

# Angle Recoding CORDIC

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## Extended EAS (EEAS) - Wu

more flexible way of decomposing the rotation angle

better

the number of iterations  
the error performance

$$S_{EAS} = \{ (\sigma \cdot \tan^{-1}(2^{-r})) : \sigma \in \{+1, 0, -1\}, r \in \{1, 2, \dots, n-1\} \}$$

$$S_{EEAS} = \{ (\sigma_1 \cdot \tan^{-1}(2^{-r_1}) + \sigma_2 \cdot \tan^{-1}(2^{-r_2})) : \\ \sigma_1, \sigma_2 \in \{+1, 0, -1\}, r_1, r_2 \in \{1, 2, \dots, n-1\} \}$$

The pseudo-rotation  
for  $i$ -th micro rotations

$$\begin{aligned}x_{i+1} &= x_i - [\sigma_1(i) \cdot 2^{-r_1(i)} + \sigma_2(i) \cdot 2^{-r_2(i)}] y_i \\y_{i+1} &= y_i + [\sigma_1(i) \cdot 2^{-r_1(i)} + \sigma_2(i) \cdot 2^{-r_2(i)}] x_i\end{aligned}$$

The pseudo-rotated vector  $[x_{R_m}, y_{R_m}]$   
after  $R_m$  (the required number of micro-rotations)

needs to be scaled by a factor  $K = \prod K_i$

$$K_i = \left[ 1 + (\sigma_1(i) \cdot 2^{-r_1(i)} + \sigma_2(i) \cdot 2^{-r_2(i)})^2 \right]^{-\frac{1}{2}}$$

$$\begin{aligned}\tilde{x}_{i+1} &= \tilde{x}_i - [k_1(i) \cdot 2^{-s_1(i)} + k_2(i) \cdot 2^{-s_2(i)}] \tilde{y}_i \\ \tilde{y}_{i+1} &= \tilde{y}_i + [k_1(i) \cdot 2^{-s_1(i)} + k_2(i) \cdot 2^{-s_2(i)}] \tilde{x}_i\end{aligned}$$

$$\tilde{x}_0 = x_{R_m}$$

$$\tilde{y}_0 = y_{R_m}$$

$$k_1, k_2 \in \{-1, 0, 1\}$$

$$s_1, s_2 \in \{1, 2, \dots, n-1\}$$

- [21] C.-S. Wu, A.-Y. Wu, and C.-H. Lin, "A high-performance/low-latency vector rotational CORDIC architecture based on extended elementary angle set and trellis-based searching schemes," *IEEE Trans. Circuits Syst. II: Anal. Digital Signal Process.*, vol. 50, no. 9, pp. 589–601, Sep. 2003.

# A Unified View for Vector Rotational CORDIC Algorithms and Architectures Based on Angle Quantization Approach

An-Yeu Wu and Cheng-Shing Wu

AR : to approximate  $\theta$   
with the combination  
of selected angle elements  
from a pre-defined EAS  
(Elementary Angle Set)

EAS : all possible values of  $\theta(j)$

$$\text{EAS } \hat{S}_1 = \{ \tan^{-1}(\alpha^* \cdot 2^{-s^*}) : \alpha^* \in \{-1, 0, +1\}, \\ s^* \in \{0, 1, \dots, N-1\} \}$$

EAS  $\hat{S}_1$  consists of  $\tan^{-1}(\text{Single signed power of two})$   
 $\tan^{-1}(\text{Single SPT})$   
 $\tan^{-1}(\alpha^* \cdot 2^{-s^*})$

SPT-based digital filter design

to increase the coefficient resolution

→ employ more SPT terms to represent filter coefficients

[12] H. Samuelli, "An improved search algorithm for the design of multiplierless FIR filters with power-of-two coefficients," *IEEE Trans. Circuits Syst.*, vol. 36, pp. 1044–1047, July 1989.

[13] Y. C. Lim, R. Yang, D. Li, and J. Song, "Signed power-of-two term allocation scheme for the design of digital filters," *IEEE Trans. Circuits Syst. II*, vol. 46, pp. 577–584, May 1999.

