BJT Amplifier
ac Model (H.10)

20170619

References

Based

[1] Floyd, Electronic Devices 7th ed
[2] Cook,

[2] en.wikipedia.org

α_{ac}	ac alpha	(I_c / I_e)
β_{ac}	ac beta	(I_c / I_b)
r'_e	ac emitter resistance	
r'_b	ac base resistance	
r'_c	ac collector resistance	

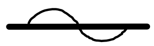
$I_c = \alpha_{ac} I_e$

$I_c = \beta_{ac} I_b$

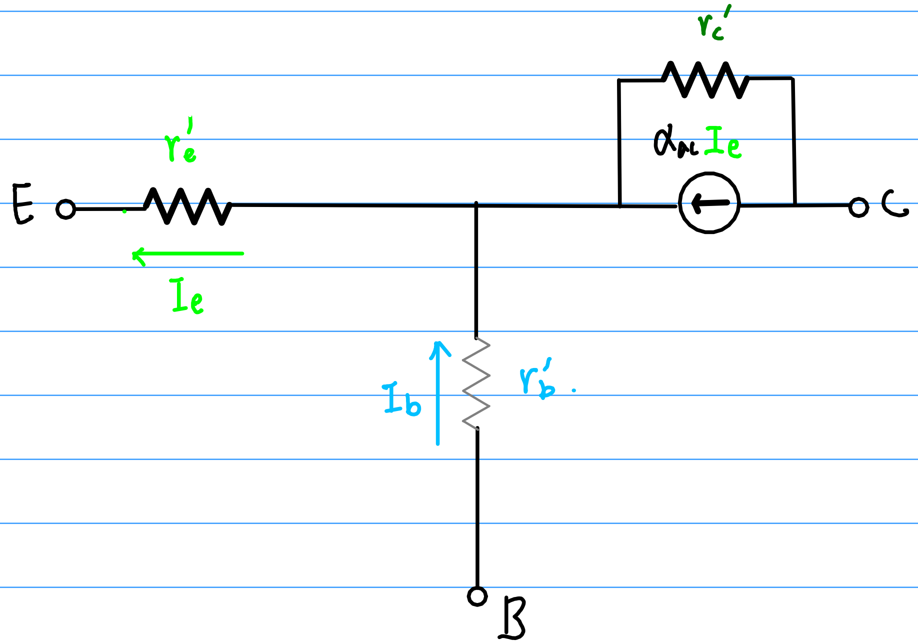
$V_{BQ} + V_b$ 

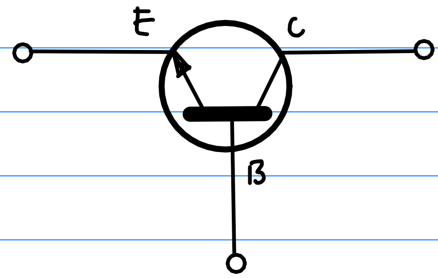
$I_B + I_b$  I_b 

$V_{CEQ} + V_{ce}$  β_{ac}

$I_{CQ} + I_c$  I_c 

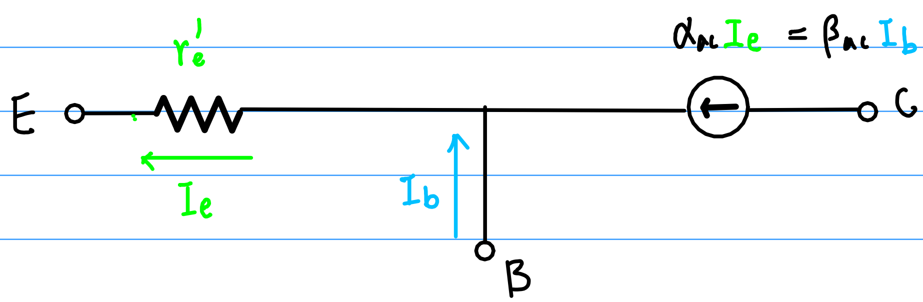
r- parameter Equivalent Circuit



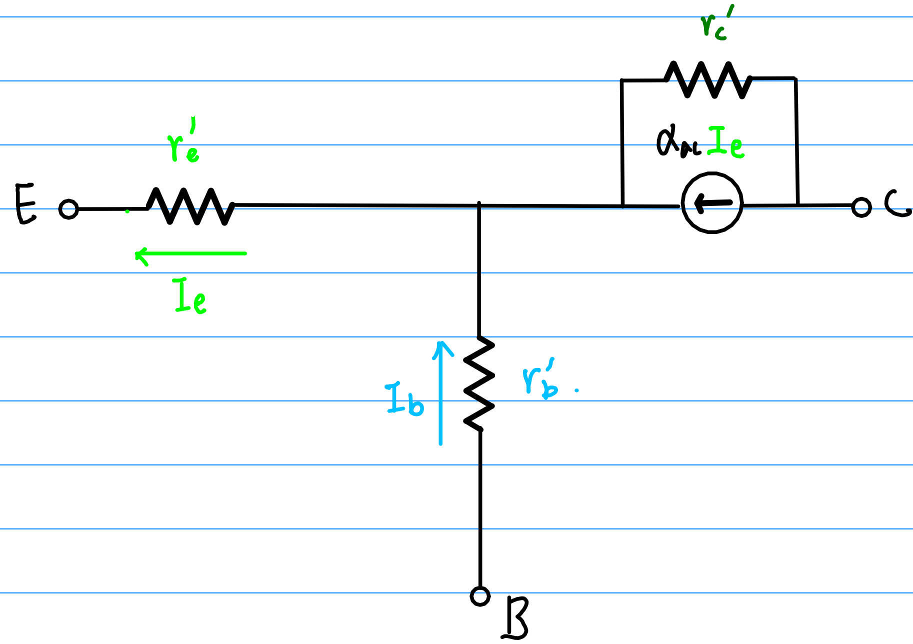


r_b' : small enough to neglect

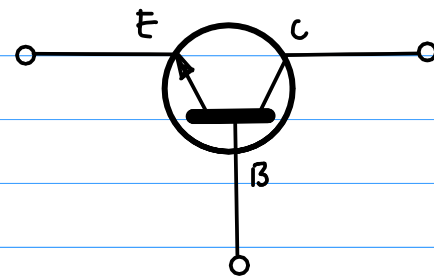
r_c' : large " " " (a few 100 kΩ)



