Complex Trig & TrigH (H.1)

20160730

real x

$$e^{ix} = \cos x + i \sin x$$

$$e^{-ix} = \cos x - i \sin x$$

$$\sin x = \frac{e^{ix} - e^{-ix}}{2i}$$

$$\cos x = \frac{e^{ix} + e^{-ix}}{2}$$

complex $z = x + iy$

$$\sin z = \frac{e^{iz} - e^{-iz}}{2i}$$

$$\cos z = \frac{e^{iz} + e^{-iz}}{2}$$

$$\tan z = \frac{\sin z}{\cos z}$$

$$\cot z = \frac{\cos z}{\sin z}$$

$$\sec z = \frac{1}{\cos z}$$

$$\csc z = \frac{1}{\sin z}$$

Analyticity

e^{iz}, e^{-iz} entire function

$$\sin z = \frac{e^{iz} - e^{-iz}}{2i}$$ entire function

$$\cos z = \frac{e^{iz} + e^{-iz}}{2}$$ entire function

$\sin z = 0$ only for real numbers $z = n\pi$

$\cos z = 0$ only for real numbers $z = (2n+1)\pi/2$

$\tan z = \dfrac{\sin z}{\cos z}$ $\quad \sec z = \dfrac{1}{\cos z}$ analytic except $z = (2n+1)\pi/2$

$\cot z = \dfrac{\cos z}{\sin z}$ $\quad \csc z = \dfrac{1}{\sin z}$ analytic except $z = n\pi$

$$\sin z = \frac{e^{iz} - e^{-iz}}{2i}$$

$$\frac{d}{dz}\sin z = \frac{ie^{iz} + ie^{-iz}}{2i} = \cos z$$

$$\cos z = \frac{e^{iz} + e^{-iz}}{2}$$

$$\frac{d}{dz}\cos z = \frac{ie^{iz} - ie^{-iz}}{2} = -\sin z$$

$$\tan z = \frac{\sin z}{\cos z}$$

$$\frac{d}{dz}\tan z = \frac{\cos^2 z + \sin^2 z}{\cos^2 z} = \sec^2 z$$

$$\cot z = \frac{\cos z}{\sin z}$$

$$\frac{d}{dz}\cot z = \frac{-\sin^2 z - \cos^2 z}{\sin^2 z} = -\csc^2 z$$

$$\sec z = \frac{1}{\cos z}$$

$$\frac{d}{dz}\sec z = \frac{\sin z}{\cos^2 z} = \sec z \tan z$$

$$\csc z = \frac{1}{\sin z}$$

$$\frac{d}{dz}\csc z = \frac{-\cos z}{\sin^2 z} = -\csc z \cot z$$

$$\sin z = \frac{e^{iz} - e^{-iz}}{2i}$$

$$\sin(-z) = \frac{e^{-iz} - e^{+iz}}{2i} = -\sin z$$

$$\cos z = \frac{e^{iz} + e^{-iz}}{2}$$

$$\cos(-z) = \frac{e^{-iz} + e^{+iz}}{2} = \cos z$$

$$\tan z = \frac{\sin z}{\cos z}$$

$$\tan(-z) = \frac{-\sin z}{\cos z} = -\tan z$$

$$\sin^2 z = \frac{e^{i2z} + e^{-i2z} - 2}{-4}$$

$$\cos^2 z = \frac{e^{i2z} + e^{-i2z} + 2}{+4}$$

$$\sin^2 z + \cos^2 z = 1$$

$$\cos(z_1+z_2) + i\sin(z_1+z_2) = e^{i(z_1+z_2)}$$

$$e^{iz_1} \cdot e^{iz_2}$$

$$= [\cos(z_1) + i\sin(z_1)][\cos(z_2) + i\sin(z_2)]$$

$$= [\cos(z_1)\cos(z_2) - \sin(z_1)\sin(z_2)] + i[\cos(z_1)\sin(z_2) + \sin(z_1)\cos(z_2)]$$

$$\cos(z_1 \pm z_2) = [\cos(z_1)\cos(z_2) \mp \sin(z_1)\sin(z_2)]$$

$$\sin(z_1 \pm z_2) = [\sin(z_1)\cos(z_2) \pm \cos(z_1)\sin(z_2)]$$

$$\sin(z+z) = \sin(z)\cos(z) + \cos(z)\sin(z)$$

$$\sin(2z) = 2\sin(z)\cos(z)$$

$$\cos(z+z) = \cos(z)\cos(z) - \sin(z)\sin(z)$$

$$\cos(2z) = \cos^2(z) - \sin^2(z)$$

$$\sinh y = \frac{e^{y} - e^{-y}}{2}$$

$$\cosh y = \frac{e^{y} + e^{-y}}{2}$$

$$\sin(z) = \sin(x+iy) = \frac{e^{i(x+iy)} - e^{-i(x+iy)}}{2i}$$

$$2 * \left[\boxed{e^{ix} e^{-y}} - \boxed{e^{-ix} e^{y}} \right]$$

$$= \left[\begin{array}{l} \boxed{e^{ix} e^{y}} + \boxed{e^{ix} e^{-y}} \\ - \boxed{e^{-ix} e^{y}} - \boxed{e^{-ix} e^{-y}} \end{array} \right] - \left[\begin{array}{l} \boxed{e^{ix} e^{y}} - \boxed{e^{ix} e^{-y}} \\ \boxed{e^{-ix} e^{y}} - \boxed{e^{-ix} e^{-y}} \end{array} \right]$$

$$= (e^{ix} - e^{-ix})(e^{y} + e^{-y}) - (e^{ix} + e^{-ix})(e^{y} - e^{-y})$$

$$\sin(x+iy) = \frac{(e^{ix} - e^{-ix})}{2i} \frac{(e^{y} + e^{-y})}{2} - \frac{(e^{ix} + e^{-ix})}{2i} \frac{(e^{y} - e^{-y})}{2}$$

$$\boxed{\sin(x+iy) = \sin(x) \cosh(y) + i \cos(x) \sinh(y)}$$

$$\sinh y = \frac{e^{y} - e^{-y}}{2}$$

$$\cosh y = \frac{e^{y} + e^{-y}}{2}$$

$$\cos(z) = \cos(x+iy) = \frac{e^{i(x+iy)} + e^{-i(x+iy)}}{2}$$

$$2 * \left[\boxed{e^{ix} e^{-y}} + \boxed{e^{-ix} e^{y}} \right]$$

$$= \left[\begin{array}{l} \boxed{e^{ix} e^{y}} + \boxed{e^{ix} e^{-y}} \\ \boxed{e^{-ix} e^{y}} + \boxed{e^{-ix} e^{-y}} \end{array} \right] - \left[\begin{array}{l} \boxed{e^{ix} e^{y}} - \boxed{e^{ix} e^{-y}} \\ - \boxed{e^{-ix} e^{y}} + \boxed{e^{-ix} e^{-y}} \end{array} \right]$$

$$= (e^{ix} + e^{-ix})(e^{y} + e^{-y}) - (e^{ix} - e^{-ix})(e^{y} - e^{-y})$$

$$\cos(x+iy) = \frac{(e^{ix} + e^{-ix})}{2} \frac{(e^{y} + e^{-y})}{2} - \frac{(e^{ix} - e^{-ix})}{2} \frac{(e^{y} - e^{-y})}{2}$$

$$\boxed{\cos(x+iy) = \cos(x) \cosh(y) - i \sin(x) \sinh(y)}$$

$$\sinh y = \frac{e^y - e^{-y}}{2} \qquad \sinh^2 y = \frac{1}{4}\left(e^{2y} + e^{-2y} - 2\right)$$

$$\cosh y = \frac{e^y + e^{-y}}{2} \qquad \cosh^2 y = \frac{1}{4}\left(e^{2y} + e^{-2y} + 2\right)$$

$$\cosh^2 y - \sinh^2 y = 1$$

$$\sin(x+iy) = \sin(x)\cosh(y) + i\cos(x)\sinh(y)$$

$$|\sin(x+iy)|^2 = \underline{\sin^2(x)\cosh^2(y)} + \underline{\cos^2(x)\sinh^2(y)}$$

$$(1-\sin^2(x))(\cosh^2 y - 1)$$

$$\cosh^2 y - \sin^2(x)\cosh^2 y - 1 + \sin^2(x)$$

$$\underline{-\sin^2(x)\cosh^2 y} + \cosh^2 y - 1 + \sin^2(x)$$

$$|\sin z|^2 = \sin^2(x) + \sinh^2(y)$$

$$\cos(x+iy) = \cos(x)\cosh(y) - i\sin(x)\sinh(y)$$

$$|\cos(x+iy)|^2 = \underline{\cos^2(x)\cosh^2(y)} + \sin^2(x)\sinh^2(y)$$

$$(1-\cos^2(x))(\cosh^2 y - 1)$$

$$\cosh^2 y - \cos^2(x)\cosh^2 y - 1 + \cos^2(x)$$

$$\underline{-\cos^2(x)\cosh^2 y} + \cosh^2 y - 1 + \cos^2(x)$$

$$|\cos z|^2 = \cos^2(x) + \sinh^2(y)$$

$$\sin(x+iy) = \sin(x)\boxed{\cosh(y)} + i\cos(x)\boxed{\sinh(y)}$$

$$\cos(x+iy) = \cos(x)\boxed{\cosh(y)} - i\sin(x)\boxed{\sinh(y)}$$

$$|\sin z|^2 = \sin^2(x) + \boxed{\sinh^2(y)}$$

$$|\cos z|^2 = \cos^2(x) + \boxed{\sinh^2(y)}$$

a complex number $z = 0 \rightleftarrows |z|^2 = 0$

$$|\sin z|^2 = \sin^2(x) + \boxed{\sinh^2(y)}$$

$$|\cos z|^2 = \cos^2(x) + \boxed{\sinh^2(y)}$$

$$\sin z = 0 \quad \rightleftarrows \quad \sin^2(x) + \sinh^2(y) = 0$$

$$\sin(x) = 0 \qquad x = n\pi$$

$$\sinh(y) = 0 \qquad y = 0$$

zero $\quad z = n\pi + i \cdot 0 = n\pi, \quad n = 0, \pm 1, \pm 2, \cdots$

$$\cos z \quad \rightleftarrows \quad \cos^2(x) + \sinh^2(y)$$

$$\cos(x) = 0 \qquad x = (2n+1)\pi$$

$$\sinh(y) = 0 \qquad y = 0$$

zero $\quad z = (2n+1)\pi + i \cdot 0 = n\pi, \quad n = 0, \pm 1, \pm 2, \cdots$

for a complex number $z = x + iy$

$$\sinh z = \frac{e^{z} - e^{-z}}{2}$$

$$\sinh y = \frac{e^{y} - e^{-y}}{2}$$

$$\cosh z = \frac{e^{z} + e^{-z}}{2}$$

$$\cosh y = \frac{e^{y} + e^{-y}}{2}$$

$$\tanh z = \frac{\sinh z}{\cosh z}$$

$$\coth z = \frac{\cosh z}{\sinh z}$$

$$\operatorname{sech} z = \frac{1}{\cosh z}$$

$$\operatorname{csch} z = \frac{1}{\sinh z}$$

$$\sinh z = \frac{e^{z} - e^{-z}}{2} \qquad \frac{d}{dz}\sinh z = \frac{e^{z} + e^{-z}}{2} = \cosh z$$