EAS  $\hat{S}_1$  consists of  $\tan^{-1}(\text{Single signed power of two})$   
 $\tan^{-1}(\text{Single SPT})$   
 $\tan^{-1}(\alpha^* \cdot 2^{-s^*})$

EAS  $\hat{S}_2$  consists of  $\tan^{-1}(\text{two signed power of two})$   
 $\tan^{-1}(\text{two SPT})$   
 $\tan^{-1}(\alpha_0^* \cdot 2^{-s_0^*} + \alpha_1^* \cdot 2^{-s_1^*})$

Two Signed - Power - of - Two terms

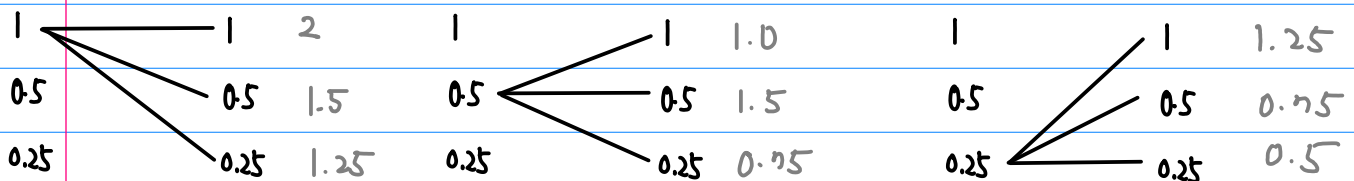
$$S_2 = \left\{ \tan^{-1} \left( \alpha_0^* \cdot 2^{-s_0^*} + \alpha_1^* \cdot 2^{-s_1^*} \right) : \right. \\ \left. \begin{array}{l} \alpha_0^*, \alpha_1^* \in \{-1, 0, +1\} \\ s_0^*, s_1^* \in \{0, 1, \dots, W-1\} \end{array} \right\}$$

$S_1$

1	$1 = 2^{-0}$	$\tan^{-1}(2^{-0})$
0.5	$\frac{1}{2} = 2^{-1}$	$\tan^{-1}(2^{-1})$
0.25	$\frac{1}{4} = 2^{-2}$	$\tan^{-1}(2^{-2})$

$S_2$

2	$1+1 = 2^{-0} + 2^{-0}$	$\pm \tan^{-1}(2^{-0} + 2^{-0})$
1.5	$1+\frac{1}{2} = 2^{-0} + 2^{-1}$	$\pm \tan^{-1}(2^{-0} + 2^{-1})$
1.25	$1+\frac{1}{4} = 2^{-0} + 2^{-2}$	$\pm \tan^{-1}(2^{-0} + 2^{-2})$
1.0	$1 = 2^{-0}$	$\pm \tan^{-1}(2^{-0})$
0.75	$\frac{1}{2}+\frac{1}{4} = 2^{-1} + 2^{-2}$	$\pm \tan^{-1}(2^{-1} + 2^{-2})$
0.5	$\frac{1}{2} = 2^{-1}$	$\pm \tan^{-1}(2^{-1})$
0.25	$\frac{1}{4} = 2^{-2}$	$\pm \tan^{-1}(2^{-2})$



$$2^{-0}, 2^{-1}, 2^{-2}$$

$$\{0, 1, 2\} = \{0, 1, w-1\}$$

$$w=3$$

$$s_0^*, s_i^* \in \{0, 1, 2\}$$

$$2^{s_0^*}, 2^{s_i^*} \in \{2^{-0}, 2^{-1}, 2^{-2}\}$$



as the wordsize  $w$  increases,  
the size of the set  $S_2$  increases exponentially

$$\theta_i = \tan^{-1} (\alpha_0(j) \cdot 2^{-s_0(j)} + \alpha_1(j) \cdot 2^{-s_1(j)})$$

$R_m$ : the number of the subangle  $N_A$

$$S_2 = \{ \theta_i \mid i = 0, 1, \dots, R_m \}$$

$$S_2 = \left\{ \tan^{-1} (\alpha_0^* \cdot 2^{-s_0^*} + \alpha_1^* \cdot 2^{-s_1^*}) : \begin{array}{l} \alpha_0^*, \alpha_1^* \in \{-1, 0, +1\} \\ s_0^*, s_1^* \in \{0, 1, \dots, w-1\} \end{array} \right\}$$

the optimization problem of the EEAS-based CORDIC algorithm

given  $\theta$  and  $R_m$

find  $\alpha_0(j)$ ,  $\alpha_1(j)$ ,  $s_0(j)$ , and  $s_1(j)$

the combination of elementary angles  
from EEAS  $S_2$

Minimize the angle quantization error

$$\left| \sum_{m, EEAS} \right| \triangleq \theta - \sum_{j=0}^{R_m-1} \tan^{-1} (\alpha_0(j) 2^{-s_0(j)} + \alpha_1(j) 2^{-s_1(j)})$$

