

Characteristics of Multiple Random Variables

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June 26, 2020

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Based on
Probability, Random Variables and Random Signal Principles,
P.Z. Peebles,Jr. and B. Shi

Outline

1 Simulation of Multiple Random Variables

Complex Random Variables (1)

N Gaussian random variables

Definition

$$Z = X + jY$$

$$E[g(Z)] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(z) f_{X,Y}(x,y) dx dy$$

$$\bar{Z} = E[Z] = E[X] + jE[Y] = \bar{X} + j\bar{Y}$$

$$\sigma_Z^2 = E[|Z - E[Z]|^2] = E[(Z - E[Z])(Z - E[Z])^*]$$

Complex Random Variables (2)

N Gaussian random variables

Definition

$$\sigma_Z^2 = E[|Z - E[Z]|^2] = E[(Z - E[Z])(Z - E[Z])^*]$$

$$g(Z) = |Z - E[Z]|^2$$

$$\tilde{\sigma}_Z^2 = E[(Z - E[Z])^2] = E[(Z - E[Z])(Z - E[Z])]$$

Complex Random Variables (3)

N Gaussian random variables

Definition

$$\sigma_X^2 = \frac{1}{2} \operatorname{Re} \{ \sigma_Z^2 - \tilde{\sigma}_Z^2 \}$$

$$\sigma_Y^2 = \frac{1}{2} \operatorname{Im} \{ \sigma_Z^2 - \tilde{\sigma}_Z^2 \}$$

$$\sigma_{XY}^2 = \sigma_{YX}^2 = \frac{1}{2} \operatorname{Im} \{ \tilde{\sigma}_Z^2 \}$$

Complex Random Variables (4)

N Gaussian random variables

Definition

$$Z_m = X_m + jY_m$$

$$Z_n = X_n + jY_n$$

$$f_{X_m Y_m X_n Y_n}(x_m, y_m, x_n, y_n) = f_{X_m Y_m}(x_m, y_m) f_{X_n Y_n}(x_n, y_n)$$

Covariance (1)

N Gaussian random variables

Definition

$$R_{Z_m Z_n} = E[Z_m Z_n^*] \quad n \neq m$$

$$C_{Z_m Z_n} = E[\{Z_m - E[Z_m]\} \{Z_n - E[Z_n]\}^*] \quad n \neq m$$

$$\tilde{R}_{Z_m Z_n} = E[Z_m Z_n] \quad n \neq m$$

$$\tilde{C}_{Z_m Z_n} = E[\{Z_m - E[Z_m]\} \{Z_n - E[Z_n]\}] \quad n \neq m$$

Covariance (2)

 N Gaussian random variables

Definition

$$C_{X_m X_n} = \frac{1}{2} \operatorname{Re} \left\{ C_{Z_m Z_n} + \tilde{C}_{Z_m Z_n} \right\}$$

$$C_{Y_m Y_n} = \frac{1}{2} \operatorname{Re} \left\{ C_{Z_m Z_n} - \tilde{C}_{Z_m Z_n} \right\}$$

$$C_{X_m Y_n} = -\frac{1}{2} \operatorname{Im} \left\{ C_{Z_m Z_n} - \tilde{C}_{Z_m Z_n} \right\}$$

$$C_{Y_m X_n} = \frac{1}{2} \operatorname{Re} \left\{ C_{Z_m Z_n} + \tilde{C}_{Z_m Z_n} \right\}$$

Covariance (3)

 N Gaussian random variables

Definition

$$R_{Z_m Z_n} = E[Z_m] E[Z_n^*] \quad n \neq m$$

$$\tilde{R}_{Z_m Z_n} = E[Z_m] E[Z_n] \quad n \neq m$$

