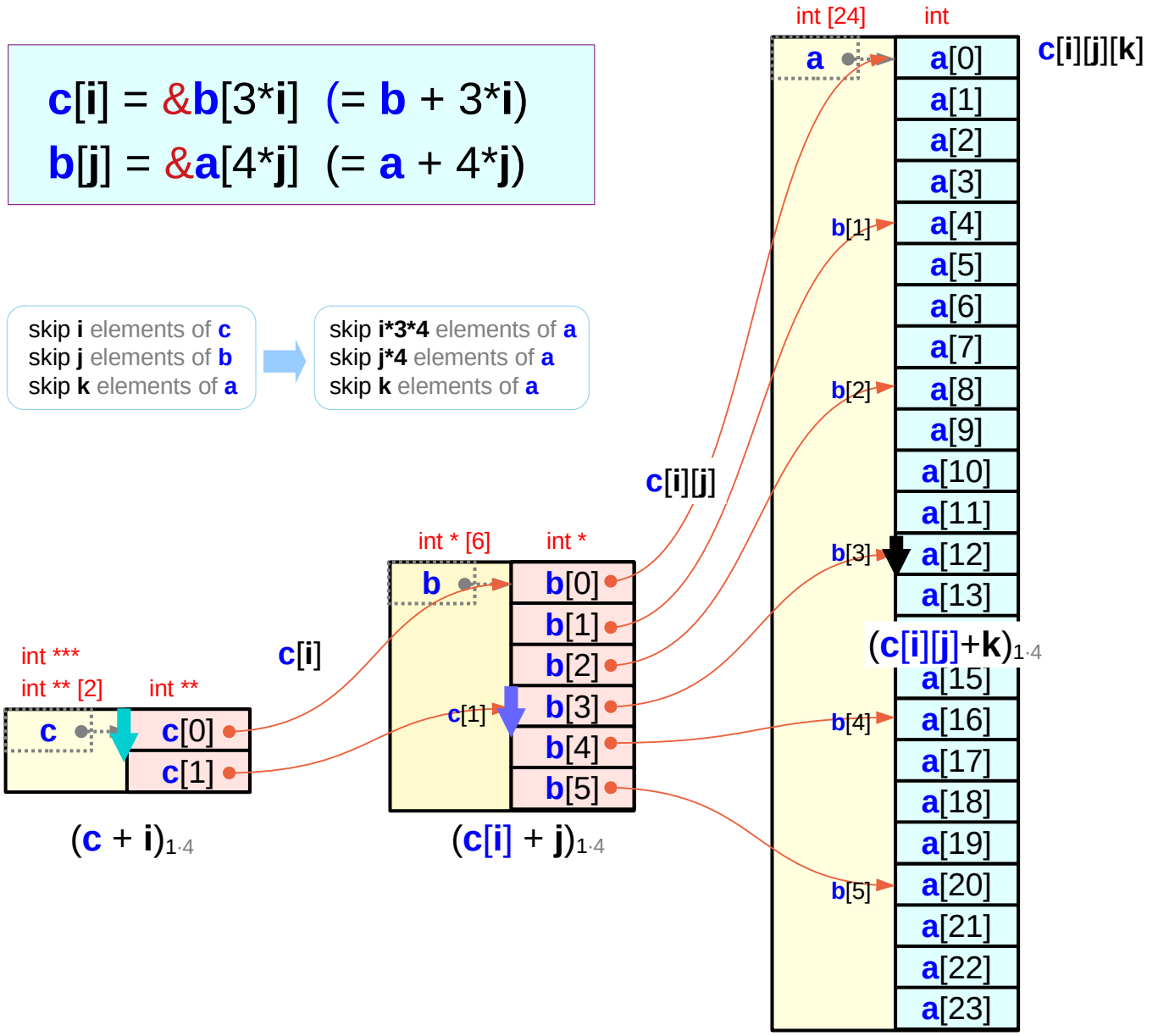
*(c[i][j]+k) = *(a+(i*3+j)*4+k);

c[i][j][j] ≡ a[(i*3+j)*4+k]

skip **i** elements of **c**
skip **j** elements of **b**
skip **k** elements of **a**

→

skip **i*3*4** elements of **a**
skip **j*4** elements of **a**
skip **k** elements of **a**