given  $\begin{bmatrix} x(0) \\ y(0) \end{bmatrix}$

$$\begin{bmatrix} x(j+1) \\ y(j+1) \end{bmatrix} = \begin{bmatrix} 1 & \alpha_0(j) 2^{-s_0(j)} + \alpha_1(j) 2^{-s_1(j)} \\ \alpha_0(j) 2^{-s_0(j)} + \alpha_1(j) 2^{-s_1(j)} & 1 \end{bmatrix} \begin{bmatrix} x(j) \\ y(j) \end{bmatrix}$$

$$\begin{bmatrix} x_f \\ y_f \end{bmatrix} = P \begin{bmatrix} x(R_m) \\ y(R_m) \end{bmatrix} = \frac{1}{\prod_{j=0}^{R_m-1} \sqrt{1 + [\alpha_0(j) \cdot 2^{-s_0(j)} + \alpha_1(j) \cdot 2^{-s_1(j)}]^2}} \begin{bmatrix} x(R_m) \\ y(R_m) \end{bmatrix}$$

Micro rotation procedure  
the scaling operation

↳ additions

increased hardware  
reduced iteration steps

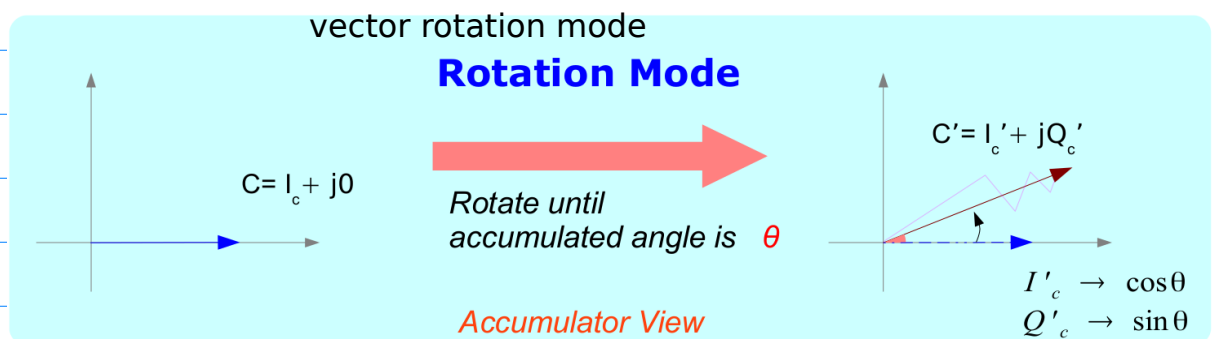
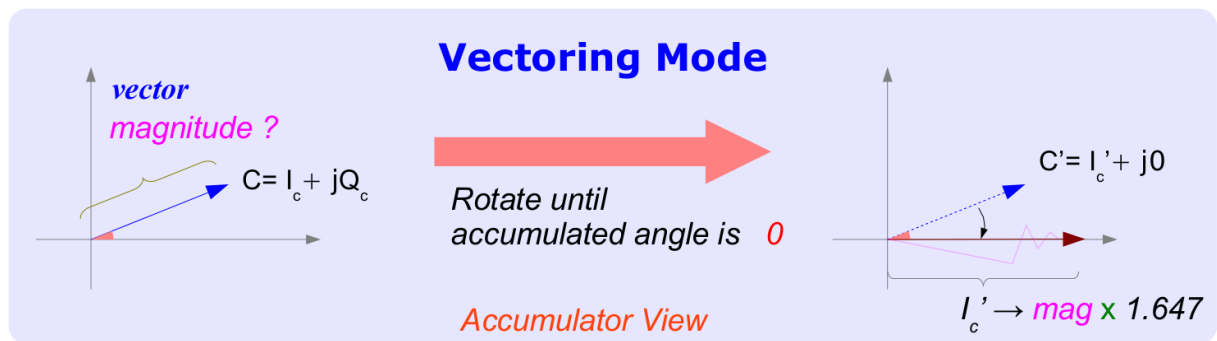
# MVR (Modified Vector Rotation)

1) Repeat of Elementary Angles  $\theta_i, \theta_i$

2) fixed total micro-rotation Number  $R_m$

\* Vector Rotation Mode

\* and the rotation angles are known in advance



# Modified Vector Rotational MVR CORDIC

- reduce the iteration number
- maintaining the SQNR performance
- modifying the basic microrotation procedure

