

Triangles (2B)

- Law of Sines - all acute angles
- Law of Cosines - all acute angles
- Law of Sines - one obtuse angle
- Law of Cosines - one obtuse angle

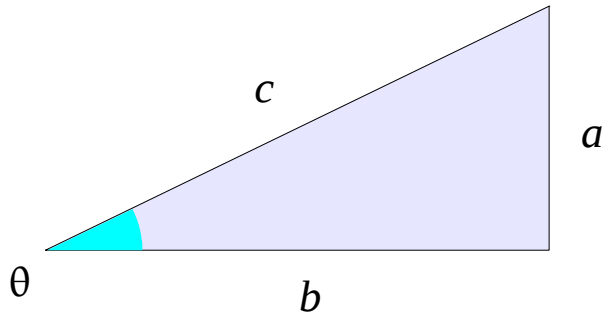
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Right Triangles (1)



$$c^2 = a^2 + b^2$$

$$\sin \theta = \frac{\textit{opposite}}{\textit{hypotenuse}}$$

$$\cos \theta = \frac{\textit{adjacent}}{\textit{hypotenuse}}$$

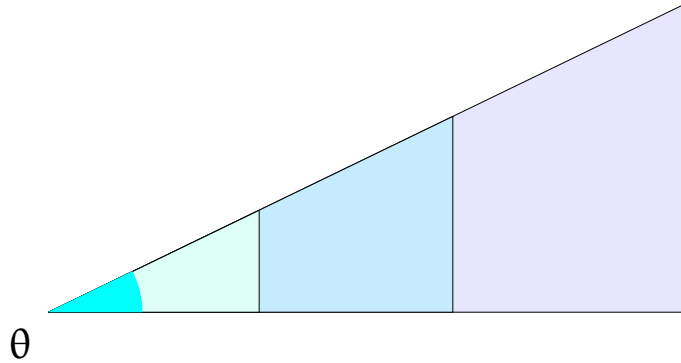
$$\tan \theta = \frac{\textit{opposite}}{\textit{adjacent}}$$

$$\sin \theta = \frac{a}{c}$$

$$\cos \theta = \frac{b}{c}$$

$$\tan \theta = \frac{b}{a}$$

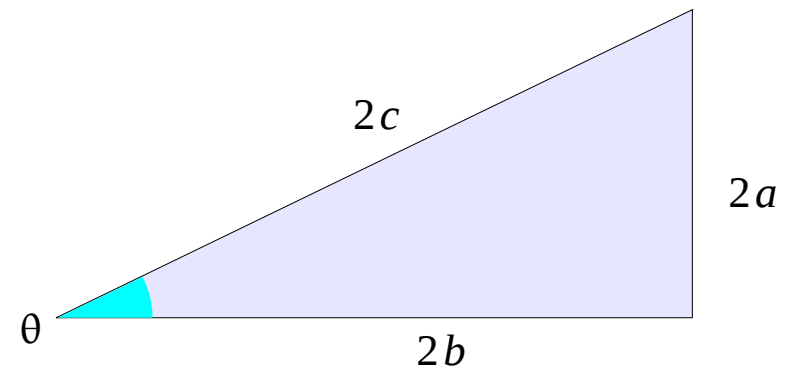
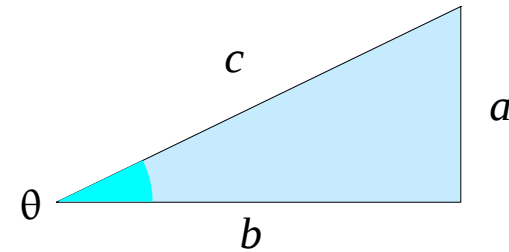
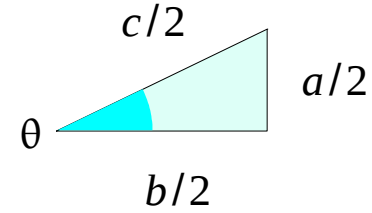
Right Triangles (2)



$$\sin \theta = \frac{a}{c}$$

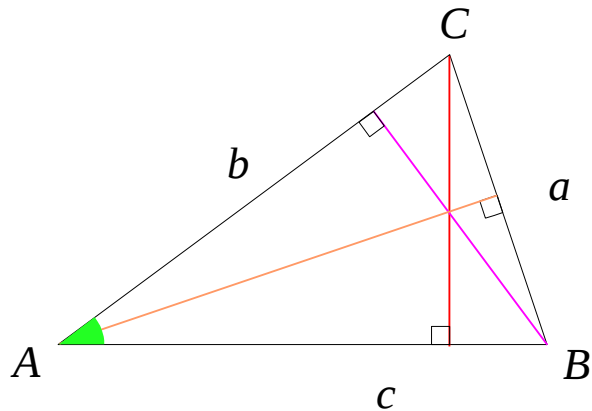
$$\cos \theta = \frac{b}{c}$$

$$\tan \theta = \frac{b}{a}$$



Oblique Triangles - (1) all acute angles

All Acute Angles



$$\begin{aligned}0^\circ < A < 90^\circ \\0^\circ < B < 90^\circ \\0^\circ < C < 90^\circ\end{aligned}$$

The Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

The Law of Cosines

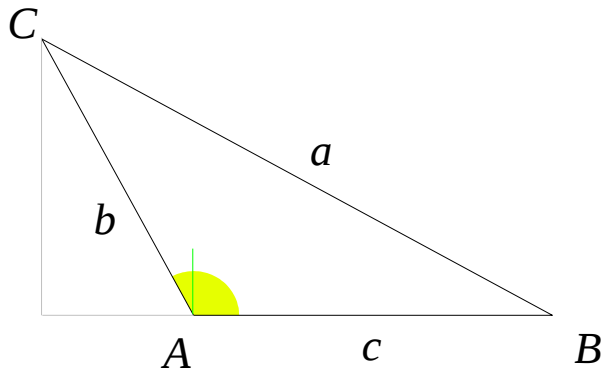
$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = c^2 + a^2 - 2ca \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

Oblique Triangles - (2) one obtuse angle

One Obtuse Angles



$$90^\circ < A < 180^\circ$$

$$0^\circ < B < 90^\circ$$

$$0^\circ < C < 90^\circ$$

The Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

The Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = c^2 + a^2 - 2ca \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

The Law of Sines - all acute angles (1) ~ (4)

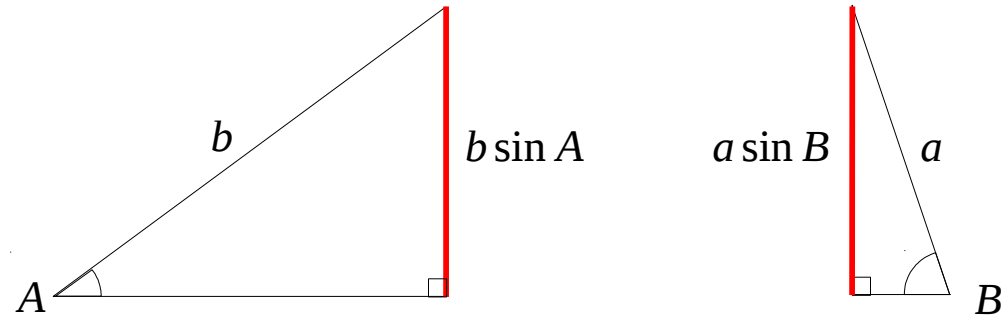
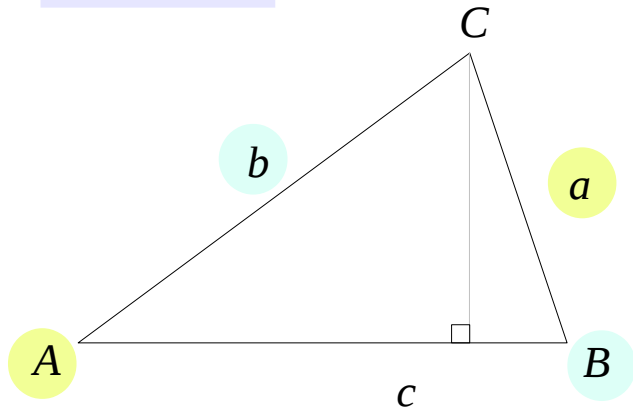
The Law of Cosines - all acute angles (1) ~ (4)

The Law of Sines - one obtuse angle (1) ~ (4)

The Law of Cosines - one obtuse angle (1) ~ (4)

Law of Sines - all acute angles (1)

Case A:

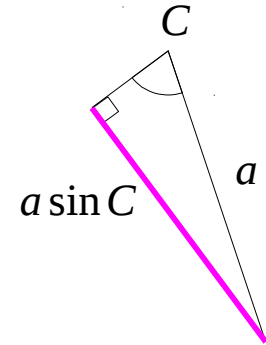
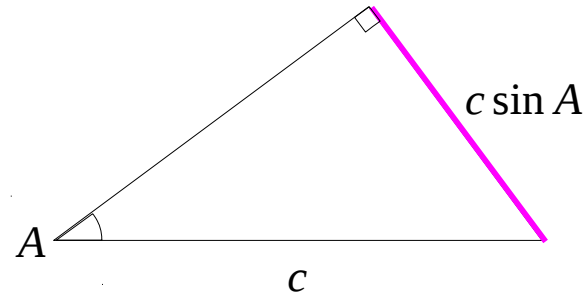
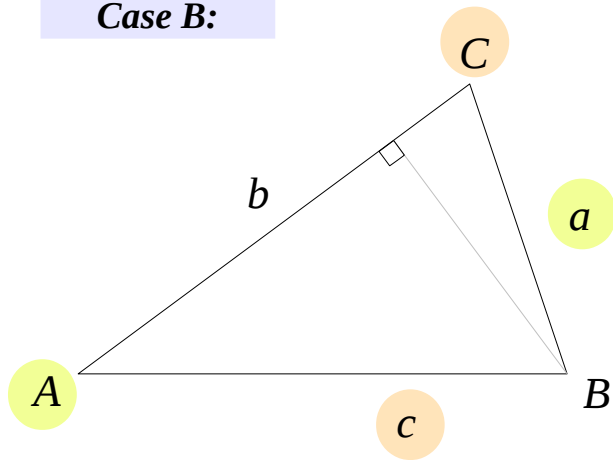


$$b \sin A = a \sin B$$

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

Law of Sines - all acute angles (2)

Case B:

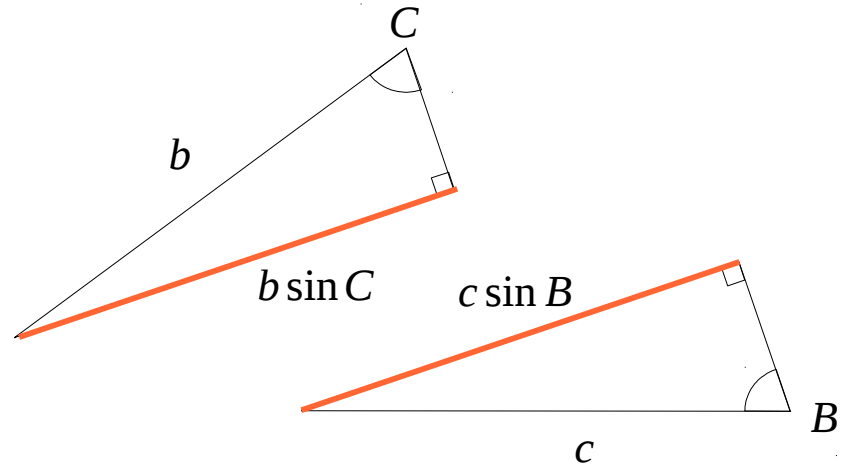
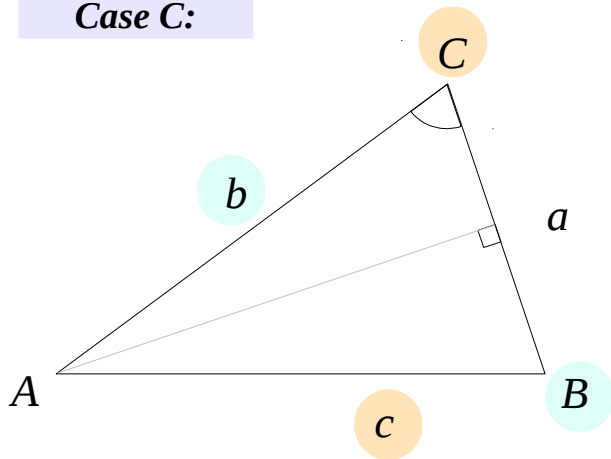


$$c \sin A = a \sin C$$

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

Law of Sines - all acute angles (3)

Case C:

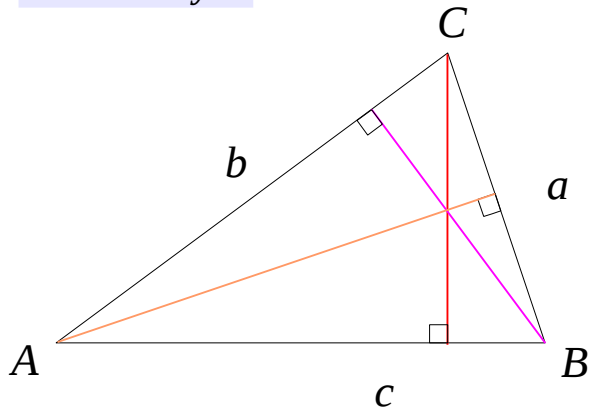


$$b \sin C = c \sin B$$

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of Sines - all acute angles (4)

Summary



The Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\begin{aligned} 0^\circ < A < 90^\circ \\ 0^\circ < B < 90^\circ \\ 0^\circ < C < 90^\circ \end{aligned}$$

The Law of Sines - all acute angles (1) ~ (4)

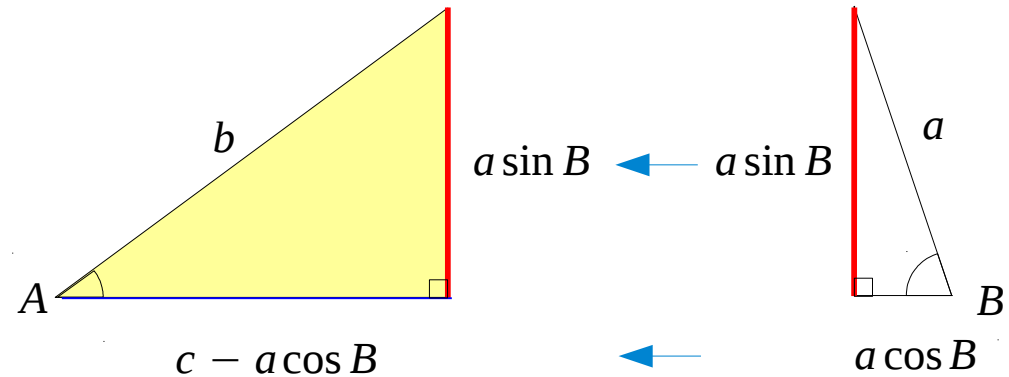
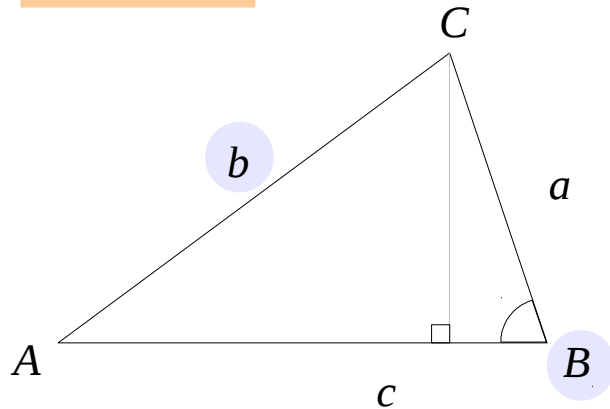
The Law of Cosines - all acute angles (1) ~ (4)