Address Calculation (3)

value(**c** + **i**) = value(**c**) + **i** * 3 * 4 * 4
value(**c[i]** + **j**) = value(**c[i]**) + **j** * 4 * 4
value(**c[i][j]** + **k**) = value(**c[i][j]**) + **k** * 4

value(**c** + **i**) = value(**c**) + **i** * sizeof(***c**)
value(**c[i]** + **j**) = value(**c[i]**) + **j** * sizeof(***c[i]**)
value(**c[i][j]** + **k**) = value(**c[i][j]**) + **k** * sizeof(***c[i][j]**)

Array Pointer Approach

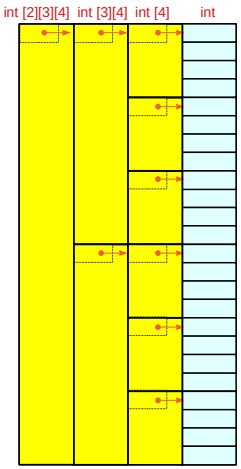
c[i] ≡ ***(c** + **i)**
c[i][j] ≡ ***(c[i]** + **j)**
c[i][j][k] ≡ ***(c[i][j]** + **k)**

&c[i] ≡ value(**c** + **i)**
&c[i][j] ≡ value(**c[i]** + **j)**
&c[i][j][k] ≡ value(**c[i][j]** + **k)**

(c + i)$_{3\cdot4\cdot4}$

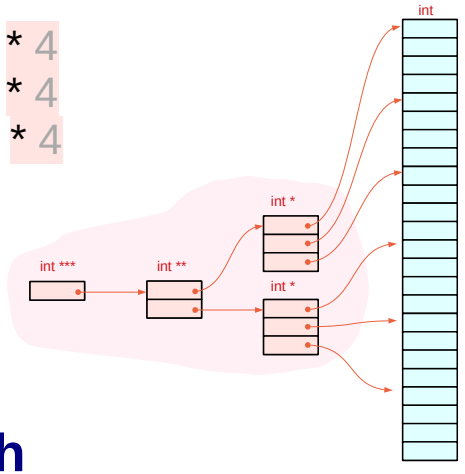
(c[i] + j)$_{4\cdot4}$

(c[i][j] + k)$_{1\cdot4}$



value(**c** + **i**) = value(**c**) + **i** * 4
value(**c[i]** + **j**) = value(**c[i]**) + **j** * 4
value(**c[i][j]** + **k**) = value(**c[i][j]**) + **k** * 4

(c + i)$_{1\cdot4}$ **(c[i] + j)**$_{1\cdot4}$ **(c[i][j] + k)**$_{1\cdot4}$



Pointer Array Approach

Subscript [ ] and dereference * notations (1a)

p[i] ≡ *(p+i)

p[i][j] ≡ *(*(p+i)+j)

p[i][j][k] ≡ *(*(*(p+i)+j)+k)

from **p**, skip **i**·M·N integers

&**p**[**i**] = value(**p** + **i**)$_{M·N·4}$
= value(**p**) + **i** * M·N·4

from **p**[**i**], skip **j**·N integers

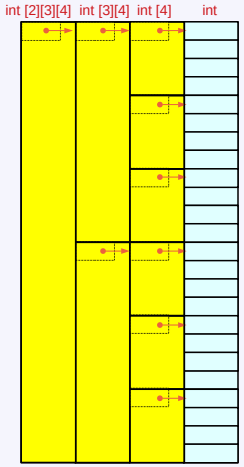
&**p**[**i**][**j**] = value(**p**[**i**] + **j**)$_{N·4}$
= value(**p**[**i**]) + **j** * N·4

from **p**[**i**][**j**], skip **k** integers

&**p**[**i**][**j**][**k**] = value(**p**[**i**][**j**]+**k**)$_{1·4}$
= value(**p**[**i**][**j**]) + **k** * 1·4

int p[L][M][N]

Array Pointer Approach



address replications

value(**p**[**i**]) = &**p**[**i**] = value(**p** + **i**)

value(**p**[**i**][**j**]) = &**p**[**i**][**j**] = value(**p**[**i**] + **j**)

value(**p**[**i**][**j**][**k**]) ≠ &**p**[**i**][**j**][**k**] = value(**p**[**i**][**j**] + **k**)

&**p**[**i**][**j**][**k**] = value(**p**) + **i** *M·N·4 + **j** *N·4 + **k** * 4

p[**i**][**j**][**k**] = ***(value(**p**) + **i** *M·N·4 + **j** *N·4 + **k** * 4)

