Boolean Algebra (8A)

Young Won Lim
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Please send corrections (or suggestions) to youngwlim@hotmail.com.

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Argument

Boolean Algebra

In mathematics and mathematical logic, **Boolean algebra** is the branch of algebra in which the values of the variables are the truth values *true* and *false*, usually denoted 1 and 0 respectively. Instead of elementary algebra where the values of the variables are numbers, and the prime operations are addition and multiplication, the main operations of Boolean algebra are the conjunction *and* denoted as ∧, the disjunction *or* denoted as ∨, and the negation *not* denoted as ¬. It is thus a formalism for describing logical relations in the same way that ordinary algebra describes numeric relations.

https://en.wikipedia.org/wiki/Boolean_algebra

Operators

x	y	$x \wedge y$	$x \vee y$
0	**0**	0	0
1	**0**	0	1
0	**1**	0	1
1	**1**	1	1

x	$\neg x$
0	1
1	0

x	y	$x \rightarrow y$	$x \oplus y$	$x \equiv y$
0	**0**	1	0	1
1	**0**	0	1	0
0	**1**	1	1	0
1	**1**	1	0	1

https://en.wikipedia.org/wiki/Boolean_algebra

Laws (1)

Associativity of $\vee$:	$x \vee (y \vee z) = (x \vee y) \vee z$	x+(y+z) = (x+y)+z
Associativity of $\wedge$:	$x \wedge (y \wedge z) = (x \wedge y) \wedge z$	x(yz) = (xy)z
Commutativity of $\vee$:	$x \vee y = y \vee x$	x+y = y+x
Commutativity of $\wedge$:	$x \wedge y = y \wedge x$	xy = yz
Distributivity of $\wedge$ over $\vee$:	$x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$	x(y+z) = xy + xz
Identity for $\vee$:	$x \vee 0 = x$	x+0=x
Identity for $\wedge$:	$x \wedge 1 = x$	x*1=x
Annihilator for $\wedge$:	$x \wedge 0 = 0$	x*0=0

https://en.wikipedia.org/wiki/Boolean_algebra

Laws (2)

Annihilator for $\vee$:	$x \vee 1 = 1$	x+1=1
Idempotence of $\vee$:	$x \vee x = x$	x+x=x
Idempotence of $\wedge$:	$x \wedge x = x$	x*x=x
Absorption 1:	$x \wedge (x \vee y) = x$	x(x+y)=x
Absorption 2:	$x \vee (x \wedge y) = x$	x+xy=x
Distributivity of $\vee$ over $\wedge$:	$x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$	x+yz=(x+y)(x+z)

Complementation 1	$x \wedge \neg x = 0$	$x\,\overline{x} = 0$
Complementation 2	$x \vee \neg x = 1$	$x + \overline{x} = 1$

De Morgan 1	$\neg x \wedge \neg y = \neg(x \vee y)$	$\overline{x}\,\overline{y} = \overline{(x+y)}$
De Morgan 2	$\neg x \vee \neg y = \neg(x \wedge y)$	$\overline{x} + \overline{y} = \overline{(x\,y)}$

https://en.wikipedia.org/wiki/Boolean_algebra

Digital Logic Gates

Digital logic is the application of the Boolean algebra of 0 and 1 to electronic hardware consisting of logic gates connected to form a circuit diagram. Each gate implements a Boolean operation, and is depicted schematically by a shape indicating the operation. The shapes associated with the gates for conjunction (AND-gates), disjunction (OR-gates), and complement (inverters) are as follows.[17]

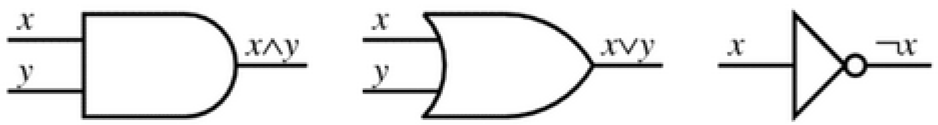
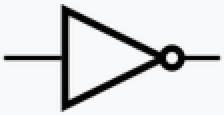
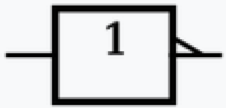


Figure 3. Logic gates

The lines on the left of each gate represent input wires or *ports*. The value of the input is represented by a voltage on the lead. For so-called "active-high" logic, 0 is represented by a voltage close to zero or "ground", while 1 is represented by a voltage close to the supply voltage; active-low reverses this. The line on the right of each gate represents the output port, which normally follows the same voltage conventions as the input ports.