The Law of Sines - one obtuse angle (1) ~ (4)

The Law of Cosines - one obtuse angle (1) ~ (4)

Law of Cosines - all acute angles (1)

Case A:



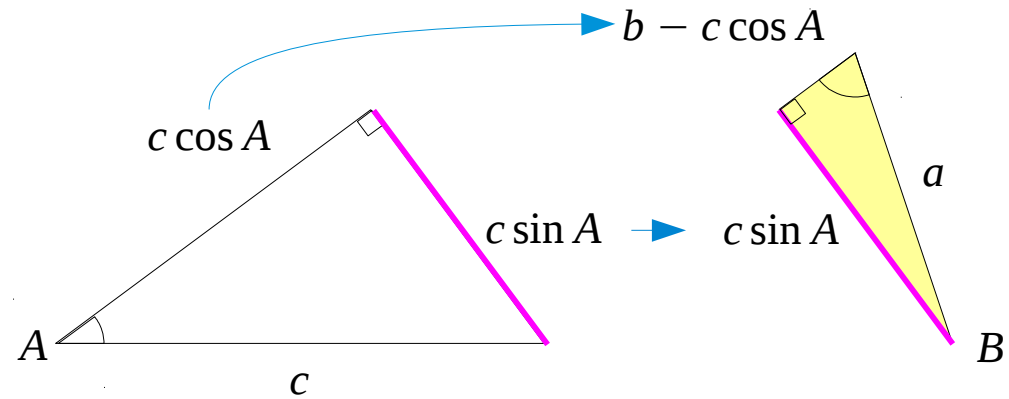
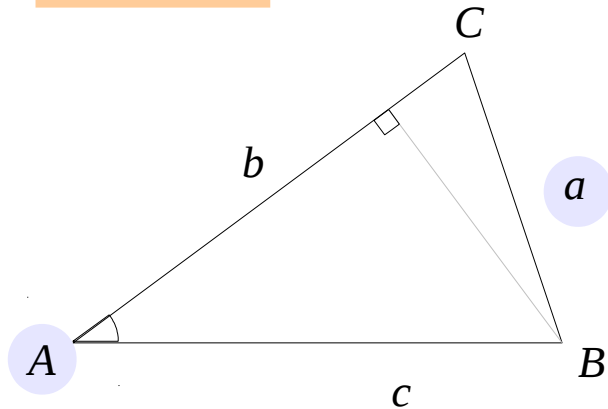
$$b^2 = (c - a \cos B)^2 + (a \sin B)^2$$

$$b^2 = c^2 - 2ca \cos B + a^2 \cos^2 B + a^2 \sin^2 B$$

$$b^2 = c^2 + a^2 - 2ca \cos B$$

Law of Cosines - all acute angles (2)

Case B:



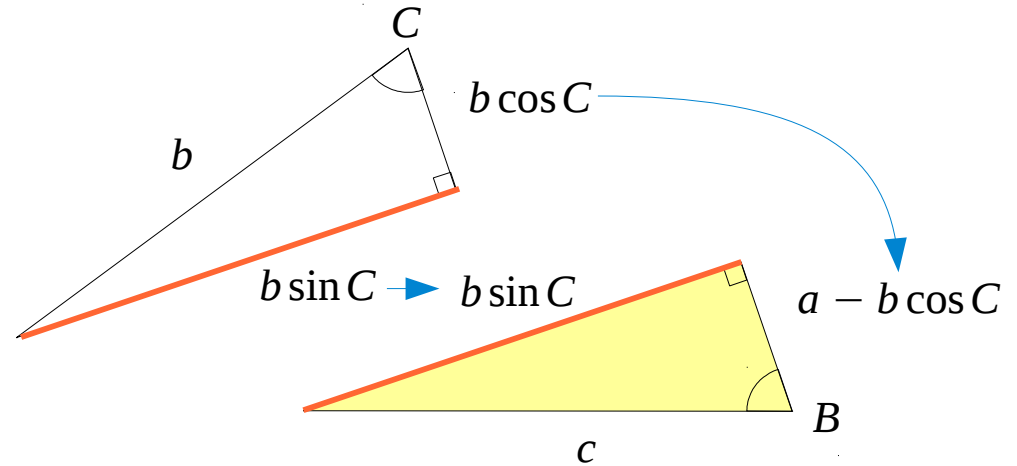
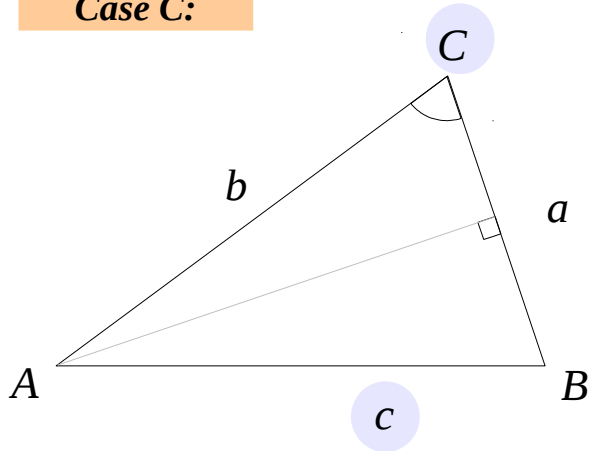
$$a^2 = (b - c \cos A)^2 + (c \sin A)^2$$

$$a^2 = b^2 - 2bc \cos A + c^2 \cos^2 A + c^2 \sin^2 A$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Law of Cosines - all acute angles (3)