$$\cosh z = \frac{e^{z} + e^{-z}}{2} \qquad \frac{d}{dz}\cosh z = \frac{e^{z} - e^{-z}}{2} = \sinh z$$

$$\tanh z = \frac{\sinh z}{\cosh z}$$

$$\frac{d}{dz}\tanh z = \frac{\cosh^2 z - \sinh^2 z}{\cosh^2 z} = \operatorname{sech}^2 z$$

$$\coth z = \frac{\cosh z}{\sinh z}$$

$$\frac{d}{dz}\coth z = \frac{\sinh^2 z - \cosh^2 z}{\sinh^2 z} = -\operatorname{csch}^2 z$$

$$\operatorname{sech} z = \frac{1}{\cosh z}$$

$$\frac{d}{dz}\operatorname{sech} z = \frac{-\sinh z}{\cosh^2 z} = -\tanh z \operatorname{sech} z$$

$$\operatorname{csch} z = \frac{1}{\sinh z}$$

$$\frac{d}{dz}\operatorname{csch} z = \frac{-\cosh z}{\sinh^2 z} = -\coth z \operatorname{csch} z$$

$$\sinh z = \frac{e^{z} - e^{-z}}{2}$$

$$\cosh z = \frac{e^{z} + e^{-z}}{2}$$

$$\sinh iz = \frac{e^{iz} - e^{-iz}}{2} = i \sin z$$

$$\cosh iz = \frac{e^{iz} + e^{-iz}}{2} = \cos z$$

$\sinh iz = i \sin z$ $\qquad$ $\sin z = -i \sinh iz$

$\cosh iz = \cos z$ $\qquad$ $\cos z = \cosh iz$

$$\sin z = \frac{e^{iz} - e^{-iz}}{2i}$$

$$\cos z = \frac{e^{iz} + e^{-iz}}{2}$$

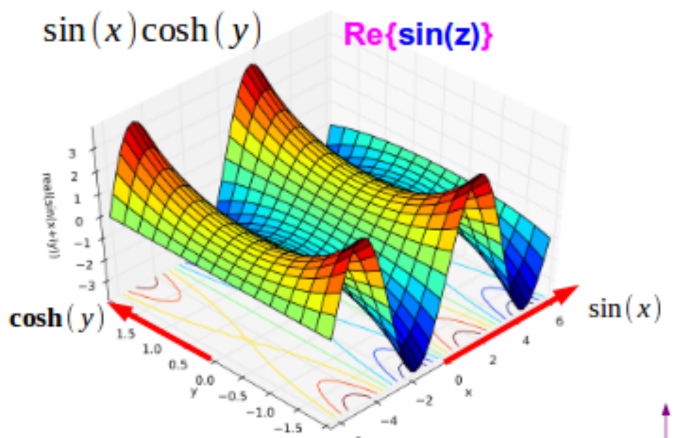
$$\sin iz = \frac{e^{-z} - e^{z}}{2i} = -\frac{1}{i} \sinh z$$

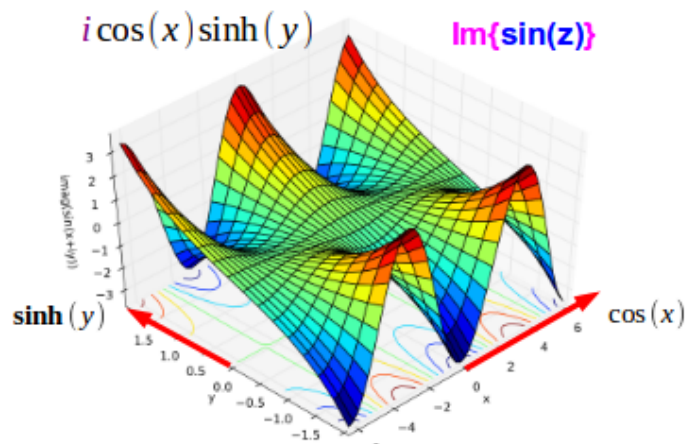
$$\cos iz = \frac{e^{-z} + e^{z}}{2} = \cosh z$$

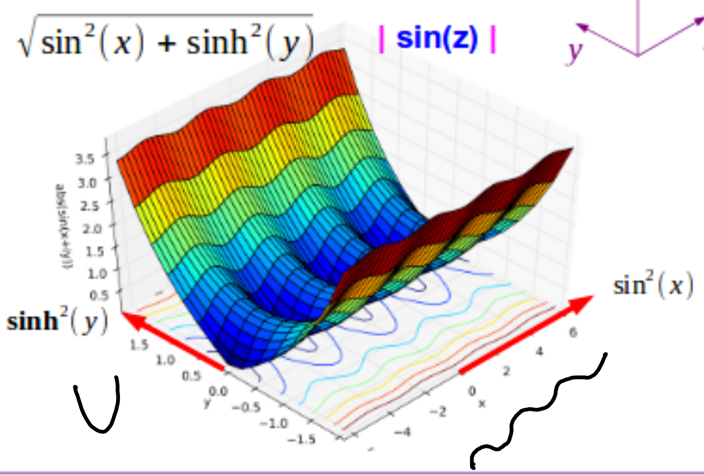
$$\sin iz = i \sinh z \qquad \sinh z = -i \sin iz$$

$$\cos iz = \cosh z \qquad \cosh z = \cos iz$$

Graphs of sin(z)







$$\sin(z) = \sin(x + i\,y)$$

$$= \sin(x)\cosh(y) + i\cos(x)\sinh(y)$$

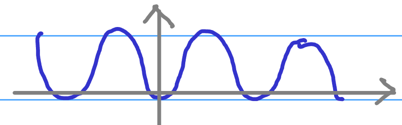
$$|\sin(z)|^2 = \sin^2(x) + \sinh^2(y)$$

http://en.wikipedia.org/

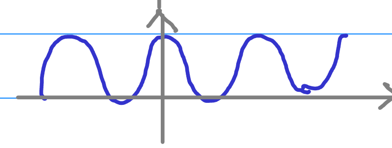
$$\sinh^2(y) = \frac{1}{4}\left(e^{+2y} + e^{-2y} - 2\right)$$

$$\tan\theta = \frac{\cos(x)\sinh(y)}{\sin(x)\cosh(y)} = \cot x \tanh y$$

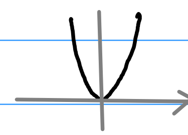
$\sin^2 x = \frac{1}{2}(1-\cos(2x))$



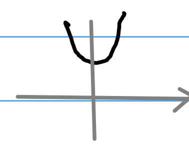
$\cos^2 x = \frac{1}{2}(1+\cos(2x))$



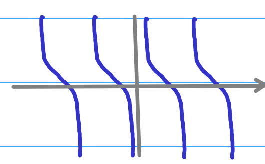
$\sinh^2(y) = \frac{1}{4}(e^{+2y}+e^{-2y}-2)$



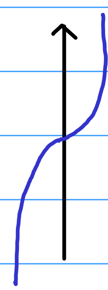
$\cosh^2(y) = \frac{1}{4}(e^{+2y}+e^{-2y}+2)$

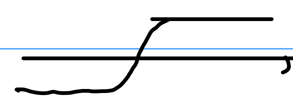


$\cot(x) =$



$\tanh(y)$



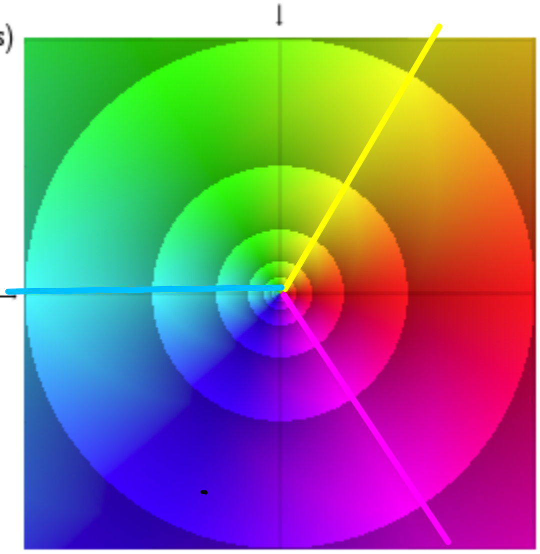


Domain Coloring

hue to phase/angle/argument legend:

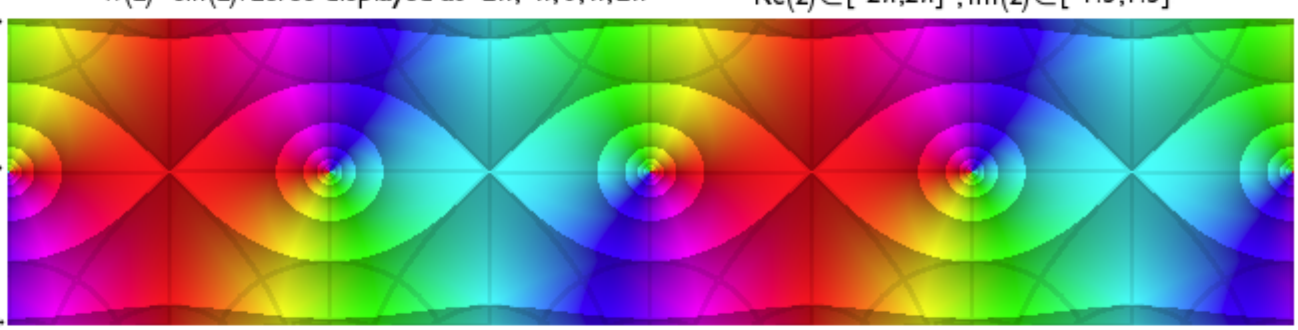
hue	phase (radians)
red	$0 \bmod 2\pi$
yellow	$\pi/3 \bmod 2\pi$
green	$2\pi/3 \bmod 2\pi$
cyan	$\pi \bmod 2\pi$
blue	$4\pi/3 \bmod 2\pi$
magenta	$5\pi/3 \bmod 2\pi$

The Unit Circle

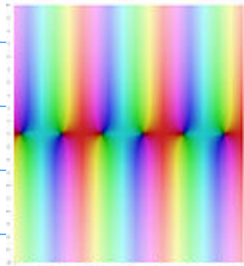


Each discontinuity in intensity occurs when $|z|=2^n$, for integer n (0,-1,-2,..)

