Continuous Time Linear Time Invariant System

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Based on
Probability, Random Variables and Random Signal Principles,
P.Z. Peebles,Jr. and B. Shi

Outline

Definition

$$y(t) = L[x(t)]$$

$$y(t) = L[x(t)] = L\left[\int_{-\infty}^{\infty} x(\xi)\delta(t-\xi)d\xi\right] = \int_{-\infty}^{\infty} x(\xi)L[\delta(t-\xi)]\,d\xi$$

$$L[\delta(t-\xi)] = h(t,\xi)$$

$$y(t) = L[x(t)] = L\left[\int_{-\infty}^{\infty} x(\xi)\delta(t-\xi)d\xi\right] = \int_{-\infty}^{\infty} x(\xi)h(t,\xi)d\xi$$

Definition

$$y(t) = L[x(t)]$$

$$y(t) = L\left[\sum_{n=1}^{N} a_n x_n(t)\right] = \sum_{n=1}^{N} a_n L[x_n(t)] = \sum_{n=1}^{N} a_n y_n(t)$$

$$x(t) = \int_{-\infty}^{\infty} x(\xi)\delta(t-\xi)d\xi$$

$$y(t) = L[x(t)] = L\left[\int_{-\infty}^{\infty} x(\xi)\delta(t-\xi)d\xi\right] = \int_{-\infty}^{\infty} x(\xi)L[\delta(t-\xi)]\,d\xi$$

Definition

$$y(t) = L[x(t)]$$

$$y(t) = \int_{-\infty}^{\infty} x(\xi) L[\delta(t-\xi)]\, d\xi$$

$$L[\delta(t-\xi)] = h(t,\xi)$$

$$y(t) = \int_{-\infty}^{\infty} x(\xi) h(t,\xi) d\xi$$

Definition

$$h(t,\xi) = h(t-\xi)$$

$$y(t) = \int_{-\infty}^{\infty} x(\xi)h(t-\xi)d\xi$$

$$y(t) = x(t) * y(t)$$

$$y(t) = \int_{-\infty}^{\infty} h(\xi)x(t-\xi)d\xi$$

Definition

$$Y(\omega) = \int_{-\infty}^{\infty} y(t) e^{-j\omega t} dt = \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} x(\xi) h(t-\xi) d\xi \right] e^{-j\omega t} dt$$

$$= \int_{-\infty}^{\infty} x(\xi) \left[\int_{-\infty}^{\infty} h(t-\xi) e^{-j\omega(t-\xi)} dt \right] e^{-j\omega\xi} d\xi$$

$$= \int_{-\infty}^{\infty} x(\xi) H(\omega) e^{-j\omega\xi} d\xi = X(\omega) H(\omega)$$

$$H(\omega) = \int_{-\infty}^{\infty} h(t-\xi) e^{-j\omega(t-\xi)} dt$$

Definition

$$x(t) = e^{j\omega t}$$

$$H(\omega) = \frac{L\left[e^{j\omega t}\right]}{e^{j\omega t}} = \frac{y(t)}{x(t)}$$

$$y(t) = L\left[e^{j\omega t}\right]$$

Definition

A linear time invariant system is said to be causal
if it does not respond prior to the application of an input signal
$y(t) = 0$ for $t < t_0$ if $x(t) = 0$ for $t < t_0$
this requires that $h(t) = 0$ for $t < 0$