Characteristics of Multiple Random Variables

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Based on
Probability, Random Variables and Random Signal Principles,
P.Z. Peebles,Jr. and B. Shi

Outline

1. Simulation of Multiple Random Variables

Complex Random Variables (1)

N Gaussian random variables

Definition

$$Z = X + jY$$

$$E[g(Z)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(z) f_{X,Y}(x,y)\,dxdy$$

$$\overline{Z} = E[Z] = E[X] + jE[Y] = \overline{X} + j\overline{Y}$$

$$\sigma_Z^2 = E[|Z - E[Z]|^2] = E[(Z - E[Z])(Z - E[Z])^*]$$

Complex Random Variables (2)

N Gaussian random variables

Definition

$$\sigma_Z^2 = E[|Z - E[Z]|^2] = E[(Z - E[Z])(Z - E[Z])^*]$$

$$g(Z) = |Z - E[Z]|^2$$

$$\tilde{\sigma}_Z^2 = E[(Z - E[Z])^2] = E[(Z - E[Z])(Z - E[Z])]$$

Complex Random Variables (3)

N Gaussian random variables

Definition

$$\sigma_X^2 = \frac{1}{2} Re\left\{\sigma_Z^2 - \tilde{\sigma}_Z^2\right\}$$

$$\sigma_Y^2 = \frac{1}{2} Im\left\{\sigma_Z^2 - \tilde{\sigma}_Z^2\right\}$$

$$\sigma_{XY}^2 = \sigma_{YX}^2 = \frac{1}{2} Im\left\{\tilde{\sigma}_Z^2\right\}$$

Complex Random Variables (4)

N Gaussian random variables

Definition

$$Z_m = X_m + jY_m$$

$$Z_n = X_n + jY_n$$

$$f_{X_m Y_m X_n Y_n}(x_m, y_m, x_n, y_n) = f_{X_m Y_m}(x_m, y_m) f_{X_n Y_n}(x_n, y_n)$$

Covariance (1)

N Gaussian random variables

Definition

$$R_{Z_m Z_n} = E\left[Z_m Z_n^{*}\right] \qquad n \neq m$$

$$C_{Z_m Z_n} = E\left[\{Z_m - E[Z_m]\}\{Z_n - E[Z_n]\}^{*}\right] \qquad n \neq m$$

$$\tilde{R}_{Z_m Z_n} = E\left[Z_m Z_n\right] \qquad n \neq m$$

$$\tilde{C}_{Z_m Z_n} = E\left[\{Z_m - E[Z_m]\}\{Z_n - E[Z_n]\}\right] \qquad n \neq m$$

Covariance (2)

N Gaussian random variables

Definition

$$C_{X_m X_n} = \frac{1}{2} Re\left\{ C_{Z_m Z_n} + \tilde{C}_{Z_m Z_n} \right\}$$

$$C_{Y_m Y_n} = \frac{1}{2} Re\left\{ C_{Z_m Z_n} - \tilde{C}_{Z_m Z_n} \right\}$$

$$C_{X_m Y_n} = -\frac{1}{2} Im\left\{ C_{Z_m Z_n} - \tilde{C}_{Z_m Z_n} \right\}$$

$$C_{Y_m X_n} = \frac{1}{2} Re\left\{ C_{Z_m Z_n} + \tilde{C}_{Z_m Z_n} \right\}$$

Covariance (3)

N Gaussian random variables

Definition

$$R_{Z_m Z_n} = E[Z_m]\,E[Z_n^*] \qquad n \neq m$$

$$\tilde{R}_{Z_m Z_n} = E[Z_m]\,E[Z_n] \qquad n \neq m$$