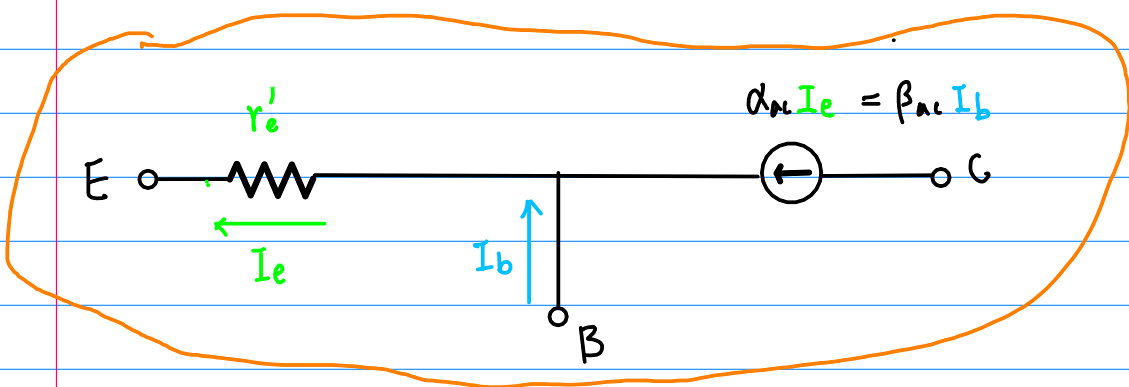
t model



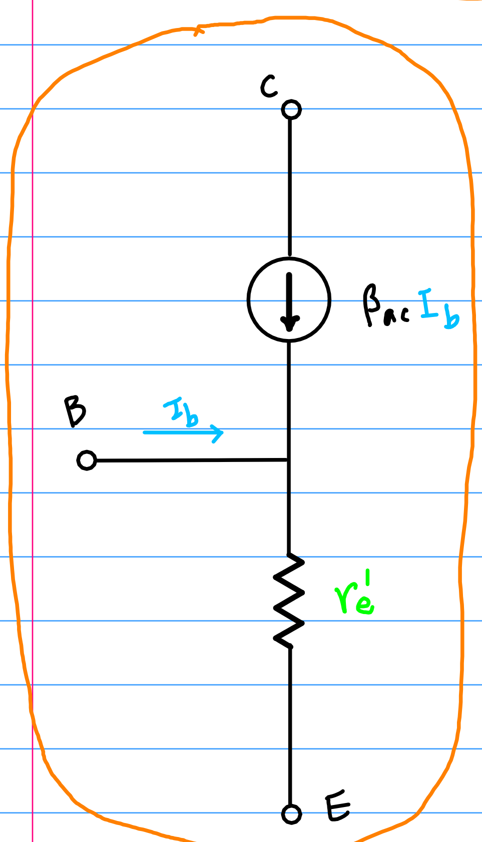
r'c
αac Ie
r'e
E
C
Ie
Ib
r'b
B



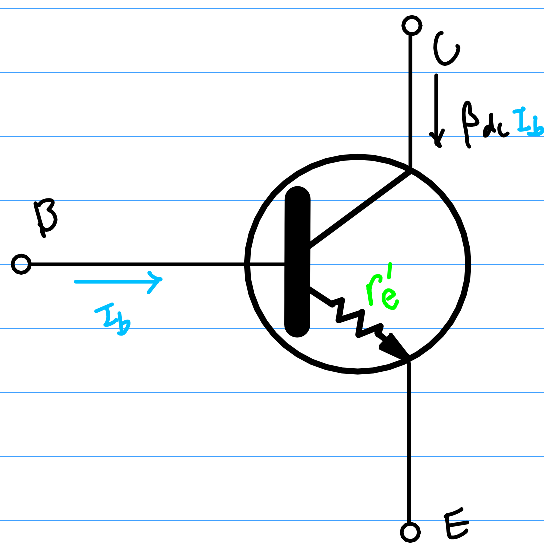
E
C
B



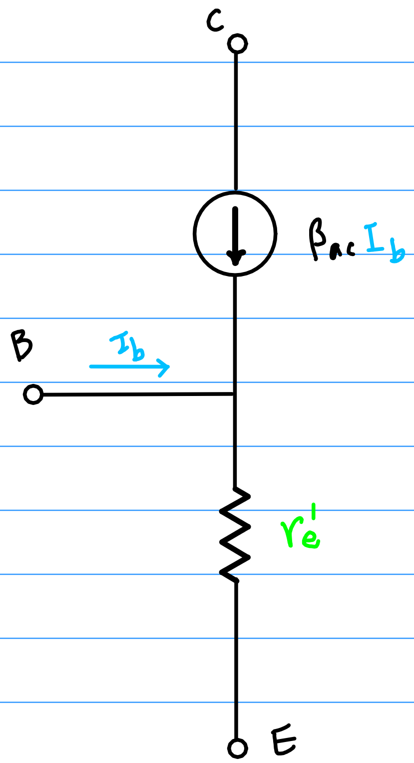
r'e
αac Ie = βac Ib
E
C
Ie
Ib
B

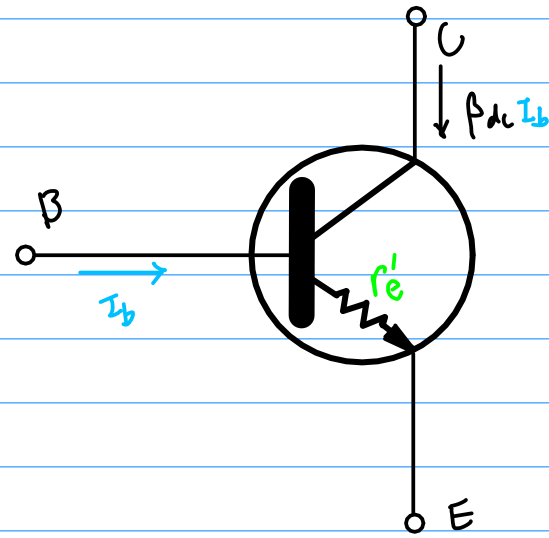


C
βac Ib
B
Ib
r'e
E



C
βdc Ib
B
Ib
r'e
E


C
βac Ib
B
Ib
r'e
E


C
βdc Ib
B
Ib
r'e
E

h parameters

h_i	Input Impedence (resistance)	Output shorted
h_r	Voltage Feedback Ratio	Input Open