https://en.wikipedia.org/wiki/Boolean_algebra

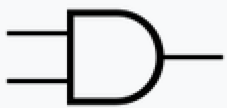
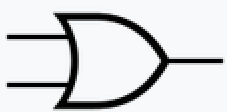
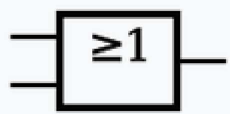
NOT Gate

Negation				
NOT			$\overline{A}$ or $\sim A$	see truth table below

INPUT	OUTPUT
A	NOT A
0	1
1	0

https://en.wikipedia.org/wiki/Logic_gate

AND, OR Gates

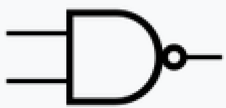
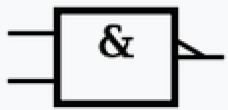
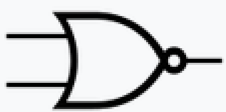
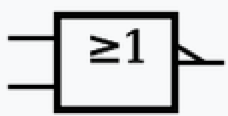
Conjunction and Disjunction				
AND			$A \cdot B$	see AND truth table below
OR			$A + B$	see OR truth table below

INPUT		OUTPUT
A	B	A AND B
0	0	0
0	1	0
1	0	0
1	1	1

INPUT		OUTPUT
A	B	A OR B
0	0	0
0	1	1
1	0	1
1	1	1

https://en.wikipedia.org/wiki/Logic_gate

NAND, NOR Gates

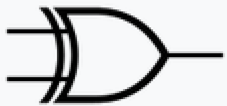
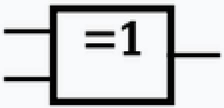
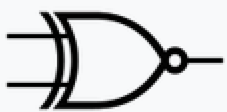
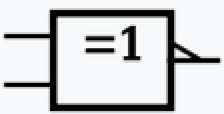
Alternative denial and Joint denial				
NAND		 	$\overline{A \cdot B}$ or $A \uparrow B$	see NAND truth table
NOR		 	$\overline{A + B}$ or $A \downarrow B$	see NOR truth table

INPUT		**OUTPUT**
A	B	A NAND B
0	0	1
0	1	1
1	0	1
1	1	0

INPUT		**OUTPUT**
A	B	A NOR B
0	0	1
0	1	0
1	0	0
1	1	0

https://en.wikipedia.org/wiki/Logic_gate

XOR, XNOR Gates

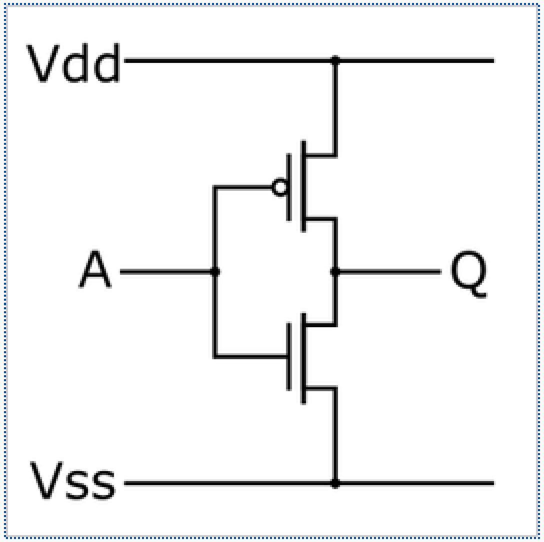
Exclusive or and Biconditional			
XOR			$A \oplus B$
XNOR			$\overline{A \oplus B}$ or $A \odot B$

INPUT		OUTPUT
A	B	A XOR B
0	0	0
0	1	1
1	0	1
1	1	0

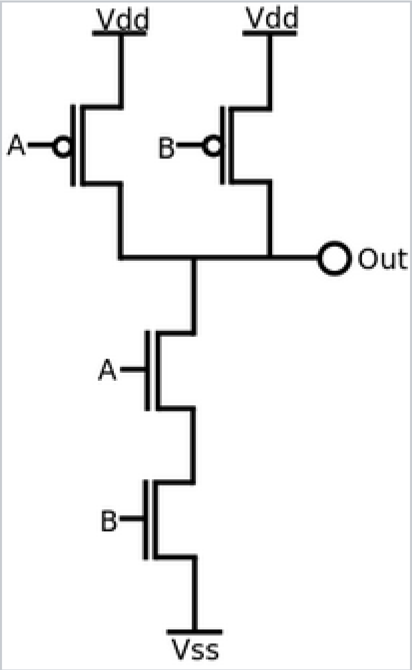
INPUT		OUTPUT
A	B	A XNOR B
0	0	1
0	1	0
1	0	0
1	1	1