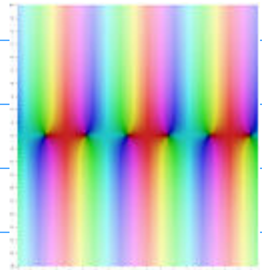
w(z)=sin(z). zeros displayed at -2π, -π, 0, π, 2π   Re(z)∈[-2π,2π] , Im(z)∈[-1.5,1.5]



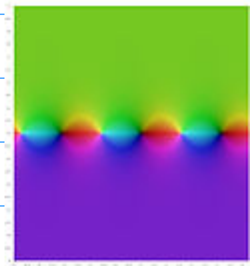
sin z



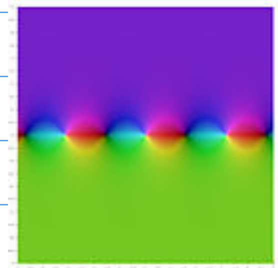
cos z



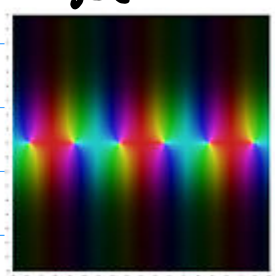
tan z



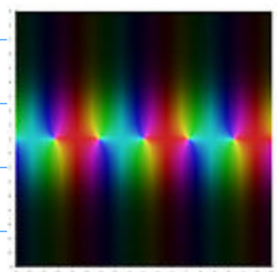
cot z



sec z



csc z



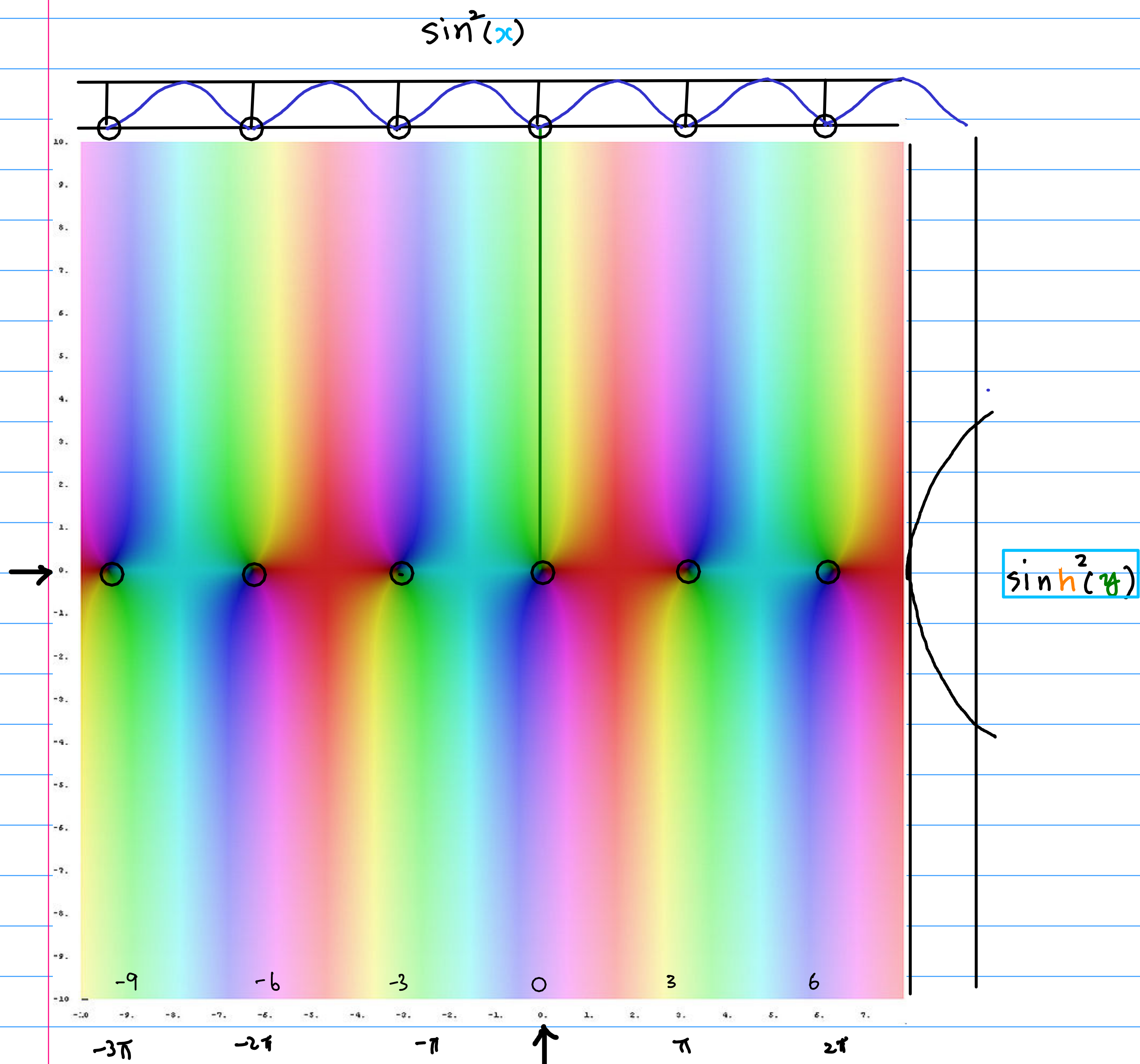
https://en.wikipedia.org/wiki/Trigonometric_functions

$|\sin z|$ brightness

$$\sin(x+iy) = \sin(x)\boxed{\cosh(y)} + i\cos(x)\boxed{\sinh(y)}$$

$$|\sin z|^2 = \sin^2(x) + \boxed{\sinh^2(y)}$$

$$\tan\theta = \cot(x)\tanh(y)$$



$\arg(\sin z)$

$\sin(x+iy) = \sin(x)\boxed{\cosh(y)} + i\cos(x)\boxed{\sinh(y)}$

$|\sin z|^2 = \sin^2(x) + \boxed{\sinh^2(y)}$

$\tan\theta = \cot(x)\tanh(y)$

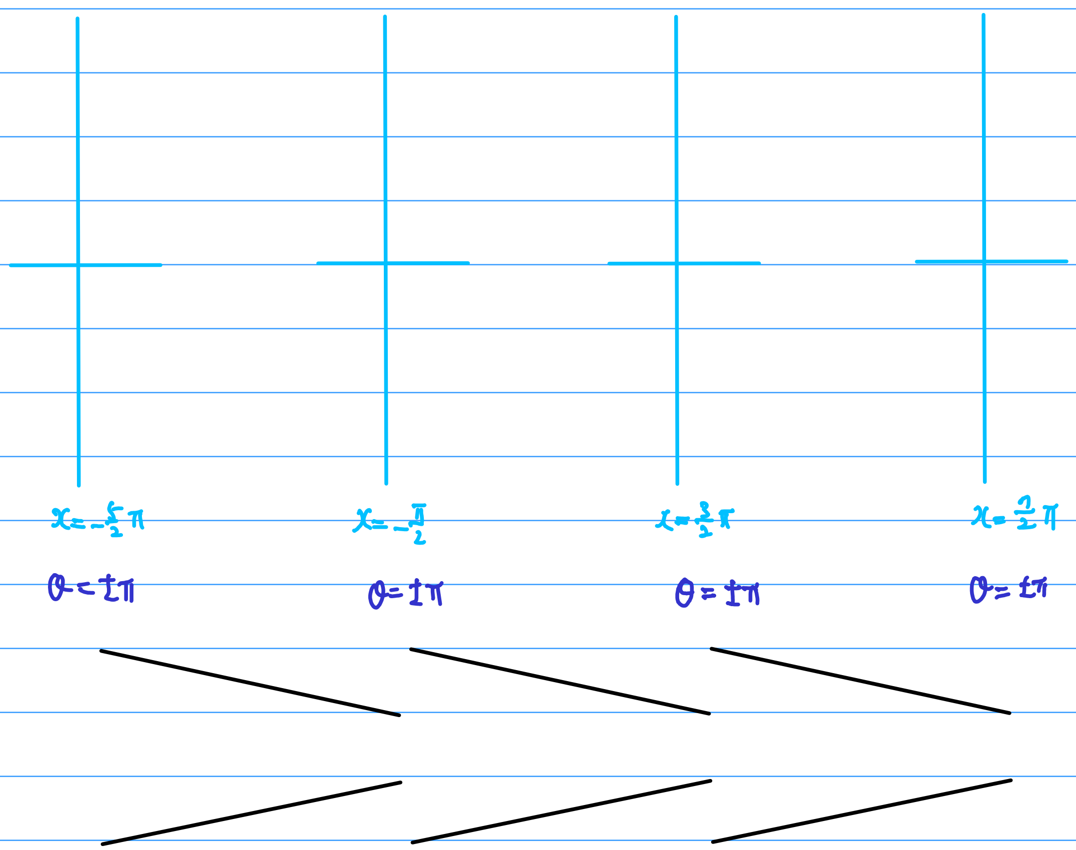
$\cot(x) = \pm\infty$ $\quad x = 0, \pm\pi, \pm 2\pi, \cdots$

$\tan\theta = \pm\infty$ $\quad \theta = \pm\frac{\pi}{2}, \pm\frac{3}{2}\pi, \cdots$

$\cot(x) = 0$ $\quad x = \pm\frac{\pi}{2}, \pm\frac{3}{2}\pi, \cdots$

$\tan\theta = 0$ $\quad \theta = 0, \pm\pi, \pm 2\pi, \cdots$

discontinuity $\quad x = \cdots -\frac{5}{2}\pi, -\frac{\pi}{2}, +\frac{3}{2}\pi, +\frac{7}{2}\pi \cdots$