Case C:



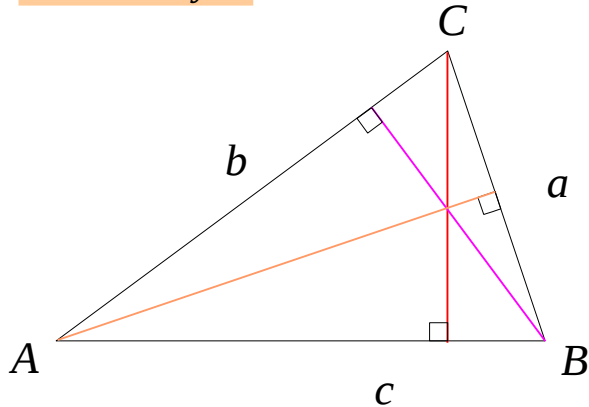
$$c^2 = (a - b \cos C)^2 + (b \sin C)^2$$

$$c^2 = a^2 - 2ab \cos C + b^2 \cos^2 C + b^2 \sin^2 C$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

Law of Cosines - all acute angles (4)

Summary



$$0^\circ < A < 90^\circ$$
$$0^\circ < B < 90^\circ$$
$$0^\circ < C < 90^\circ$$

The Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = c^2 + a^2 - 2ca \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

The Law of Sines - all acute angles (1) ~ (4)

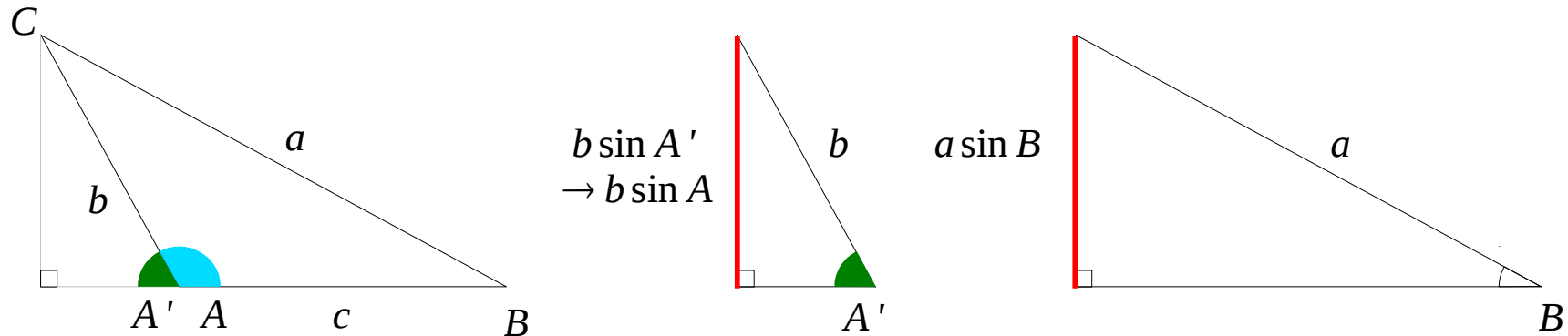
The Law of Cosines - all acute angles (1) ~ (4)

The Law of Sines - one obtuse angle (1) ~ (4)

The Law of Cosines - one obtuse angle (1) ~ (4)

Law of Sines - one obtuse angle (1)

Case A:



$$\sin A' = \sin A,$$

$$\sin(\pi - A) = \sin A$$

$$b \sin A' = b \sin A$$

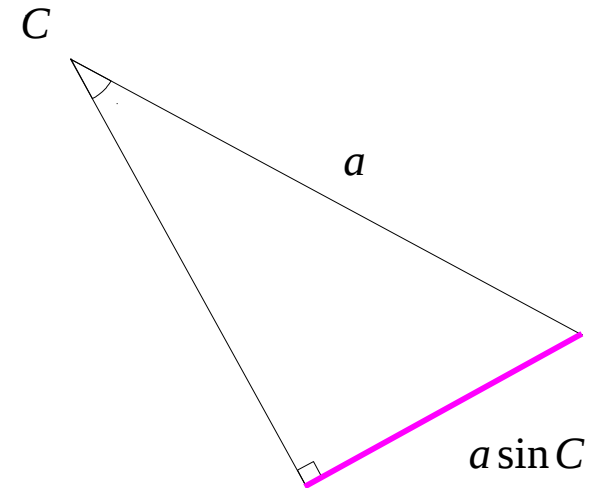
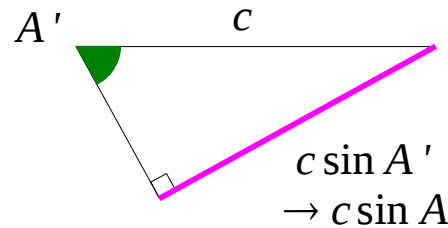
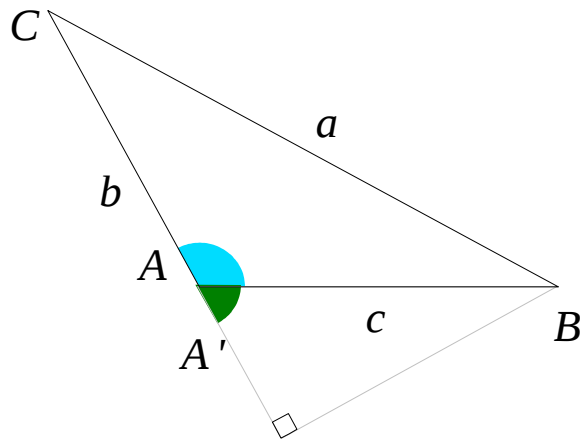


$$b \sin A = a \sin B$$

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

Law of Sines - one obtuse angle (2)