h_f	Forward Current Gain	Output Shorted
h_o	Output Admittance (conductance)	Input Open

Common Emitter	Common Base	Common Collector
h_{ie}	h_{ib}	h_{ic}
h_{re}	h_{rb}	h_{rc}
h_{fe}	h_{fb}	h_{fc}
h_{oe}	h_{ob}	h_{oc}

$$\alpha_{ac} = h_{fb}$$

$$\beta_{ac} = h_{fe}$$

$$r_e' = \frac{h_{re}}{h_{oe}}$$

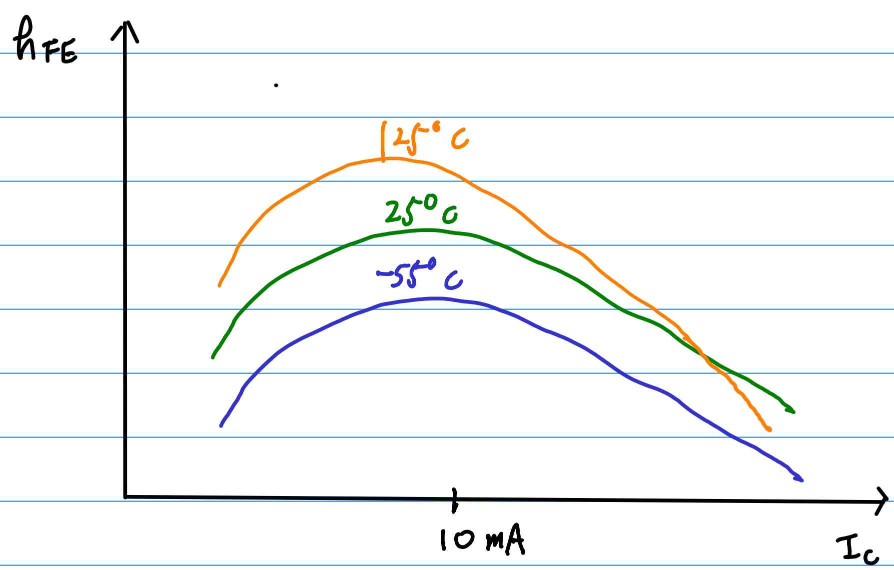
$$r_c' = \frac{h_{re} + 1}{h_{oe}}$$

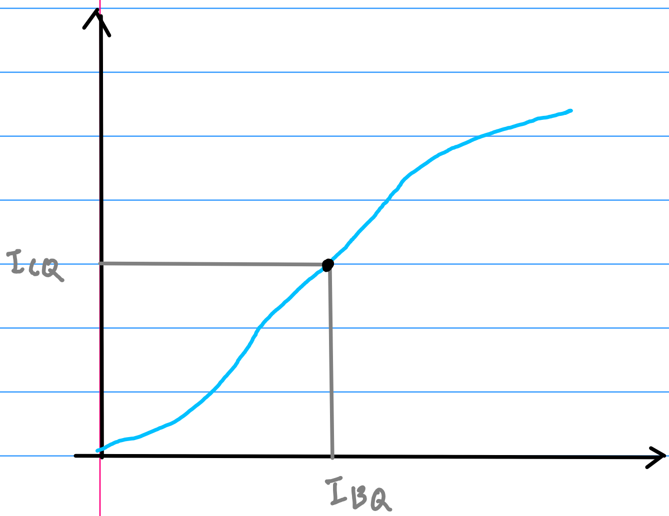
$$r_b' = h_{ie} - \frac{h_{re}}{h_{oe}}(1 + h_{fe})$$