## Three Searching Algorithm

- ① the selective prerotation
- ② the selective scaling
- ③ iteration-tradeoff scheme

# Angle Quantization

Quantization process on the rotational angle  $\theta$

decompose  $\theta$  into several subangles  $\theta_i$ 's

the angle quantization error

$$\xi_m \triangleq \theta - \sum_{i=0}^{N_A-1} \theta_i$$

$N_A$  the number of subangles  
 $\theta_0, \theta_1, \dots, \theta_{N_A-1}$

$$\theta = \theta_0 + \theta_1 + \dots + \theta_{N_A-1} + \xi_m$$

data :  $W$ -bit word length

the iteration number :  $N$   $N \leq W$

the restricted iteration number :  $R_m$   $R_m \ll W$

# AQ Process : 2 Design Issues

① need to determine the sub-angles  $\theta_i$

② select / combine sub angles  
to minimize the angle quantization angle  $\xi_m$

# CSD (Canonical Signed Digit) Quantization

digital filter designs

coefficients are recoded

in terms of SPT (Signed Power of Two) terms

multiplication can be easily realized  
with shift-and-add operations

$$h_2 = (-0.156249)_{10} \Rightarrow (0.0\bar{1}011)_2$$

$w=8$  , 3 non-zero digits

- ① CSD quantization decomposes coefficients into several SPT terms (sub-coefficients)
- ② the multiplication of a coefficient can be reformed through the combination of the non-zero SPT sub-coefficients

Quantize the rotation angle  $\theta$

decompose the rotation angle  $\theta$   
into several sub-angles  $\theta_i$ 's

the rotational operation of each  $\theta_i$   
should be easily realized

If each  $\theta_i$  can be realized  
using only shift-and-add operations

the rotation of  $\theta$  can be performed  
through successive applications of  
sub-angle rotations  
in a cost-effective way



Approximation target	Coefficient $r_i$	Rotation angle $\theta$
Basic Element	Non-zero digit $2^{-i}$	Sub-angle $\alpha(i) = \tan^{-1}(2^{-i})$
Basic Operation	shift-and-add operation	2 shift-and-add operations
Approximation Equation	$r_i \approx \sum_{j=0}^{N_D-1} g_j \cdot 2^{-d_j}$	$\theta \approx \sum_{j=0}^{N_A-1} \alpha(j) \cdot a(s_j)$
	$g_j \in \{-1, 0, +1\}$ $d_j \in \{0, 1, \dots, w-1\}$	
	$N_D =$ the number of non-zero digits	$N_A =$ the number of sub-angles

# Vector Rotation CORDIC Family

① Conventional CORDIC

② AR

③ MVR

④ EEAS

## ① Conventional CORDIC

elementary angle  $\alpha(i) = \tan^{-1}(2^{-i})$

the number of elementary angles  $N$

the rotation sequence  $\mu(i) = \{-1, +1\}$   
 $+1, -1, -1, +1, +1, \dots$

the  $i$ -th rotation angle  $\alpha(i)$

the  $w$ -bit word length

the iteration number  $N \leq w$

the angle quantization error

$$\xi_{m, \text{CORDIC}} \equiv \theta - \sum_{i=0}^{M-1} \mu(i) \alpha(i)$$

# ① AR [Hu]

skip certain micro rotations

the rotation sequence  $\mu(i) = \{-1, 0, +1\}$

$$\mu(i) = 0 \rightarrow \text{skip}$$

desire to minimize

$$\sum_{i=0}^N |\mu(i)|$$

so that the total number of CORDIC iterations can be minimized

Angle Recoding  $\leftarrow$  Multiplier Recoding

angle recoding method for efficient implementation of the CORDIC algorithm  
Hu & Naganathan, ISCAS 89

Greedy algorithm

$$\theta(0) = \theta, \{\mu(i) = 0, 0 \leq i \leq N-1\}, k=0$$

repeat until  $|\theta(k)| < \epsilon(N-1)$  Do

choose  $i_k, 0 \leq i_k \leq N-1$

$$| |\theta(k)| - a(i_k) | = \text{Min}_{0 \leq i \leq N-1} | |\theta(k)| - a(i) |$$

$$\theta(k+1) = \theta(k) - \mu(i_k) a(i_k)$$

$$\mu(i_k) = \text{Sign}(\theta(k))$$

try to approach the target rotation angle  $\theta$   
step by step

decisions are made in each step  
by choosing the best combination of  $\alpha(i)$   $a(s(i))$