https://en.wikipedia.org/wiki/Trigonometric_functions

$\arg(\sin z)$

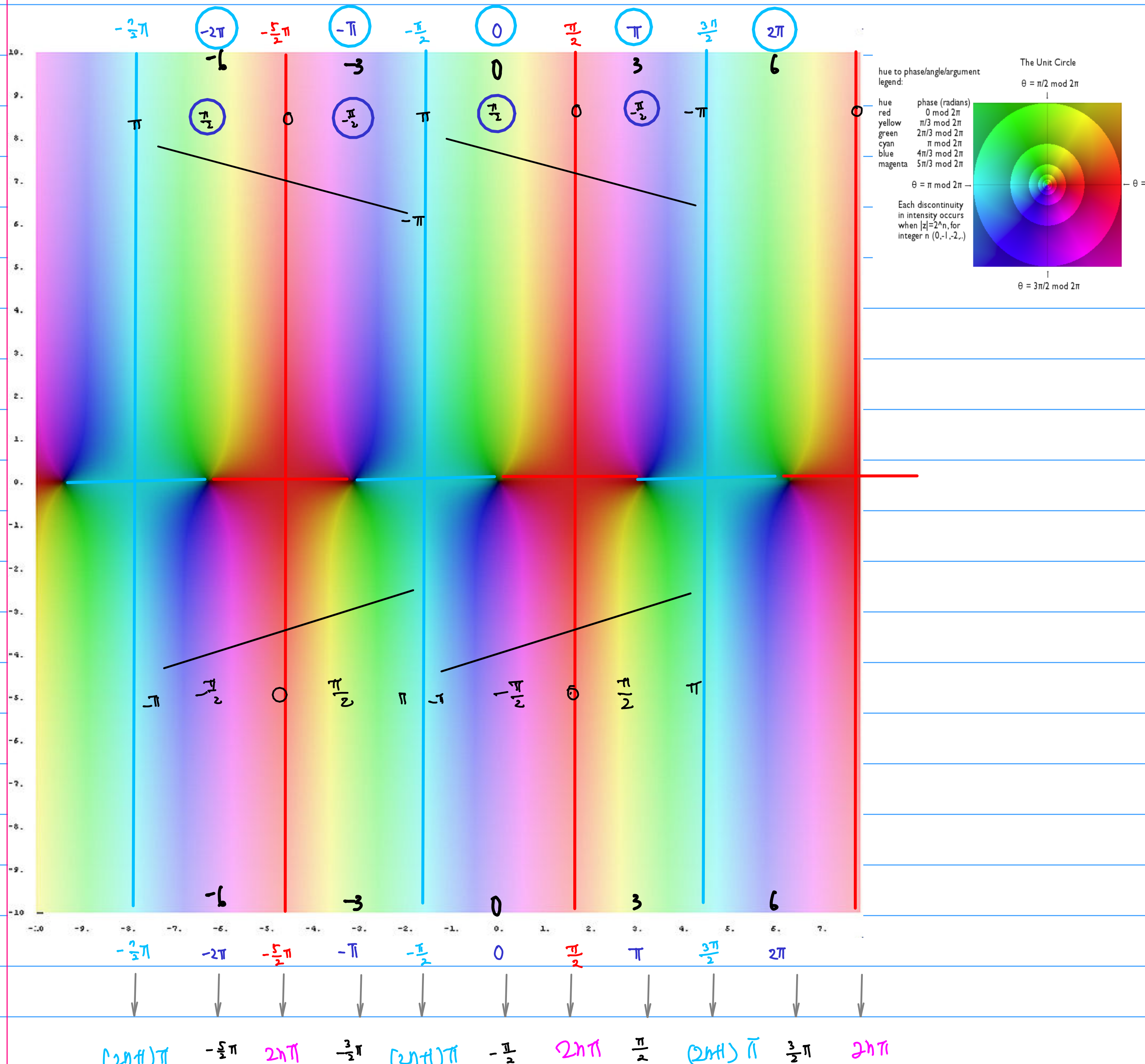
$$\tan\theta = \cot(x)\tanh(y)$$

$\cot(x) = \pm\infty \qquad x = 0, \pm\pi, \pm 2\pi, \cdots$

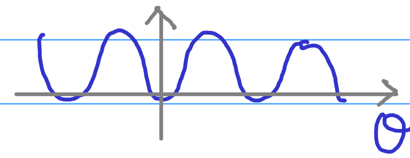
$\tan\theta = \pm\infty \qquad \theta = \pm\frac{\pi}{2}, \pm\frac{3}{2}\pi,$

$\cot(x) = 0 \qquad x = \pm\frac{\pi}{2}, \pm\frac{3}{2}\pi, \cdots$

$\tan\theta = 0 \qquad \theta = 0, \pm\pi, \pm 2\pi, \cdots$



$\sin^2\theta = \frac{1}{2}(1-\cos(2\theta))$



arg $\quad \theta = 0, 2\pi \rightarrow \sin^2\theta = 0$ dominantly real

$\theta = \frac{\pi}{2}, \frac{3}{2}\pi \rightarrow \sin^2\theta = 1$ dominantly imag

http://functions.wolfram.com/ElementaryFunctions/Sin/visualizations/5/

red $\quad \theta = \pm 2n\pi$

cyan $\quad \theta = \pm(2n+1)\pi$

the square of the sine of the argument of sin(z)

plot $\quad \boxed{\sin^2\theta}$, $\quad \tan\theta = \cot(x)\tanh(y)$

$$\theta = \arg\{\sin(z)\} = \tan^{-1}\{\cot(x)\tanh(y)\}$$

https://en.wikipedia.org/wiki/Trigonometric_functions

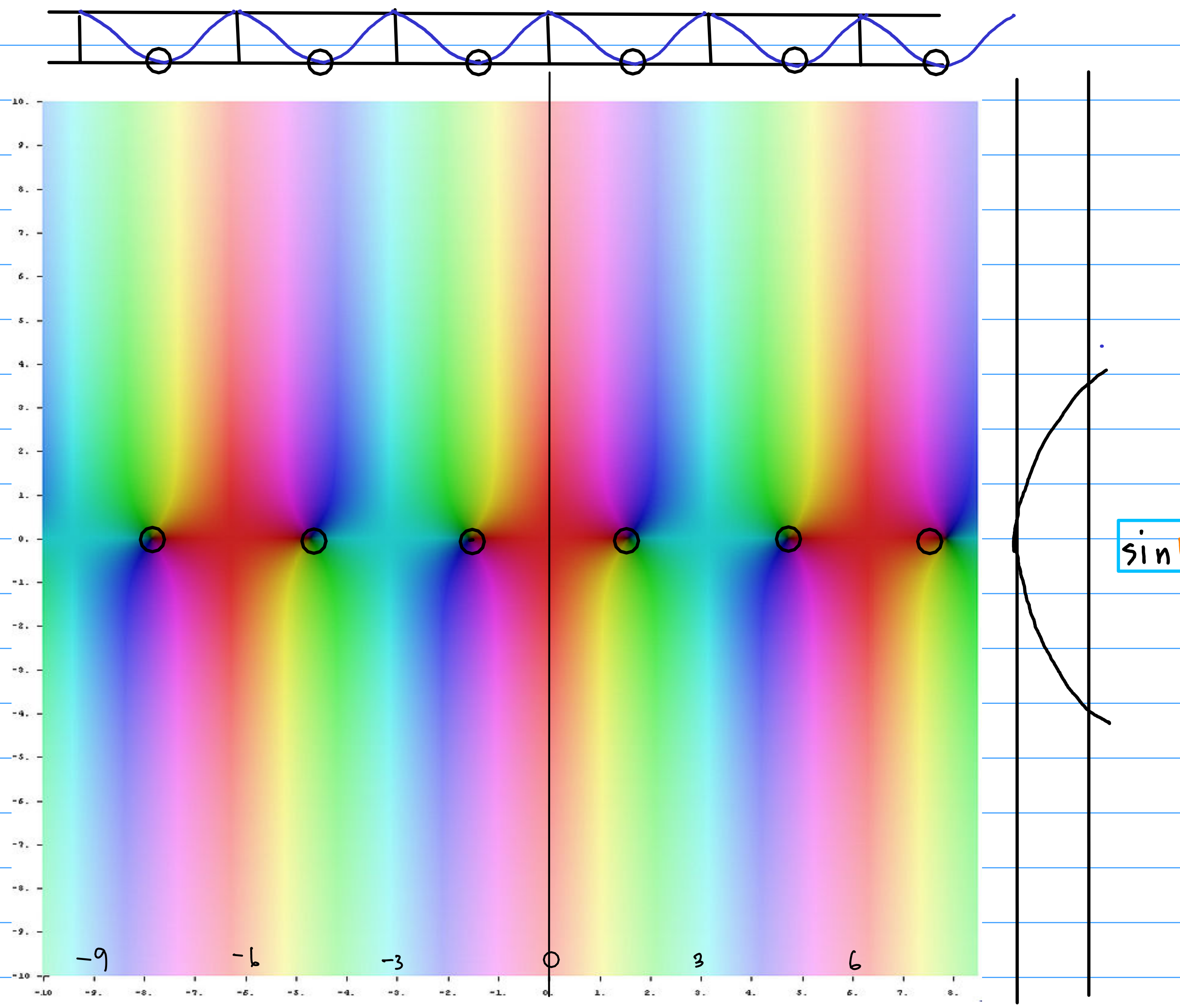
$|\cos z|$

$$\cos(x+iy) = \cos(x)\boxed{\cosh(y)} - i\sin(x)\boxed{\sinh(y)}$$

$$|\cos z|^2 = \cos^2(x) + \boxed{\sinh^2(y)}$$

$$\tan\theta = -\tan(x)\tanh(y)$$

$\cos^2(x)$



https://en.wikipedia.org/wiki/Trigonometric_functions

$\arg(\sin z)$

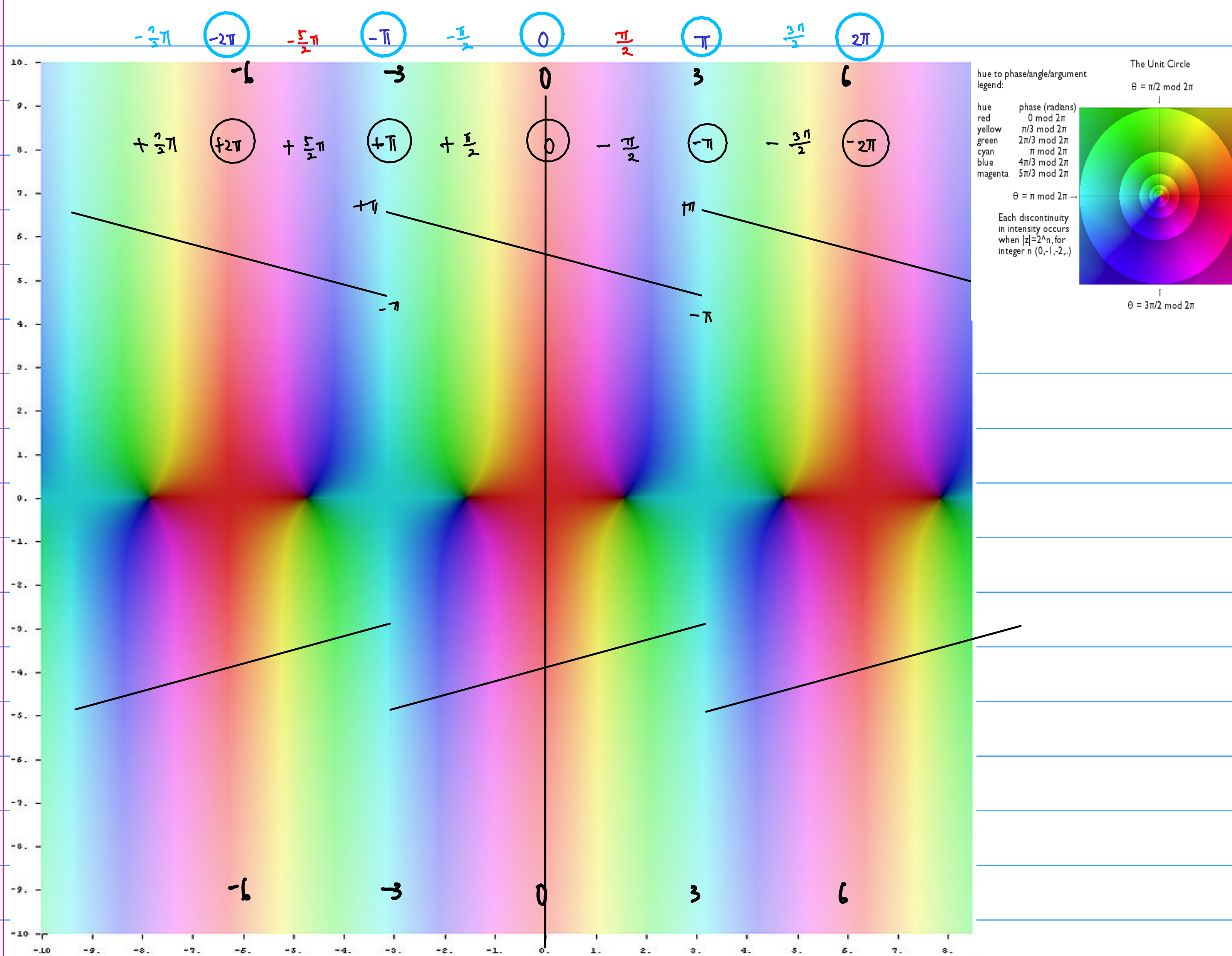
$$\tan\theta = -\tan(x)\,\tanh(y)$$

$$\tan(x) = \pm\infty \qquad x = \pm\frac{\pi}{2}, \pm\frac{3}{2}\pi, \cdots$$

$$\tan\theta = \mp\infty \qquad \theta = \mp\frac{\pi}{2}, \mp\frac{3}{2}\pi, \cdots$$

$$\tan(x) = 0 \qquad x = 0, \pm\pi, \pm 2\pi, \cdots$$

$$\tan\theta = 0 \qquad \theta = 0, \pm\pi, \pm 2\pi, \cdots$$



$$\sin(x+iy) = \sin(x)\boxed{\cosh(y)} + i\cos(x)\boxed{\sinh(y)}$$

$$\cos(x+iy) = \cos(x)\boxed{\cosh(y)} - i\sin(x)\boxed{\sinh(y)}$$

$$\tan(x+iy) = \frac{\sin(x+iy)}{\cos(x+iy)} = \frac{\sin(x)\boxed{\cosh(y)} + i\cos(x)\boxed{\sinh(y)}}{\cos(x)\boxed{\cosh(y)} - i\sin(x)\boxed{\sinh(y)}}$$