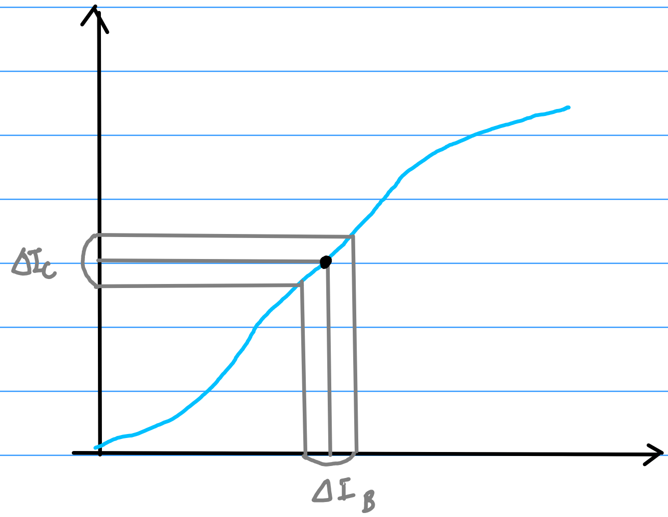
Variations in h_{FE} (Current Gain)

$\beta_{DC} = h_{FE}$ 100 ~ 300

- individual transistors
- temperature
- I_C

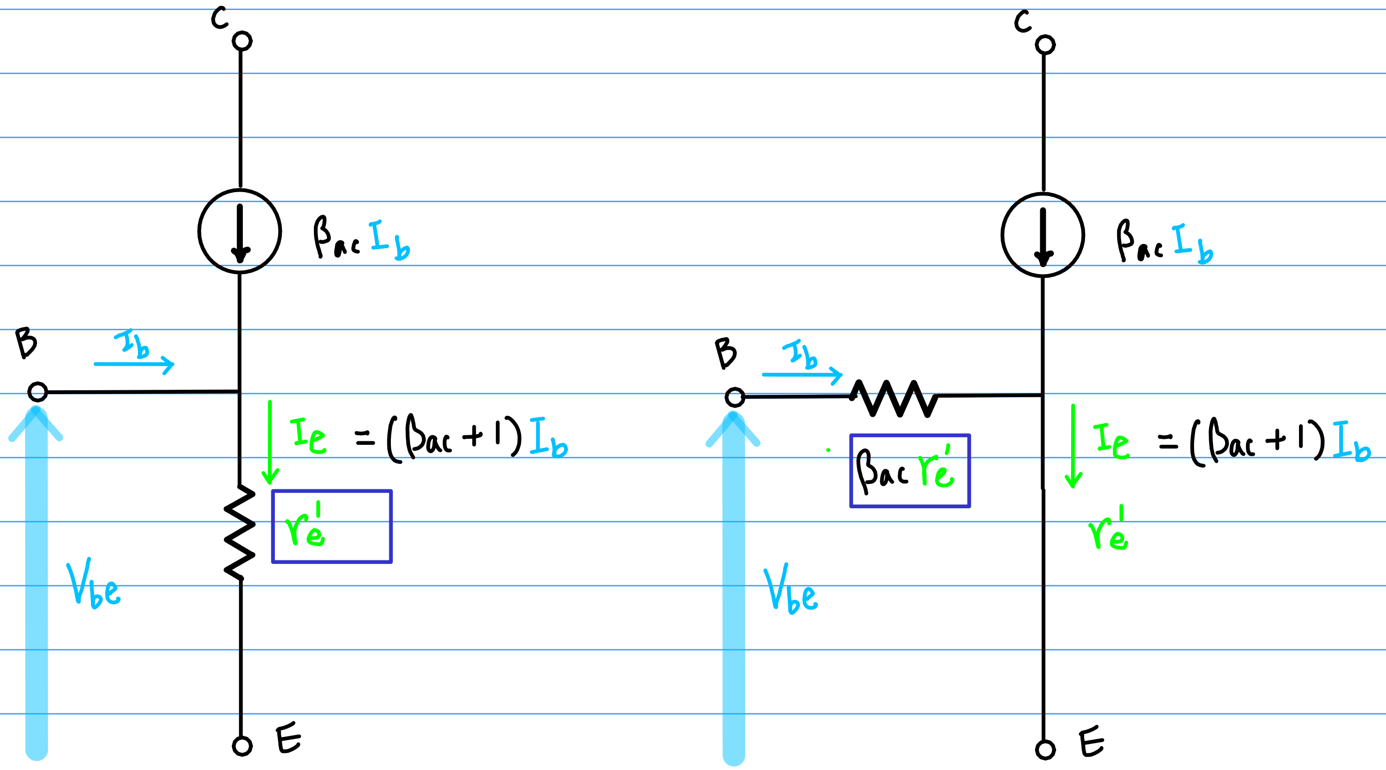




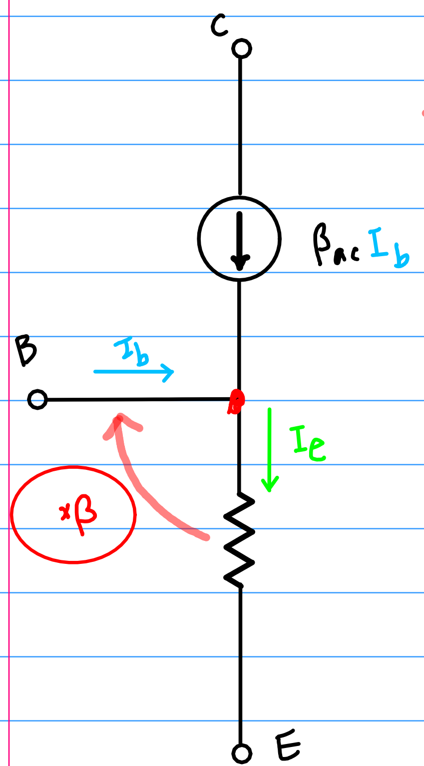


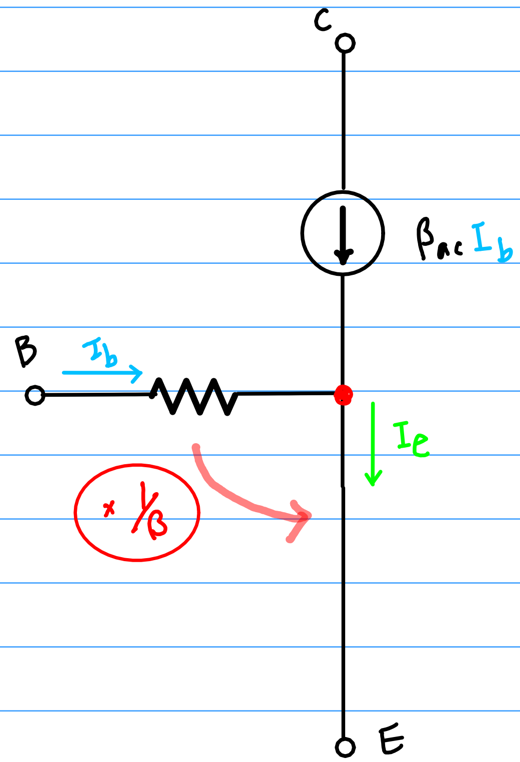
