Applications of Array Pointers (1A)

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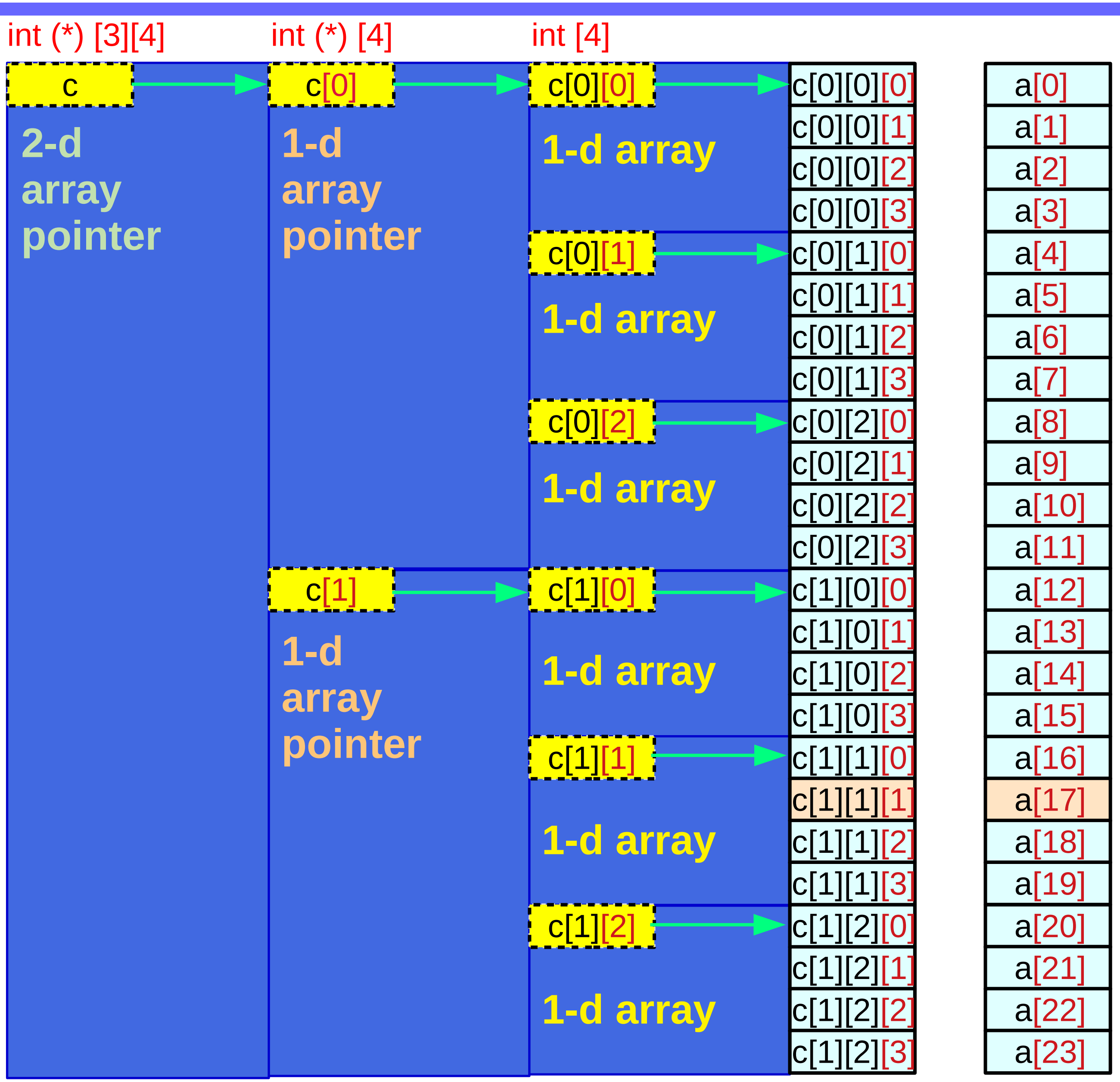
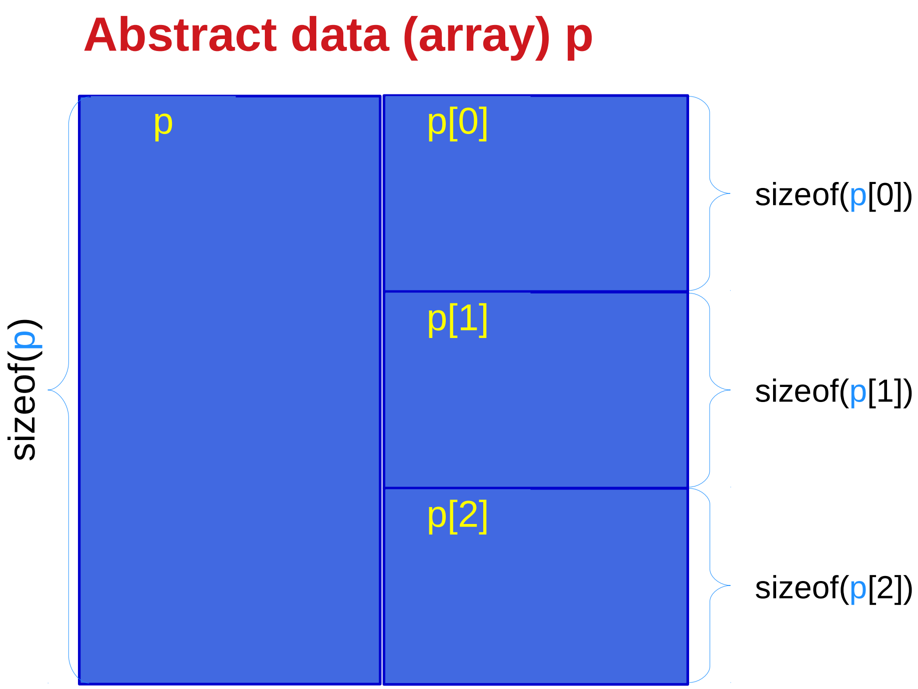
Virtual Array Pointers in Multi-dimensional Arrays

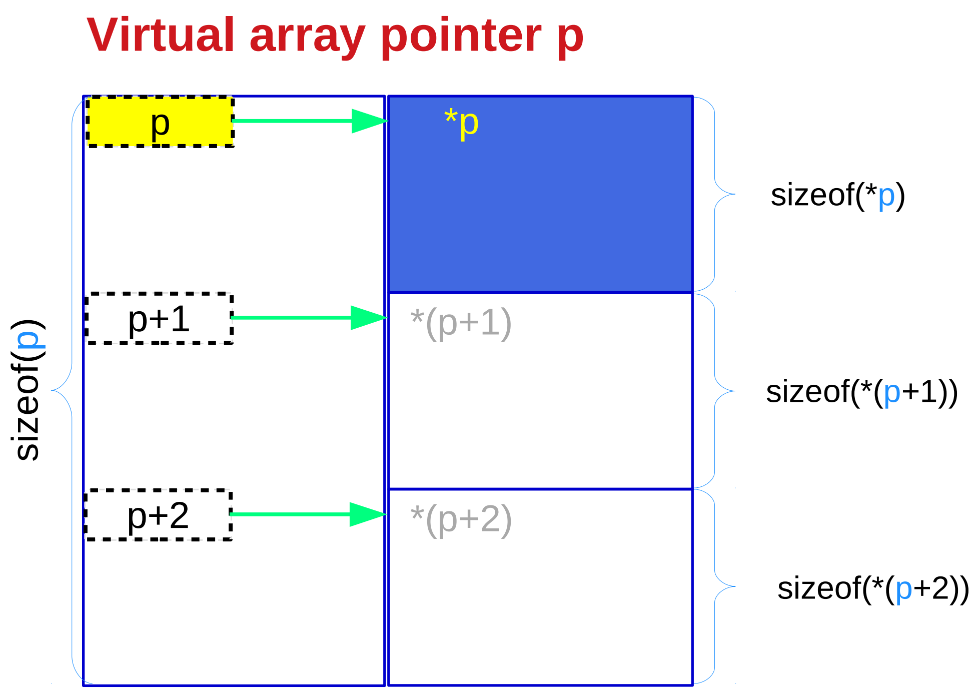
3-d array structure

- Hierarchical
- Nested Structure
- Virtual Array Pointers over
  - Contiguous
  - Linear Layout



Array **p** and virtual array pointer **p**

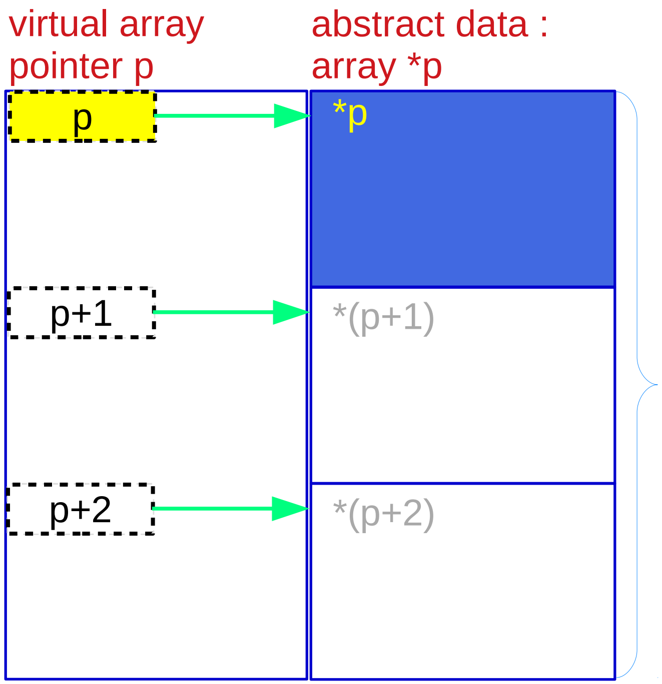


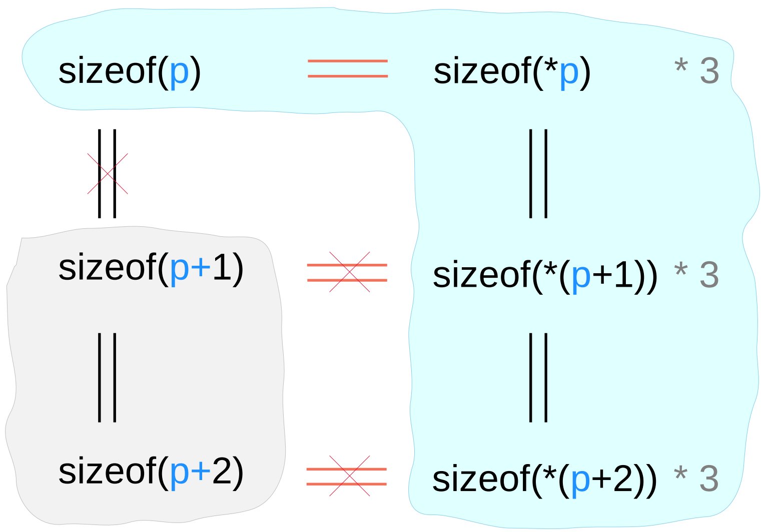


p is the name of an array and
has a array pointer type but
has a size of the array

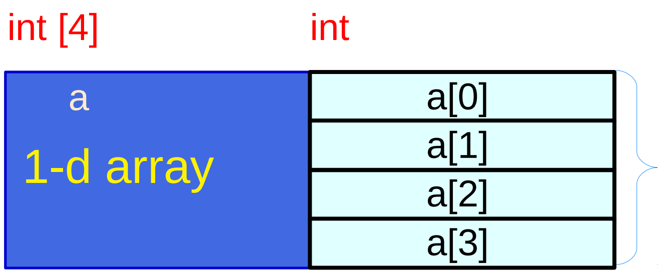
p is a virtual array pointer

Virtual array pointer to abstract data



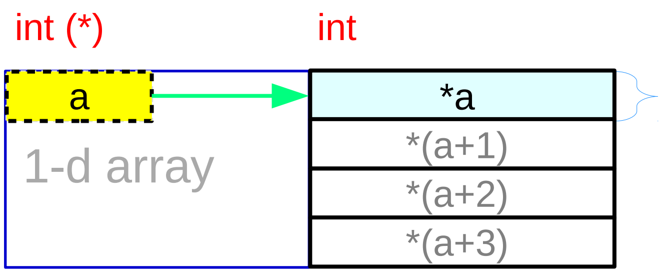


Array **a** and pointer **a**



1-d array a specific array type

sizeof(a)



pointer a general pointer type

sizeof(a) = sizeof(*a) * 4

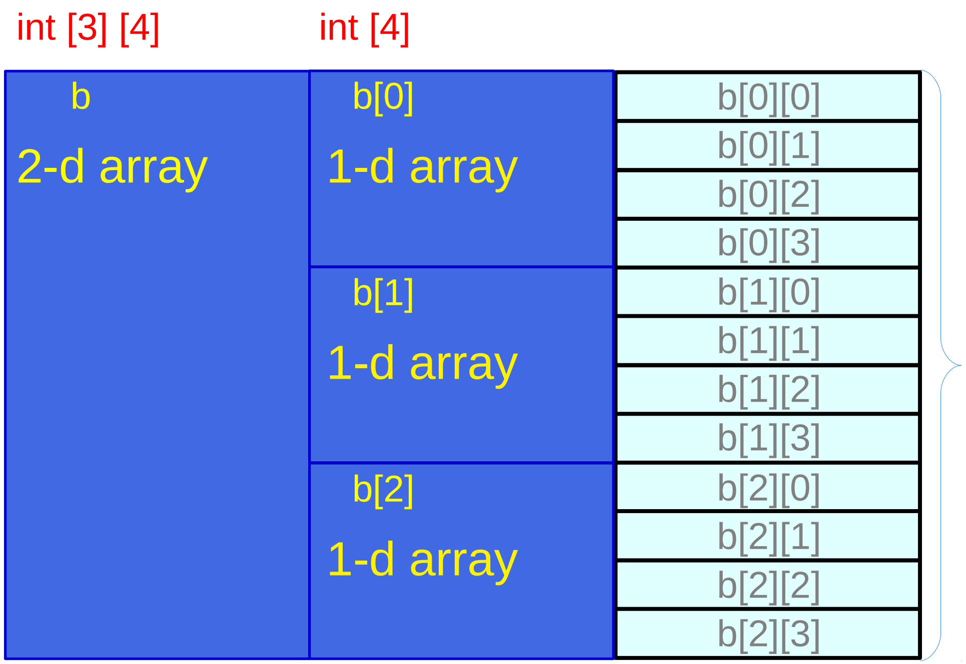
a is the name of a 1-d array and
has a pointer type but
has a size of the array

a is a virtual array pointer

Array **b** and pointer **b**

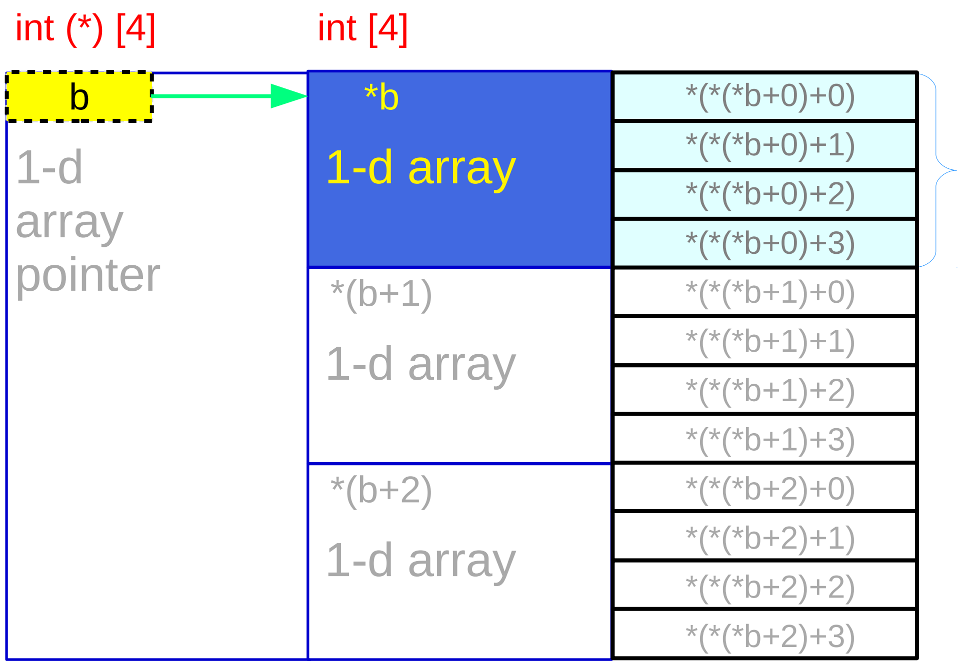
2-d array b specific array type

sizeof(b)



1-d array pointer b general pointer type

sizeof(b) = sizeof(*b) * 3



b is the name of a 2-d array and has a 1-d array pointer type but has a size of the array

b is a virtual array pointer

Array **c**

3-d array c

specific array type

sizeof(c)

c is the name of a 3-d array and has a 2-d array pointer type but has a size of the array

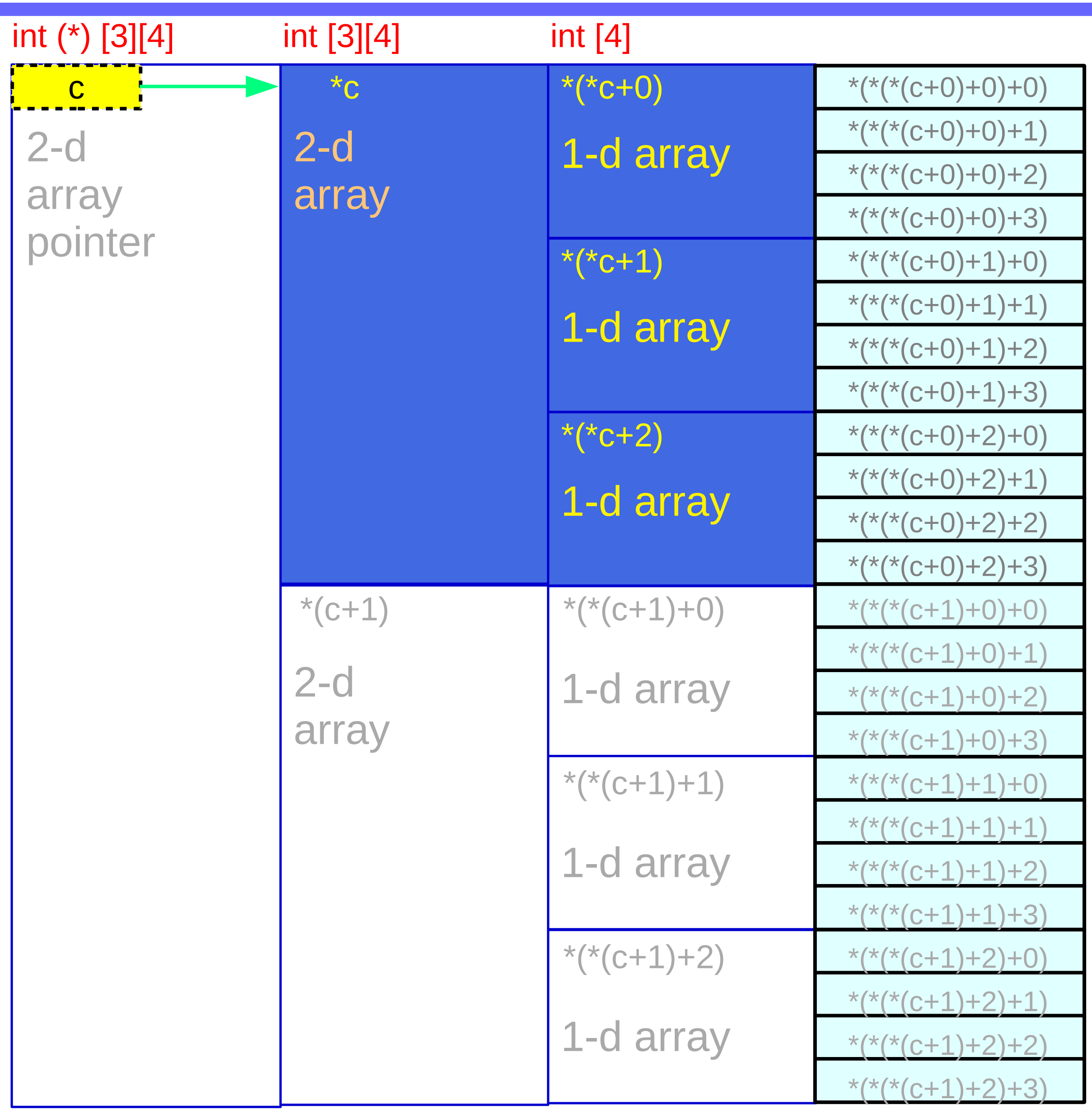
c is a virtual array pointer

Pointer **c**

2-d array pointer c

general pointer type

sizeof(c) = sizeof(*c) * 2

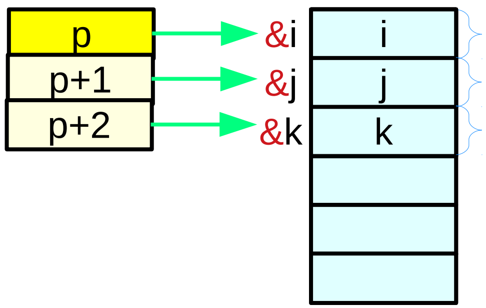


c is the name of a 3-d array and has a 2-d array pointer type but has a size of the array

