

# Measurement of Correlation Functions

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October 13, 2020

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Based on  
Probability, Random Variables and Random Signal Principles,  
P.Z. Peebles, Jr. and B. Shi



# Measuring a correlation function

$N$  Gaussian random variables

in the real world, we can never measure the true correlation functions

of two random processes  $X(t)$  and  $Y(t)$

- each sample realization of a process is a time function
- collecting many independent sample functions costs a lot
- cannot have all sample functions of the ensemble
- can have only a portion of one sample function from each process

# Measuring a correlation function

$N$  Gaussian random variables

- can determine time averages based on finite time portion of single sample functions
- because we are able to work only with time functions, we are forced to presume that the given processes satisfy appropriate **ergodic theorems**

# Measuring a correlation function

$N$  Gaussian random variables

a possible system for measuring  
the approximate time cross-correlation function  
of two cross-correlation-ergodic and  
stationary random processes  $X(t)$  and  $Y(t)$

- sample functions  $x(t)$  and  $y(t)$  are delayed by amounts  $T$  and  $T - \tau$ , respectively
- the product of the delayed waveforms
- integrate to form the output which equals the integral at time  $t_1 + 2T_s$ , where  $t_1$  is arbitrary and  $2T$  is the integration period

# Measuring a correlation function

$N$  Gaussian random variables

- sample functions  $x(t)$  and  $y(t)$  are delayed by amounts  $T$  and  $T - \tau$ , respectively  
 $x(t + T)$  and  $y(t + T - \tau)$
- the product of the delayed waveforms
- integrate to form the output which equals the integral at time  $t_1 + 2T_s$ , where  $t_1$  is arbitrary and  $2T$  is the integration period

$$R_o(t_1 + 2T) = \frac{1}{2T} \int_{t_1}^{t_1 + 2T} x(t + T)y(t + T + \tau)dt$$

# Time Cross-correlation function measurement system

$N$  Gaussian random variables

## Definition

assume  $x(t)$  and  $y(t)$  exist at least during the interval  $t \geq -T$  and that  $t_1 \geq 0$  is an arbitrary time instant and that  $\tau \leq T$  then the output is as follows

$$\begin{aligned} R_o(t_1 + 2T) &= \frac{1}{2T} \int_{t_1}^{t_1+2T} x(t+T) \cdot y(t+T-\tau) dt \\ &= \frac{1}{2T} \int_{t_1-T}^{t_1+T} x(t)y(t+\tau) dt \end{aligned}$$



# Time Cross-correlation function measurement system

$N$  Gaussian random variables

## Definition

$$R_o(t_1 + 2T) = \frac{1}{2T} \int_{t_1 - T}^{t_1 + T} x(t)y(t + \tau)dt$$

if we choose  $t_1 = 0$  and assume  $T$  is large

$$R_o(2T) = \frac{1}{2T} \int_{-T}^{+T} x(t)y(t + \tau)dt \approx R_{XY}(\tau)$$

# Time Cross-correlation function measurement system

$N$  Gaussian random variables

## Definition

for cross-correlation ergodic processes,  
approximately measuring cross-correlation  
 $\tau$  is varied to obtain the complete function

$$R_o(2T) = \frac{1}{2T} \int_{-T}^{+T} x(t)y(t+\tau)dt \approx R_{XY}(\tau)$$

# Time Cross-correlation function measurement system

$N$  Gaussian random variables

## Definition

measuring auto-correlation functions

$$R_o(2T) = \frac{1}{2T} \int_{-T}^{+T} x(t)y(t+\tau)dt = R_{XY}(\tau)$$

by applying  $x(t)$  and  $x(t)$

$$R_o(2T) = \frac{1}{2T} \int_{-T}^{+T} x(t)x(t+\tau)dt = R_{XX}(\tau)$$

by applying  $y(t)$  and  $y(t)$

$$R_o(2T) = \frac{1}{2T} \int_{-T}^{+T} y(t)y(t+\tau)dt = R_{YY}(\tau)$$