https://en.wikipedia.org/wiki/Logic_gate

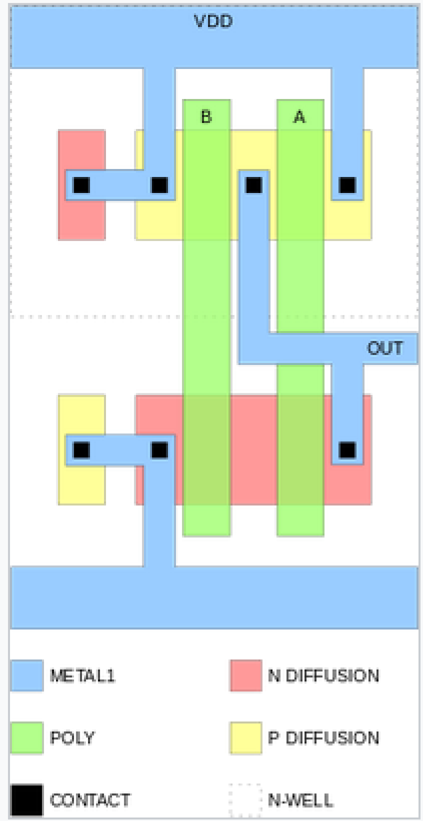
CMOS Logic Gates



CMOS inverter (NOT logic gate)