c is a <u>virtual</u> array pointer

Pointers to primitive data

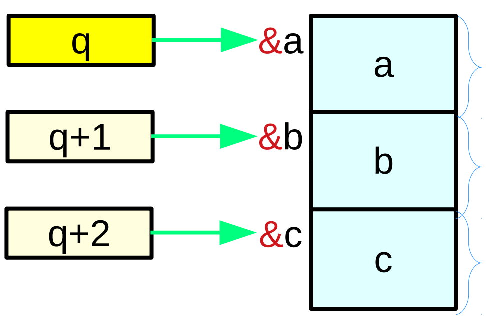
int *p; **int i, j, k;**



size = sizeof(i) = sizeof(*p) ≠ sizeof(p)
size = sizeof(j) = sizeof(*(p+1)) ≠ sizeof(p+1)
size = sizeof(k) = sizeof(*(p+2)) ≠ sizeof(p+2)

pointer size
4 or 8 bytes

double *q; **double a, b, c;**

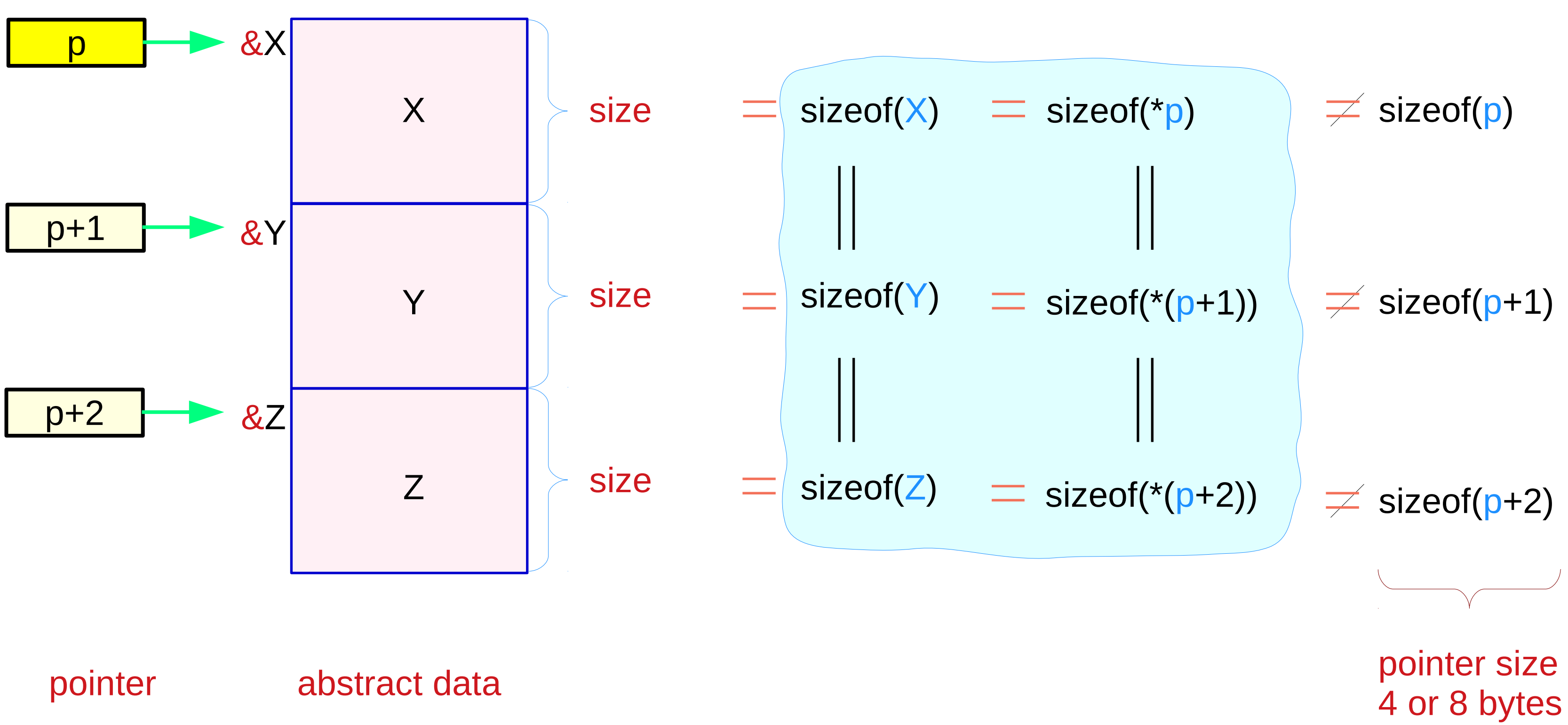


size = sizeof(a) = sizeof(*q) ≠ sizeof(q)
size = sizeof(b) = sizeof(*(q+1)) ≠ sizeof(q+1)
size = sizeof(c) = sizeof(*(q+2)) ≠ sizeof(q+2)

pointer size
4 or 8 bytes

Pointers to abstract data

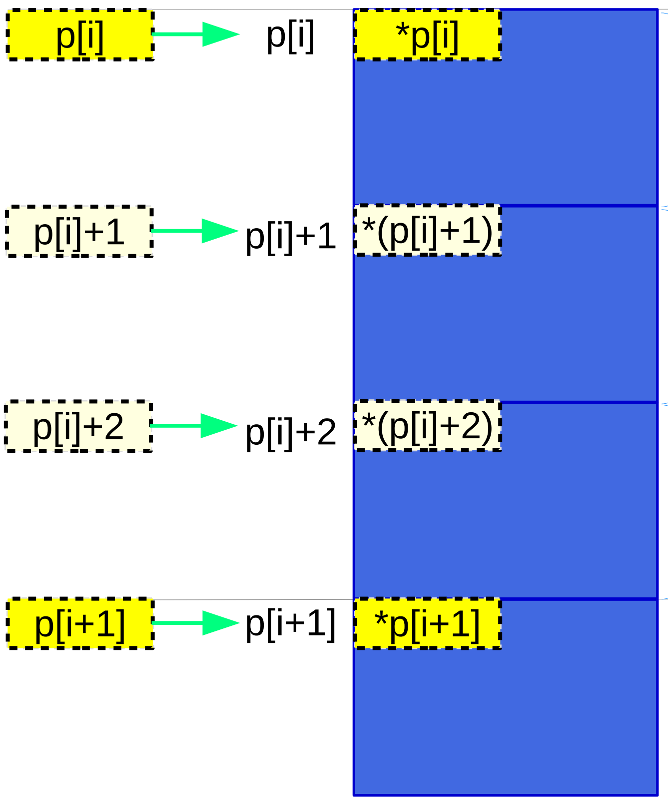
T *p; **T X, Y, Z;**



type -------- array
value -------- start address
increment size -------- size

Virtual pointers in a multi-dimensional array

p[i] :: T1 ***p[i], *p[i+1] :: T2**



size = sizeof(*p[i]) =
sizeof(p[i][0])

||

size = sizeof(*(p[i]+1)) =
sizeof(p[i][1])

||

size = sizeof(*(p[i]+2)) =
sizeof(p[i][2])

sizeof(p[i]) = sizeof(*p[i]) * N
sizeof(p[i][0]) * N

p[i] is a virtual pointer
to the abstract data *p[i] = p[i][0]
p[i] is also an abstract data

sizeof(p[i+1]) = sizeof(*p[i+1]) * N
sizeof(p[i+1][0]) * N

T1		T2		
int (*) [4]	→	int [4]	⊂	int (*)
int (*) [3][4]	→	int [3][4]	⊂	int (*)[4]

Virtual pointers in a multi-dimensional array

		T1		**T2**		
int [3] [4]	⊂	int (*) [4]	→	int [4]	⊂	int (*)
int [2] [3][4]	⊂	int (*) [3][4]	→	int [3] [4]	⊂	Int (*) [4]
specific		general		specific		general

```
typedef int (*T1) [4];          typedef int T2[4];
typedef int (*T1) [3][4];       typedef int T2[3][4];
```

```
T1    a;                        T2    b;
```

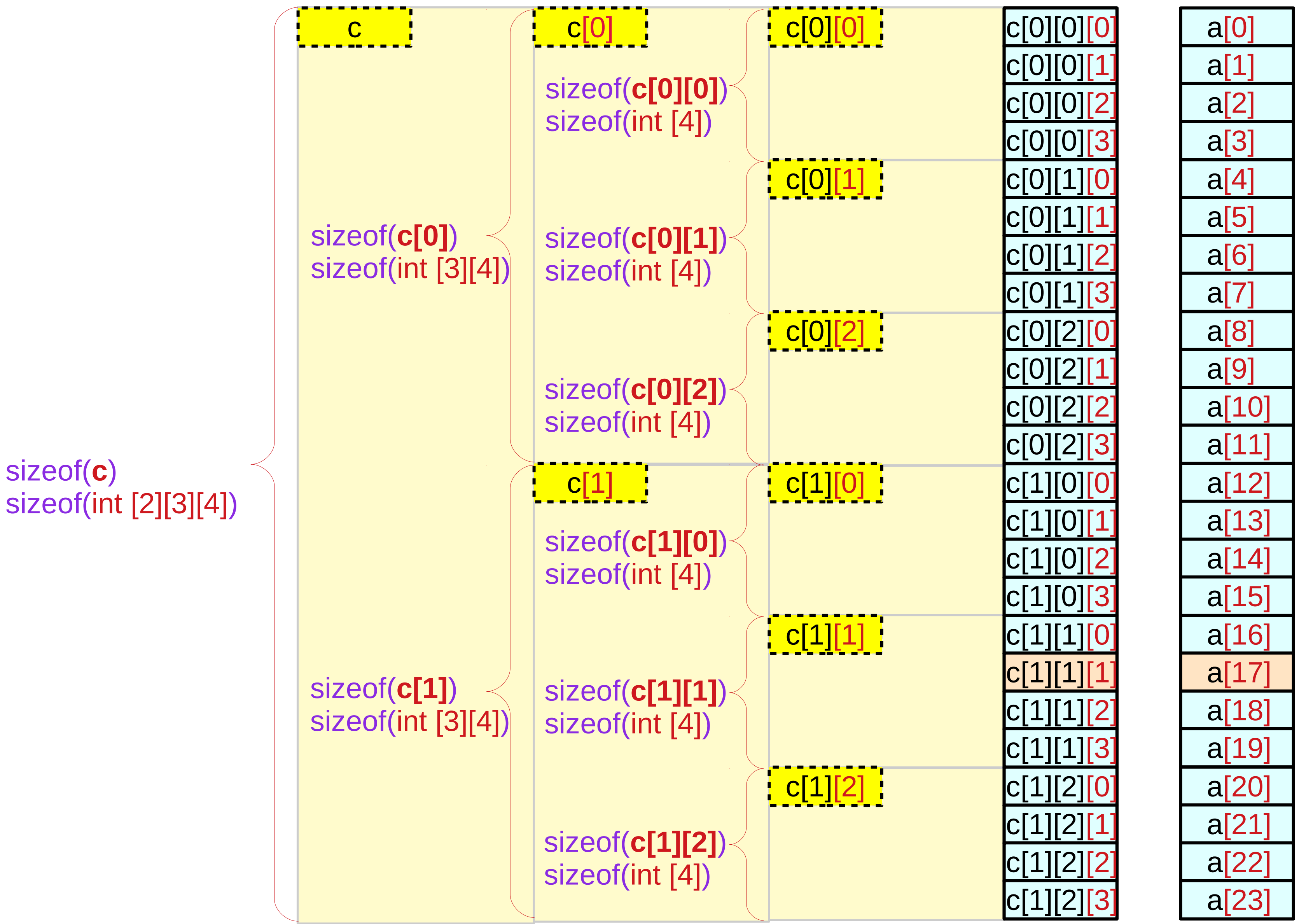
T1 references T2
T2 is a dereference of T1

T1 is a pointer type
T2 is an array type
T1 has one more dimension than T2

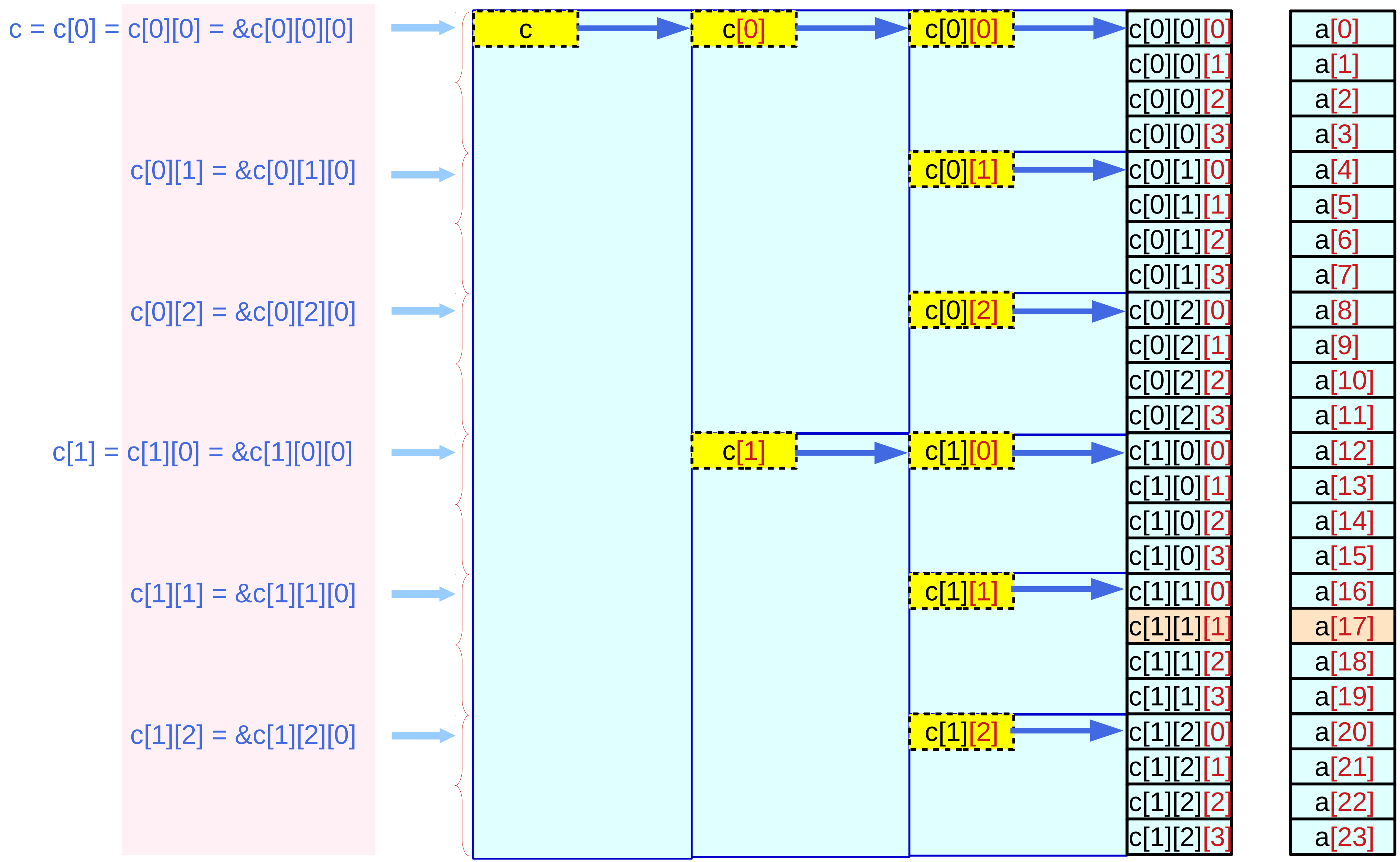
Virtual array pointers – types, sizes, and values

int c[2][3][4];	**c[i][j]**	**c[i][j][0]**	
type	int [4] int (*)	int int	• abstract data type • array pointer type
size	sizeof(c[i][j]) =	sizeof(c[i][j][0]) * **4**	= sizeof(int) * **4**
value (address)	c[i][j] =	&c[i][j][0]	
int c[2][3][4];	**c[i]**	**c[i][0]**	
type	int [3][4] int (*)[4]	int [4] int (*)	• abstract data type • array pointer type
size	sizeof(c[i]) =	sizeof(c[i][0]) * **3**	= sizeof(int) * **4** * **3**
value (address)	c[i] =	&c[i][0]	
int c[2][3][4];	**c**	**c[0]**	
type	int [2][3][4] int (*)[3][4]	int [3][4] int (*)[4]	• abstract data type • array pointer type
size	sizeof(c) =	sizeof(c[0]) * **2**	= sizeof(int) * **4** * **3** * **2**
value (address)	c =	&c[0]	

virtual array pointers c, c[i], c[i][j] – sizes



Virtual array pointer c, c[i], c[i][j] – values (addresses)



Virtual array pointer c, c[i], c[i][j] – vertical displacement

c = c[0] = c[0][0] = &c[0][0][0]

For address values

Horizontal displacements
are <u>not</u> counted

Only vertical displacements
are considered

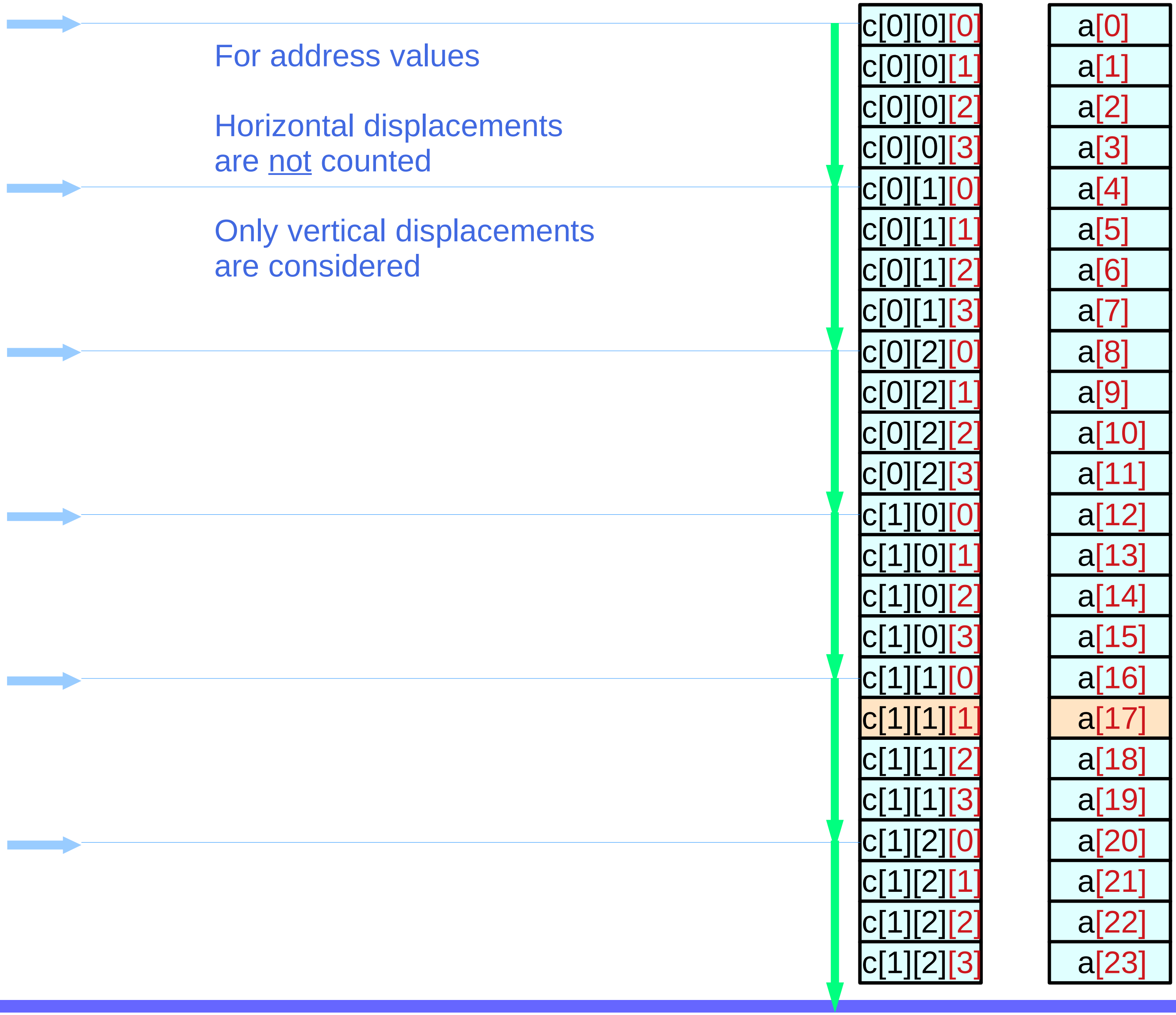
c[0][1] = &c[0][1][0]

c[0][2] = &c[0][2][0]

c[1] = c[1][0] = &c[1][0][0]

c[1][1] = &c[1][1][0]

c[1][2] = &c[1][2][0]