So as to minimize  $|\xi_m|$

$\alpha(i)$ ,  $a(i)$  are determined such that  
the error function is minimized

$$J(i) = |\theta(i) - \alpha(i)a(s(i))|$$

$$\theta(i) = \theta - \sum_{m=0}^{i-1} \alpha(m) a(s(m))$$

terminated if no further improvement can be found

$$J(i) \geq J(i-1)$$

or  $\alpha(R_m-1)$  and  $s(R_m-1)$   
are determined at the end

$$\xi_{m, \text{CORDIC}} \equiv \theta - \sum_{i=0}^M \mu(i) \alpha(i) \quad \mu(i) = \{-1, 0, +1\}$$

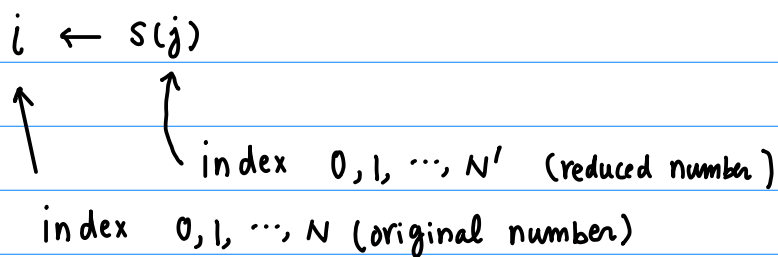
$$= \theta - \sum_{j=0}^{N'} \tilde{\theta}(j)$$

$$N' \equiv \sum_{i=0}^{N-1} |\mu(i)| \quad \{+1, 0, +1\}$$

the effective iteration number  $N'$

$S(j)$  the rotational sequence

determines the micro-rotation angle in the  $j$ -th iteration



$$\mu(S(j)) \leftarrow \alpha(j)$$

$$\downarrow \qquad \uparrow$$

$$\{-1, +1\}$$

$$\mu(i) = \begin{cases} \mu(S(j)) & i = S(j) \\ 0 & i \neq S(j) \text{ --- reduced index} \end{cases}$$

er

$$\begin{aligned}
 i &= 0, \overset{\text{see}}{\boxed{1, 2}}, 3, \dots, N-1 \\
 s(j) &= 0, \boxed{1, 2}, 3, \dots, N-1 && \text{rotational sequence} \\
 \alpha(j) &= -1, \boxed{0, 0}, +1, \dots, -1 && \text{directional sequence} \\
 j &= 0, -, -, 1, \dots, N'-1 && \text{effective iteration number} \\
 N' &= N-2
 \end{aligned}$$

the  $j$ -th micro-rotation of  $a(s(j))$

elementary angle

$$a(i) = \tan^{-1}(2^{-i})$$

$$a(s(j)) = \tan^{-1}(2^{-s(j)})$$

$$\alpha(j) a(s(j)) = \alpha(j) \tan^{-1}(2^{-s(j)}) \quad \alpha(j) \in \{-1, +1\}$$

$$\Leftrightarrow \mu(i) a(i) \quad \mu(i) \in \{-1, 0, +1\}$$



$$\sum_{m, \text{CORDIC}} \equiv \theta - \sum_{i=0}^{N-1} \mu(i) a(i) \quad \mu(i) \in \{-1, 0, +1\}$$

$$= \theta - \left[ \sum_{j=0}^{N'} \tilde{\theta}(j) \right]$$

$$= \theta - \left[ \sum_{j=0}^{N'} \tan^{-1}(\alpha(j) \cdot 2^{-s(j)}) \right] \quad \alpha(j) \in \{-1, +1\}$$

$$\tilde{\theta}(j) = \alpha(j) \tan^{-1}(2^{-s(j)})$$

$$= \tan^{-1}(\alpha(j) \cdot 2^{-s(j)})$$

$$S_1 = \{ \tan^{-1}(\alpha \cdot 2^{-s}) \mid \alpha \in \{-1, 0, +1\}, s \in \{0, 1, 2, \dots, N-1\} \}$$

## ② MVR (Modified Vector Rotational)

two modifications

① **repetition** of elementary angles

each micro-rotation of elementary angle  
can be performed repeatedly

- more possible combinations
- smaller  $\xi_m$

② **confinement** of total micro-rotation number

confine the iteration number  
in the micro-rotation phase  
to  $R_m$  ( $R_m \ll W$ )