$$\beta_{DC} = \frac{I_C}{I_B}$$

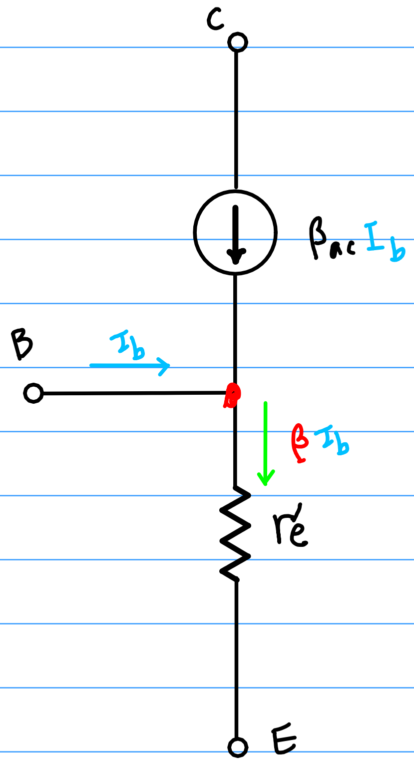
$$\beta_{AC} = \frac{\Delta I_C}{\Delta I_B}$$

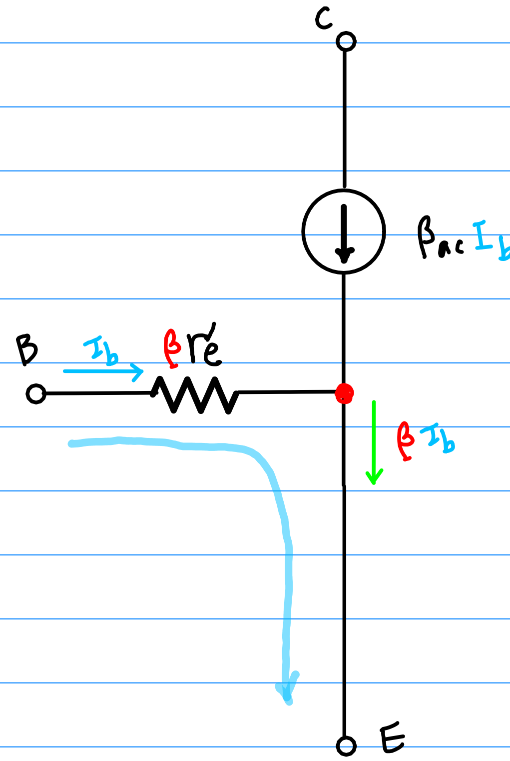


$$V_{be} = I_e r_e' = (\beta_{ac}+1) I_b r_e'$$
$$= \beta_{ac} I_b r_e'$$
$$= I_b (\beta_{ac} r_e')$$