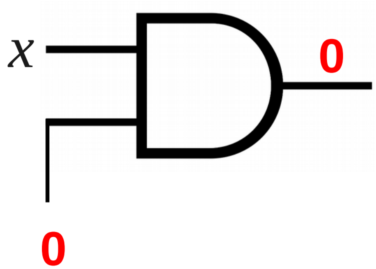


NAND gate in CMOS logic

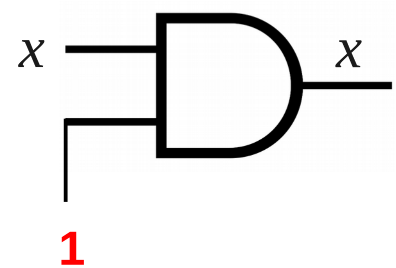


https://en.wikipedia.org/wiki/CMOS

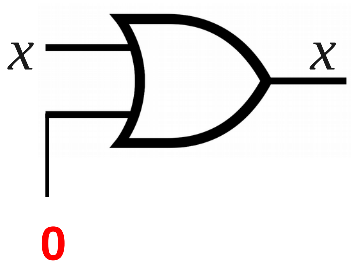
Identity and Null Element Theorem



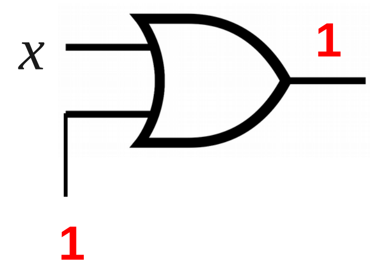
$$x \cdot 0 = 0$$



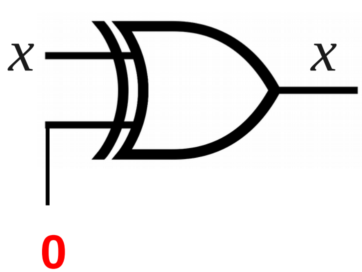
$$x \cdot 1 = x$$



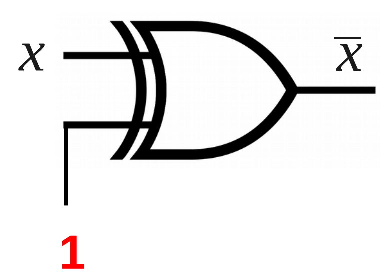
$$x + 0 = x$$



$$x + 1 = 1$$



$$x \oplus 0 = x$$



$$x \oplus 1 = \bar{x}$$

https://en.wikiversity.org/wiki/Discrete_Mathematics_in_plain_view#Algorithms

Distributive

$$x \cdot (y + z) = x \cdot y + x \cdot z \neq x \cdot y + z$$

This parenthesis cannot be deleted

$$x + (y \cdot z) = (x + y) \cdot (x + z) = x + y \cdot z$$

This parenthesis can be deleted

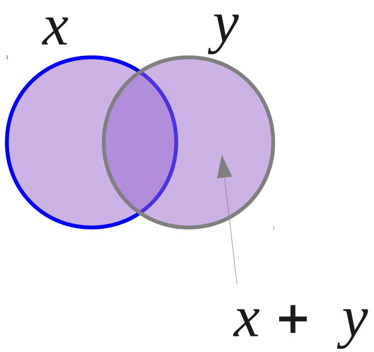
Operator precedence : $\cdot > +$

https://en.wikiversity.org/wiki/Discrete_Mathematics_in_plain_view#Algorithms

Inclusion

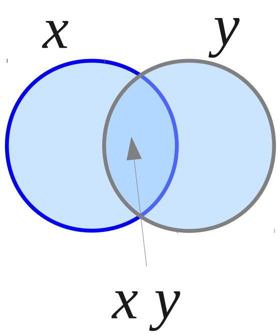
$$x \cdot (x + y) = x$$

$$x \cdot (x + y) = x \cdot x + x \cdot y$$
$$= x + x \cdot y$$
$$= x \cdot (1 + y)$$
$$= x$$



$$x + xy = x$$

$$x + xy = x \cdot 1 + x \cdot y$$
$$= x \cdot (1 + y)$$
$$= x$$

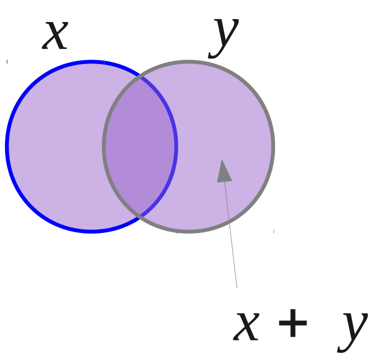


https://en.wikiversity.org/wiki/Discrete_Mathematics_in_plain_view#Algorithms

Inclusion

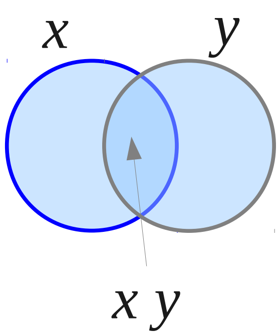
$$x \cdot (x + y) = x$$

$$x \cdot (x + y) = x \cdot x + x \cdot y$$
$$= x + x \cdot y$$
$$= x \cdot (1 + y)$$
$$= x$$



$$x + xy = x$$

$$x + xy = x \cdot 1 + x \cdot y$$
$$= x \cdot (1 + y)$$
$$= x$$

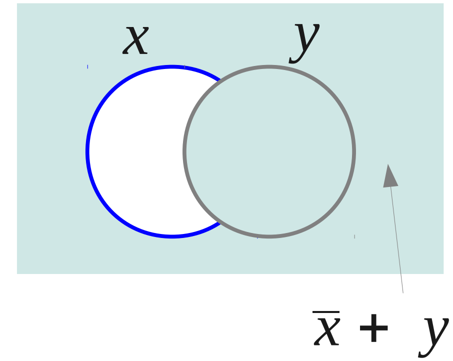


https://en.wikiversity.org/wiki/Discrete_Mathematics_in_plain_view#Algorithms

Eliminate

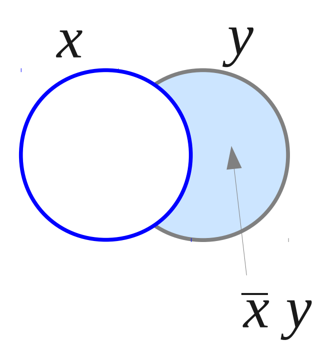
$$x \cdot (\overline{x} + y) = x\,y$$

$$x \cdot (\overline{x} + y) = x \cdot \overline{x} + x \cdot y$$
$$= 0 + x \cdot y$$
$$= x \cdot y$$



$$x + \overline{x}\,y = x + y$$

$$x + \overline{x}\,y = (x + \overline{x}) \cdot (x + y)$$
$$= 1 \cdot (x + y)$$
$$= x + y$$



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References

[1] http://en.wikipedia.org/
[2]