The role of  $R_m$  is quite similar  
to the **number of non-zero digit**  
 $N_D$  in CSD recoding scheme

$$\sum_{m, \text{MVR}}^x \triangleq \theta - \sum_{i=0}^{R_m-1} \alpha(i) a(s(i))$$

the rotational sequence

$$s(i) \in \{0, 1, \dots, W-1\}$$

the micro-rotation angle

in the  $i$ -th iteration

the directional sequence

$$\alpha(i) \in \{-1, 0, +1\}$$

the direction of the  $i$ -th

micro-rotation of  $a(s(i))$

$$\alpha(i) a(s(i)) = \tilde{\theta}(j)$$

$$\xi_{m, MVR} \cong \theta - \sum_{j=0}^{R_m-1} \alpha(j) a(s(j))$$

the rotational sequence  $s(j)$

$$j = 0, 1, 2, \dots, R_m-1$$



$$s(j) \in \{0, 1, \dots, W-1\}$$

determines the micro-rotation angle  $a(s(j))$   
in the  $j$ -th iteration

the directional sequence  $\alpha(j)$

$$\alpha(j) \in \{-1, 0, +1\}$$

controls the direction of the  $j$ -th  
micro-rotation of  $a(s(j))$

$$\alpha(j) a(s(j)) = \tilde{\theta}(j)$$

$i = 0, 1, 2, 3, \dots, W-1$	
$s(j) = 0, 1, 2, 3, \dots, W-1$	rotational sequence
$\alpha(j) = -1, 0, 0, +1, \dots, -1$	directional sequence
$j = 0, \dots, R_m-1$	effective iteration number
$R_m \ll W$	

sub-angle  $(\alpha(j) a(s(j))) \sim \tilde{\theta}(j)$

$$\xi_{m,AR} = \theta - \left[ \sum_{j=0}^{N'-1} \tan^{-1}(\alpha(j) \cdot 2^{-s(j)}) \right]$$
$$= \theta - \left[ \sum_{j=0}^{N'-1} \tilde{\theta}(j) \right], \quad \tilde{\theta}(j) = \tan^{-1}(\alpha(j) \cdot 2^{-s(j)})$$

$$N' \triangleq \sum_{j=0}^{N-1} |\mu(j)| \quad \text{the effective iteration number}$$

EAS formed by MVR-CORDIC  
is the same as AR  
also performs AQ

The major difference

1) the total number of sub-angles  $N_A$

the total iteration number

in the micro-rotation phase

is kept fixed to a pre-defined value of  $R_m$

$$N_A = R_m$$

2) the sub-angle  $\theta_j$  corresponds to  $\alpha(j) a(s(j))$

$$\theta_j = \alpha(j) a(s(j)) = \tilde{\theta}_j$$

# Optimization Problem

EAS point of view

Given  $\theta$ , find the combination of  $R_m$  elementary angles from EAS  $S_i$ , such that the angle quantization error  $|\xi_{m, \text{MUR}}|$  is minimized.

Semi-greedy algorithm  
trade offs between computational complexities  
and performance

key issue in the MVR-CORDIC

is to find the best sequences of

$s(i)$  and  $\alpha(i)$  to minimize  $|\xi_m|$

subject to the constraint that

the total iteration number is confined to  $R_m$

1) Greedy Algorithm

2) Exhaustive Algorithm

3) Semigreedy Algorithm

# 1) Greedy Algorithm

try to approach the target rotation angle,  $\theta$ , step by step in each step, decisions are made on  $\alpha(i)$  and  $s(i)$  by choosing the best combination of  $\alpha(i)$  and  $s(i)$  so as to minimize  $|\xi_m|$

$\alpha(i)$  and  $s(i)$  are determined such that the error function  $J(i) = |\theta(i) - \alpha(i) a(s(i))|$  is minimized

$\theta(i)$  : the residue angle in the  $i$ -th step

$$\theta(i) = \theta - \sum_{m=0}^{i-1} \alpha(m) a(s(m))$$

the searching is terminated

if no further improvements can be found

$$J(i) \geq J(i-1)$$

$\alpha(R_m-1)$  and  $s(R_m-1)$  are determined

at the end of the searching

the greedy algorithm terminates

Only when the residue angle error cannot be further reduced.