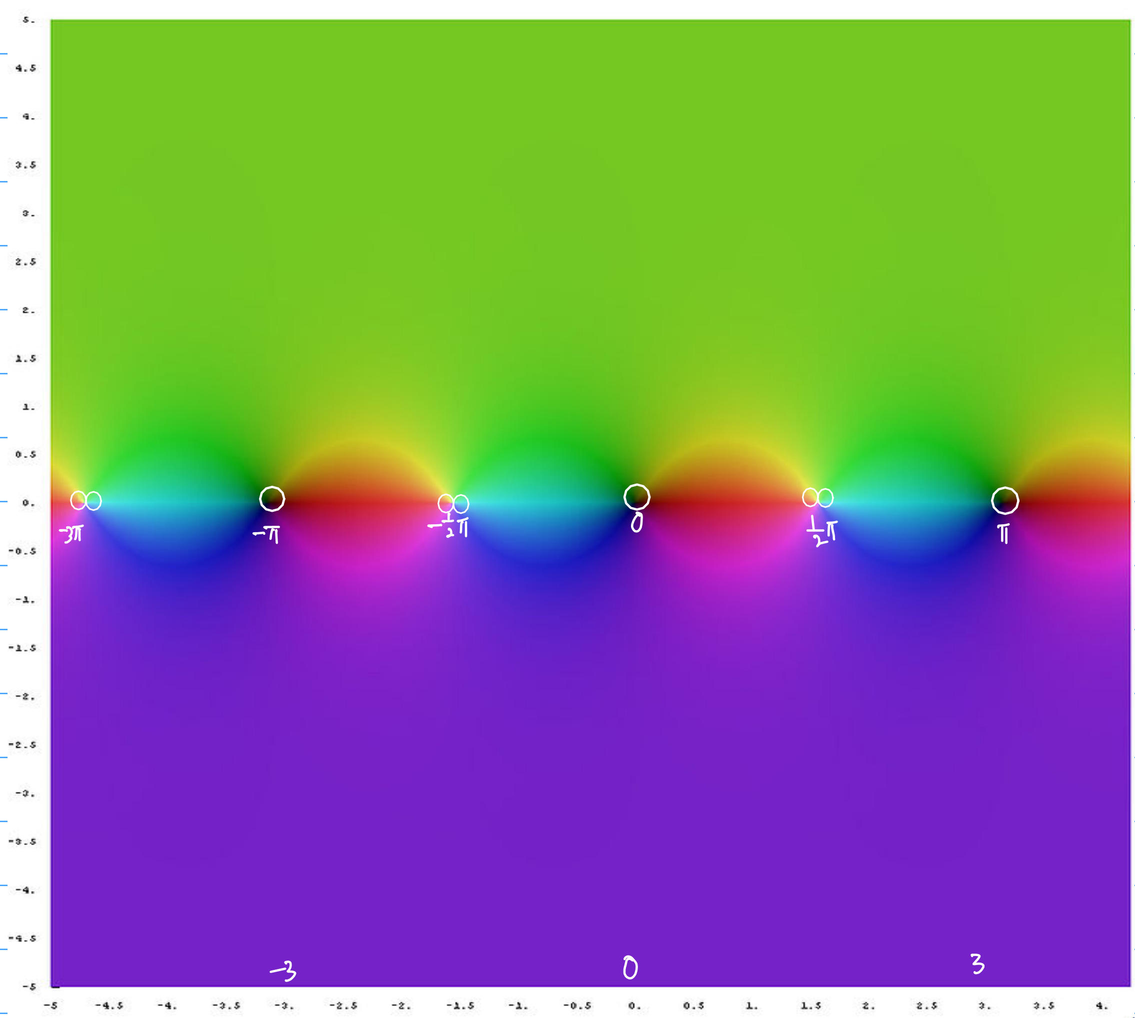
$$|\tan z|^2 = \frac{|\sin z|^2}{|\cos z|^2} = \frac{\sin^2(x) + \boxed{\sinh^2(y)}}{\cos^2(x) + \boxed{\sinh^2(y)}}$$

zeros $\quad y=0$ & $x = 0, \pm\pi, \pm 2\pi, \cdots$

$\infty \quad y=0$ & $x = \pm\frac{\pi}{2}, \pm\frac{3}{2}\pi, \cdots$

https://en.wikipedia.org/wiki/Trigonometric_functions

$|\tan z|$



https://en.wikipedia.org/wiki/Trigonometric_functions

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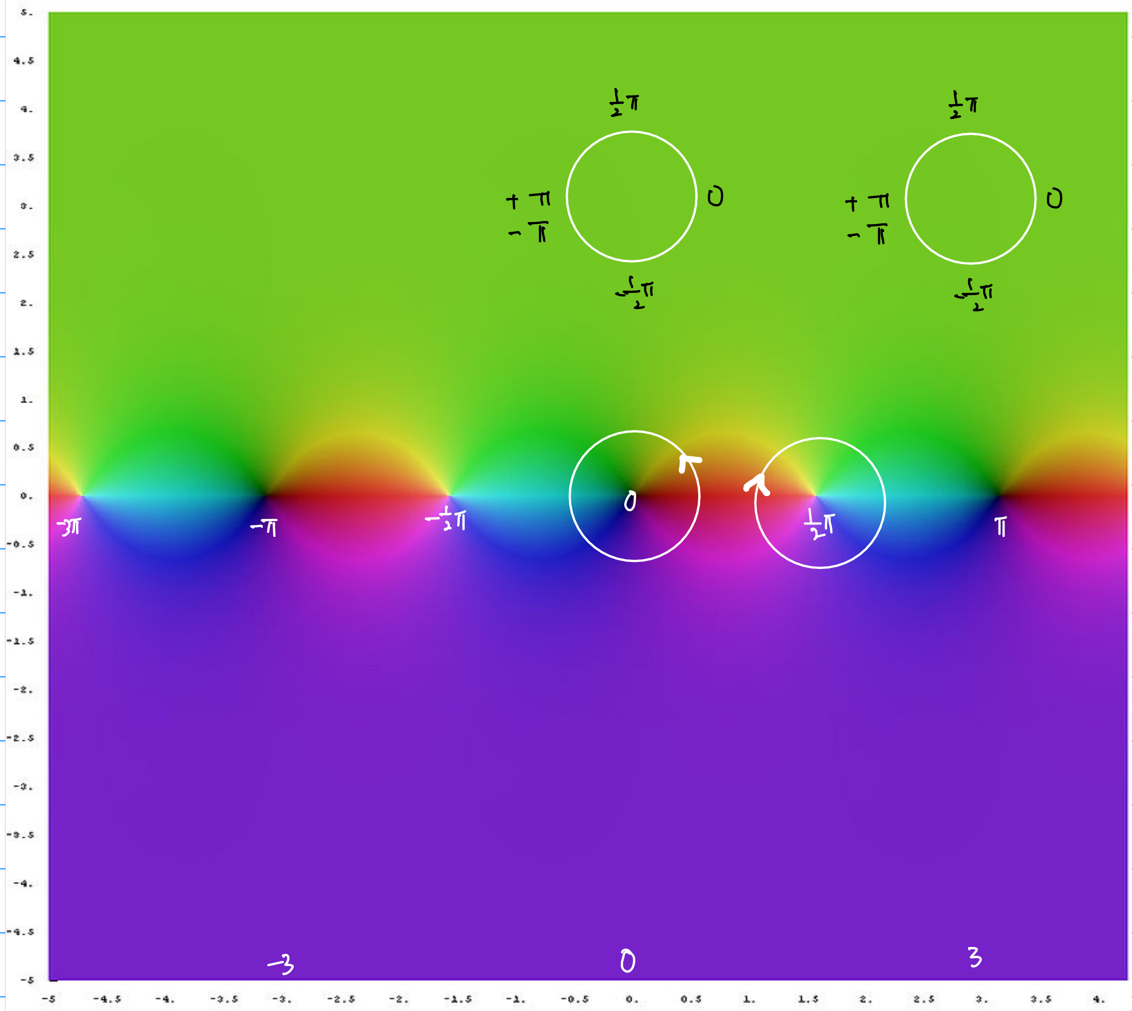