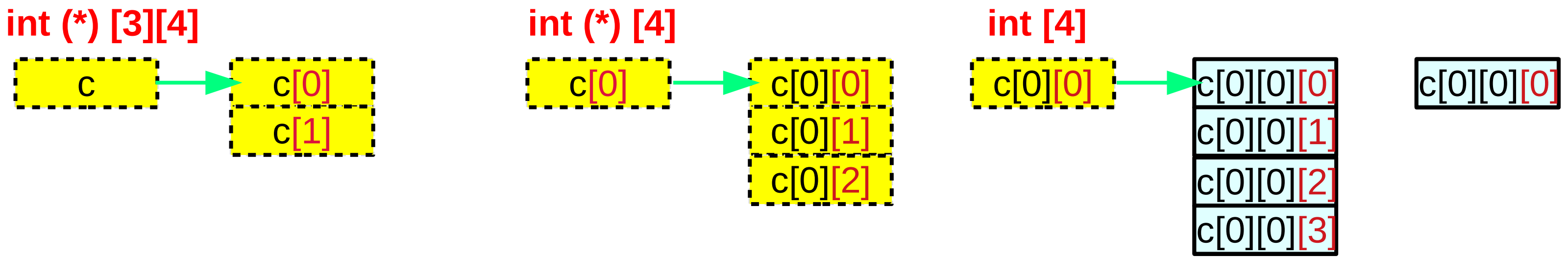


Virtual array pointer c, c[i], c[i][j] – values and types

c = c[0] = c[0][0] = &c[0][0][0]	means	value(c) = value(c[0]) = value(c[0][0]) = value(&c[0][0][0]) type(c) ≠ type(c[0]) ≠ type(c[0][0]) = type(&c[0][0][0]) int (*) [3][4] int (*) [4] int * int *
c[0][1] = &c[0][1][0]	means	value(c[0][1]) = value(&c[0][1][0]) type(c[0][1]) = type(&c[0][1][0]) int * int *
c[0][2] = &c[0][2][0]	means	value(c[0][2]) = value(&c[0][2][0]) type(c[0][2]) = type(&c[0][2][0]) int * int *
c[1] = c[1][0] = &c[1][0][0]	means	value(c[1]) = value(c[1][0]) = value(&c[1][0][0]) type(c[1]) ≠ type(c[1][0]) = type(&c[1][0][0]) int (*) [4] int * int *
c[1][1] = &c[1][1][0]	means	value(c[1][1]) = value(&c[1][1][0]) type(c[1][1]) = type(&c[1][1][0]) int * int *
c[1][2] = &c[1][2][0]	means	value(c[1][2]) = value(&c[1][2][0]) type(c[1][2]) = type(&c[1][2][0]) int * int *

Virtual array pointer c, c[0], c[0][0] – types and sizes

Types – array pointers



Sizes – abstract data

sizeof(**c**)
sizeof(**c[0]**) * 2
sizeof(**c[0][0]**) * 2 * 3
sizeof(**c[0][0][0]**) * 2 * 3 * 4

sizeof(**int [2][3][4]**)

sizeof(**c[0]**)
sizeof(**c[0][0]**) * 3
sizeof(**c[0][0][0]**) * 3 * 4

sizeof(**int [3][4]**)

sizeof(**c[0][0]**)
sizeof(**c[0][0][0]**) * 4

sizeof(**int [4]**)

sizeof(**c[0][0][0]**)

sizeof(**int**)

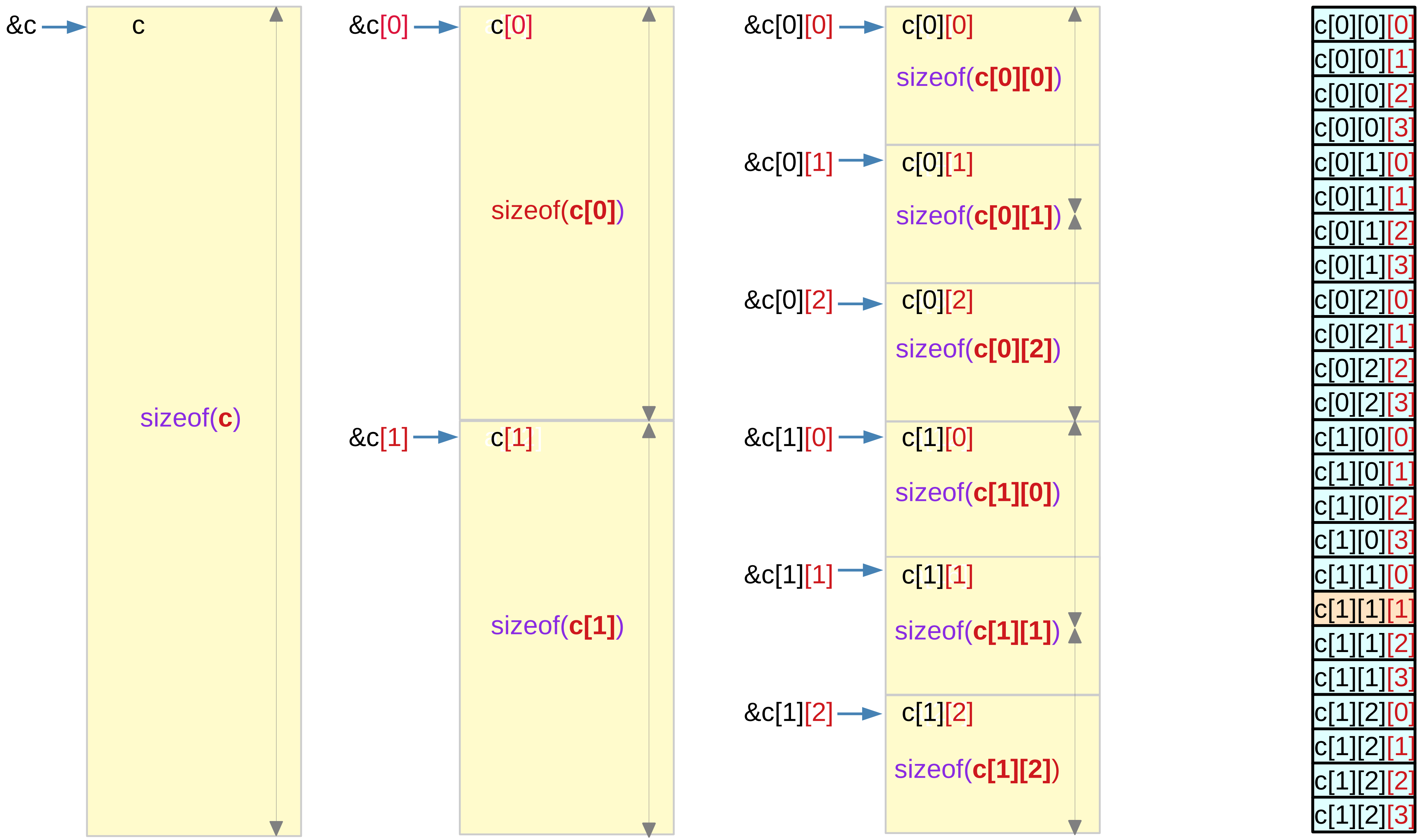
sizeof(**int [2][3][4]**) = 96
sizeof(**int (*)[3][4]**) = 4 / 8

sizeof(**int [3][4]**) = 48
sizeof(**int (*)[4]**) = 4 / 8

sizeof(**int [4]**) = 16
sizeof(**int (*)**) = 4 / 8

sizeof(**int**) = 4

Abstract Data c, c[i], c[i][j] – start addresses and sizes



Types in a multi-dimensional array

```
int     c [2][3][4];
```

abstract data int [2] [3][4]
array pointer int (*) [3][4]

c → *c (x 2)

int [3] [4] abstract data
int (*) [4] array pointer

abstract data int [3] [4]
array pointer int (*) [4]

c[i] → *c[i] (x 3)

int [4] abstract data
int (*) array pointer

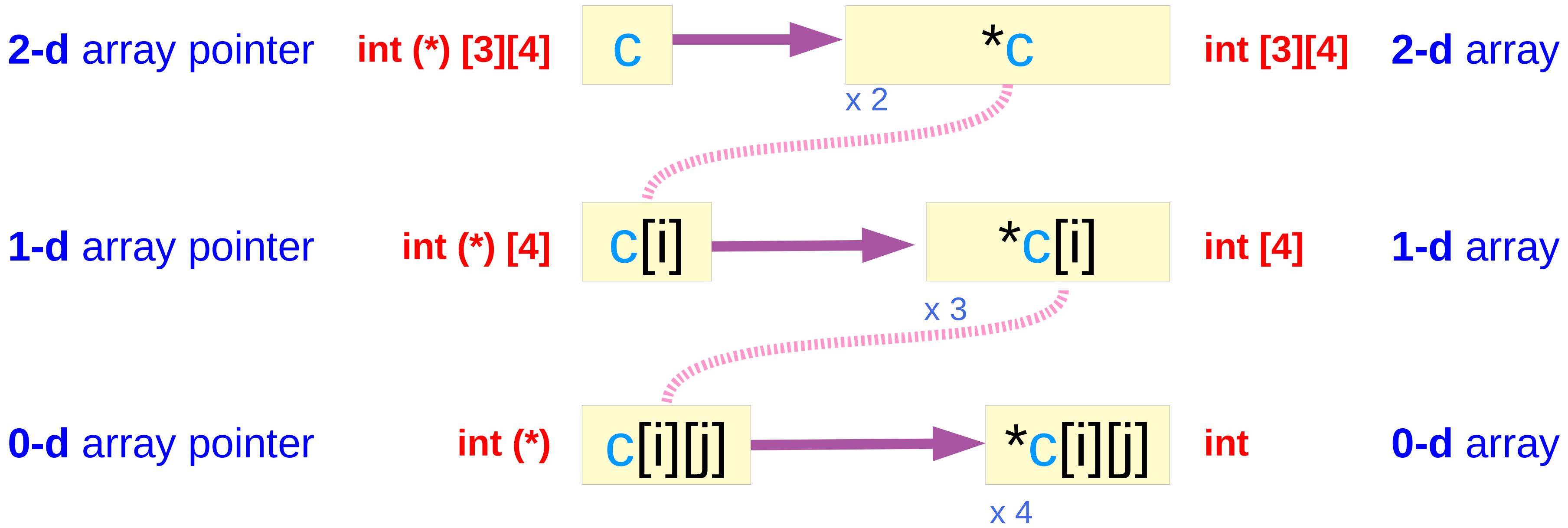
abstract data int [4]
array pointer int (*)

c[i][j] → *c[i][j] (x 4)

int primitive data

Virtual array pointers and abstract data

```
int     c [2][3][4];
```



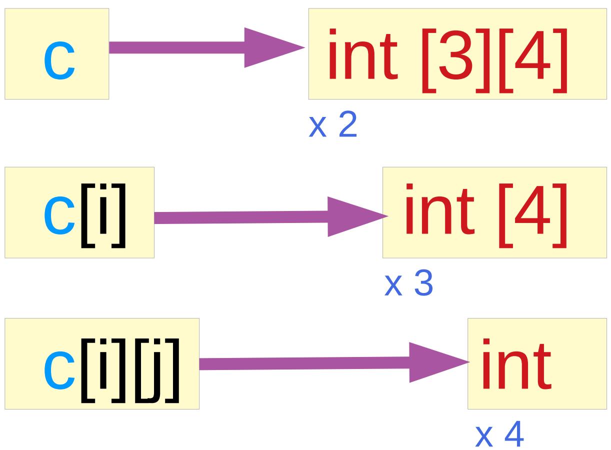
all these pointers are virtual, and take no actual memory locations

exploiting the **contiguity** of allocated memory locations

Abstract Data Sizes

```
int     c [2][3][4];
```

the size of a pointer type is fixed
Here, the sizes of virtual pointers are shown
i.e, the sizes of different abstract data types




```
sizeof( c)          = sizeof(int [2][3][4])
sizeof(*c)          = sizeof(int [3][4])

sizeof( c[i])       = sizeof(int [3][4])
sizeof(*c[i])       = sizeof(int [4])

sizeof( c[i][j])    = sizeof(int [4])
sizeof(*c[i][j])    = sizeof(int)
```

all are sizes of arrays

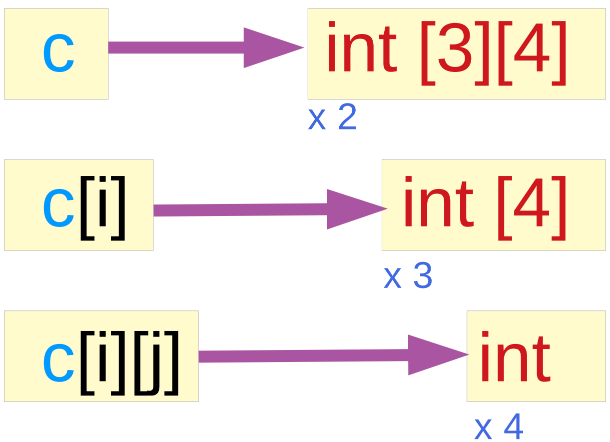
c, c[i], c[i][j] are virtual array pointers
and they are also abstract data (arrays)

when sizes are considered,
view them as abstract data (arrays)

Virtual array pointer sizes and abstract data sizes

int c [2][3][4];

size of a virtual array pointer = size of the pointed abstract data type * the number of such types



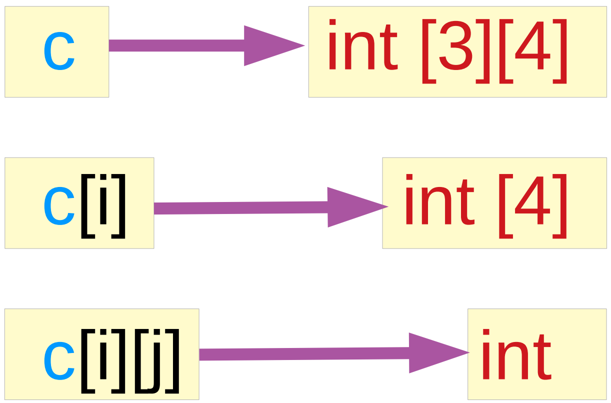
sizeof(c) = sizeof(*c) * **2**

sizeof(c[i]) = sizeof(*c[i]) * **3**

sizeof(c[i][j]) = sizeof(*c[i][j]) * **4**

Sizes of array pointer types

int c [2][3][4];



not real array pointers
virtual array pointers

c int (*)[3][4]
sizeof(int (*) [3][4]) = pointer size ≠ sizeof(c)

c[i] int (*) [4]
sizeof(int (*) [4]) = pointer size ≠ sizeof(c[i])

c[i][j] int [4]
sizeof(int [4]) = pointer size ≠ sizeof(c[i][j])

4 bytes for 32-bit machines
8 bytes for 64-bit machines

Hierarchical nested array pointers

```
int     c [2][3][4];
```

c points to a **2-d** array
increment size: sizeof(int)***2***3*4

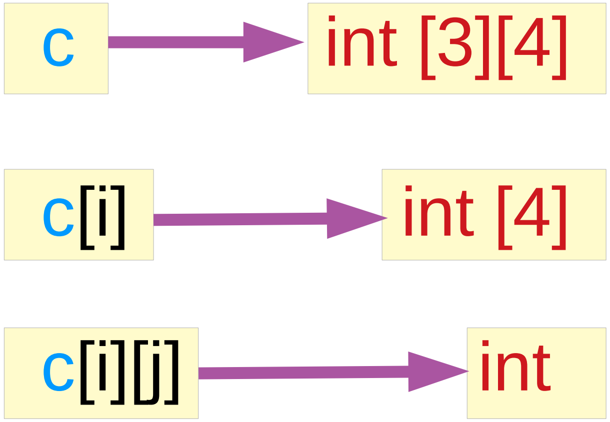
int (*) [3][4]

c[i] points to an **1-d** array
increment size: sizeof(int)***3***4

int (*) [4]

c[i][j] points to an integer
increment size: sizeof(int)***4**

int (*)



Sub-array properties in multi-dimensional arrays

int c [2][3][4]; ➡ 3-d access c **[i][j][k]**

2-d array pointer	c	int (*) [3][4]
1-d array pointers	c[i]	int (*) [4]
0-d array pointers	c[i][j]	int (*)

Hierarchical Sub-arrays in a 3-d array

```
int     c [L][M][N];
```

c [i][j][k]

left-to-right associativity

c	[i]	[j][k]
c[i]	[j]	[k]
c[i][j]	[k]	

Array Names and Types		Pointers to hierarchical sub-arrays	
c	3-d array names	int (*) [M][N]	2-d array pointer
c[i]	2-d array names	int (*) [N]	1-d array pointer
c[i][j]	1-d array names	int (*)	0-d array pointer

General requirements for accessing **c[i][j][k]**

c [**i**][**j**][**k**]

↑

&c[i][j][k] = c[i][j]+k	&c[i][j][0] = c[i][j]
&c[i][j] = c[i]+j	&c[i][0] = c[i]
&c[i] = c+i	&c[0] = c
c[i][j][k] = *(c[i][j]+k)	c[i][j][0] = *(c[i][j])
c[i][j] = *(c[i]+j)	c[i][0] = *(c[i])
c[i] = *(c+i)	c[0] = *(c)

3-d access pattern **c[i][j][k]**

General requirements

c [i][j][k]

&c[i][j][k]	= c[i][j]+k
&c[i][j]	= c[i]+j
&c[i]	= c+i

&c[i][j][0]	= c[i][j]
&c[i][0]	= c[i]
&c[0]	= c

Pointer array approach

```
int** c[2];
int*  b[2*3];
int   c[2*3*4];
```

c[i][j][k]	:: int
c[i][j]	:: int *
c[i]	:: int **

c[i]	⟸	&b[i*3]
b[j]	⟸	&a[j*4]

Explicit
Arrays of pointers with
Multiple Indirection

N-dim Array approach

```
int c[2][3][4];
```

c[i][j][k]	:: int
c[i][j]	:: int [4]
c[i]	:: int (*) [4]

c[i][j]	⟸	&c[i][j][0]
c[i]	⟸	&c[i][0][0]
c	⟸	&c[0][0][0]

Implicit
Nested
Virtual Array Pointers

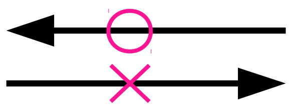
3-d access pattern **c[i][j][k]** – array pointer approach