Initialization:

given  $\theta$ ,  $w$ ,  $R_m$

$$\theta^{(i)} = \theta - \sum_{m=0}^{i-1} \alpha^{(m)} a(s^{(m)})$$

Select  $\alpha^{(i)} \in \{-1, 0, +1\}$   
 $s^{(i)} \in \{0, 1, 2, \dots, w-1\}$   
to minimize  $J^{(i)} = \theta^{(i)} - \alpha^{(i)} a(s^{(i)})$

N  
 $J^{(i)} < J^{(i-1)}$

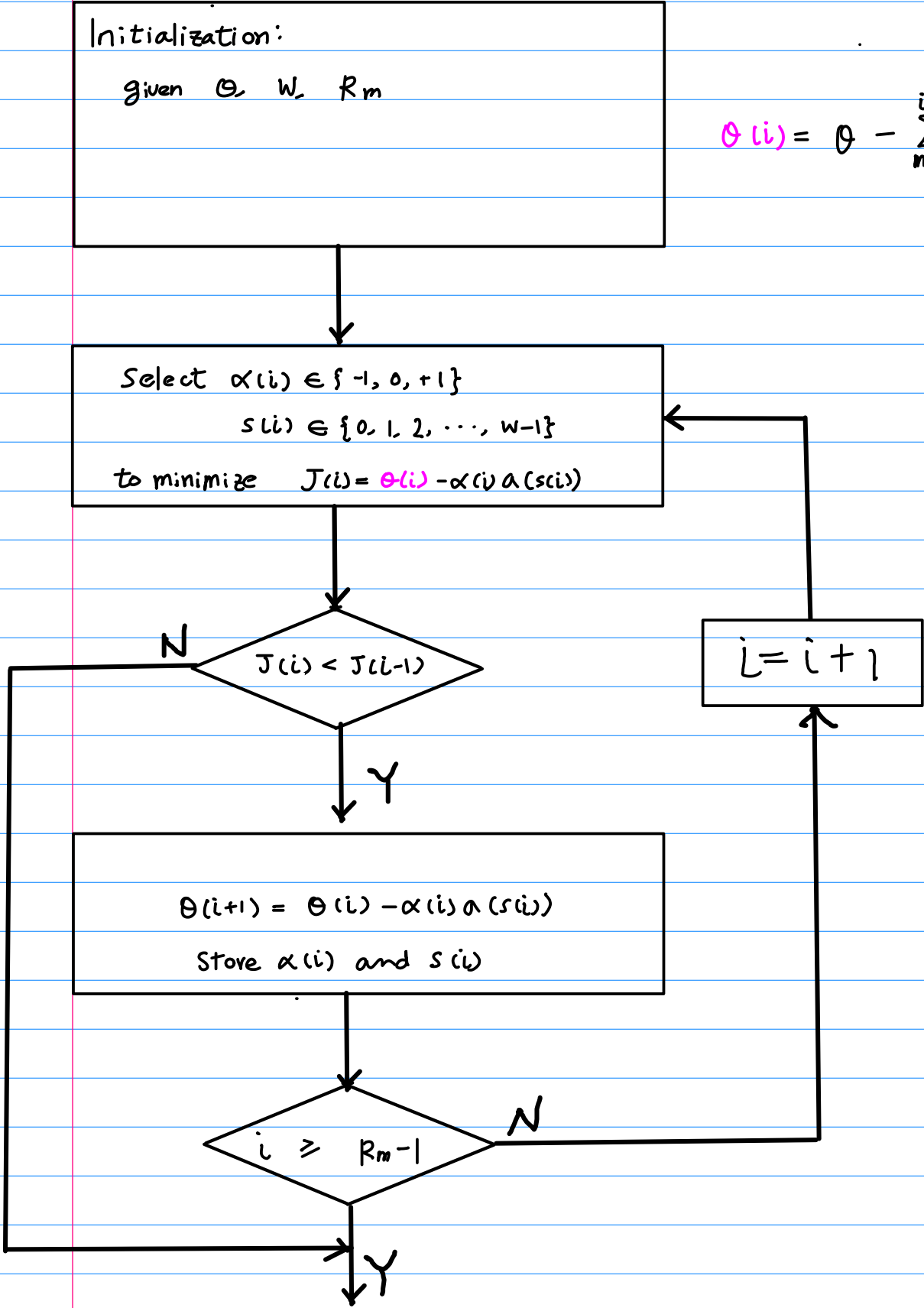
Y

$\theta^{(i+1)} = \theta^{(i)} - \alpha^{(i)} a(s^{(i)})$   
Store  $\alpha^{(i)}$  and  $s^{(i)}$

