

Continuous Time Linear Time Invariant System

Young W Lim

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Based on
Probability, Random Variables and Random Signal Principles,
P.Z. Peebles, Jr. and B. Shi

Outline

General Linear System

N Gaussian random variables

Definition

$$y(t) = L[x(t)]$$

$$y(t) = L[x(t)] = L \left[\int_{-\infty}^{\infty} x(\xi) \delta(t - \xi) d\xi \right] = \int_{-\infty}^{\infty} x(\xi) L[\delta(t - \xi)] d\xi$$

$$L[\delta(t - \xi)] = h(t, \xi)$$

$$y(t) = L[x(t)] = L \left[\int_{-\infty}^{\infty} x(\xi) \delta(t - \xi) d\xi \right] = \int_{-\infty}^{\infty} x(\xi) h(t, \xi) d\xi$$

System response of an impulse input

N Gaussian random variables

Definition

$$y(t) = L[x(t)]$$

$$y(t) = L \left[\sum_{n=1}^N a_n x_n(t) \right] = \sum_{n=1}^N a_n L[x_n(t)] = \sum_{n=1}^N a_n y_n(t)$$

$$x(t) = \int_{-\infty}^{\infty} x(\xi) \delta(t - \xi) d\xi$$

$$y(t) = L[x(t)] = L \left[\int_{-\infty}^{\infty} x(\xi) \delta(t - \xi) d\xi \right] = \int_{-\infty}^{\infty} x(\xi) L[\delta(t - \xi)] d\xi$$

Impulse Response $h(t, \xi)$

N Gaussian random variables

Definition

$$y(t) = L[x(t)]$$

$$y(t) = \int_{-\infty}^{\infty} x(\xi) L[\delta(t - \xi)] d\xi$$

$$L[\delta(t - \xi)] = h(t, \xi)$$

$$y(t) = \int_{-\infty}^{\infty} x(\xi) h(t, \xi) d\xi$$

Linear Time Invariant Systems

N Gaussian random variables

Definition

$$h(t, \xi) = h(t - \xi)$$

$$y(t) = \int_{-\infty}^{\infty} x(\xi) h(t - \xi) d\xi$$

$$y(t) = x(t) * y(t)$$

$$y(t) = \int_{-\infty}^{\infty} h(\xi) x(t - \xi) d\xi$$

LTI System Transfer Functions

N Gaussian random variables

Definition

$$Y(\omega) = \int_{-\infty}^{\infty} y(t) e^{-j\omega t} dt = \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} x(\xi) h(t - \xi) d\xi \right] e^{-j\omega t} dt$$

$$= \int_{-\infty}^{\infty} x(\xi) \left[\int_{-\infty}^{\infty} h(t - \xi) e^{-j\omega(t - \xi)} dt \right] e^{-j\omega\xi} d\xi$$

$$= \int_{-\infty}^{\infty} x(\xi) H(\omega) e^{-j\omega\xi} d\xi = X(\omega) H(\omega)$$

$$H(\omega) = \int_{-\infty}^{\infty} h(t - \xi) e^{-j\omega(t - \xi)} dt$$

Computing Transfer Functions

N Gaussian random variables

Definition

$$x(t) = e^{j\omega t}$$

$$H(\omega) = \frac{L[e^{j\omega t}]}{e^{j\omega t}} = \frac{y(t)}{x(t)}$$

$$y(t) = L[e^{j\omega t}]$$

Definition

A linear time invariant system is said to be causal if it does not respond prior to the application of an input signal
 $y(t) = 0$ for $t < t_0$ if $x(t) = 0$ for $t < t_0$
this requires that $h(t) = 0$ for $t < 0$

