Monad P3 : Types (1A)

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This document was produced by using LibreOffice.

Based on

Haskell in 5 steps

https://wiki.haskell.org/Haskell_in_5_steps

data

data - creates new algebraic type with value constructors

- Can have several **value constructors**
- **Value constructors** are lazy
- **Values** can have several fields
- Affects both compilation and runtime, have runtime overhead
- Created type is a distinct new type
- Can have its own **type class instances**
- When pattern matching against **value constructors**,
  WILL be evaluated at least to weak head normal form (**WHNF**) *
- Used to create *new data type*
  (example: Address { zip :: String, street :: String })

https://www.reddit.com/r/haskell/comments/6xri4d/whats_the_difference_between_newtype_type_and_data/

newtype

newtype - creates new “decorating” type with value constructor

- Can have only one **value constructor**
- **Value constructor** is strict
- **Value** can have only one field
- Affects only compilation, no runtime overhead
- Created type is a distinct new type
- Can have its own **type class instances**
- When pattern matching against **value constructor**,
  CAN be not evaluated at all *
- Used to create *higher level concep*t based on existing type
  with distinct set of supported operations or
  that is not interchangeable with original type
  (example: Meter, Cm, Feet is Double)

https://www.reddit.com/r/haskell/comments/6xri4d/whats_the_difference_between_newtype_type_and_data/

type

type - creates an alternative name (synonym) for a type (like typedef in C)

- No value constructors
- No fields
- Affects only compilation, no runtime overhead
- No new type is created (only a new name for existing type)
- Can NOT have its own **type class instances**
- When pattern matching against **data constructor**,
  behaves the same as original type
- Used to create higher level concept based on existing type
  with the same set of supported operations
  (example: String is [Char])

https://www.reddit.com/r/haskell/comments/6xri4d/whats_the_difference_between_newtype_type_and_data/

data, newtype, type

data: zero or more **constructors**,
each can contain zero or more **values**.

newtype: similar to above
but exactly one **constructor**
and only one **value** in that constructor,
and has the exact **same runtime representation**
as the value that it stores.

type: **type synonym**, compiler more or less
forgets about it once it is expanded.

https://www.reddit.com/r/haskell/comments/6xri4d/whats_the_difference_between_newtype_type_and_data/

data (lazy), newtype (strict)

Both **newtype** and the single-constructor **data**

introduce a single **data constructor**,

but the **data constructor** introduced by **newtype** is **strict**

and the **data constructor** introduced by **data** is **lazy**.

```
data      D = D Int    -- lazy
newtype   N = N Int    -- strict
```

Then **N undefined** is equivalent to **undefined**

and causes an **error** when **evaluated**.

But **D undefined** is not equivalent to **undefined**,

and it can be **evaluated** as long as you don't try to peek inside.

https://www.reddit.com/r/haskell/comments/6xri4d/whats_the_difference_between_newtype_type_and_data/

Data definition without data constructors (1)

a **data definition** without **data constructors**

cannot be instantiated

data B

a new **type constructor B**,

but no **data constructors** to produce **values** of **type B**

In fact, such a data type is declared in the Haskell base: **Void**

ghci> **import Data.Void**

ghci> **:i Void**

data Void -- Defined in ‘Data.Void’

https://stackoverflow.com/questions/45385621/data-declaration-with-no-data-constructor-can-it-be-instantiated-why-does-it-c

Data definition without data constructors (2)

Being able to have **uninhabited types** turns out

to be useful in some areas

passing an **uninhabited type** as a **type parameter**

to another **type constructor**

https://stackoverflow.com/questions/45385621/data-declaration-with-no-data-constructor-can-it-be-instantiated-why-does-it-c

Data definition with data constructors

data B = String

defines

a **type constructor B** and

a **data constructor String**,

both taking no arguments.

Note that the **String** you define is in the **value namespace**, so is different from the usual **String type constructor**.

ghci> data B = String

ghci> x = String

ghci> :t x

x :: B

https://stackoverflow.com/questions/45385621/data-declaration-with-no-data-constructor-can-it-be-instantiated-why-does-it-c

Type class instances

Type classes allow us

to declare which types are **instances** of which class, and

to provide **definitions** of the **overloaded operations**
associated with a **class**.

https://www.haskell.org/tutorial/classes.html

Type class instances

For example, let's define a **type class** containing an **equality operator**:

```
class Eq a where
  (==)          :: a -> a -> Bool
```

Eq is the **name** of the **class** being defined,
== is the single **operation** in the **class**.

a **type a** is an **instance** of the **class Eq**
if there is an (**overloaded**) **operation ==**,
of the appropriate **type**, defined on it.