C
$\beta_{ac} I_b$
B
I_b
I_e
×β
E


C
$\beta_{ac} I_b$
B
I_b
I_e
× 1/β
E





KVL $\beta I_b r_e'$

constrained by

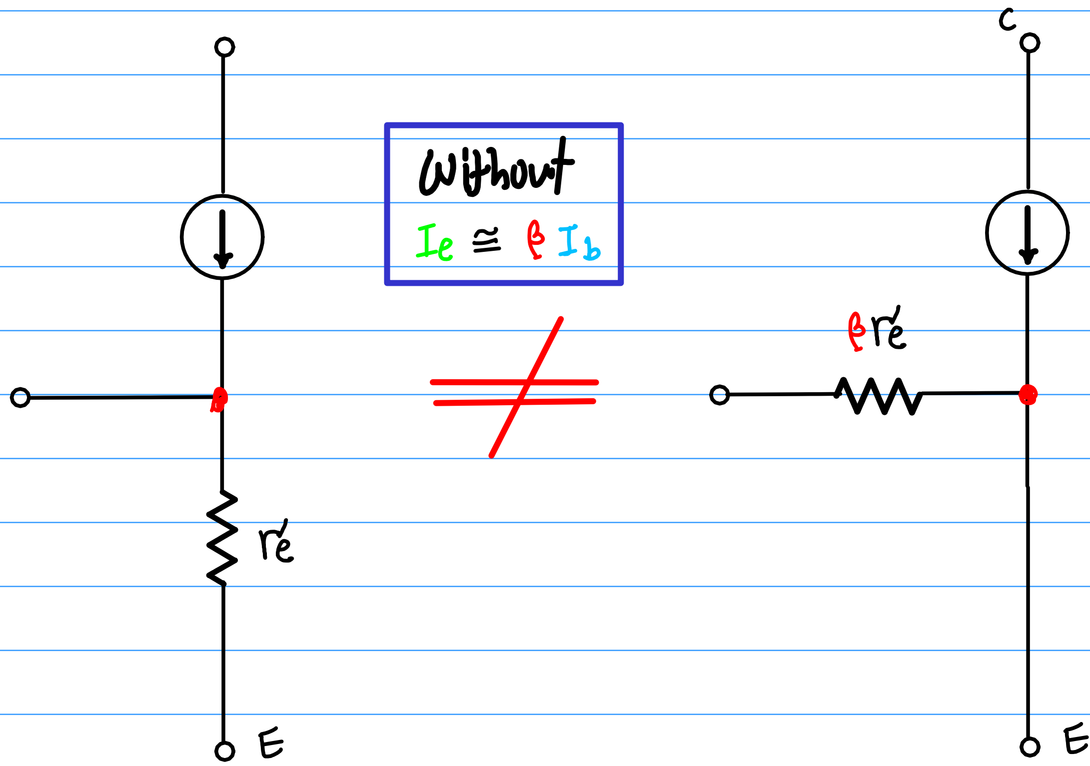
$I_e \cong \beta I_b$

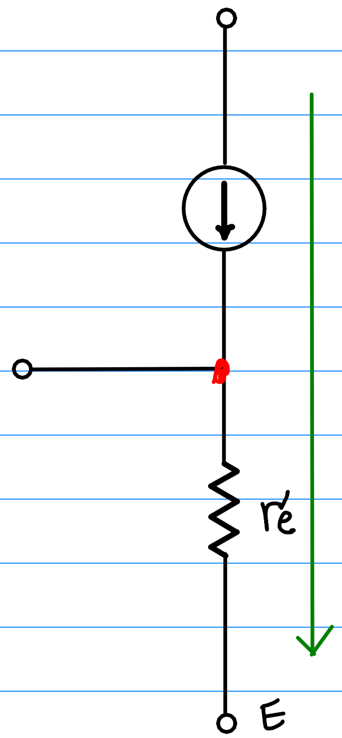
=

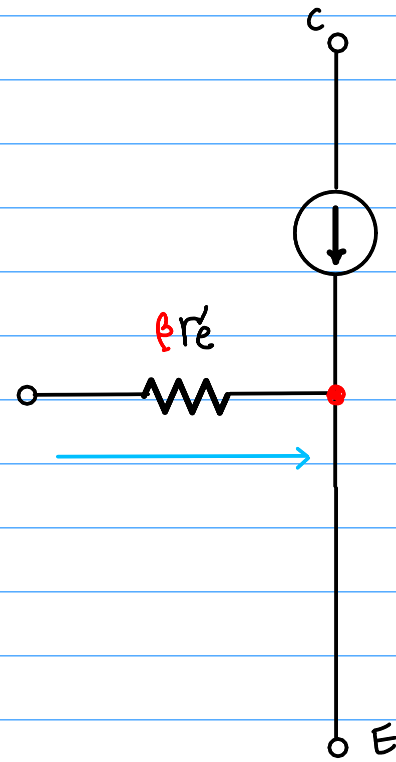
KVL $I_b \beta r_e'$

constrained by

$I_e \cong \beta I_b$


Without
Ie ≅ β Ib
r'e
E
C
βr'e
E



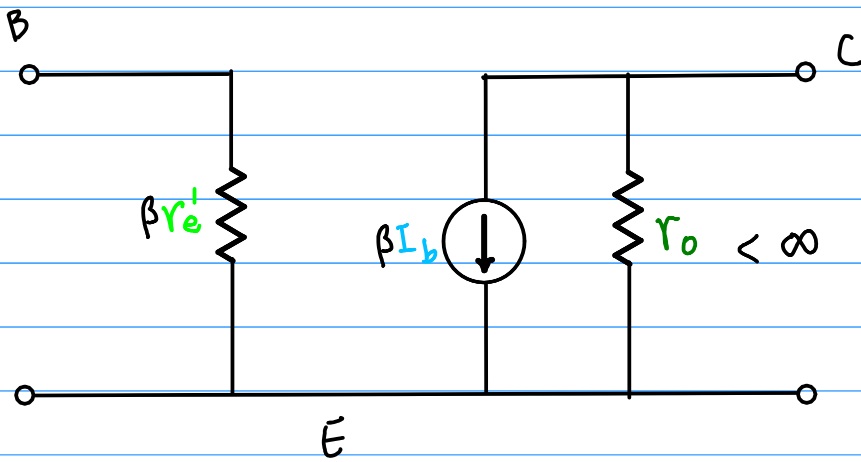


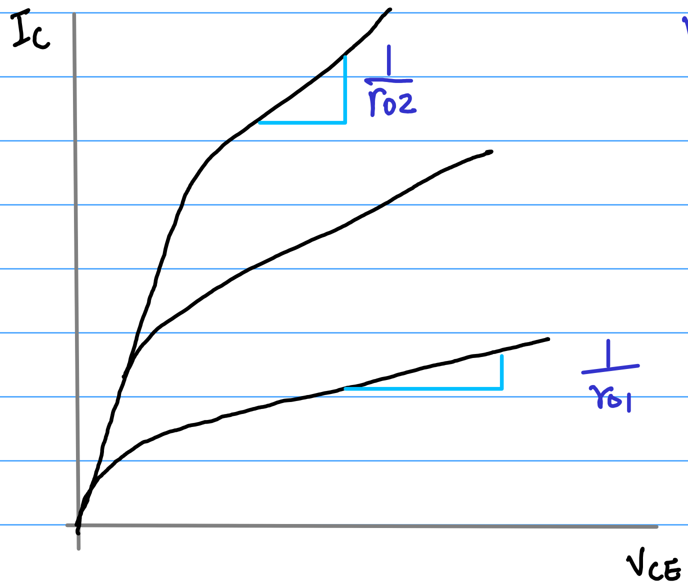
Z_o involves I_c, I_e

→ use the (t) model

Z_i involves I_b

→ use the (π) model





$r_{o1} > r_{o2}$

$\frac{r_{o1} + r_{o2}}{2}$ mean

r_o

$$I = I_S \left(e^{\frac{V_D}{n V_T}} - 1 \right)$$

I is the diode current,

I_S is the reverse bias saturation current (or scale current),

V_D is the voltage across the diode,

V_T is the thermal voltage, and

n is the *ideality factor*, also known as the *quality factor* or sometimes *emission coefficient*. The ideality factor *n* typically varies from 1 to 2 (though can in some cases be higher), depending on the fabrication process and semiconductor material and is set equal to 1 for the case of an "ideal" diode (thus the n is sometimes omitted). The ideality factor was added to account for imperfect junctions as observed in real transistors. The factor mainly accounts for carrier recombination as the charge carriers cross the depletion region.

https://en.wikipedia.org/wiki/Diode

The thermal voltage V_T is approximately 25.85 mV at 300 K, a temperature close to "room temperature" commonly used in device simulation software. At any temperature it is a known constant defined by:

$$V_{\mathrm{T}} = \frac{kT}{q},$$

where k is the Boltzmann constant, T is the absolute temperature of the p–n junction, and q is the magnitude of charge of an electron (the elementary charge).

The reverse saturation current, I_S, is not constant for a given device, but varies with temperature; usually more significantly than V_T, so that V_D typically decreases as T increases.

https://en.wikipedia.org/wiki/Diode

```
(%i1) Is : 1.36 * 10^-15;
```

(%o1) $1.36\ 10^{-15}$

```
(%i2) Vt : 25.85 * 10^-3;
```

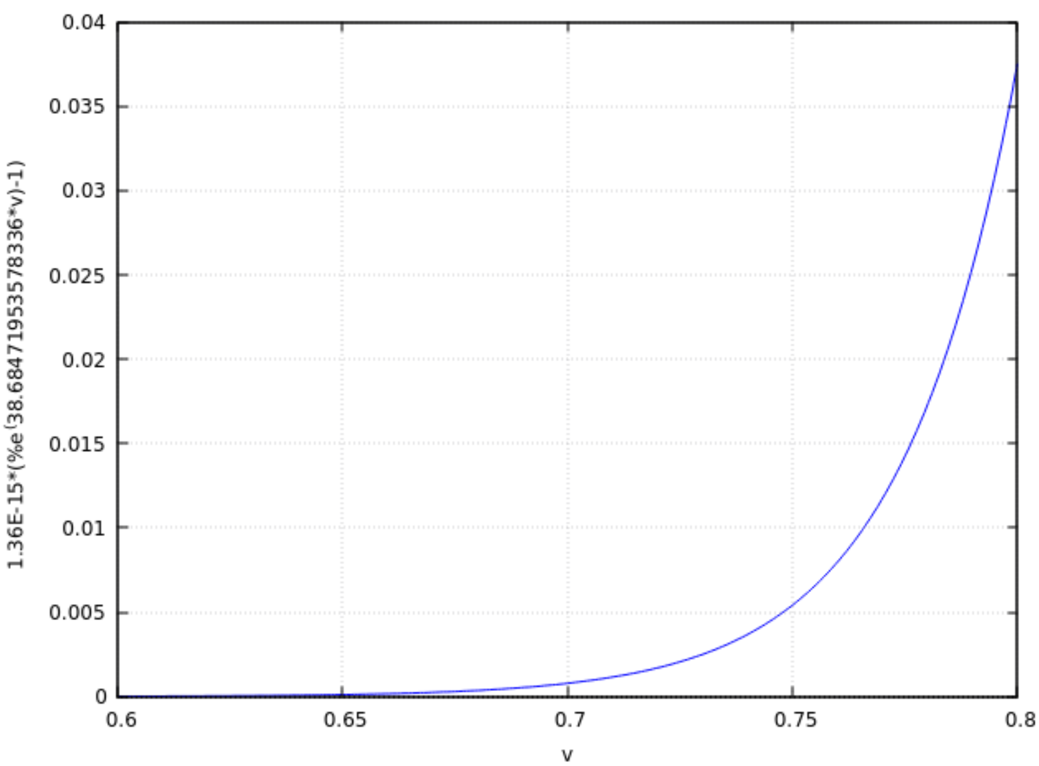
(%o2) 0.02585

```
(%i5) I(v) := Is * (%e^(v/Vt) - 1);
```