Subscript [ ] and dereference * notations (1b)

$$p[i] \equiv *(p+i)$$

$$p[i][j] \equiv *(*(p+i)+j)$$

$$p[i][j][k] \equiv *(*(*(p+i)+j)+k)$$

skip **i** pointers from **p**

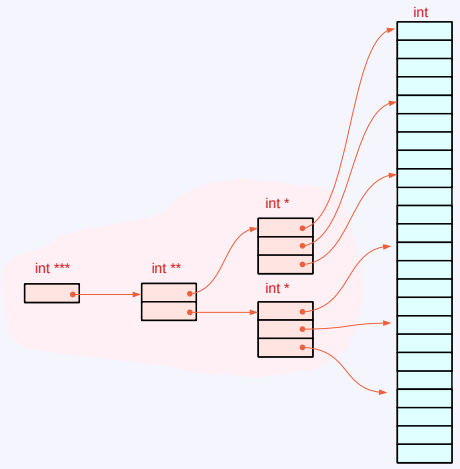
$\&p[i] = value(p + i)_{1\cdot4}$
$= value(p) + i * 4$

skip **j** pointers from **p[i]**

$\&p[i][j] = value(p[i] + j)_{1\cdot4}$
$= value(p[i]) + j * 4$

skip **k** integers from **p[i][j]**

$\&p[i][j][k] = value(p[i][j] + k)_{1\cdot4}$
$= value(p[i][j]) + k * 4$



int ** p [L] ;
int * q [L·M] ;
int r [L·M·N] ;

Pointer Array Approach

address dereferences

value(p[i]) = *(&p[i]) = *value(p + i)

value(p[i][j]) = *(&p[i][j]) = *value(p[i] + j)

value(p[i][j][k]) = *(&p[i][j][k]) = *value(p[i][j] + k)

p[i][j][k] = *value(*value(*value(p+i)+j)+k)

&p[i][j][k] = value(*value(*value(p+i)+j)+k)

Subscript [ ] and dereference * notations (1a)

p[i][j][k] = *value(*value(*value(p+i)+j)+k)

address replications

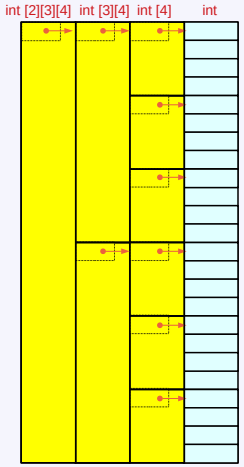
= value(value(value(p + i)$_{3\cdot4\cdot4}$) + j)$_{4\cdot4}$ + k)$_4$
= value(value(value(p) + i *3·4·4) + j *4·4) + k *4)
= value(value(p) + i *3·4·4 + j *4·4) + k *4)
= value(p) + i *3·4·4 + j *4·4 + k *4)

= *value(*value(*value(p + i)$_4$) + j)$_4$ + k)$_4$
= *value(*value(*(value(p) + i *4) + j *4) + k *4)
= *value(*(*(value(p) + i *4) + j *4) + k *4)
= *(*(*(value(p) + i *4) + j *4) + k *4)

Subscript [ ] and dereference * notations (1a)

int p[L][M][N]

Array Pointer Approach



address replications

value(p[i]) = &p[i] = value(p + i)

value(p[i][j]) = &p[i][j] = value(p[i] + j)

value(p[i][j][k]) ≠ &p[i][j][k] = value(p[i][j] + k)

&p[i][j][k] = value(p + i *M·N·4 + j *N·4 + k * 4)

p[i][j][k] = *value(p + i *M·N·4 + j *N·4 + k * 4)

abstract data	int [3][4]	value(p[i])	= &p[i]	= value(p + i)	int (*)[3][4]	virtual pointer
abstract data	int [4]	value(p[i][j])	= &p[i][j]	= value(p[i] + j)	int (*)[4]	virtual pointer
primitive data	int	value(p[i][j][k])	≠ &p[i][j][k]	= value(p[i][j] + k)	int (*)	virtual pointer

p[i][j][k] = *value(*value(*value(p+i)+j)+k)

address replications

= value(value(value(p + i)$_{3\cdot4\cdot4}$) + j)$_{4\cdot4}$ + k)$_4$

= value(p + i *3·4·4 + j *4·4 + k *4)

Subscript [ ] and dereference * notations (2)

p[i]	≡	*(p+i)
p[i][j]	≡	*(*(p+i)+j)
p[i][j][k]	≡	*(*(*(p+i)+j)+k)