General requirements

c [i][j][k]

&c[i][j][k] = c[i][j]+k
&c[i][j] = c[i]+j
&c[i] = c+i

&c[i][j][0] = c[i][j]
&c[i][0] = c[i]
&c[0] = c



N-dim array approach

int c[2][3][4];

c[i][j][k] :: int
c[i][j] :: int [4]
c[i] :: int (*) [4]
c :: int (*) [3][4]

c[i][j] ⟸ &c[i][j][0]
c[i] ⟸ &c[i][0][0]
c ⟸ &c[0][0][0]

Implicit
Nested
Virtual Array Pointers

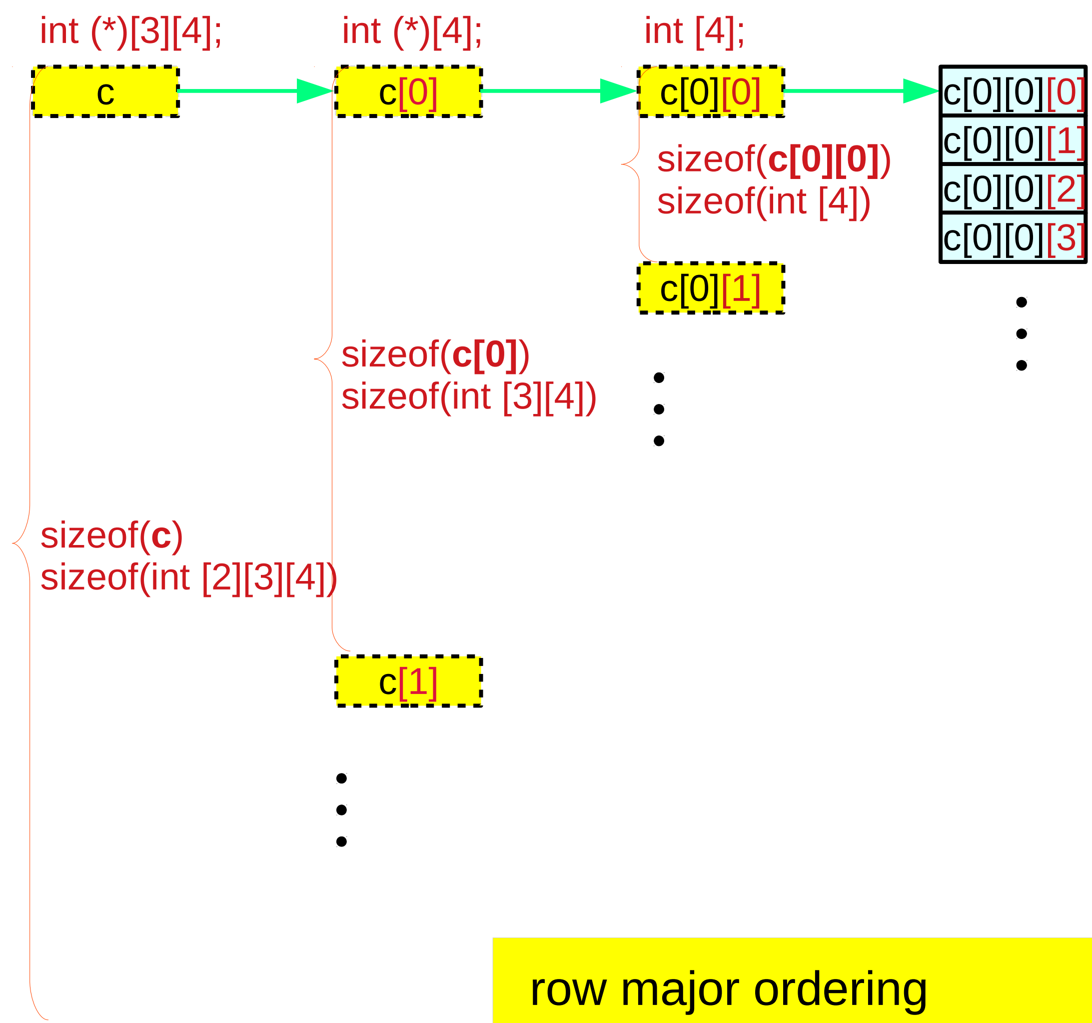
Using N-dimensional arrays

```
int     c [2][3][4];
```

```
        c [i][j][k];
```

constraints

c	⇐	&c[0][0][0]
c[i]	⇐	&c[i][0][0]
c[i][j]	⇐	&c[i][j][0]



row major ordering
contiguous linear layout

Types of **c[i]** and **c[i][j]**

c [i][j][k];

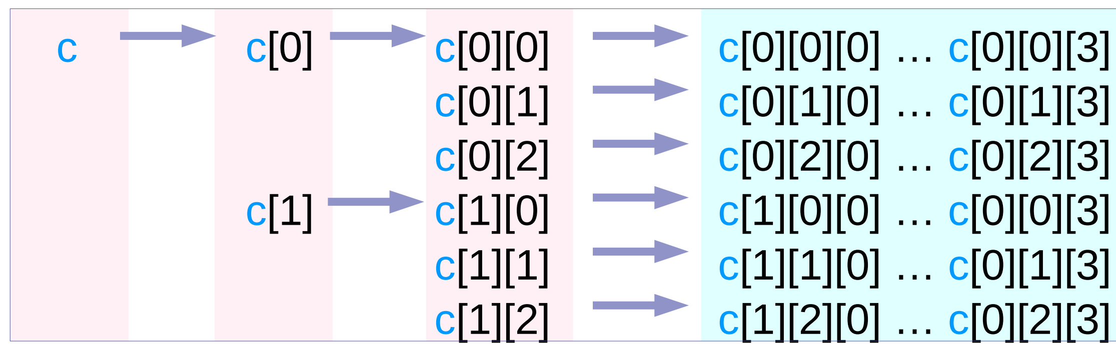
&c[i][j][0] = c[i][j]
&c[i][0] = c[i]
&c[0] = c

&c[i][j][k] = c[i][j]+k
&c[i][j] = c[i]+j
&c[i] = c+i

int c [2][3][4];

c[i] virtual array pointer of the type int (*) [4]
c[i][j] : the name of 1-d array with 4 integers int [4]

c[i][j] (virtual array) pointer of the type int (*)
c[i][j][k] : an element of a 4-integer array int



int [2] [3][4]	int [3] [4]	int [4]	int	…	int
int (*) [3][4]	int (*) [4]	int (*)	int	…	int

pointers to a 2-d array	pointers to a 1-d array	1-d array names	leading element of 4-integer array

Values of **c[i]** and **c[i][j]**

c [i][j][k];

&c[i][j][0] = c[i][j]
&c[i][0] = c[i]
&c[0] = c

&c[i][j][k] = c[i][j]+k
&c[i][j] = c[i]+j
&c[i] = c+i

int c [2][3][4];

virtual array pointers
in each row in the following figure
have the same value (address value)

c ≡	c[0] ≡	c[0][0] ≡	&c[0][0][0]
		c[0][1] ≡	&c[0][1][0]
		c[0][2] ≡	&c[0][2][0]
	c[1] ≡	c[1][0] ≡	&c[1][0][0]
		c[1][1] ≡	&c[1][1][0]
		c[1][2] ≡	&c[1][2][0]

Horizontal displacements are not counted
only **vertical displacements** are considered
for address values

c[i][j] = &c[i][j][0]
c[i] = &c[i][0][0]
c = &c[0][0][0]

Finding address values of **c**, **c[i]**, **c[i][j]**

c [i][j][k];

&c[i][j][0] = c[i][j]
&c[i][0] = c[i]
&c[0] = c

&c[i][j][k] = c[i][j]+k
&c[i][j] = c[i]+j
&c[i] = c+i

int c [2][3][4];

c[i][j] = &c[i][j][0]
c[i] = &c[i][0][0]
c = &c[0][0][0]

append [0] to the right

int (*) [3][4]		int (*) [4]		int [4]		int
c	+[0] =	c[0]	+[0] =	c[0][0]	+[0] =	&c[0][0][0]
				c[0][1]	+[0] =	&c[0][1][0]
				c[0][2]	+[0] =	&c[0][2][0]
		c[1]	+[0] =	c[1][0]	+[0] =	&c[1][0][0]
				c[1][1]	+[0] =	&c[1][1][0]
				c[1][2]	+[0] =	&c[1][2][0]

c[i][j][0] : leading elements of c[i][j]

&c[0][0][0]
&c[0][1][0]
&c[0][2][0]
&c[1][0][0]
&c[1][1][0]
&c[1][2][0]

c[i][0][0] : leading elements of c[i]

&c[0][0][0]
&c[1][0][0]

c[0][0][0] : leading elements of c

&c[0][0][0]

Finding sub arrays for the leading elements **c[i][j][0]**

c [i][j][k];

&c[i][j][0] = c[i][j]
&c[i][0] = c[i]
&c[0] = c

&c[i][j][k] = c[i][j]+k
&c[i][j] = c[i]+j
&c[i] = c+i

int c [2][3][4];

c[i][j] = &c[i][j][0]
c[i] = &c[i][0][0]
c = &c[0][0][0]

delete [0] from the right

&c[0][0][0]	-[0] =	c[0][0]	-[0] =	c[0]	-[0] =	c
&c[0][1][0]	-[0] =	c[0][1]				
&c[0][2][0]	-[0] =	c[0][2]				
&c[1][0][0]	-[0] =	c[1][0]	-[0] =	c[1]		
&c[1][1][0]	-[0] =	c[1][1]				
&c[1][2][0]	-[0] =	c[1][2]				
int		int [4]		int (*) [4]		int (*) [3][4]

c[0][0][0] is the leading element of c[0][0], c[0], c
c[0][1][0] is the leading element of c[0][1]
c[0][2][0] is the leading element of c[0][2]
c[1][0][0] is the leading element of c[1][0], c[1]
c[1][1][0] is the leading element of c[1][1]
c[1][2][0] is the leading element of c[1][2]

multi-dimensional arrays

```
c[i][j] = &c[i][j][0]
c[i]    = &c[i][0][0]
c       = &c[0][0][0]
```

⇒

```
&c[i][j][0]  = c[i][j]
&c[i][0]     = c[i]
&c[0]        = c
```

Pointer reference and dereference relationship

c [i][j][k];

&c[i][j][0] = c[i][j]
&c[i][0] = c[i]
&c[0] = c

&c[i][j][k] = c[i][j]+k
&c[i][j] = c[i]+j
&c[i] = c+i

int c [2][3][4];

c →	c[0] →	c[0][0] →	c[0][0][0]
		c[0][1] →	c[0][1][0]
		c[0][2] →	c[0][2][0]
	c[1] →	c[1][0] →	c[1][0][0]
		c[1][1] →	c[1][1][0]
		c[1][2] →	c[1][2][0]

c[i][j] = &c[i][j][0]

c →	c[0] →	c[0][0]	═	&c[0][0][0]
		c[0][1]	═	&c[0][1][0]
		c[0][2]	═	&c[0][2][0]
	c[1] →	c[1][0]	═	&c[1][0][0]
		c[1][1]	═	&c[1][1][0]
		c[1][2]	═	&c[1][2][0]

c[i] = &c[i][0]

c →	c[0]	═	&c[0][0]
	c[1]	═	&c[1][0]

c = &c[0]

c ═ c[0]

General requirements for **c[i][j][k]**

c [i][j][k];

&c[i][j][0] = c[i][j]
&c[i][0] = c[i]
&c[0] = c

&c[i][j][k] = c[i][j]+k
&c[i][j] = c[i]+j
&c[i] = c+i

int c [2][3][4];

c[i][j] virtual array pointer of the type int (*)
c[i][j][0] : leading element of a 4-integer array int

*(c[0][0]+0) = c[0][0][0]
*(c[0][1]+0) = c[0][1][0]
*(c[0][2]+0) = c[0][2][0]
*(c[1][0]+0) = c[1][0][0]
*(c[1][1]+0) = c[1][1][0]
*(c[1][2]+0) = c[1][2][0]

c[0][0] is the address of c[0][0][0]
c[0][1] is the address of c[0][1][0]
c[0][2] is the address of c[0][2][0]
c[1][0] is the address of c[1][0][0]
c[1][1] is the address of c[1][1][0]
c[1][2] is the address of c[1][2][0]

c[i] virtual array pointer of the type int (*) [4]
c[i][j] : a 4-element 1-d array name int [4]

*(c[0]+0) = c[0][0]
*(c[1]+0) = c[1][0]

c[0] is the address of c[0][0]
c[1] is the address of c[1][0]

c[i][j] = &c[i][j][0]
c[i] = &c[i][0][0]
c = &c[0][0][0]

→

&c[i][j][0] = c[i][j]
&c[i][0] = c[i]
&c[0] = c

multi-dimensional arrays

c[i][j] = &c[i][j][0] c[i] = &c[i][0][0] c = &c[0][0][0]	←	&c[i][j][0] = c[i][j] &c[i][0] = c[i] &c[0] = c

c[0] = c[0][0] relation

c [i][j][k];

&c[i][j][0] = c[i][j]
&c[i][0] = c[i]
&c[0] = c

&c[i][j][k] = c[i][j]+k
&c[i][j] = c[i]+j
&c[i] = c+i

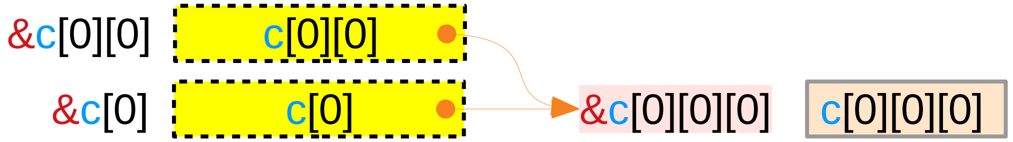
int c [2][3][4];

c ═ c[0] ═ c[0][0] ═ &c[0][0][0]

value(c[0]) = &c[0][0][0]
value(c[0][0]) = &c[0][0][0]

type(c[0]) = int (*)[4]
type(c[0][0]) = int [4]

c[0] = c[0][0] *means*
value(c[0]) = value(c[0][0])



c[i][j] = &c[i][j][0]
c[i] = &c[i][0][0]
c = &c[0][0][0]

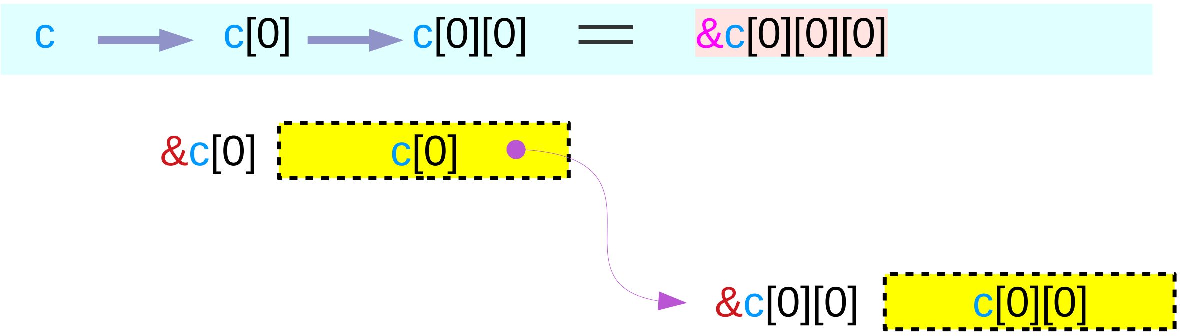
Addresses and Values of **c[0]** and **c[0][0]**

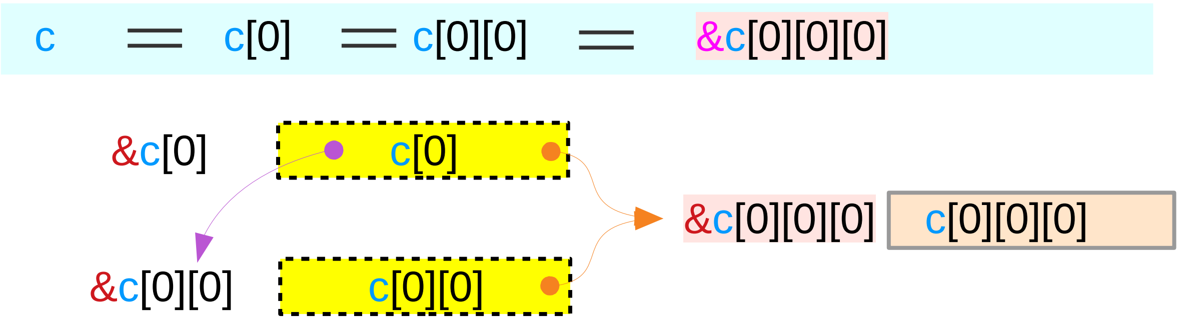
c [i][j][k];

&c[i][j][0] = c[i][j]
&c[i][0] = c[i]
&c[0] = c

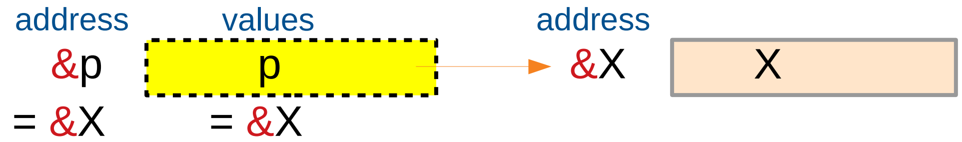
&c[i][j][k] = c[i][j]+k
&c[i][j] = c[i]+j
&c[i] = c+i

int c [2][3][4];





A virtual pointer's address and value are the same



c[0] and c[0][0] point to the same c[i][0][0]

c [i][j][k];

&c[i][j][0] = c[i][j]
&c[i][0] = c[i]
&c[0] = c

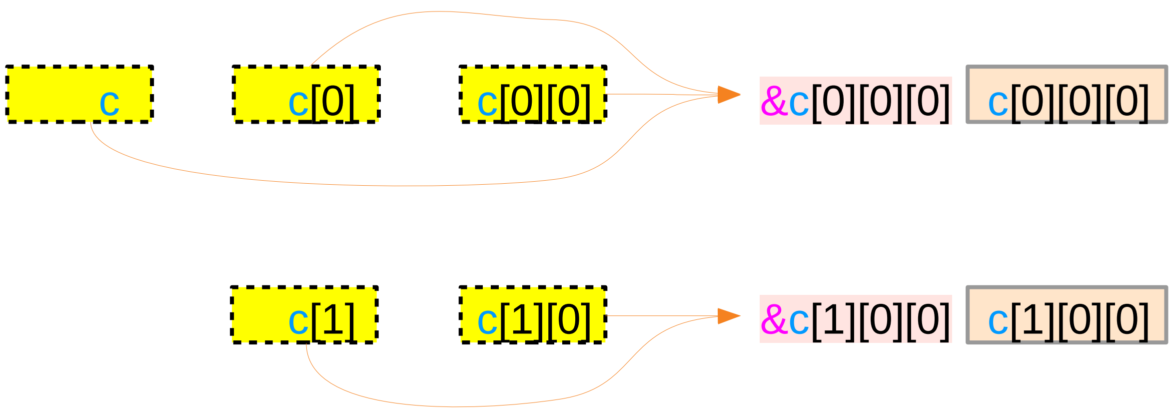
&c[i][j][k] = c[i][j]+k
&c[i][j] = c[i]+j
&c[i] = c+i

int c [2][3][4];

c[i][j] = &c[i][j][0]
c[i] = &c[i][0][0]
c = &c[0][0][0]

c =	c[0] =	c[0][0] =	&c[0][0][0]	⟸ value
int(*)[3][4]	int(*)[4]	int(*)	int	⟸ type

c[1] =	c[1][0] =	&c[1][0][0]	⟸ value
int(*)[4]	int(*)	int	⟸ type



These virtual pointers have different types
but the same value (address)

&c[i][0] and &c[i][0][0] – equivalence relations

c [i][j][k];

&c[i][j][0] = c[i][j]
&c[i][0] = c[i]
&c[0] = c

&c[i][j][k] = c[i][j]+k
&c[i][j] = c[i]+j
&c[i] = c+i

int c [2][3][4];

c[i][j] = &c[i][j][0]
c[i] = &c[i][0][0]
c = &c[0][0][0]

int(*)[3][4] int(*)[4] int(*) int *

c = c[0] = c[0][0] = &c[0][0][0]