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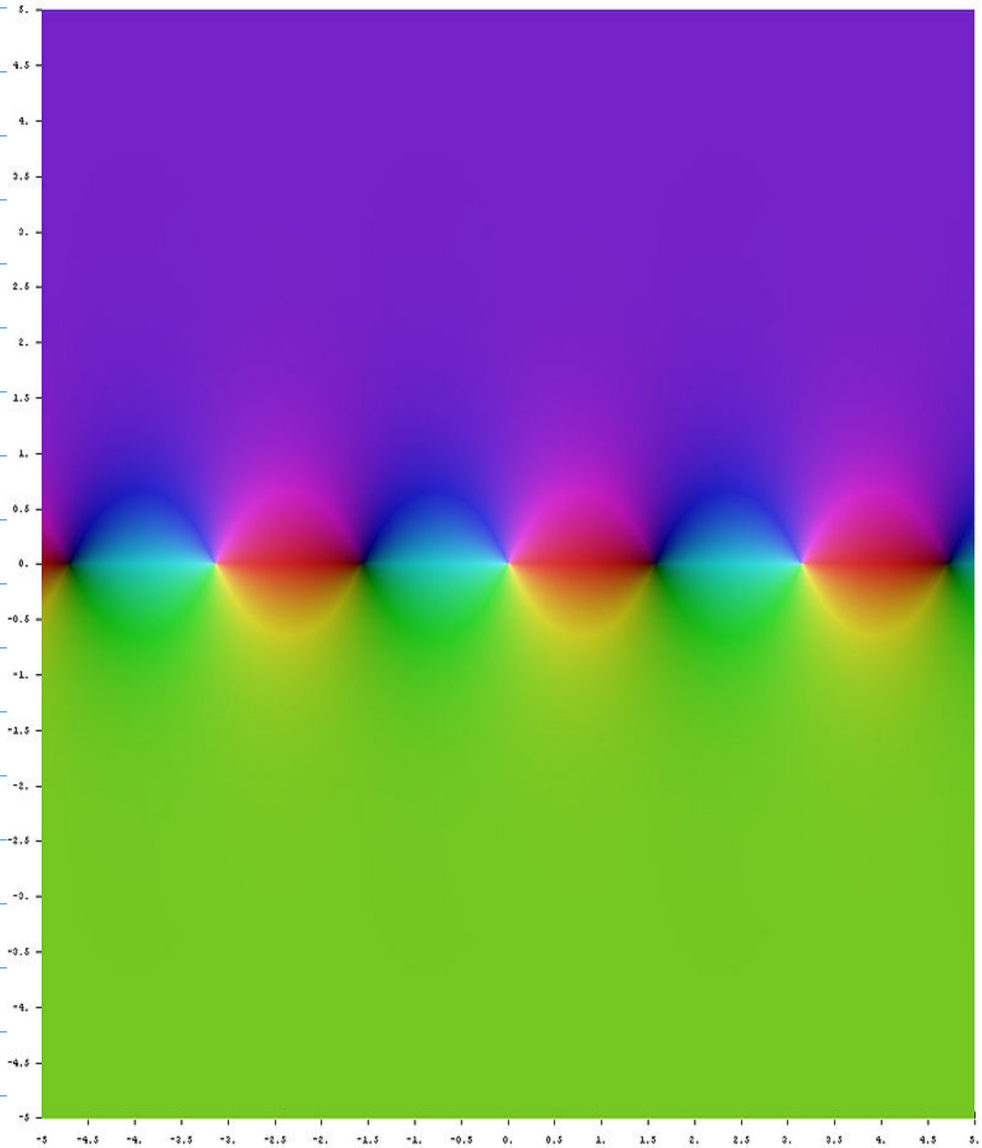

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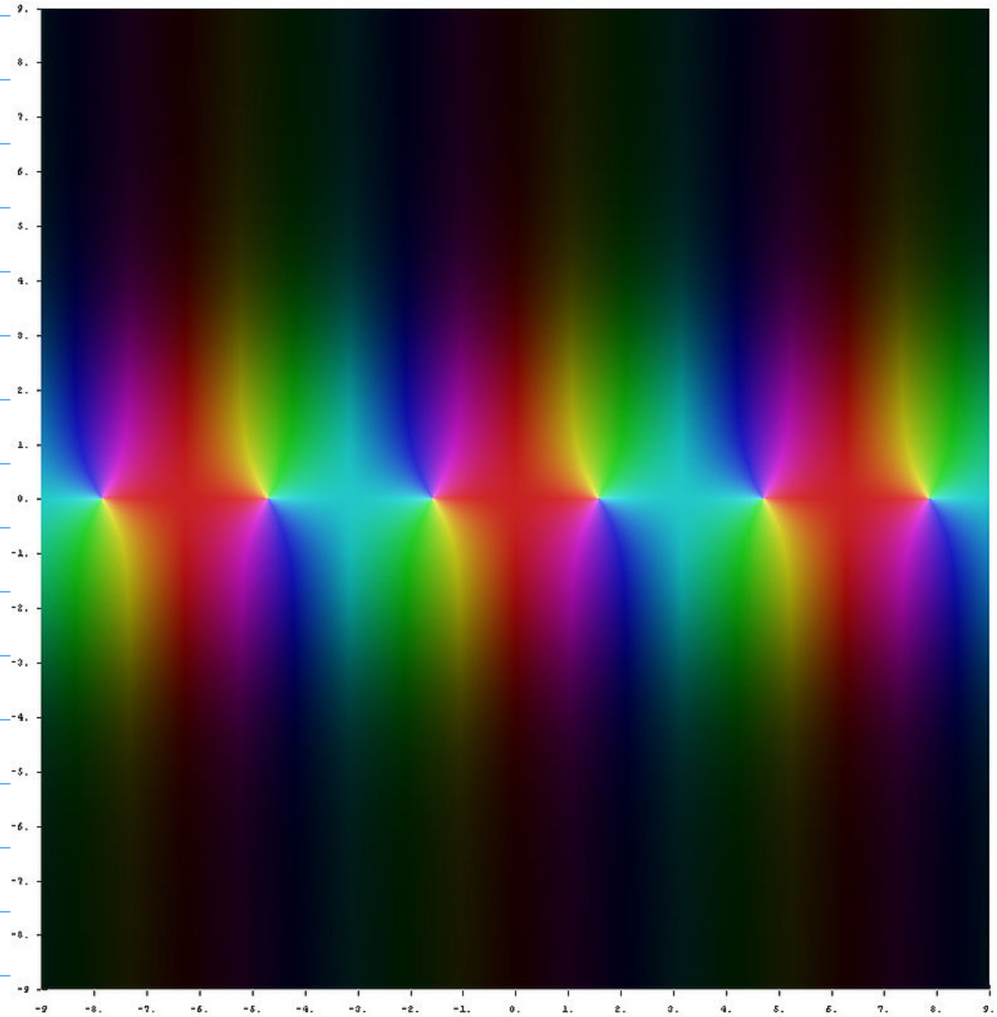

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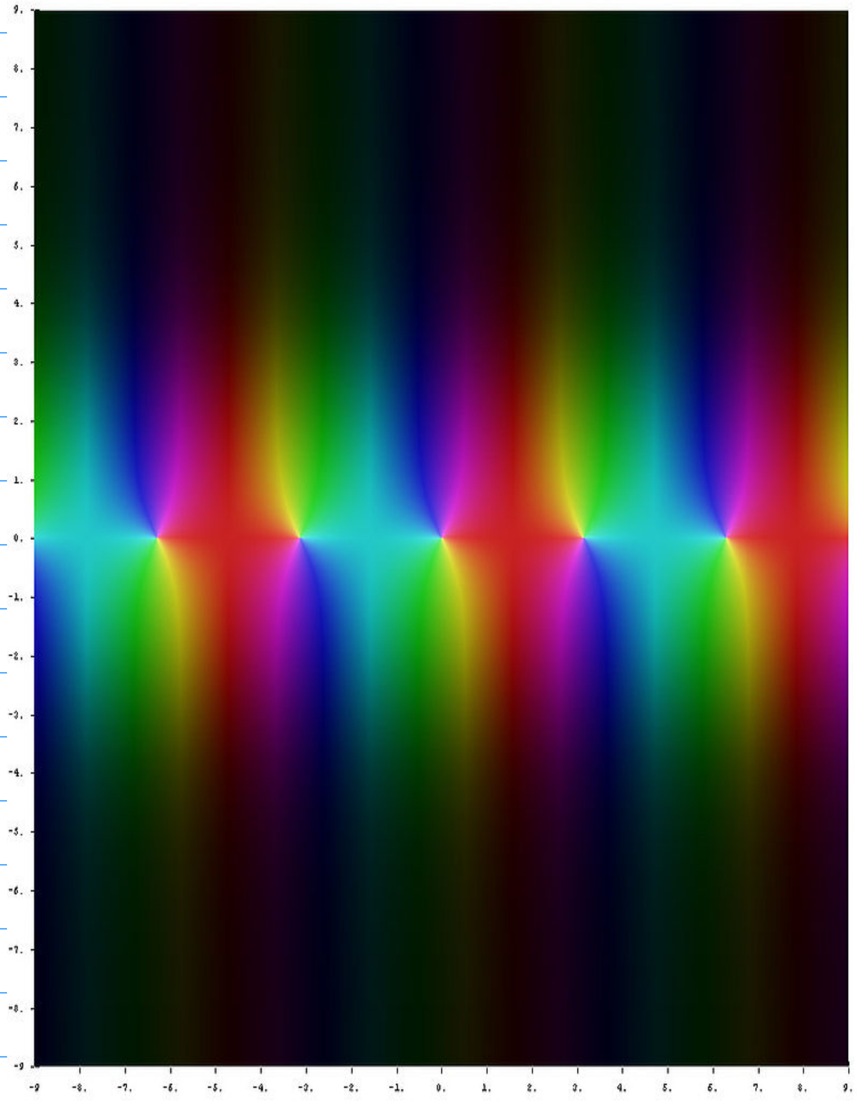

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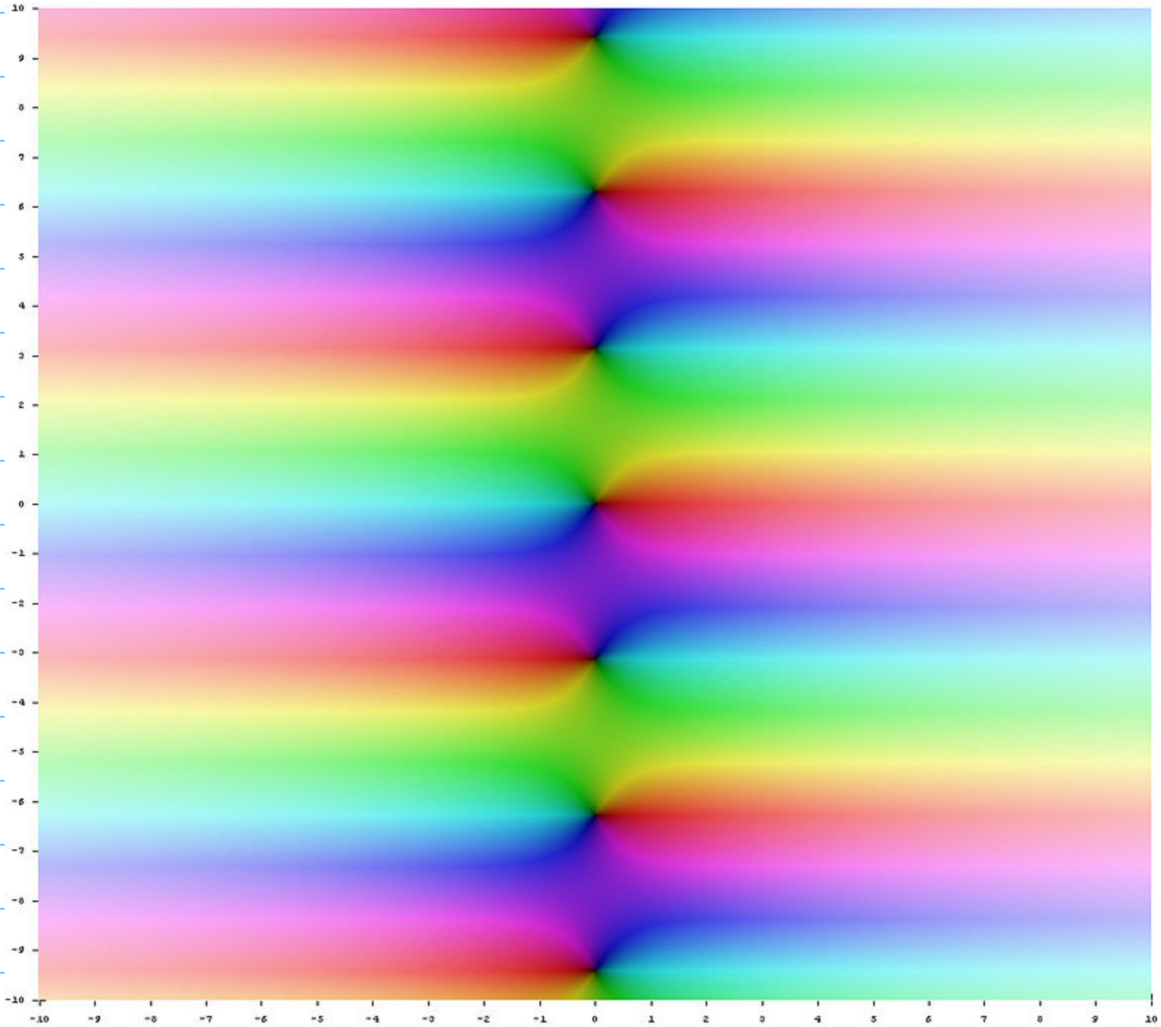
https://en.wikipedia.org/wiki/Hyperbolic_function