(%o5) $\mathrm{I}(v) := Is\left(\%e^{v/Vt} - 1\right)$

```
-->
```

```
(%i7) plot2d([I(v)], [v,0.6,0.8],
       [plot_format, gnuplot],
       [gnuplot_postamble, "set grid;"])$
```



$$I = I_S \left(e^{\frac{v}{V_T}} - 1 \right) \cong I_S \, e^{\frac{v}{V_T}}$$

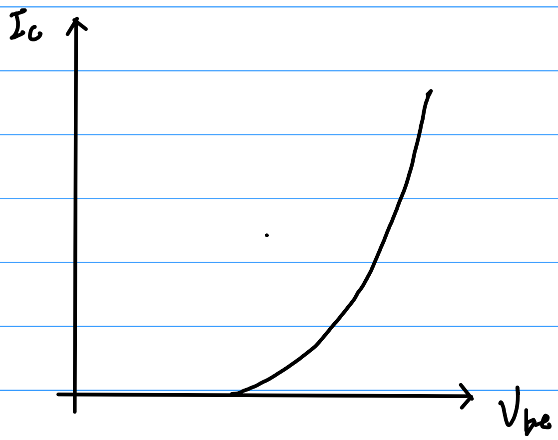
$$\frac{dI}{dv} = \frac{I_S}{V_T} \cdot e^{\frac{v}{V_T}} = \frac{1}{V_T} I$$

v : V_{be}

I : I_c

$$\frac{1}{r'_e} = \frac{I_c}{25.85\,mV}$$

$$r'_e = \frac{25.85\,mV}{I_c}$$

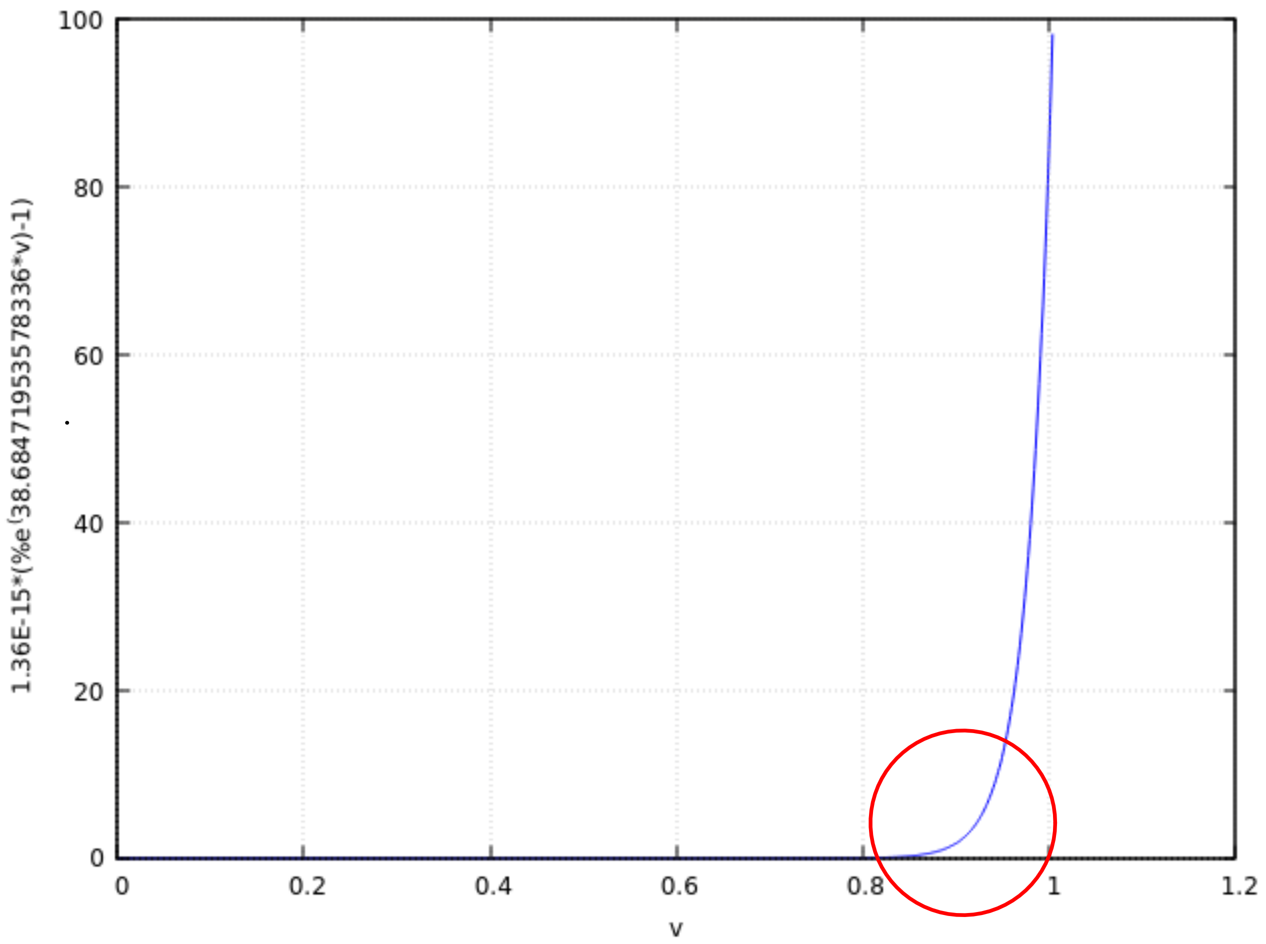


$$\frac{25\,mV}{I_c = 1\,mA} = 25\,\Omega$$

$$\frac{25\,mV}{I_c = 2\,mA} = 12.5\,\Omega$$

$$\frac{25\,mV}{I_c = \frac{1}{2}\,mA} = 50\,\Omega$$

http://users.physics.harvard.edu/~horowitz/aoe/sm/smlitlre.htm



in this small voltage range
the slope of a derivative
changes rapidly

slope = 0 ~ slope=infinity