

Quantization

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October 10, 2013

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Amplitude Quantization

Definition

the process of transforming the sample amplitude $m(nT_s)$ of a baseband signal $m(t)$ at time $t = nT_s$ into a **discrete amplitude** $v(nT_s)$ taken from a finite set of possible levels

- instantaneous quantization (at time $t = nT_s$)
- memoryless quantization (not affected by previous samples)

Quantizer Index

$$m \equiv m(nT_s)$$

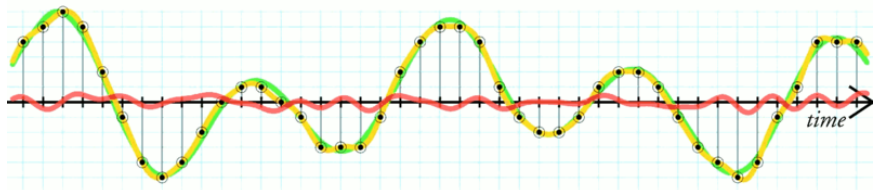
- the signal amplitude $m(t)$ is specified by the index k if it lies in the interval I_k
- $I_k : \{m_k < m \leq m_{k+1}\}$ $k = 1, 2, \dots, L$
- L : the total number of amplitude levels used in the quantizer
- $\{m_1, m_2, \dots, m_L\}$: decision levels (decision thresholds)

Quantizer Output

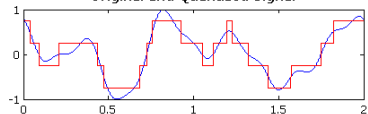
- at the quantizer output, the index k is transformed into an amplitude v_k
- $\{v_1, v_2, \dots, v_L\}$: representation levels (reconstruction levels)
- $(v_{k+1} - v_k)$: the spacing between two adjacent representation levels : quantum (step-size)
- if the input signal sample m belongs to the interval I_k , then the quantizer output becomes v_k

Quantization Error

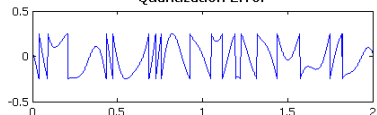
original signal
quantized signal
quantization noise



Original and Quantized Signal



Quantization Error



Types of Quantizers

Uniform / Non-uniform

- the representation levels are uniformly spaced (a uniform quantizer)
- the representation levels are non-uniformly spaced (a non-uniform quantizer)

Midtread / Midrise

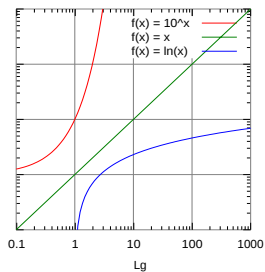
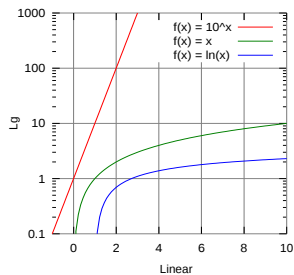
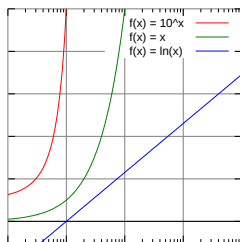
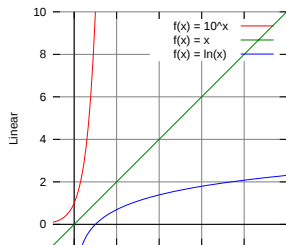
- ... $-\Delta$, 0 , $+\Delta$... (Midtread)
- ... $-\Delta/2$, $+\Delta/2$... (Midrise)

Non-Uniform Quantizer

the ratio of peaks of loud voice to that of weak voice
is in the order of 1000

- Uniform Quantizer:
relatively large step size over the weak voice
- Non-uniform Quantizer:
smaller step size for the weak voice (fine resolution)
larger step size for the loud voice (coarse resolution)

Logarithmic Scale



Mu-law

After a Compressor, then a uniform quantizer

- μ -law

$$|v| = \frac{\ln(1 + \mu|m|)}{\ln(1 + \mu)}$$

inear if $\mu|m| \ll 1$

logarithmic if $\mu|m| \gg 1$

- A-law

$$|v| = \frac{A|m|}{(1+A)}, \quad (1 \leq |m| \leq 1/A)$$

$$|v| = \frac{1 + \ln(A|m|)}{(1 + \ln(A))}, \quad (1/A \leq |m| \leq 1)$$

Reference

[1] S. Haykin, M Moher, “Introduction to Analog and Digital Communications”, 2ed