Case B:



$$\sin A' = \sin A,$$

$$\sin(\pi - A) = \sin A$$

$$c \sin A' = c \sin A$$

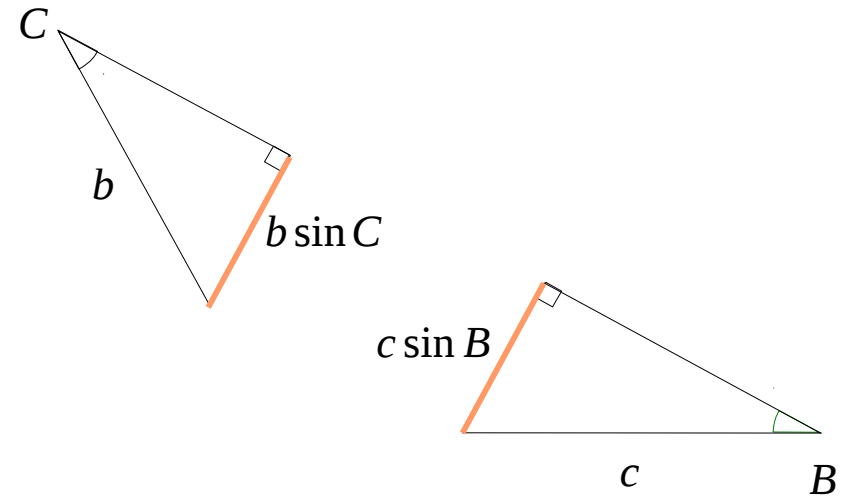
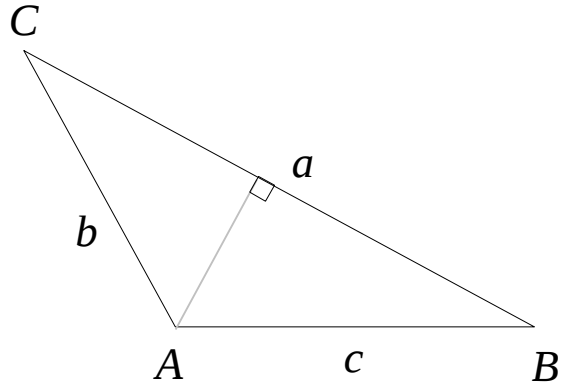


$$a \sin C = c \sin A$$

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

Law of Sines - one obtuse angle (3)

Case C:

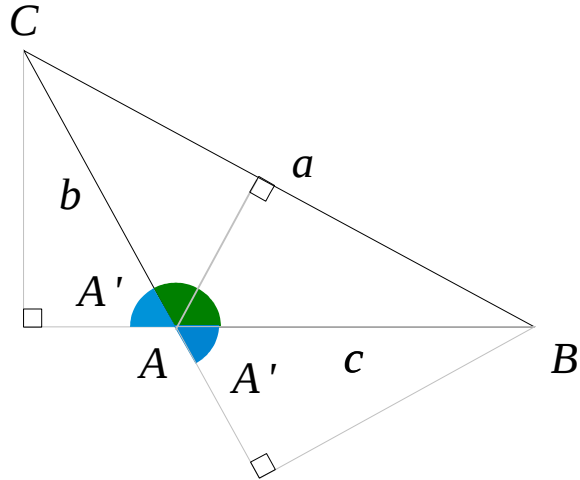


$$b \sin C = c \sin B$$

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of Sines - one obtuse angle (4)

Summary



$$90^\circ < A < 180^\circ$$

$$0^\circ < B < 90^\circ$$

$$0^\circ < C < 90^\circ$$

$$\sin A' = \sin A,$$

$$\sin(\pi - A) = \sin A$$



The Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

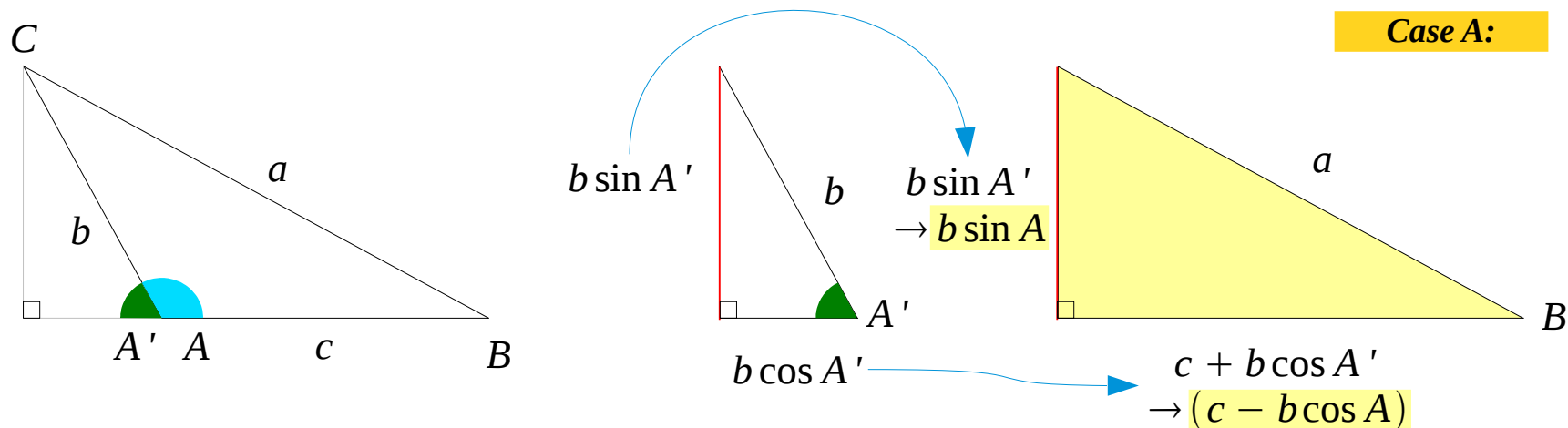
The Law of Sines - all acute angles (1) ~ (4)