$i \geq R_m - 1$   
N

Y

$i = i + 1$



## 2) Exhaustive Algorithm

search for the entire solution space

all possible combinations of

$$\sum_{i=0}^{R_m-1} \alpha(i) a(s(i))$$

in a single step

decisions for  $\alpha(i)$  and  $s(i)$ ,  $0 \leq i \leq R_m-1$

by minimizing the error function

$$J = \left| 0 - \sum_{i=0}^{R_m-1} \alpha(i) a(s(i)) \right|$$

global optimal solution

Initialization:

given  $\Theta, W, R_m$

let  $\theta^{(0)} = \Theta,$

$i = 0$

$J^{(-1)} = \infty$

Select  $\alpha^{(i)} \in \{-1, 0, +1\}$

$s^{(i)} \in \{0, 1, 2, \dots, W-1\}$

for  $0 \leq i \leq R_m - 1$

to minimize  $J^{(i)} = \Theta - \sum_{i=0}^{R_m-1} \alpha^{(i)} a(s^{(i)})$

$$(3 \cdot W) \cdot (3 \cdot W) \dots (3 \cdot W) \\ = 3^{R_m} \cdot W^{R_m}$$

Store  $\alpha^{(i)}$  and  $s^{(i)}$

for  $0 \leq i \leq R_m - 1$



in the  $i$ -th block

decision of  $\alpha(k)$  and  $s(k)$  for  $iD \leq k \leq (i+1)D-1$

$$\text{minimizes } J = \left| \theta(i) - \sum_{k=iD}^{(i+1)D-1} \alpha(k) a(s(k)) \right|$$

$$\text{where } \theta(i) = \theta - \sum_{m=0}^{i-1} \left[ \sum_{k=mD}^{(m+1)D-1} \alpha(k) a(s(k)) \right]$$

the residue angle in the  $i$ -th step

$$s = \left\lceil \frac{R_m}{D} \right\rceil$$

$$\theta(i) = \theta - \left[ \sum_{k=0D}^{1D-1} \alpha(k) a(s(k)) + \sum_{k=1D}^{2D-1} \alpha(k) a(s(k)) + \dots + \sum_{k=(i-1)D}^{iD-1} \alpha(k) a(s(k)) \right]$$

Initialization:

given  $\theta, W, R_m$

let  $\theta(0) = \theta,$

$i = 0$

$J(-1) = \infty$

Select  $\alpha(k) \in \{-1, 0, +1\}$

$s(k) \in \{0, 1, 2, \dots, W-1\}$

for  $iD \leq k \leq (i+1)D - 1$

to minimize  $J(i) = \theta(i) - \sum_{k=iD}^{(i+1)D-1} \alpha(k) a(s(k))$

N  
 $J(i) < J(i-1)$

$\theta(i+1) = \theta(i) - \sum_{k=iD}^{(i+1)D-1} \alpha(k) a(s(k))$

Store  $\alpha(k), s(k)$

$i \geq \lceil \frac{R_m}{D} \rceil - 1$   
N

Y

$i = i + 1$

### ③ Extended EAS-based CORDIC

$$\mathcal{S}_2 = \left\{ \tan^{-1} \left( \alpha_0^* \cdot 2^{-s_0^*} + \alpha_1^* \cdot 2^{-s_1^*} \right) : \right. \\ \left. \alpha_0^*, \alpha_1^* \in \{-1, 0, +1\}, s_0^*, s_1^* \in \{0, 1, \dots, W-1\} \right\}$$

$$\theta_i = \tan^{-1} \left( \alpha_0(j) \cdot 2^{-s_0(j)} + \alpha_1(j) \cdot 2^{-s_1(j)} \right)$$

$$\left| \xi_{m, \text{EAS}} \right| \triangleq$$

$$\left| \theta - \sum_{j=0}^{R_m-1} \tan^{-1} \left( \alpha_0(j) \cdot 2^{-s_0(j)} + \alpha_1(j) \cdot 2^{-s_1(j)} \right) \right|$$

# Generalized EEAS Scheme

$$S_d = \left\{ \tan^{-1} \left( \alpha_0^* \cdot 2^{-s_0^*} + \alpha_1^* \cdot 2^{-s_1^*} + \dots + \alpha_{d-1}^* \cdot 2^{-s_{d-1}^*} \right) : \right. \\ \left. \begin{array}{l} \alpha_0^*, \alpha_1^*, \dots, \alpha_{d-1}^* \in \{-1, 0, +1\}, \\ s_0^*, s_1^*, \dots, s_{d-1}^* \in \{0, 1, \dots, W-1\} \end{array} \right\}$$