Measurement of Correlation Functions

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Based on
Probability, Random Variables and Random Signal Principles,
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Outline

in the real world, we can never measure the true correlation functions
of two random processes $X(t)$ and $Y(t)$

- each sample realization of a process is a time function
- collecting many independent sample functions costs a lot
- cannot have all sample functions of the ensemble
- can have only a portion of one sample function from each process

- can determine time averages based on finite time portion of single sample functions
- because we are able to work only with time functions, we are forced to presume that the given processe satisfy appropriate **ergodic theorems**

a possible system for measuring
the approximate time cross-correlation function
of two cross-correlation-ergodic and
stationary random processes $X(t)$ and $Y(t)$

- sample functions $x(t)$ and $y(t)$ are delayed by amounts T and $T-\tau$, respectively
- the product of the delayed waveforms
- integrate to form the output which equals the integral at time t_1+2T_s, where t_1 is arbitrary and $2T$ is the integration period

- sample functions $x(t)$ and $y(t)$ are delayed by amounts T and $T-\tau$, respectively $x(t+T)$ and $y(t+T-\tau)$
- the product of the delayed waveforms
- integrate to form the output which equals the integral at time t_1+2T_s, where t_1 is arbitrary and $2T$ is the integration period

$$R_o(t_1+2T)=\frac{1}{2T}\int_{t_1}^{t_1+2T} x(t+T)y(t+T+\tau)dt$$

Definition

assume $x(t)$ and $y(t)$ exist at least during the interval $t \geq -T$ and that $t_1 \geq 0$ is an arbitrary time instant and that $\tau \leq T$ then the output is as follows

$$R_o(t_1 + 2T) = \frac{1}{2T} \int_{t_1}^{t_1+2T} x(t+T) \cdot y(t+T-\tau) dt$$

$$= \frac{1}{2T} \int_{t_1-T}^{t_1+T} x(t) y(t+\tau) dt$$

Definition

$$R_o(t_1+2T) = \frac{1}{2T}\int_{t_1-T}^{t_1+T} x(t)y(t+\tau)dt$$

if we choose $t_1 = 0$ and assume T is large

$$R_o(2T) = \frac{1}{2T}\int_{-T}^{+T} x(t)y(t+\tau)dt \approx R_{XY}(\tau)$$

Definition

for cross-correlation ergodic processes,
approximately measuring cross-correlation
τ is varied to obtain the complete function

$$R_o(2T) = \frac{1}{2T}\int_{-T}^{+T} x(t)y(t+\tau)dt \approx R_{XY}(\tau)$$

Definition

measuring auto-correlation functions

$$R_o(2T) = \frac{1}{2T} \int_{-T}^{+T} x(t) y(t+\tau) dt = R_{XY}(\tau)$$

by applying $x(t)$ and $x(t)$

$$R_o(2T) = \frac{1}{2T} \int_{-T}^{+T} x(t) x(t+\tau) dt = R_{XX}(\tau)$$

by applying $y(t)$ and $y(t)$

$$R_o(2T) = \frac{1}{2T} \int_{-T}^{+T} y(t) y(t+\tau) dt = R_{YY}(\tau)$$