&

&c &c[0] &c[0][0]

c[1] = c[1][0] = &c[1][0][0]

&

&c[1] &c[1][0]

Horizontal displacements are not counted
only vertical displacements are considered
for address values

equivalences

c ≡ &c[0],
c[0] ≡ &c[0][0]
c[0][0] ≡ &c[0][0][0]

equivalences

c[1] ≡ &c[1][0]
c[1][0] ≡ &c[1][0][0]

equivalences

c ≡ &c[0],
c[i] ≡ &c[i][0]
c[i][0] ≡ &c[i][0][0]

c[i] = &c[i] and c[i][0] = &c[i][0]

c [i][j][k];

&c[i][j][0] = c[i][j]
&c[i][0] = c[i]
&c[0] = c

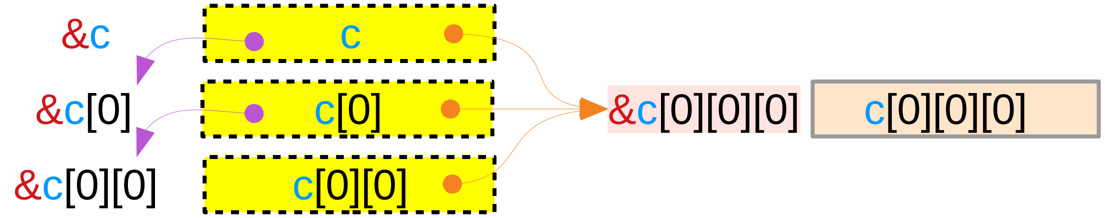
&c[i][j][k] = c[i][j]+k
&c[i][j] = c[i]+j
&c[i] = c+i

int c [2][3][4];

c[i][j] = &c[i][j][0]
c[i] = &c[i][0][0]
c = &c[0][0][0]

c = c[0] = c[0][0]= &c[0][0][0]
|| || ||
&c = &c[0] = &c[0][0]

c[1] = c[1][0] = &c[1][0][0]
|| ||
&c[1] = &c[1][0]



c[i] = &c[i] and c[i][0] = &c[i][0]

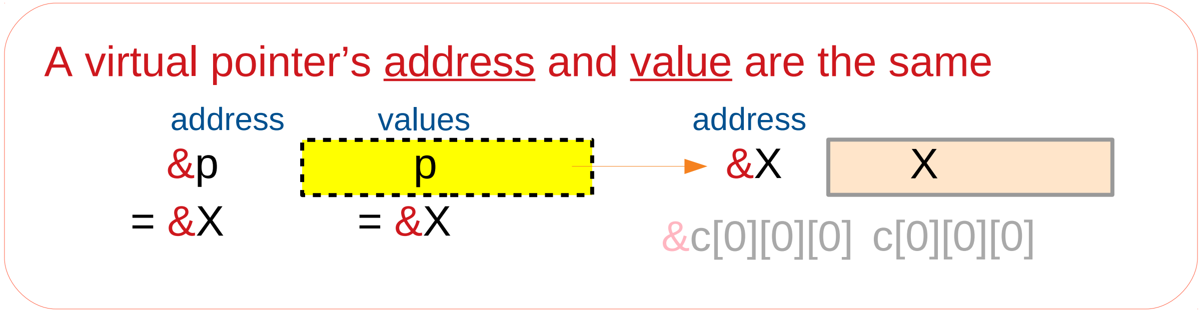
c [i][j][k];

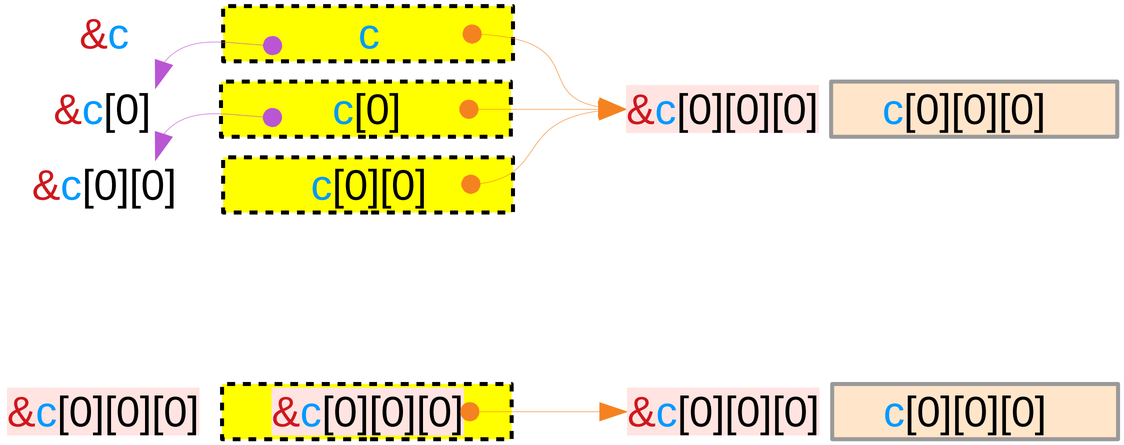
&c[i][j][0] = c[i][j]
&c[i][0] = c[i]
&c[0] = c

&c[i][j][k] = c[i][j]+k
&c[i][j] = c[i]+j
&c[i] = c+i

int c [2][3][4];

c[i][j] = &c[i][j][0]
c[i] = &c[i][0][0]
c = &c[0][0][0]





Leading elements and array pointers

c[0][0][0] is the leading element of c[0][0], c[0], c
c[0][1][0] is the leading element of c[0][1]
c[0][2][0] is the leading element of c[0][2]
c[1][0][0] is the leading element of c[1][0], c[1]
c[1][1][0] is the leading element of c[1][1]
c[1][2][0] is the leading element of c[1][2]

Array Pointers to **c[i][0][0]**

c [i][j][k];

&c[i][j][0] = c[i][j]
&c[i][0] = c[i]
&c[0] = c

&c[i][j][k] = c[i][j]+k
&c[i][j] = c[i]+j
&c[i] = c+i

int c [2][3][4];

c[i][j] = &c[i][j][0]
c[i] = &c[i][0][0]
c = &c[0][0][0]

&c[i][0][0] ≡ c[i][0]

&c[i][0] ≡ c[i]

&c[i] ≡ c+i

virtual pointers:
the address of a pointer is
the same as its value

= c + i*sizeof(*c)

= &c[0][0][0] + i*3*4

delete [0] from the right

&c[0][0][0]	-[0] ≡	c[0][0]	-[0] ≡	c[0]	-[0] ≡	c
&c[1][0][0]	-[0] ≡	c[1][0]	-[0] ≡	c[1]		

Array Pointers to **c[i][j][0]**

c [i][j][k];

&c[i][j][0] = c[i][j]
&c[i][0] = c[i]
&c[0] = c

&c[i][j][k] = c[i][j]+k
&c[i][j] = c[i]+j
&c[i] = c+i

int c [2][3][4];

c[i][j] = &c[i][j][0]
c[i] = &c[i][0][0]
c = &c[0][0][0]

&c[i][j][0] ≡ c[i][j]

||

&c[i][j] ≡ c[i] + j

= c[i] + j*sizeof(*c[i])

= c + i*sizeof(*c) + j*4

= &c[0][0][0] + i*3*4 + j*4

delete [0] from the right

&c[0][0][0]	$\overset{-[0]}{=}$	c[0][0]	$\overset{-[0]}{=}$	c[0]	$\overset{-[0]}{=}$	c
&c[0][1][0]	$\overset{-[0]}{=}$	c[0][1]				
&c[0][2][0]	$\overset{-[0]}{=}$	c[0][2]				
&c[1][0][0]	$\overset{-[0]}{=}$	c[1][0]	$\overset{-[0]}{=}$	c[1]		
&c[1][1][0]	$\overset{-[0]}{=}$	c[1][1]				
&c[1][2][0]	$\overset{-[0]}{=}$	c[1][2]				

Contiguity Constraints

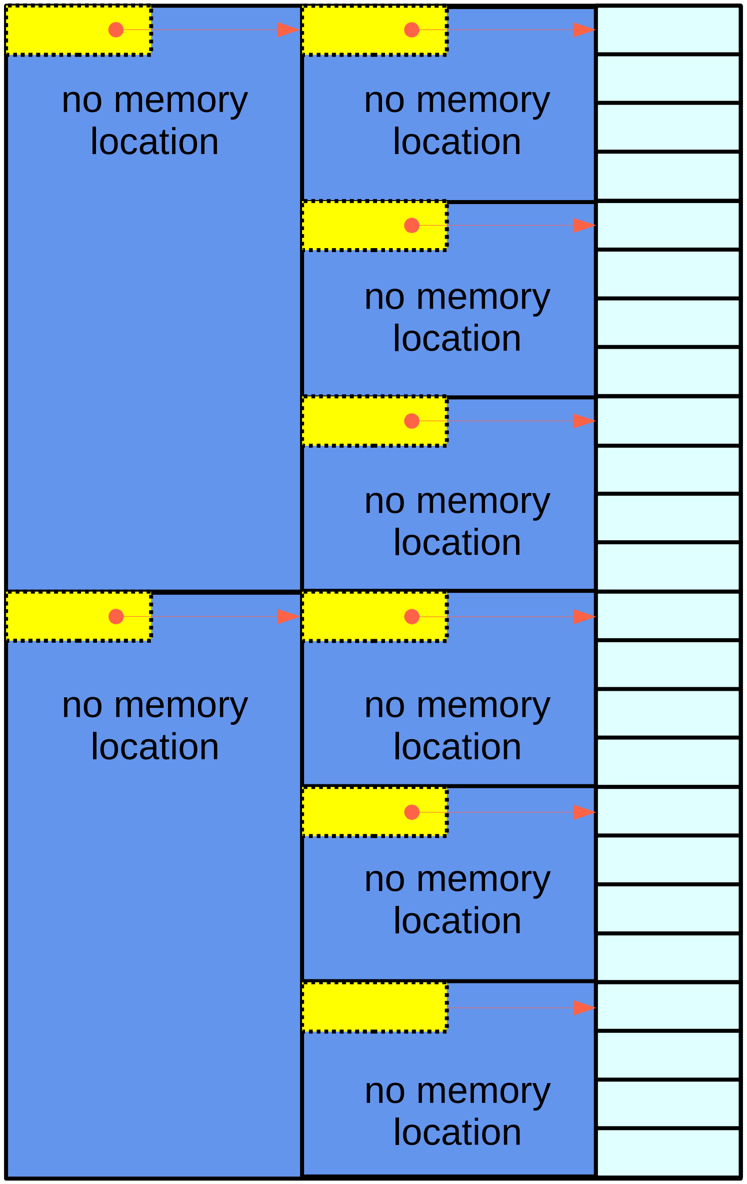
c **[i][j][k]**;

Virtual Array Pointers and Contiguity

Using array pointers

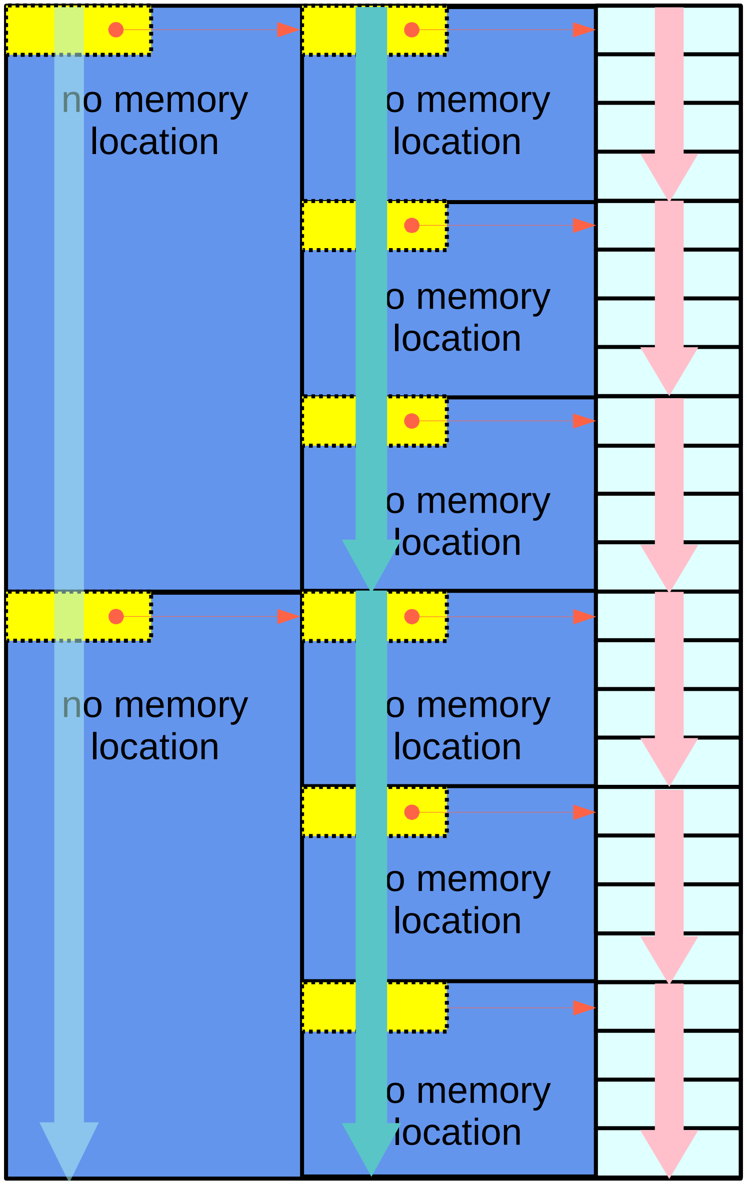
int (*) [N], int (*) [M][N], int (*) [L][M][N], …

Array pointer approach for 3-d access patterns



Array Pointer Approach (pointer to arrays)

Array pointer approach – contiguity constraints



Array Pointer Approach (pointer to arrays)

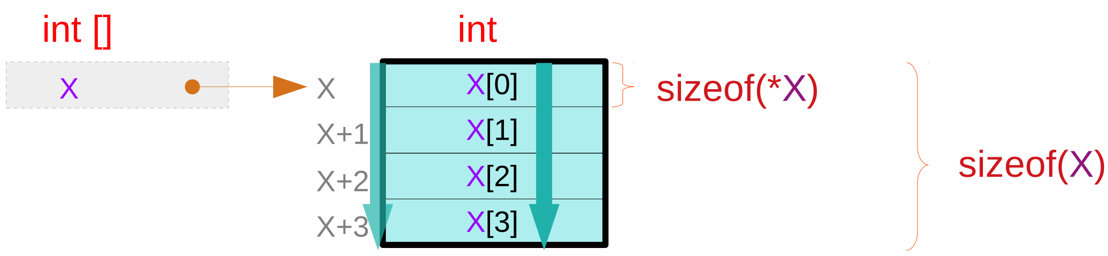
Equivalence and contiguity (1)

consecutive address

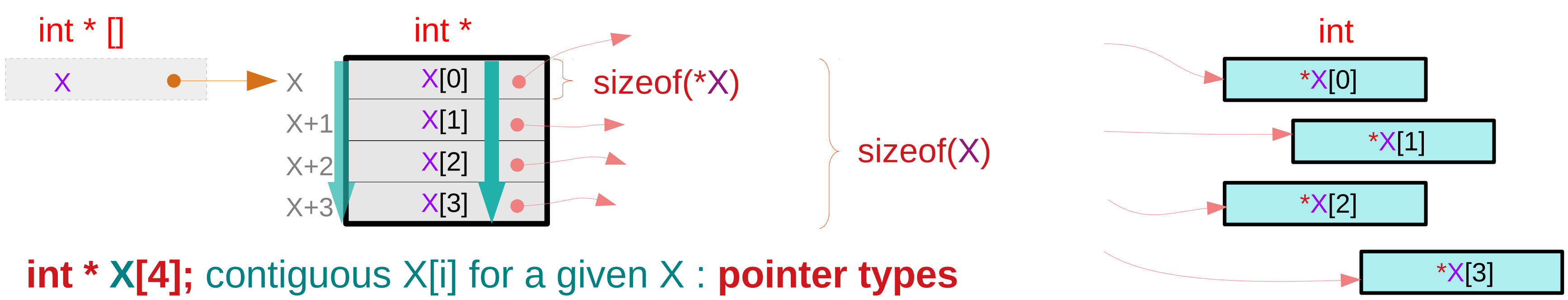
consecutive data

$*(X+n) \equiv X[n]$

contiguous index : n



int X[4]; contiguous X[i] for a given X : **primitive types**



int * X[4]; contiguous X[i] for a given X : **pointer types**

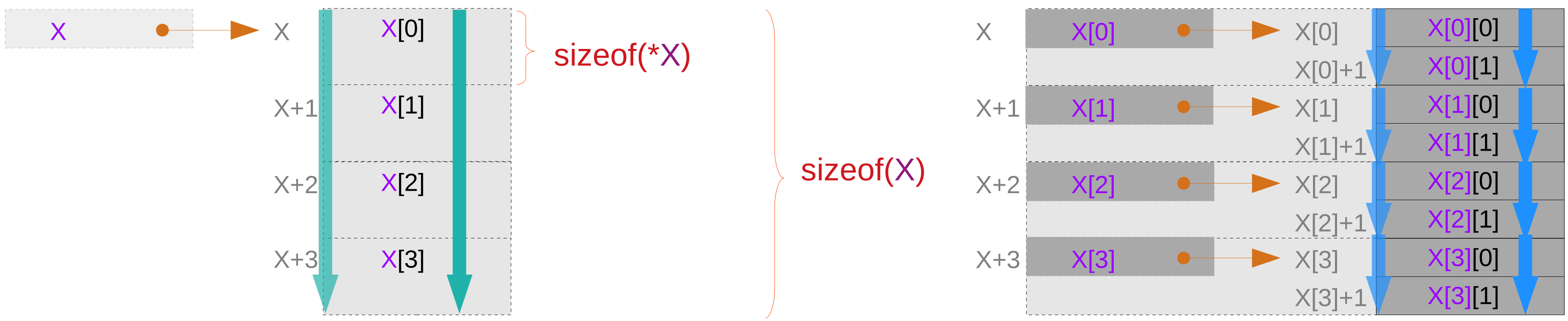
Equivalence and contiguity (2)

consecutive address — consecutive data

$*(X+n) \equiv X[n]$

contiguous index : n

can be recursively applied



atype * X[4]; contiguous X[i] for a given X : **abstract data types**

Recursive applications of equivalences

By definition, contiguous memory locations are assumed

consecutive address		consecutive data
*(X+n)	≡	X[n]

contiguous index : n

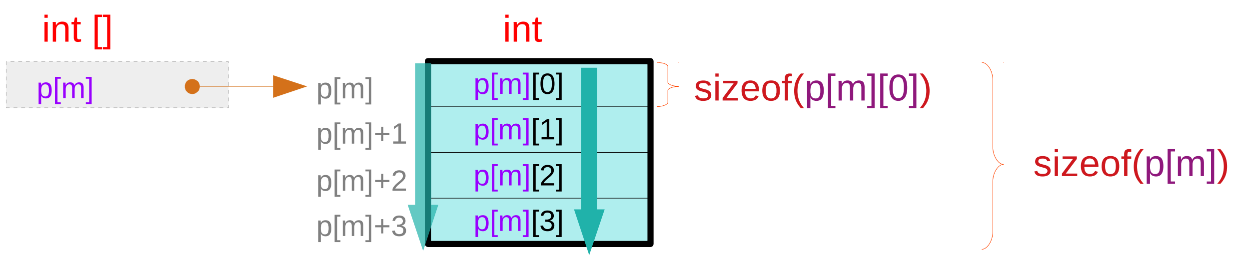
*(p[m]+n)	⟷	p[m][n]	X = p[m]	contiguous index : **n**
(*(p+m))[n];	⟷	p[m][n];	X = p	contiguous index : **m**

Equivalence for a given p[m] (1)