The Law of Cosines - all acute angles (1) ~ (4)

The Law of Sines - one obtuse angle (1) ~ (4)

The Law of Cosines - one obtuse angle (1) ~ (4)

Law of Cosines - one obtuse angle (1)



$$\sin A' = \sin A$$

$$\cos A' = -\cos A$$

$$\sin(\pi - A) = \sin A$$

$$\cos(\pi - A) = -\cos A$$

$$b \sin A' = b \sin A$$

$$b \cos A' = -b \cos A$$



$$a^2 = b^2 \sin^2 A + (c - b \cos A)^2$$

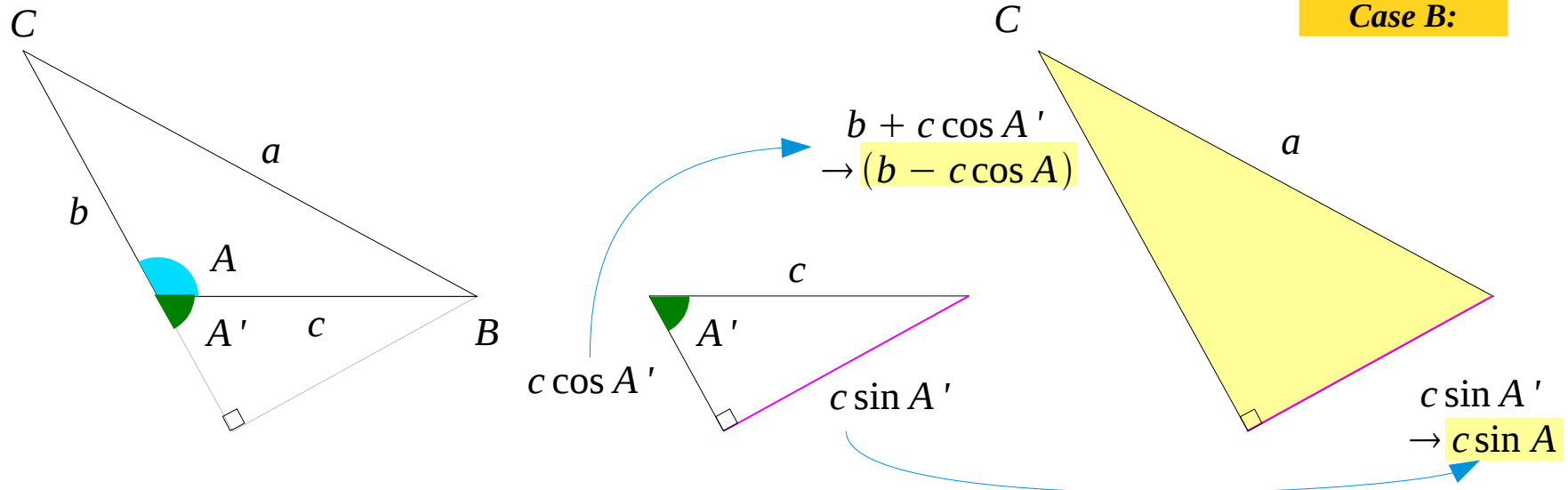
$$a^2 = b^2 \sin^2 A + c^2 - 2bc \cos A + b^2 \cos^2 A$$

$$a^2 = b^2(\sin^2 A + \cos^2 A) + c^2 - 2bc \cos A$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Law of Cosines - one obtuse angle (2)