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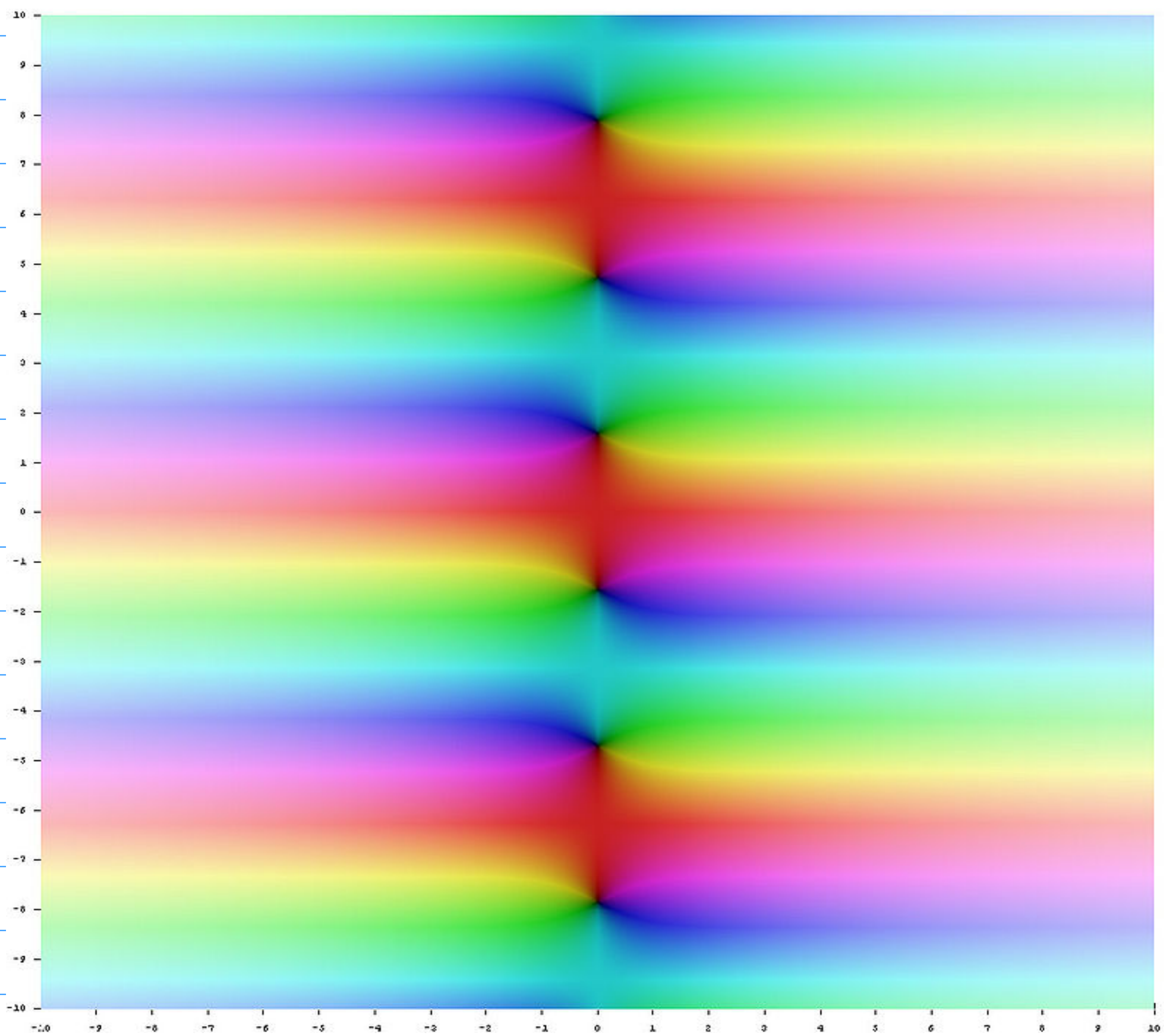

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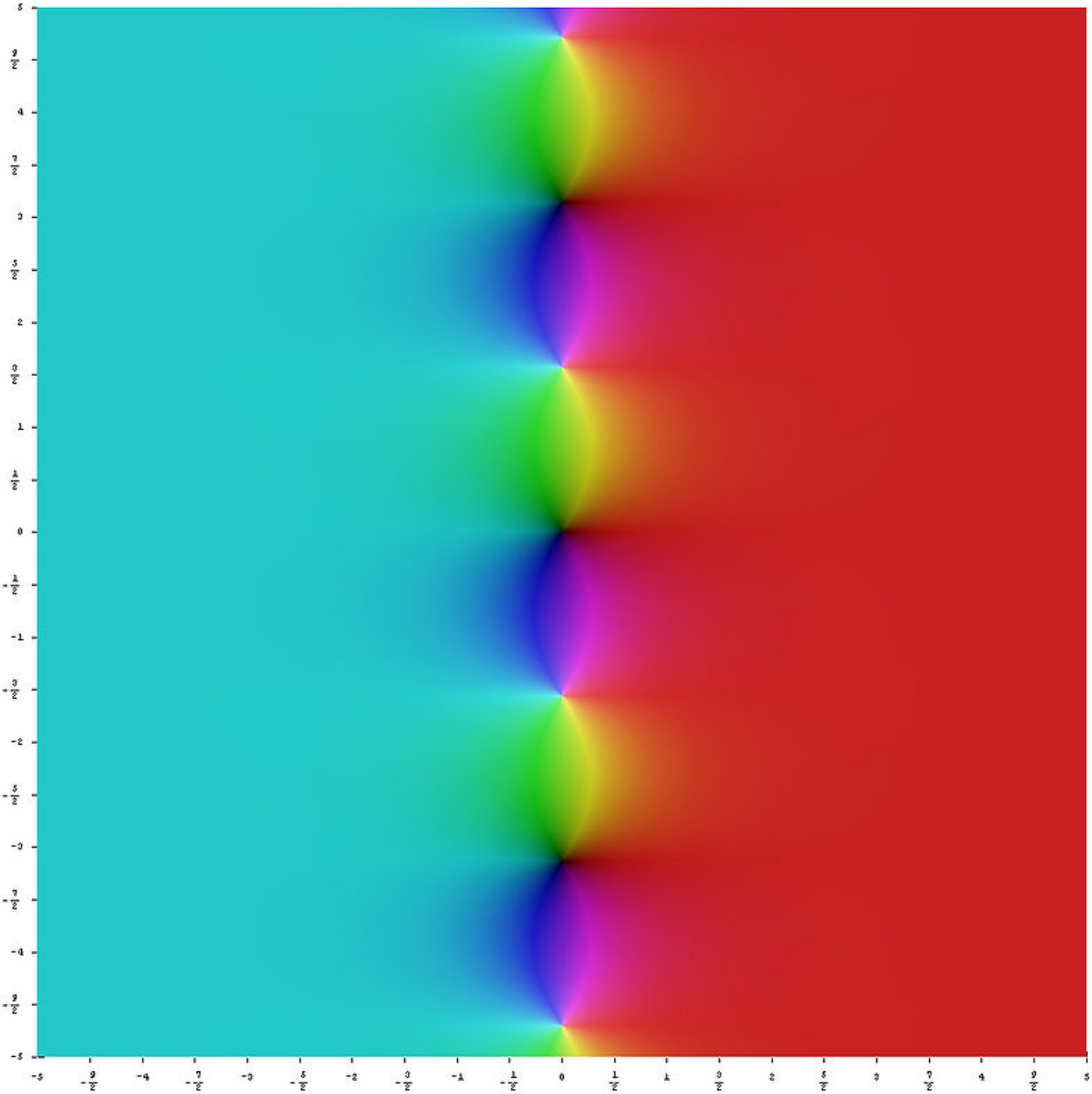

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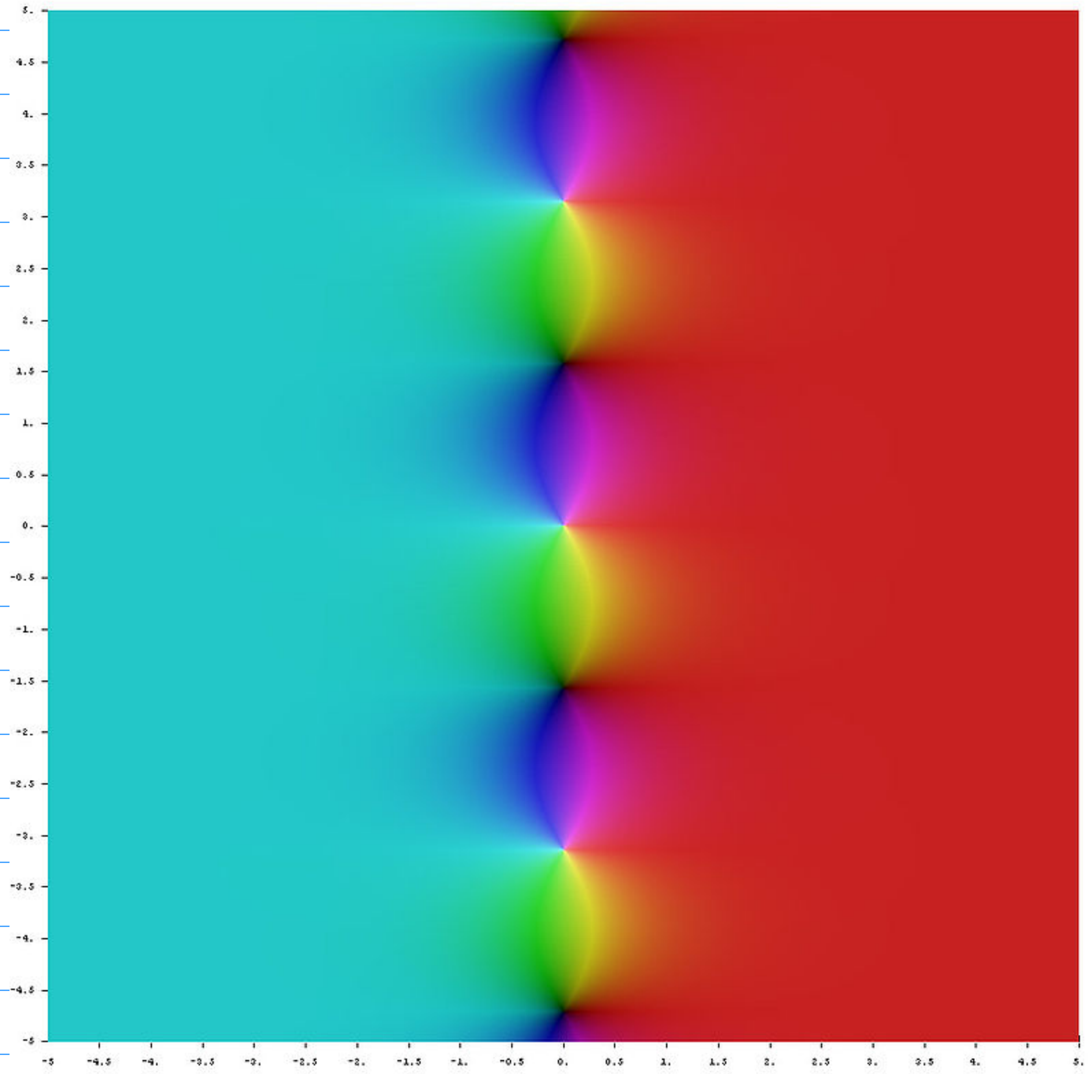