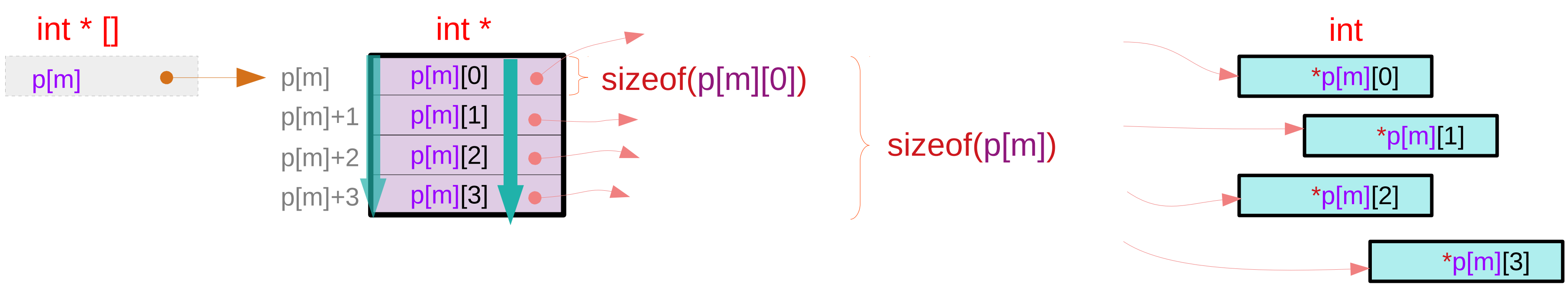
*(p[m]+n) ⟺ p[m][n]

for a given p[m]   contiguous index : **n**

int p[M][4]; contiguous p[m][n] for a given p[m] : **primitive types**   m = 0, 1, …, M-1



int * p[M][4]; contiguous p[m][n] for a given p[m] : **pointer types**   m = 0, 1, …, M-1

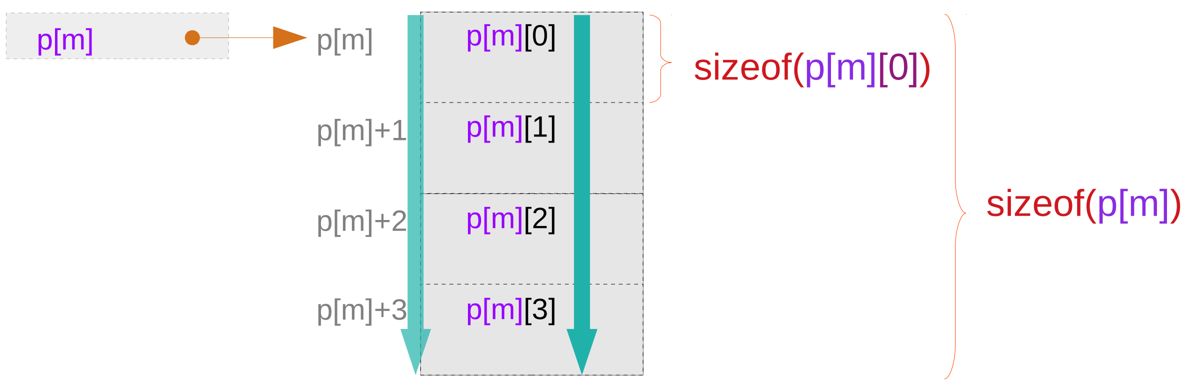


Equivalence for a given p[m] (2)

*(p[m]+n) ⟺ p[m][n]

for a given p[m]   contiguous index : **n**

atype * p[M][4]; contiguous p[m][n] for a given p[m] : **abstract data types**   m = 0, 1, …, M-1



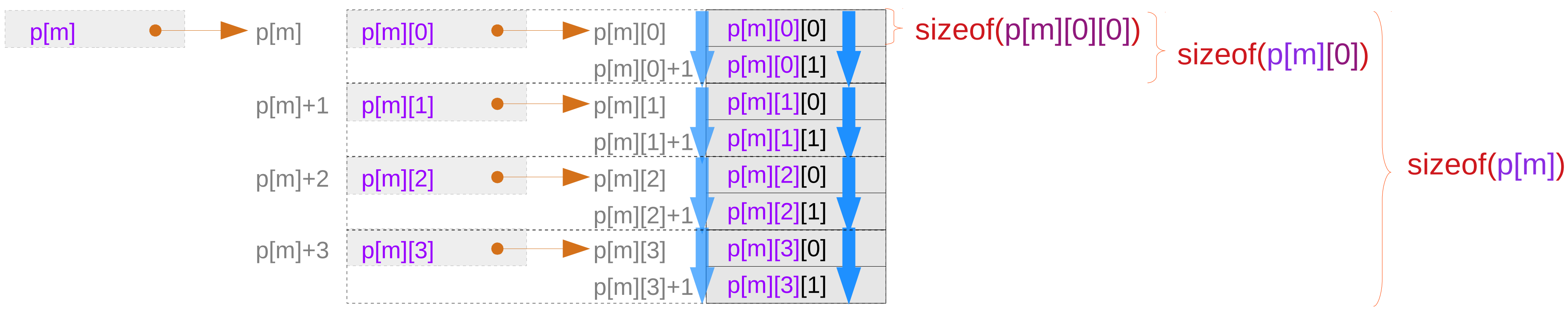
can be recursively applied

Equivalence for a given p[m][n]

*(p[m][n]+k) ⟺ p[m][n][k]

for a given p[m][n] contiguous index : **k**

atype * p[M][4][2]; contiguous p[m][n][k] for a given p[m][n] : **abstract data types** m = 0, 1, …, M-1

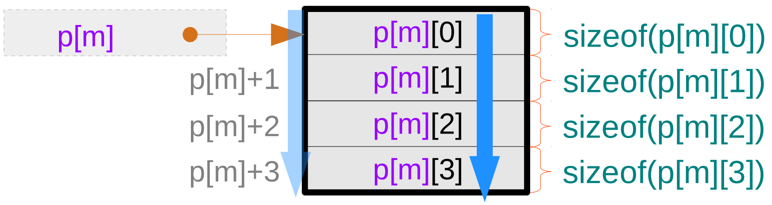


Contiguity constraints in multi-dimensional arrays

*(p[m]+n) ⟺ p[m][n]

for a given p[m], thus for a given p and m,
p[m][n]'s must be contiguous for all n.
p[m][0], p[m][1], … , p[m][N-1]

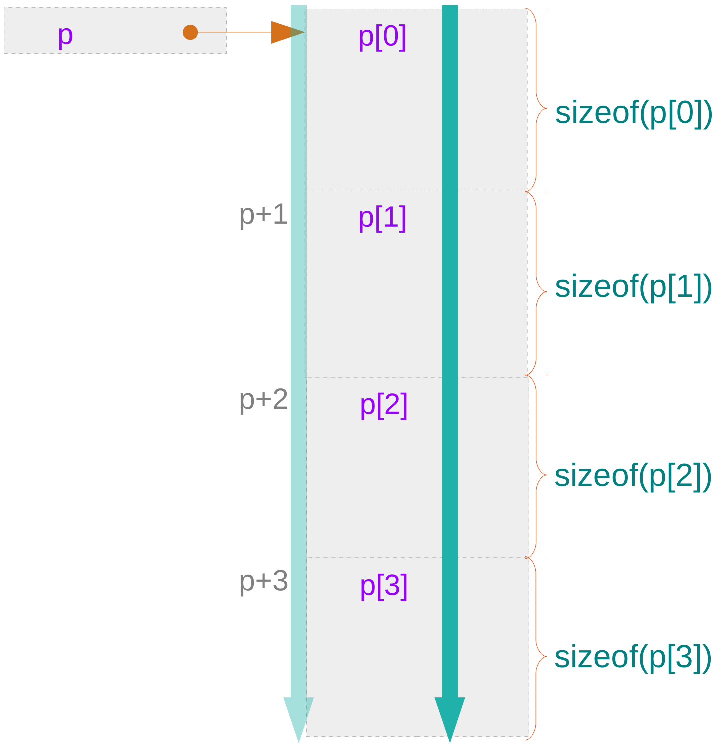
contiguous index : n



*(p+m) ⟺ p[m]

for a given p,
p[m]'s must be contiguous for all m.
p[0], p[1], … , p[M-1]

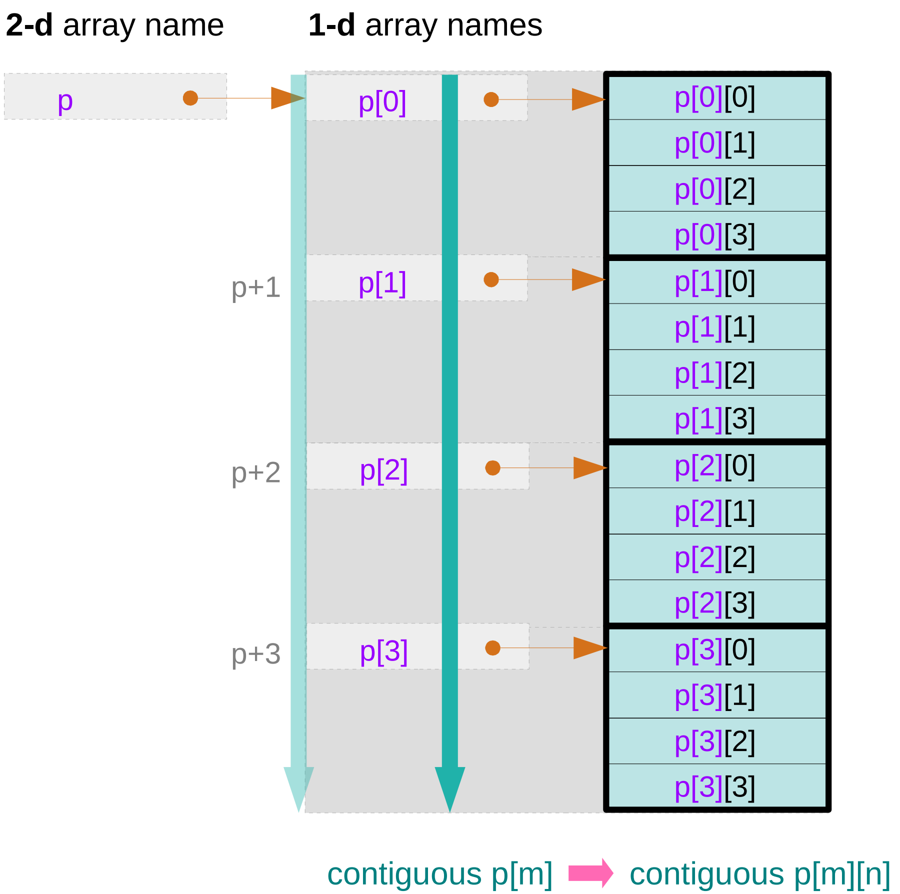
contiguous index : m

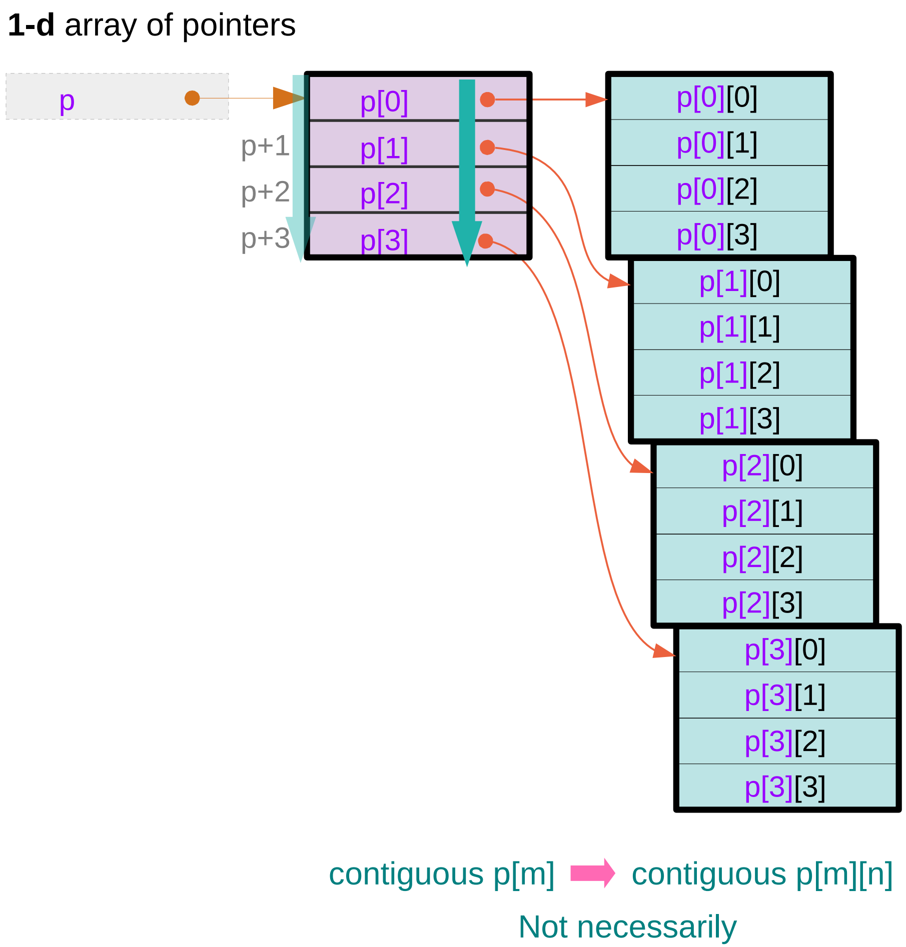


Contiguity constraints for p

*(p+m) ⟺ p[m]

for a given p   contiguous index : **m**



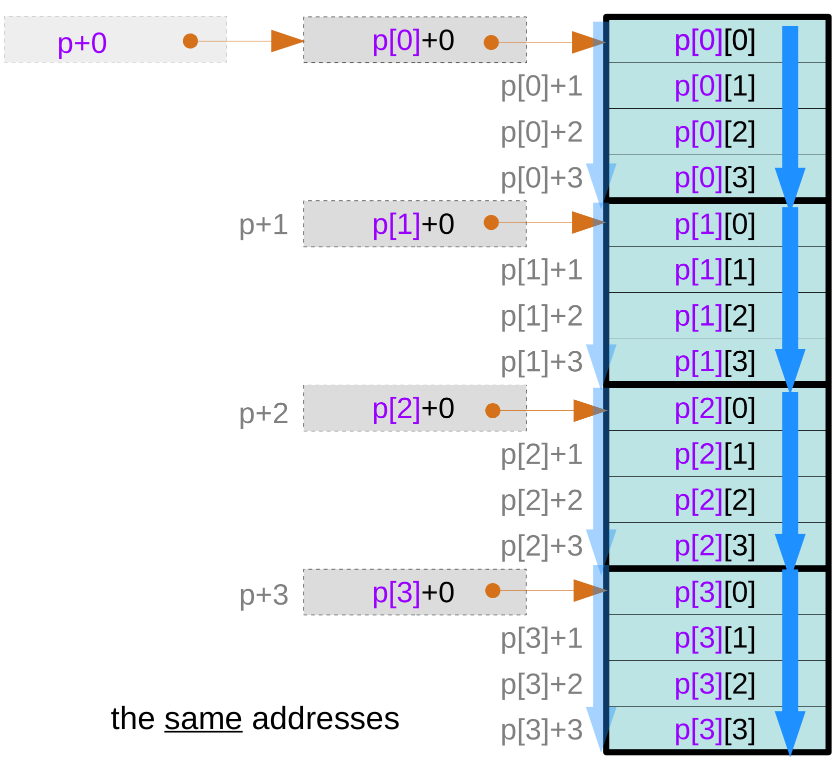


Contiguity constraints for p[m] – using array pointers

*(p[m]+n) ⟺ p[m][n]

for a given p[m] contiguous index : **n**

2-d array name **1-d** array names



p[0][0] = *(p[0]+0) addr ⇒ &p[0][0] = p[0] addr ⇒ p+0

p[1][0] = *(p[1]+0) addr ⇒ &p[1][0] = p[1] addr ⇒ p+1

p[2][0] = *(p[2]+0) addr ⇒ &p[2][0] = p[2] addr ⇒ p+2

p[3][0] = *(p[3]+0) addr ⇒ &p[3][0] = p[3] addr ⇒ p+3

the same addresses

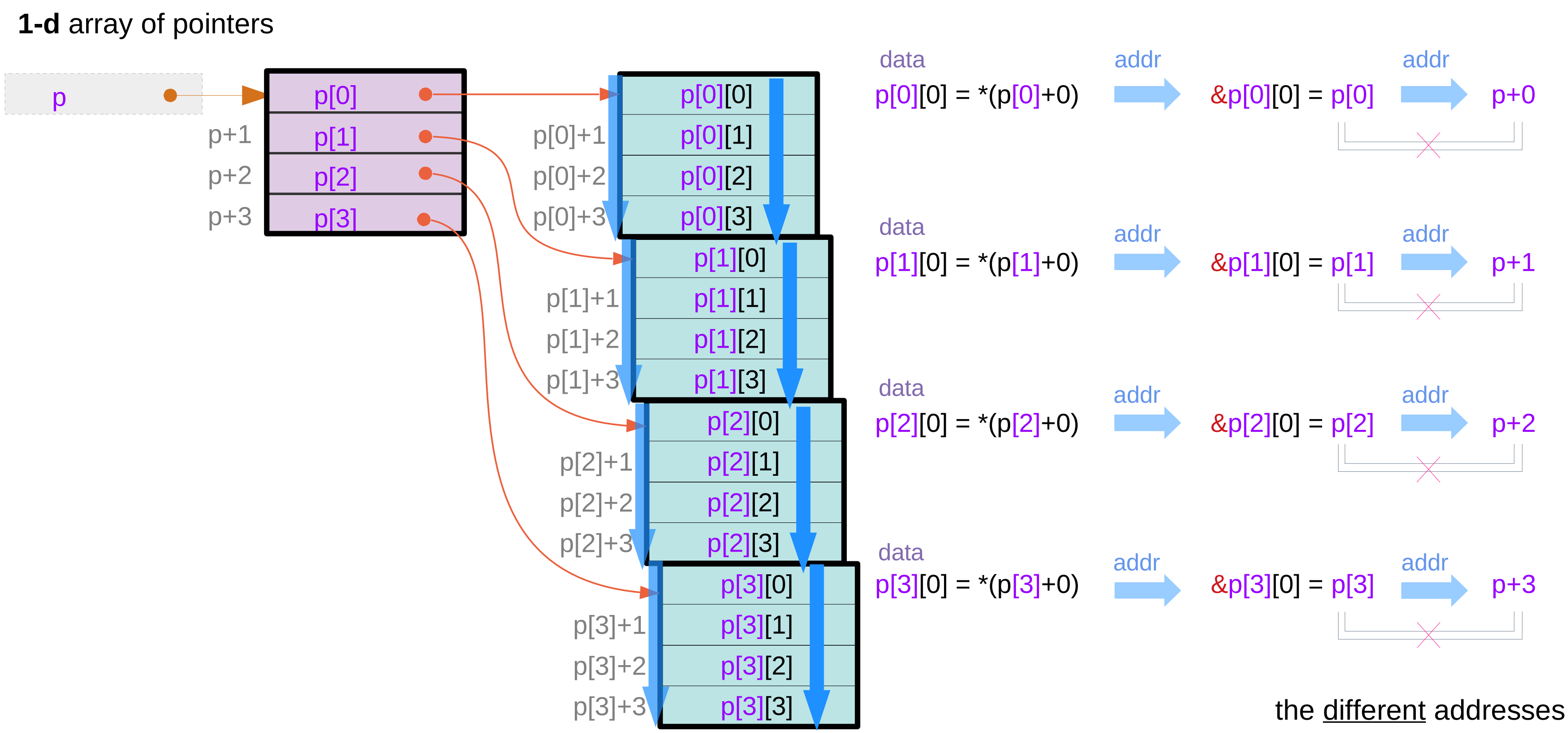
contiguous p[m] ⇒ contiguous p[m][n]

virtual array pointer ⟺ no real memory locations

Contiguity constraints for p[m] – using pointer arrays

*(p[m]+n) ⟺ p[m][n]

for a given p[m] contiguous index : **n**



contiguous p[m] ➡ contiguous p[m][n]

Not necessarily

Contiguity constraints for 2-d arrays

int a[M][N] ;

*(a+m) ⟺ a[m]	

a[0], a[1], … , a[M-1]
are contiguous

*(a[m]+n) ⟺ a[m][n]

a[m][0], a[m][1], … , a[m][N-1]
are contiguous

int (*b)[N] ;

