### Temporal Characteristics of Random Processes

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Based on Probability, Random Variables and Random Signal Principles, P.Z. Peebles.Jr. and B. Shi

#### Outline

1 Joint Distributions, Independence, and Moments

# First Order Distribution Function

N Gaussian random variables

#### Definition

For one particular time  $t_1$ , the distribution function associated with the random variable  $X_1 = X(t_1)$ 

$$F_X(x_1;t_1) = P\{X(t_1) \le x_1\}$$

the density function

$$f_X(x_1; t_1) = dF_X(x_1; t_1)/dx_1$$

## Second Order Distribution Function

N Gaussian random variables

#### Definition

For one particular time  $t_1$ ,  $t_2$ , the distribution function associated with the random variables  $X_1 = X(t_1)$  and  $X_2 = X(t_2)$ 

$$F_X(x_1,x_2;t_1,t_2) = P\{X(t_1) \le x_1, X(t_2) \le x_2\}$$

the density function

$$f_X(x_1, x_2; t_1, t_2) = \partial^2 F_X(x_1, x_2; t_1, t_2) / \partial x_1 \partial x_2$$

### N-th Order Distribution Function

N Gaussian random variables

#### Definition

For one particular time  $t_1$ ,  $t_2$ , ...,  $t_N$ , the distribution function associated with the random variables  $X_1 = X(t_1)$ ,  $X_2 = X(t_2)$ , ...,  $X_N = X(t_N)$ 

$$F_X(x_1,...,x_N;t_1,...,t_N) = P\{X(t_1) \le x_1,...,X(t_N) \le x_N\}$$

the density function

$$f_X(x_1,\ldots,x_N;t_1,\ldots,t_N) = \partial^N F_X(x_1,\ldots,x_N;t_1,\ldots,t_N)/\partial x_1\cdots\partial x_N$$