$$\sin A' = \sin A$$

$$\cos A' = -\cos A$$

$$\sin(\pi - A) = \sin A$$

$$\cos(\pi - A) = -\cos A$$

$$c \sin A' = c \sin A$$

$$c \cos A' = -c \cos A$$



$$a^2 = c^2 \sin^2 A + (b - c \cos A)^2$$

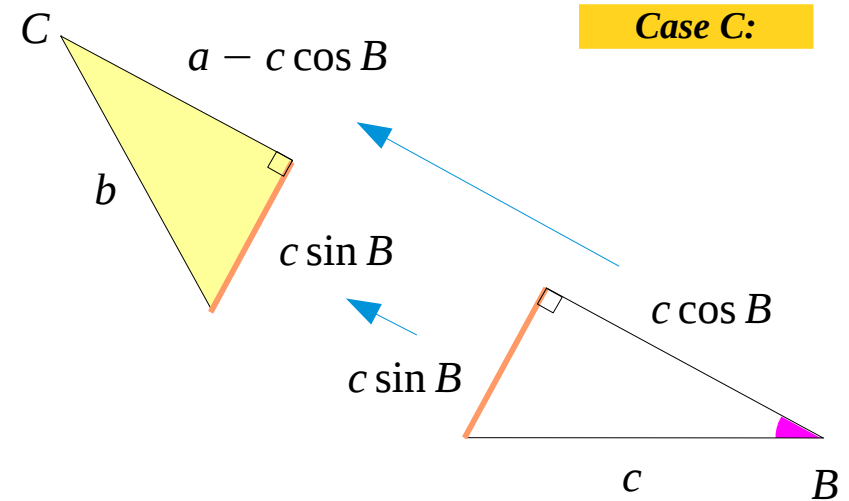
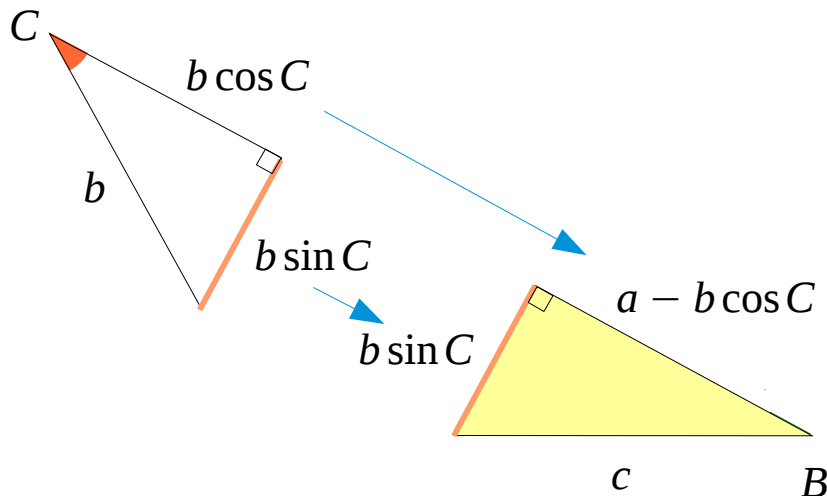
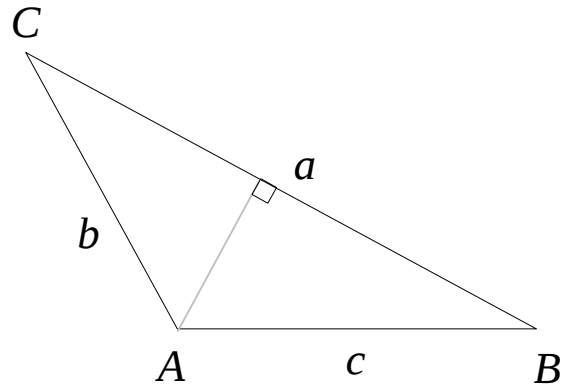
$$a^2 = c^2 \sin^2 A + b^2 - 2bc \cos A + c^2 \cos^2 A$$

$$a^2 = b^2 + c^2 (\sin^2 A + \cos^2 A) - 2bc \cos A$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Law of Sines - one obtuse angle (3)



$$b^2 = (c \sin B)^2 + (a - c \cos B)^2$$

$$b^2 = c^2 \sin^2 B + a^2 - 2ac \cos B + c^2 \cos^2 B$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

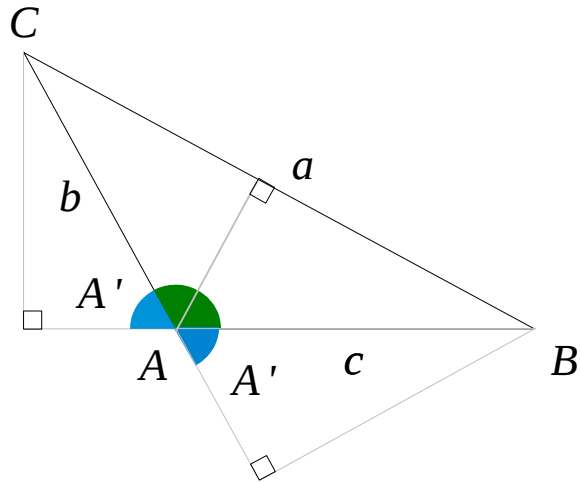
$$c^2 = (b \sin C)^2 + (a - b \cos C)^2$$

$$c^2 = b^2 \sin^2 C + a^2 - 2ab \cos C + b^2 \cos^2 C$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

Law of Cosines - one obtuse angle (4)

Summary



$$90^\circ < A < 180^\circ$$

$$0^\circ < B < 90^\circ$$

$$0^\circ < C < 90^\circ$$

$$\sin A' = \sin(\pi - A) = \sin A$$

$$\cos A' = \cos(\pi - A) = -\cos A$$



The Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = c^2 + a^2 - 2ca \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

References

- [1] <http://en.wikipedia.org/>
- [2] <http://planetmath.org/>
- [3] Blitzer, R. "Algebra & Trigonometry." 3rd ed, Prentice Hall
- [4] Smith, R. T., Minton, R. B. "Calculus: Concepts & Connections," Mc Graw Hill
- [5] 홍성대, "기본/실력 수학의 정석," 성지출판