*(b+m) ⟺ b[m]

b[0], b[1], … , b[M-1]
are contiguous

*(b[m]+n) ⟺ b[m][n]

b[m][0], b[m][1], … , b[m][N-1]
are contiguous

int * c[M] ;

*(c+m) ⟺ c[m]

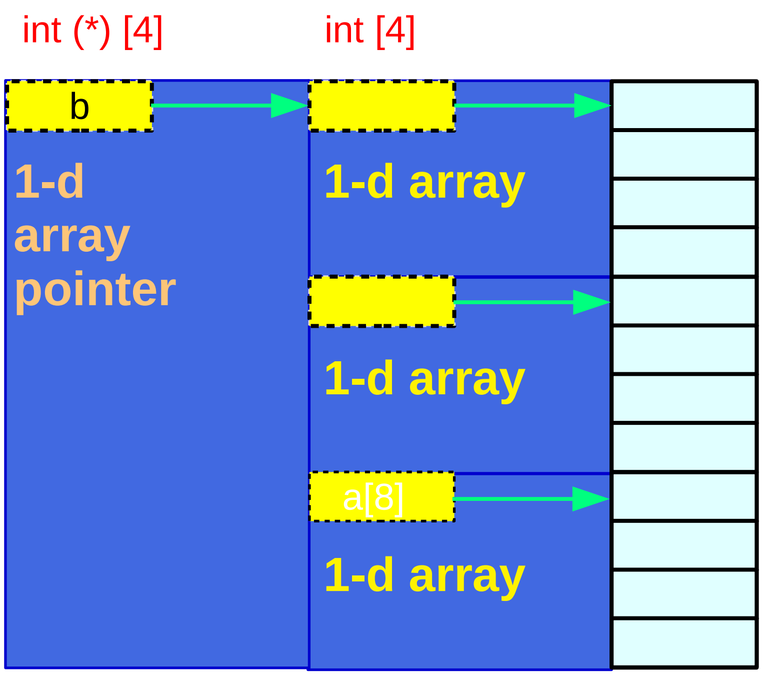
c[0], c[1], … , c[M-1]
are contiguous

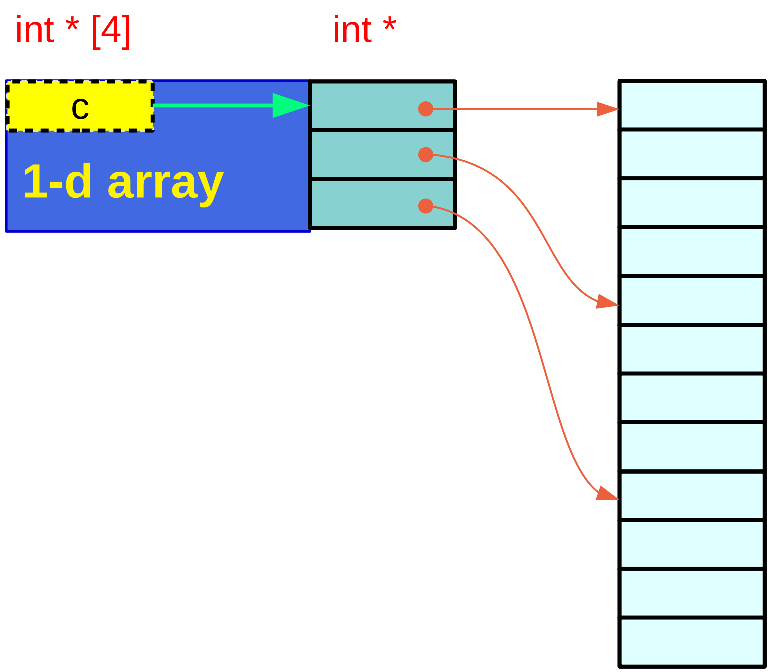
*(c[m]+n) ⟺ c[m][n]

c[m][0], c[m][1], … , c[m][N-1]
are contiguous

a set of assignments of pointers
are necessary for this contiguity

Pointer Arrays vs Array Pointers





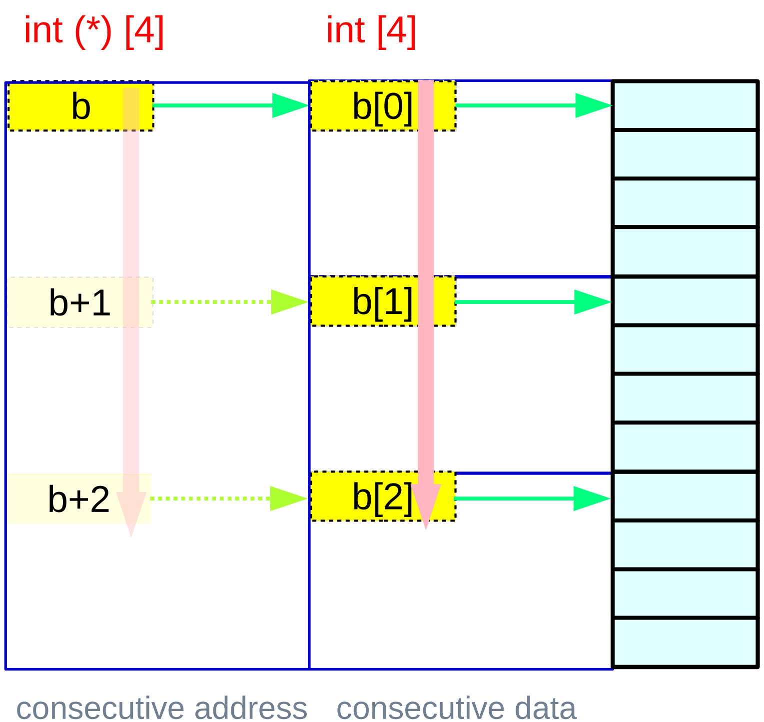
int (*b)[N] ;

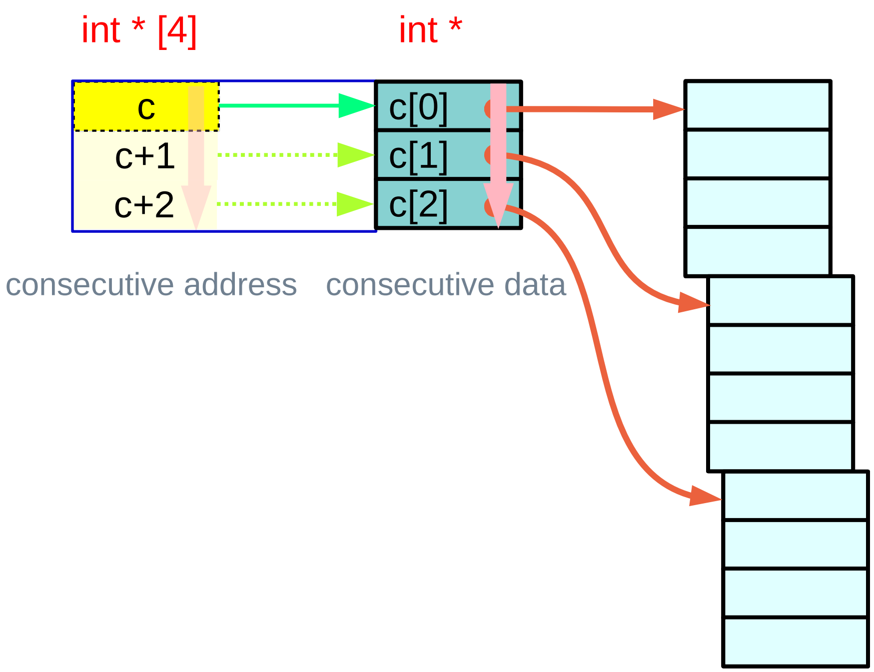
*(b+m) ⟺ b[m]
*(b[m]+n) ⟺ b[m][n]

int * c[M] ; with proper assignments

*(c+m) ⟺ c[m] or
*(c[m]+n) ⟺ c[m][n]

Pointer Arrays vs Array Pointers





int (*b)[N] ;

int * c[M] ; with proper assignments

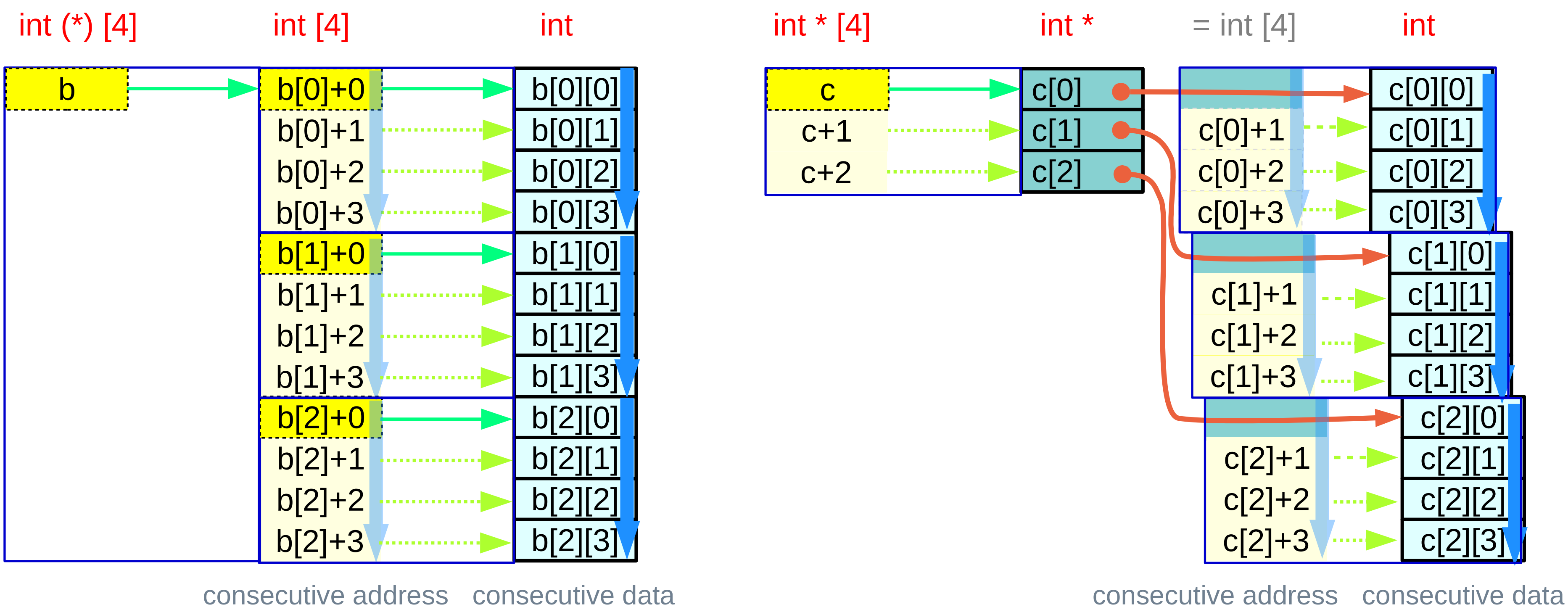
*(b+m) ⟺ b[m]

*(b[m]+n) ⟺ b[m][n]

*(c+m) ⟺ c[m] or

*(c[m]+n) ⟺ c[m][n]

Pointer Arrays vs Array Pointers



int (*b)[N] ;

*(b+m) ⟺ b[m]

*(b[m]+n) ⟺ b[m][n]

int * c[M] ;   with proper assignments

*(c+m) ⟺ c[m]

*(c[m]+n) ⟺ c[m][n]

Three contiguity constraints for 3-d arrays

Pointer Array Approach (array of pointers)

c[i][j][k]	➡	*(c[i][j] +k)
*(c[i][j] +k)	➡	*(*(c[i] +j) +k)
((c[i] +j) +k)	➡	*(*(*(c +i) +j) +k)

contiguous **int**	int
contiguous pointers to **int**	int *
contiguous double pointers to **int**	int **

the contiguity constraints are satisfied by allocating arrays of pointers

Array Pointer Approach (pointer to arrays)

c[i][j][k]	➡	*(c[i][j] +k)
*(c[i][j] +k)	➡	*(*(c[i] +j) +k)
((c[i] +j) +k)	➡	*(*(*(c +i) +j) +k)

contiguous **0-d** arrays	int	int
contiguous **1-d** arrays	int [4]	int *
contiguous **2-d** arrays	int [3][4]	int (*) [4]

The contiguity constraints are satisfied by row major ordered linear data layout

Types of array pointers in 3-d arrays

```
int c[2][3][4];
```

	array type (name)	array pointer type
c[i][j][k]	:: int	int
c[i][j]	:: int [4] [k]	int (*) [k]
c[i]	:: int [3][4] [j] [k]	int (*)[4] [j] [k]
c	:: int [2][3][4] [i] [j] [k]	int (*)[3][4] [i] [j] [k]

Sizes of array pointers in 3-d arrays

```
int c[2][3][4];
```

sizeof(**c[i][j][k]**) = sizeof(int)

sizeof(**c[i][j]**) = sizeof(int) * 4
[k]

sizeof(**c[i]**) = sizeof(int) * 3 * 4
[j] [k]

sizeof(**c**) = sizeof(int) * 2 * 3 * 4
[i] [j] [k]

Address values of array pointers in 3-d arrays

```
int c[2][3][4];
```

c[i][j][k] = &c[i][j][k]

c[i][j]+k = &c[i][j][0] + k * sizeof(c[i][j][k])

sizeof(*c[i][j])
= sizeof(c[i][j][0]) = sizeof(int)

c[i]+j = &c[i][0][0] + j * sizeof(c[i][j])

sizeof(*c[i])
= sizeof(c[i][0]) = sizeof(int) * 4
[k]

c+i = &c[0][0][0] + i * sizeof(c[i])

sizeof(*c)
= sizeof(c[0]) = sizeof(int) * 3 * 4
[j] [k]

c[0][0][0] = *(c[0][0] + 0)
c[0][0][1] = *(c[0][0] + 1)
c[0][0][2] = *(c[0][0] + 2)
c[0][0][3] = *(c[0][0] + 3)
c[0][1][0] = *(c[0][1] + 0)
c[0][1][1] = *(c[0][1] + 1)
c[0][1][2] = *(c[0][1] + 2)
c[0][1][3] = *(c[0][1] + 3)

⋮ ⋮

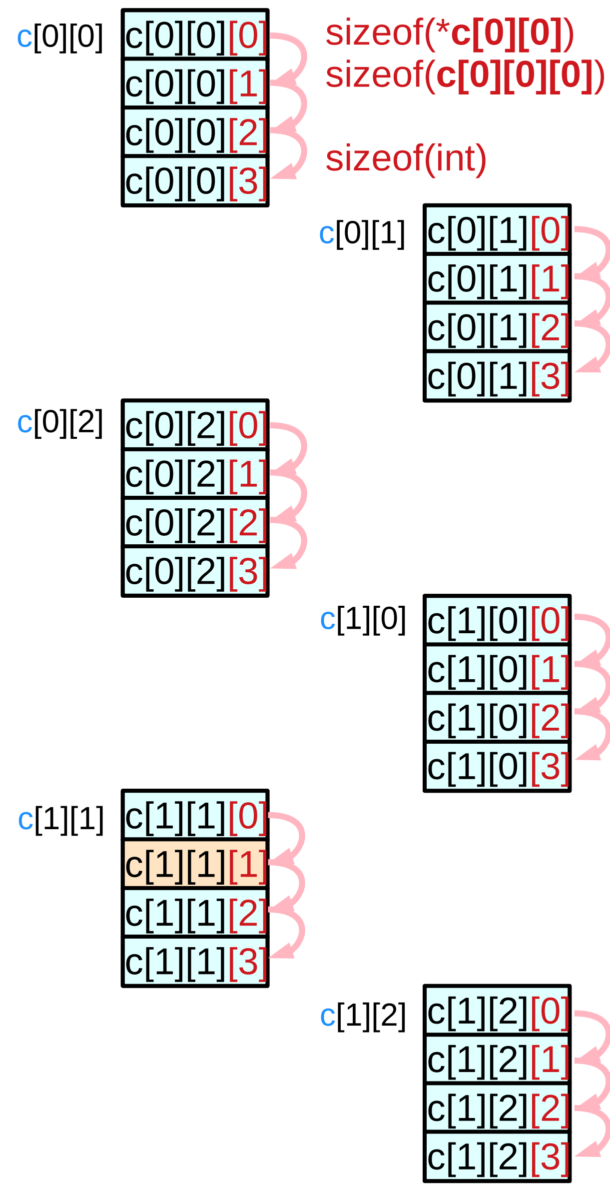
contiguous 1-d
array elements

c[i][j]	
int [4]	**4** contiguous 0-d arrays
int *	points to the 1st 0-d array
int	0-d array

sizeof(**c[i][j]**) [k]
sizeof(**c[i][j][k]**) * 4
sizeof(**int**) * 4

int c[2][3][4];

Address Value
c[i][j] + k
&c[i][j][0] + k * sizeof(*c[i][j])
&c[i][j][0] + k * sizeof(c[i][j][0])
&c[i][j][0] + k * 4



c[i][j] ≡ *(c[i] +j)   sizes and address values

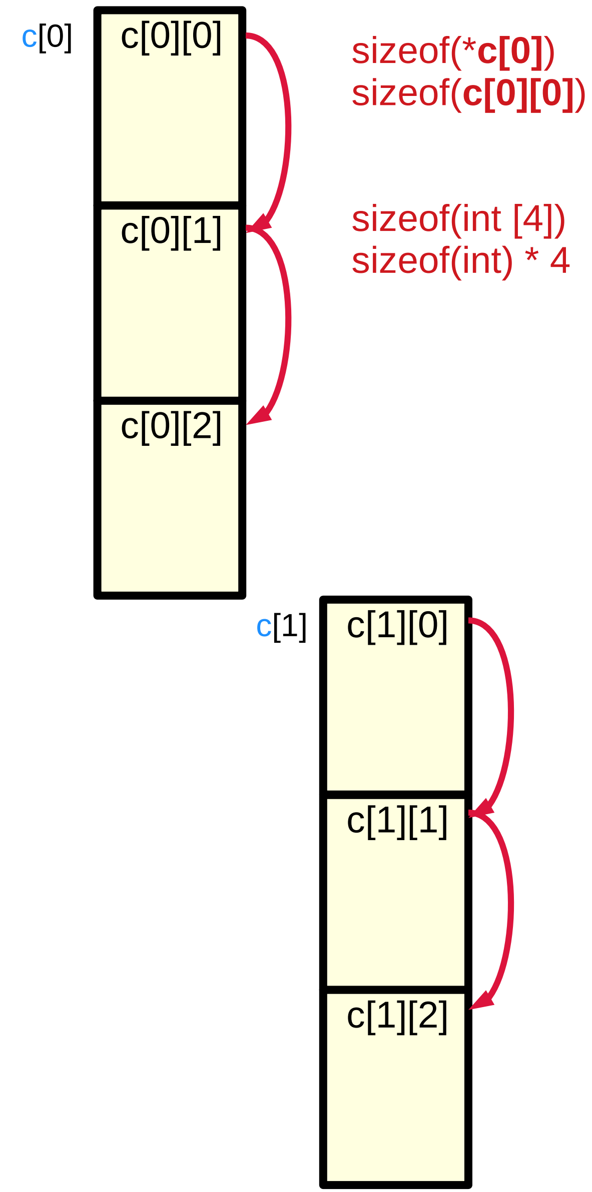
c[0][0] = *(c[0] + 0)
c[0][1] = *(c[0] + 1)
c[0][2] = *(c[0] + 2)
c[1][0] = *(c[1] + 0)
c[1][1] = *(c[2] + 1)
c[1][2] = *(c[3] + 2)

c[i]
int [3][4]   **3** contiguous 1-d arrays
int (*) [4]   points to the 1st 1-d array
int [4]   1-d array

sizeof(**c[i]**)   [j] [k]
sizeof(**c[i][j][k]**) * 3 * 4
sizeof(**int**) * 3 * 4

int c[2][3][4];