C Expressions

&p[i]	≡	value(p+i)
&p[i][j]	≡	value(*(p+i)+j)
&p[i][j][k]	≡	value(*(*(p+i)+j)+k)

C Expressions

int p [L][M][N] ;

value(&**X**) = value(**X**) (address replication)

p[i]	→	$*(p+i)_{M\cdot N\cdot 4}$
p[i][j]	→	$*(*(p+i)_{M\cdot N\cdot 4}+j)_{N\cdot 4}$
p[i][j][k]	→	$*(*(*(p+i)_{M\cdot N\cdot 4}+j)_{N\cdot 4}+k)_{1\cdot 4}$

Math Expressions

&p[i]	→	$value(p+i)_{M\cdot N\cdot 4}$
&p[i][j]	→	$value((p+i)_{M\cdot N\cdot 4}+j)_{N\cdot 4}$
&p[i][j][k]	→	$value(((p+i)_{M\cdot N\cdot 4}+j)_{N\cdot 4}+k)_{1\cdot 4}$

Math Expressions

int ** p[L], * q[L·M], r[L·M·N] ;

*value(**X**) = ***X**

p[i]	→	$*(p+i)_{1\cdot 4}$
p[i][j]	→	$*(*(p+i)_{1\cdot 4}+j)_{1\cdot 4}$
p[i][j][k]	→	$*(*(*(p+i)_{1\cdot 4}+j)_{1\cdot 4}+k)_{1\cdot 4}$

Math Expressions

&p[i]	→	$value(p+i)_{1\cdot 4}$
&p[i][j]	→	$value(*(p+i)_{1\cdot 4}+j)_{1\cdot 4}$
&p[i][j][k]	→	$value(*(*(p+i)_{1\cdot 4}+j)_{1\cdot 4}+k)_{1\cdot 4}$

Math Expressions

Subscript [ ] and dereference * notations (3)

int p [L][M][N] ; value(&X) = value(X) (address replication)

&p[i] = value((p + i)$_{M\cdot N\cdot 4}$) = value(p + i * M·N·4)

&p[i][j] = value((p[i] + j)$_{N\cdot 4}$) = value(p[i] + j * N·4)

&p[i][j][k] = value((p[i][j]+k)$_{1\cdot 4}$) = value(p[i][j] + k * 1·4)

= value(p + i * M·N·4 + j * N·4 + k * 4)

&p[i] → value(p+i)$_{M\cdot N\cdot 4}$

&p[i][j] → value((p+i)$_{M\cdot N\cdot 4}$+j)$_{N\cdot 4}$

&p[i][j][k] → value(((p+i)$_{M\cdot N\cdot 4}$+j)$_{N\cdot 4}$+k)$_{1\cdot 4}$

Math Expressions

int ** p[L], * q[L·M], r[L·M·N] ; *value(X) = *X

&p[i] = value(p + I)$_{1\cdot 4}$ = value(p + i * 1·4)

*

&p[i][j] = value(p[i] + j)$_{1\cdot 4}$ = value(p[i] + j * 1·4)

*

&p[i][j][k] = value(p[i][j] + k)$_{1\cdot 4}$ = value(p[i][j] + k * 1·4)

= value(*value(*value(p +i*4) +j*4) +k*4)

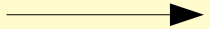
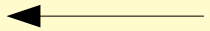
&p[i] → value(p+i)$_{1\cdot 4}$

&p[i][j] → value(*(p+i)$_{1\cdot 4}$+j)$_{1\cdot 4}$

&p[i][j][k] → value(*(*(p+i)$_{1\cdot 4}$+j)$_{1\cdot 4}$+k)$_{1\cdot 4}$

Math Expressions

Operator Precedence

Precedence	Operator	Description	Associativity
1	++ -- () [] . -> (type){list}	Suffix/postfix increment and decrement Function call Array subscripting Structure and union member access member access through pointer Compound literal(C99)	Left-to-right (((x[m])[n])[p]) 
2	++ -- + - ! ~ (type) * & sizeof _Alignof	Prefix increment and decrement Unary plus and minus Logical NOT and bitwise NOT Type cast Indirection (dereference) Address-of Size-of Alignment requirement(C11)	Right-to-left *(*(*(*x))) 

https://en.cppreference.com/w/c/language/operator_precedence

Limitations

No index Range Checking
Array Size must be a constant expression
    Variable Array Size
Arrays cannot be Copied or Compared
Aggregate Initialization and Global Arrays
Precedence Rule
Index Type Must be Integral

References

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