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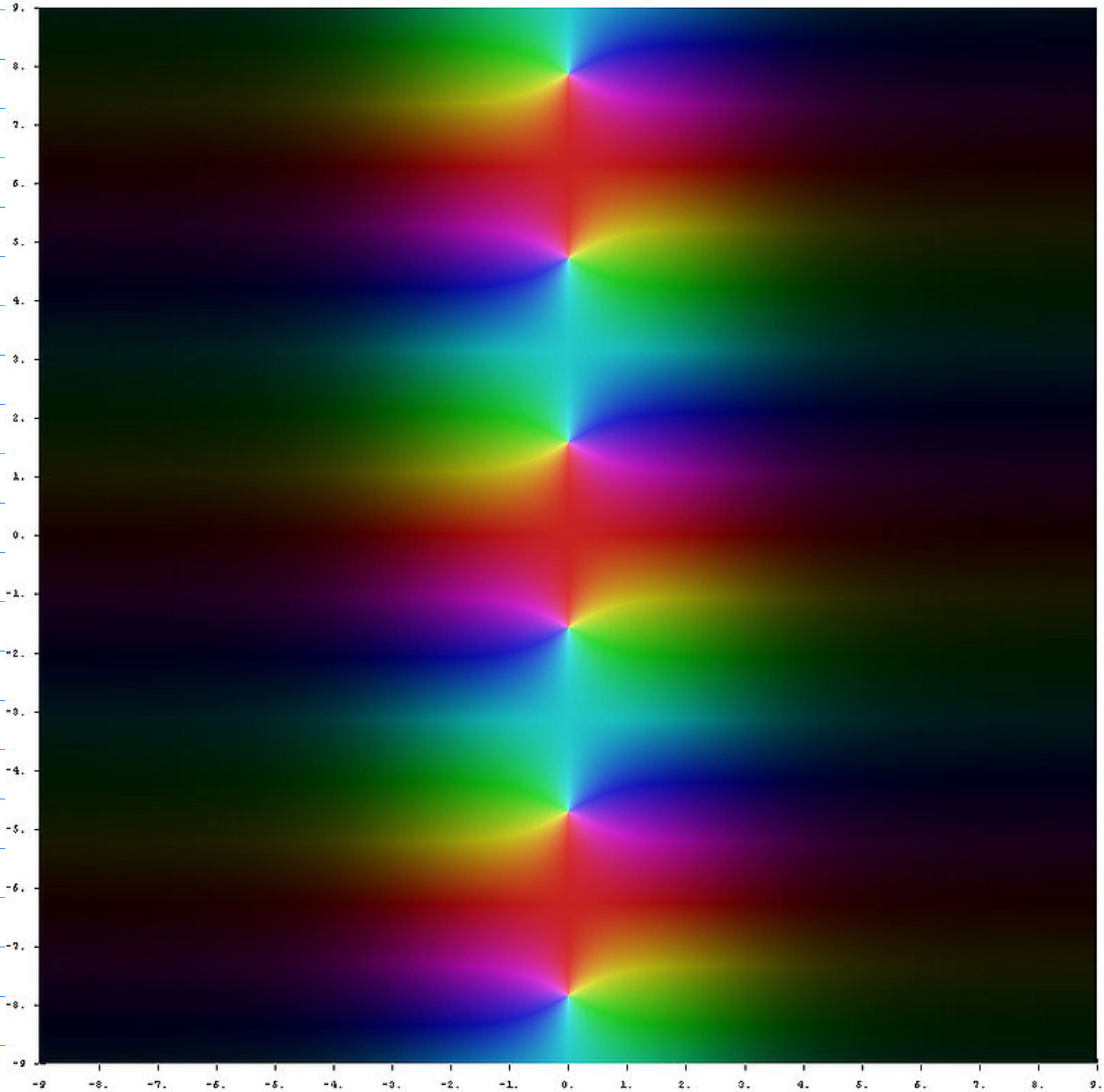

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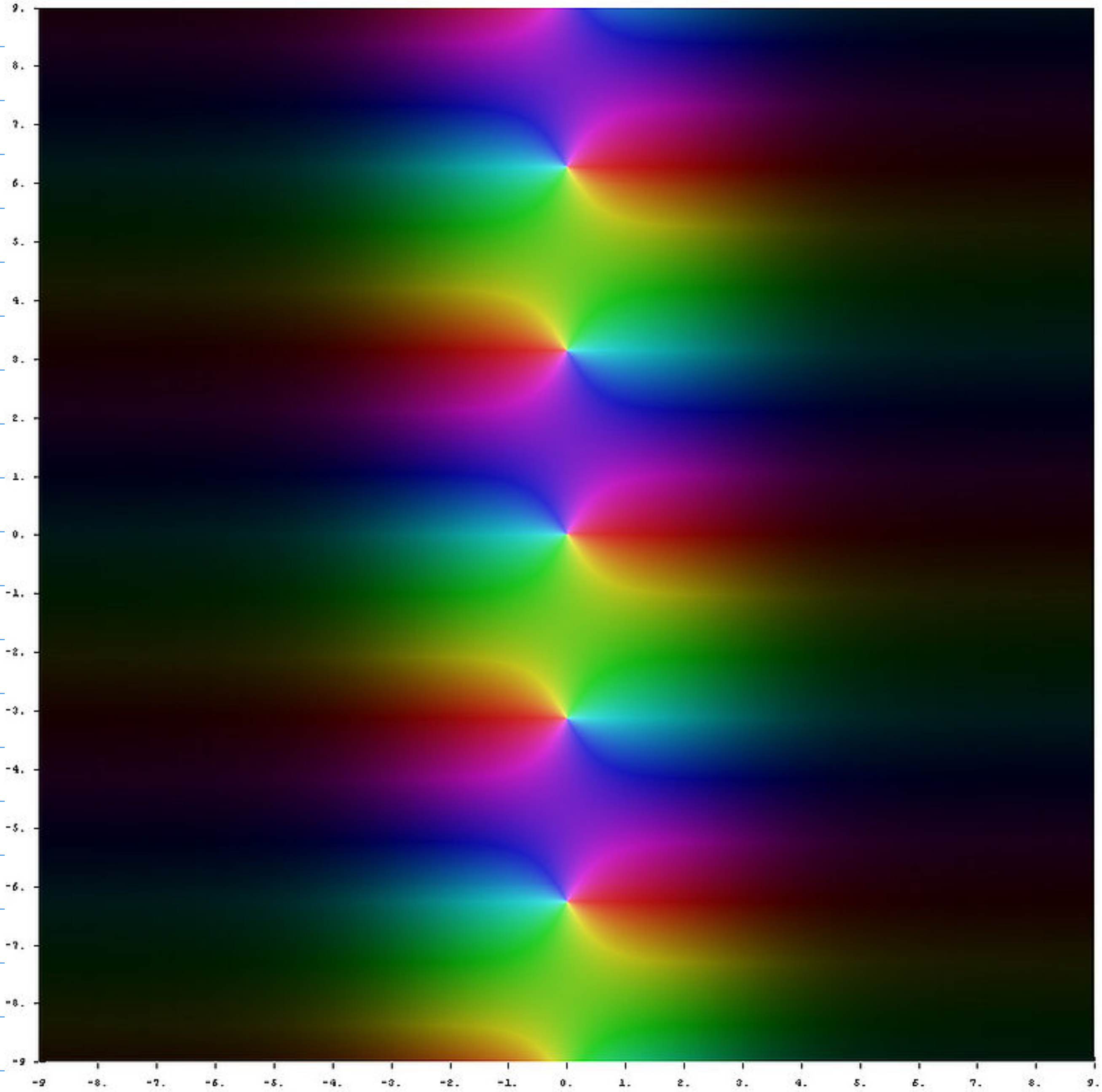

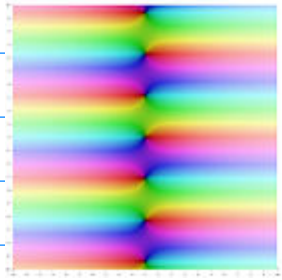
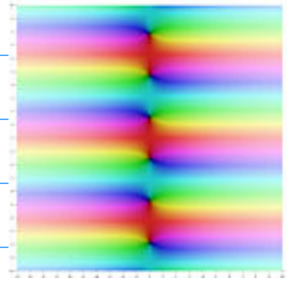

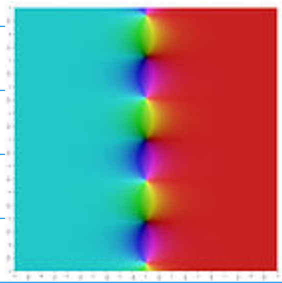
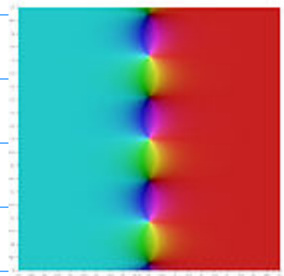

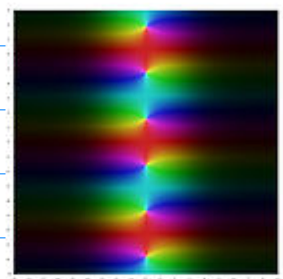
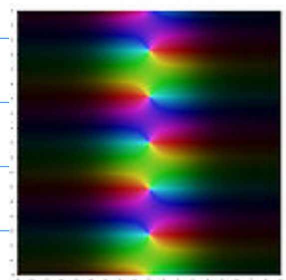

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