Address Value
c[i] + j
&c[i][0][0] + j * sizeof(*c[i])
&c[i][0][0] + j * sizeof(c[i][0])
&c[i][0][0] + j * 4 * 4



c[i] ≡ *(c +i) sizes and address values

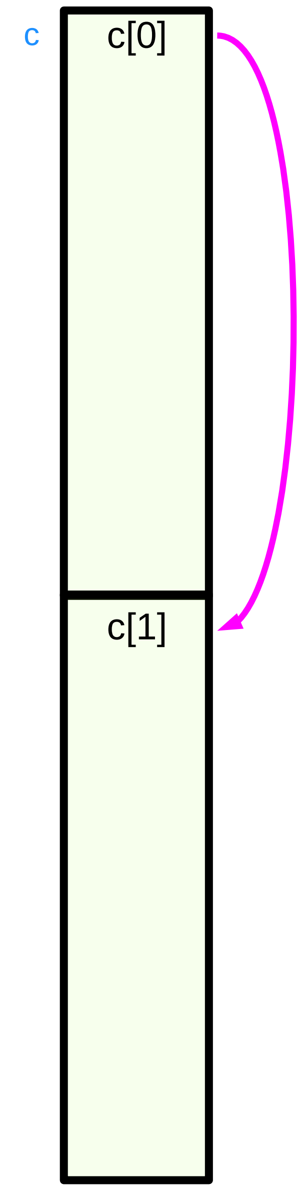
c[0] = *(c + 0)	
c[1] = *(c + 1)	

c
int [2][3][4] — **2** contiguous 2-d arrays
Int (*) [3][4] — points to the 1st 2-d array
int [3][4] — 2-d array

sizeof(**c**)
[i] [j] [k]
sizeof(**c[i][j][k]**) * 2 * 3 * 4
sizeof(**int**) * 2 * 3 * 4

```
int c[2][3][4];
```

Address Value
c + i
&c[0][0][0] + i * sizeof(*c)
&c[0][0][0] + i * sizeof(c[0])
&c[0][0][0] + i * 4 * 3 * 4



sizeof(*c)
sizeof(**c[0]**)

sizeof(int [3][4])
sizeof(int) * 3 * 4

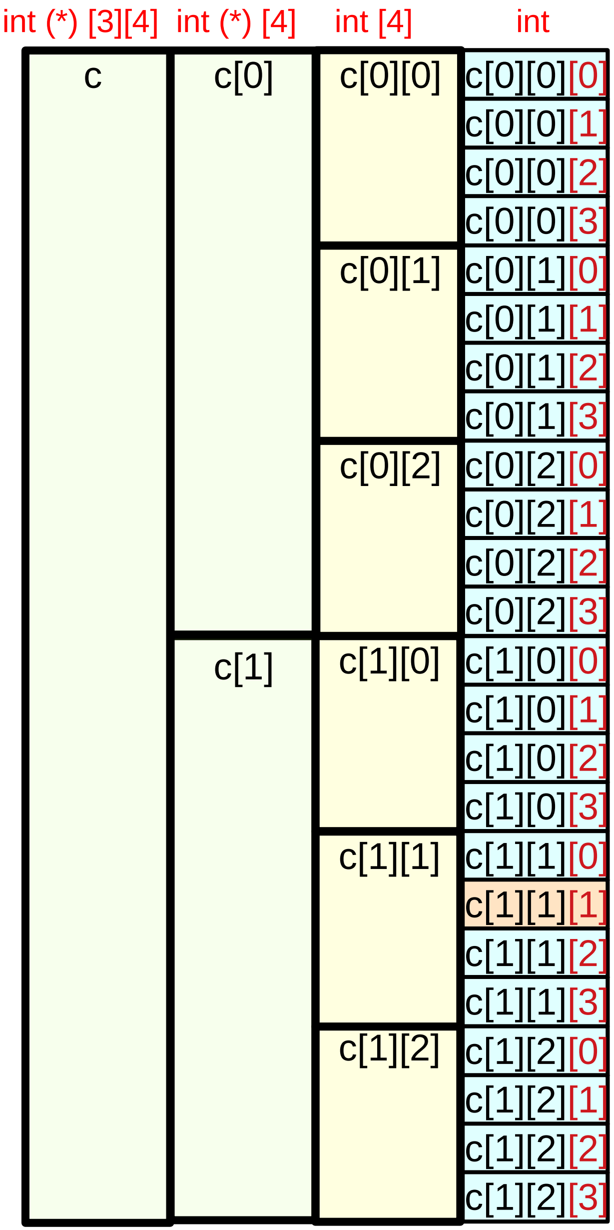
c[i][j][k] ≡ *(c[i][j] +k)

c[i][j][k]	➡	*(c[i][j] +k)
*(c[i][j] +k)	➡	*(*(c[i] +j) +k)
((c[i] +j) +k)	➡	*(*(*(c +i) +j) +k)

int c[2][3][4];

contiguous **1-d** array elements	int	int
contiguous **1-d** array names	int (*)	int [4]
contiguous **1-d** array pointers	int (*) [4]	int [3][4]

The contiguity constraints are satisfied by row major ordered linear data layout



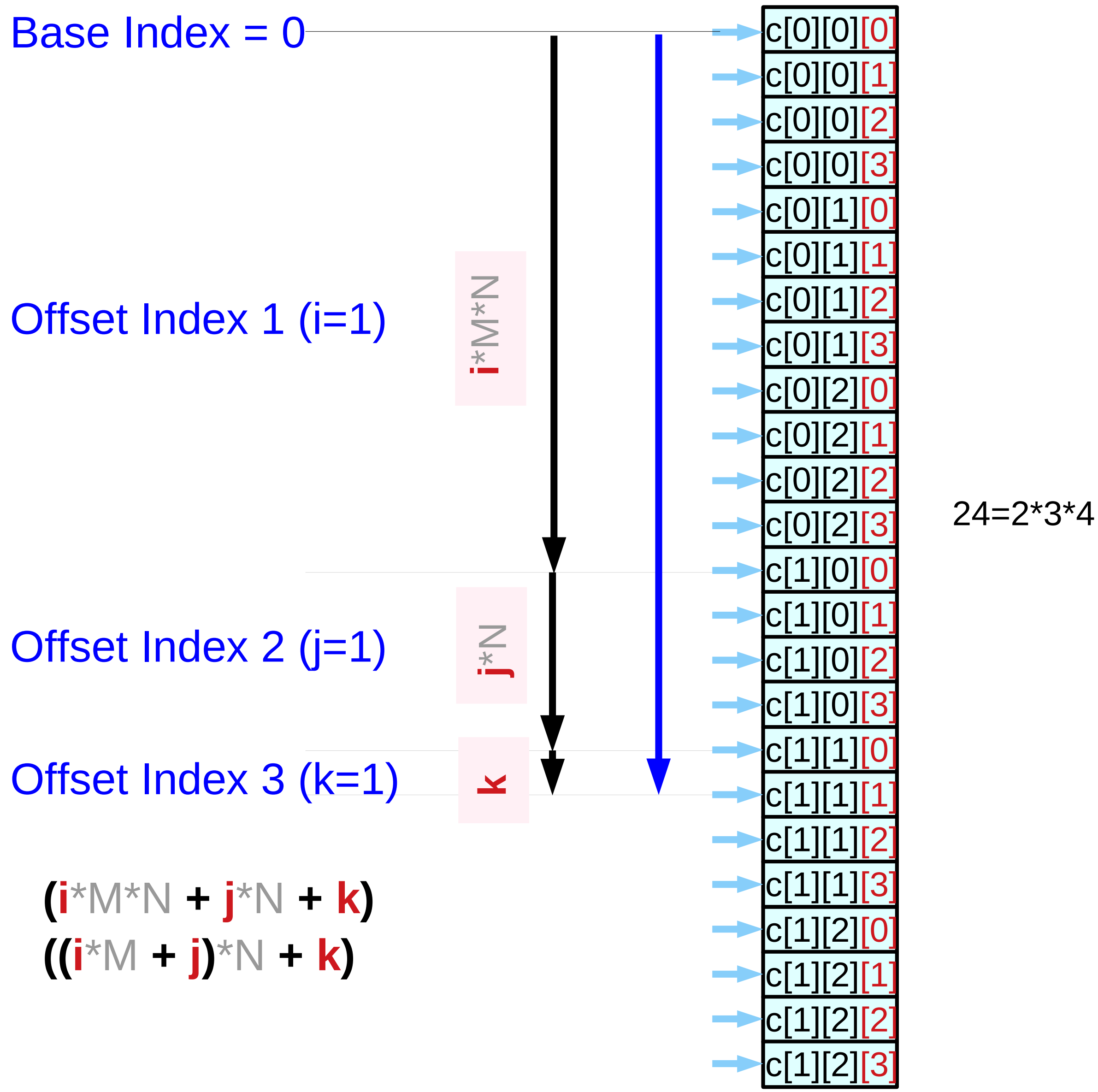
Contiguity Constraints

c[i] ≡ *(c +i)	c[i][j] ≡ *(c[i] +j)	c[i][j][k]≡ *(c[i][j] +k)
int (*) [3][4] 2-d array pointer c int [2] [3][4] 3-d array name c	int (*) [4] 1-d array pointers c[i] Int [3] [4] 2-d array names c[i]	int (*) 0-d array pointers c[i][j] int [4] 1-d array names c[i][j]
address value c + i &c[0][0][0] + i * sizeof(*c) &c[0][0][0] + i * sizeof(c[0]) &c[0][0][0] + i * 4 * 3 * 4	**address value** c[i] + j &c[i][0][0] + j * sizeof(*c[i]) &c[i][0][0] + j * sizeof(c[i][0]) &c[i][0][0] + j * 4 * 4	**address value** c[i][j] + k &c[i][j][0] + k * sizeof(*c[i][j]) &c[i][j][0] + k * sizeof(c[i][j][0]) &c[i][j][0] + k * 4
leading elements c[0][0][0]	**leading elements** c[0][0][0] c[1][0][0]	**leading elements** c[0][0][0] c[0][1][0] c[0][2][0] c[1][0][0] c[1][1][0] c[1][2][0]

Contiguous linear layout

```
int     c [L][M][N];
```

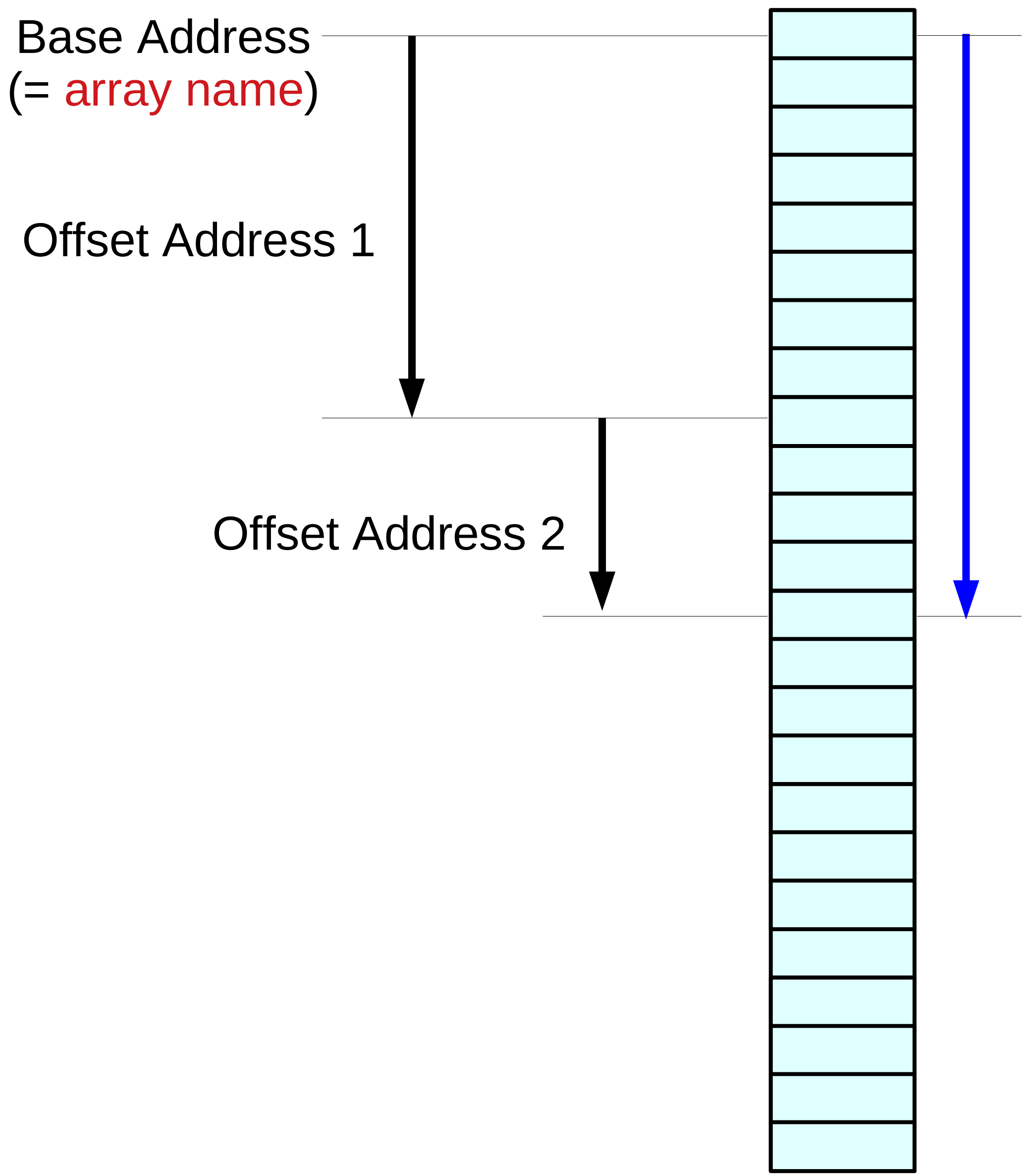
L	M	N
i	j	k
i*M*N	j*N	k

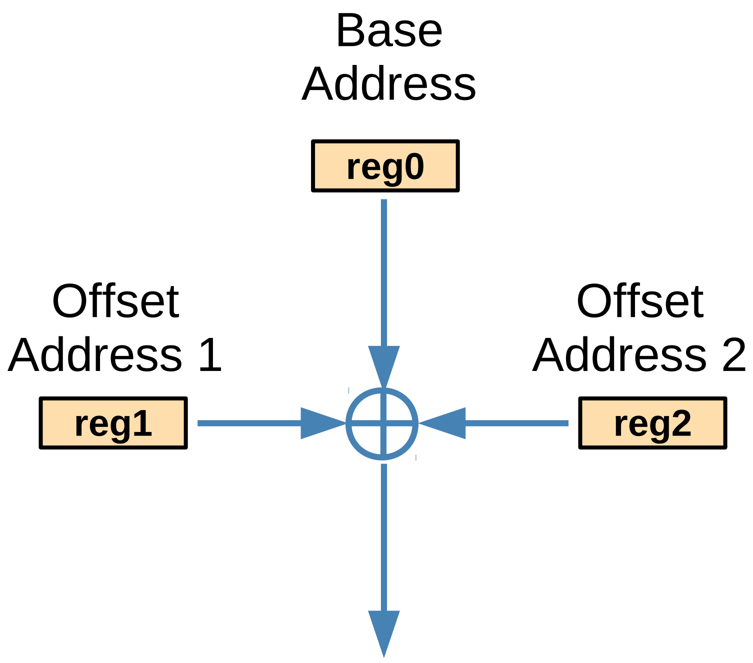


(i*M*N + j*N + k)

((i*M + j)*N + k)

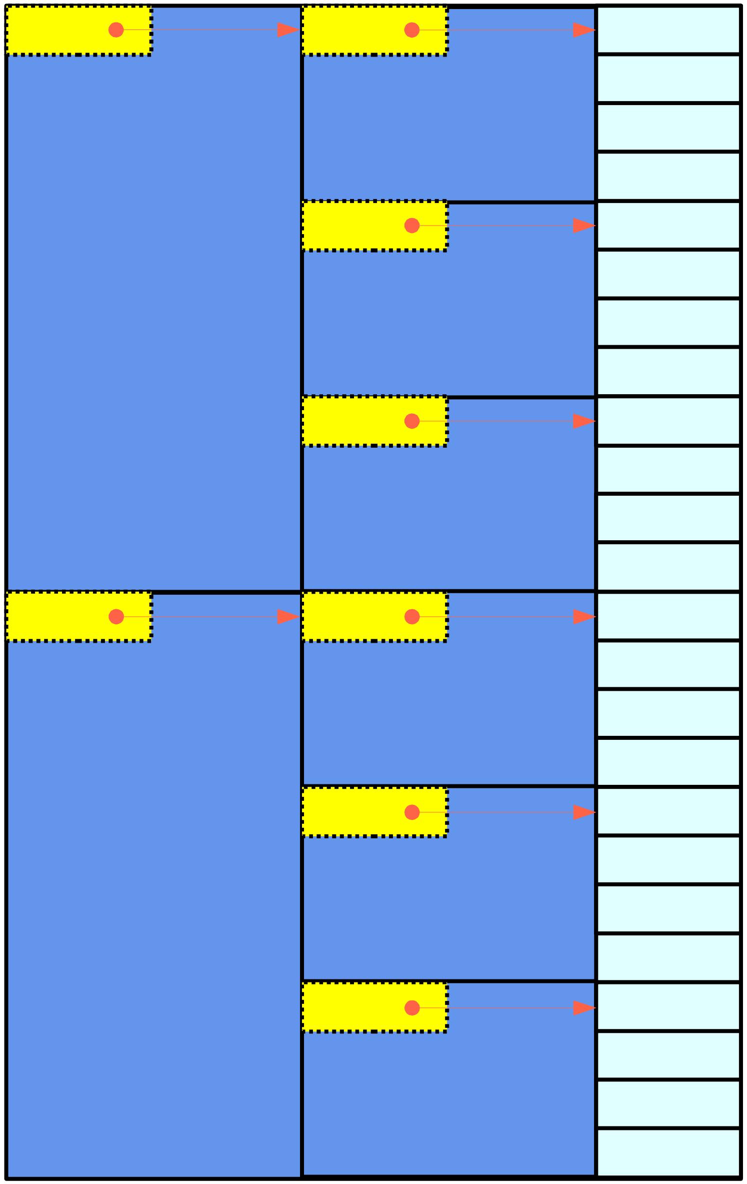
Base and Offset Addressing

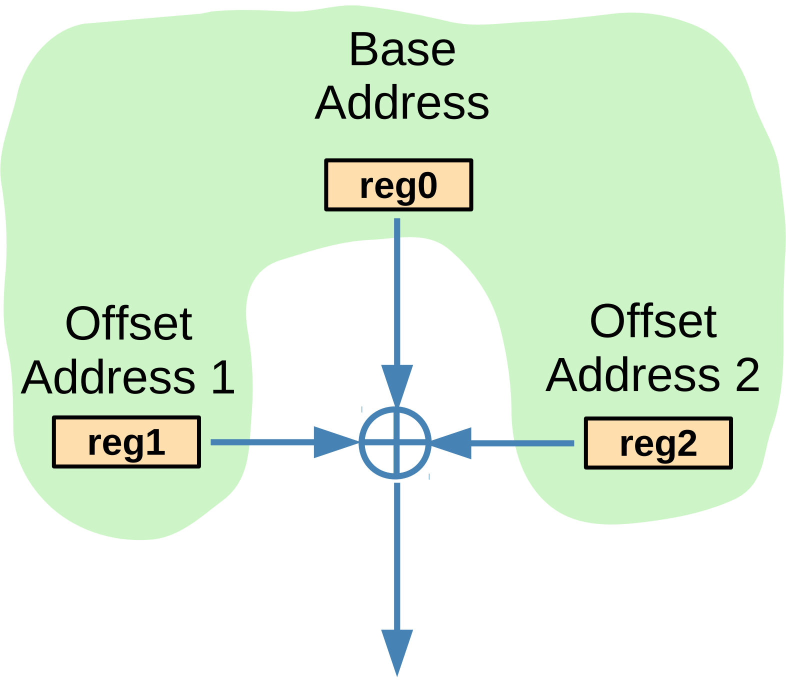




compiler
assembly instruction
registers in the CPU

Array Pointer Approach





register based address **computations**
eliminate the pointer arrays – by a compiler

Array Pointer Approach
(pointer to arrays)

References

[1] Essential C, Nick Parlante
[2] Efficient C Programming, Mark A. Weiss
[3] C A Reference Manual, Samuel P. Harbison & Guy L. Steele Jr.
[4] C Language Express, I. K. Chun