(Note that == is only defined on pairs of objects of the same type.)

		type
class	**Eq**	**a**
	class name	class instance

https://www.haskell.org/tutorial/classes.html

Type class instances

Eq a expresses a **constraint** that

a **type a** must be an **instance** of the **class Eq**

Eq a

is not a **type expression**

expresses a **constraint** on a **type**

called a **context**

placed at the front of **type expressions**

https://www.haskell.org/tutorial/classes.html

Type class instances

For example, the effect of the above class declaration
is to assign the following type to ==:

(==) :: (Eq a) => a -> a -> Bool

for every **type a** that is an **instance** of the **class Eq**,
== has type **a->a->Bool**

elem :: (Eq a) => a -> [a] -> Bool

for every **type a** that is an **instance** of the **class Eq**,
elem has type **a->[a]->Bool**

https://www.haskell.org/tutorial/classes.html

Type class instances

An **instance declaration** specifies

which types are **instances** of the **class Eq**, and

the <u>actual</u> <u>behavior</u> of == on each of those **types**

```
instance Eq Integer where
x == y        =  x `integerEq` y
```

the **definition** of **==** is called a **method**.

integerEq happens to be the **primitive function**

in general, any valid expression for a function definition

type

instance Eq integer

class name

class instance

https://www.haskell.org/tutorial/classes.html

Type class instances

```
instance Eq Integer where
x == y        =  x `integerEq` y
```

the **type Integer** is an **instance** of the **class Eq**

the definition of the **method ==**

```
instance Eq Float where
x == y        =  x `floatEq` y
```

the **type Float** is an **instance** of the **class Eq**

the definition of the **method ==**

https://www.haskell.org/tutorial/classes.html

Type class instances

simply substituting **type class** for **class**, and **type** for **object**,
yields a valid summary of Haskell's **type class mechanism**:

"**Classes** capture common sets of operations.
A particular **object** may be an **instance** of a **class**,
and will have a **method** corresponding to each **operation**.
Classes may be arranged **hierarchically**,
forming notions of **superclasses** and **sub classes**,
and permitting **inheritance** of operations/methods.
A **default method** may also be associated with an operation."

Haskell	**OOP**
type class	**class**
type	**object**

https://www.haskell.org/tutorial/classes.html

Type class instances

In contrast to OOP, it should be clear that **types** are <u>not</u> **objects**, and in particular there is <u>no notion</u> of an object's or type's **internal mutable state**.

An advantage over some OOP languages is that **methods** in Haskell are completely <u>type-safe</u>: any attempt to apply a **method** to a **value** whose **type** is not in the required **class** will be <u>detected</u> <u>at compile time</u> instead of at runtime.

In other words, **methods** are <u>not</u> "looked up" <u>at runtime</u> but are simply passed as **higher-order functions**.

https://www.haskell.org/tutorial/classes.html

Type class instances

parametric polymorphism is useful in defining families of types by universally quantifying over all types.

Sometimes, however, it is necessary
to quantify over some smaller set of types,
eg. those types whose elements can be compared for equality.

https://www.haskell.org/tutorial/classes.html

Type class instances

type classes can be seen as providing a structured way
to quantify over a constrained set of types

Indeed, we can think of **parametric polymorphism**
as a kind of **overloading** too!

an **overloading** occurs implicitly over all types
a **type class** for a constrained set of types

https://www.haskell.org/tutorial/classes.html

Polymorphic Types

types that are universally quantified in some way over all types.

polymorphic type expressions essentially describe families of types.

For example, **(forall a) [a]** is the family of types
consisting of, for every **type a**, the **type of lists of a**.

Lists of integers (e.g. **[1,2,3]**), lists of characters (**['a','b','c']**),
even lists of lists of integers, etc., are all members of this family.

(Note, however, that [2,'b'] is not a valid example,
since there is *no single type* that contains both 2 and 'b'.)

https://www.haskell.org/tutorial/goodies.html

Polymorphic Types

Identifiers such as **a** above are called **type variables**,
and are uncapitalized to distinguish them
from specific types such as **Int**.

since Haskell has only universally quantified **types**,
there is no need to explicitly write out the symbol
for **universal quantification**,
and thus we simply write **[a]** in the example above.

In other words, all **type variables** are implicitly universally quantified

https://www.haskell.org/tutorial/goodies.html

Polymorphic Types

Lists are a commonly used data structure in functional languages,
and are a good vehicle for explaining the principles of polymorphism.

The list **[1,2,3]** in Haskell is actually shorthand for the list **1:(2:(3:[]))**,
where **[]** is the **empty list** and **:** is the **infix operator**
that adds its first argument to the front of its second argument (a list).

Since **:** is right associative, we can also write this list as **1:2:3:[]**.

https://www.haskell.org/tutorial/goodies.html

Polymorphic Types

```
length              :: [a] -> Integer
length []           =  0
length (x:xs)       =  1 + length xs

length [1,2,3]      =>  3
length ['a','b','c'] =>  3
length [[1],[2],[3]] =>  3
```

an example of a polymorphic function.

It can be applied to a list containing elements of any type,

for example **[Integer]**, **[Char]**, or **[[Integer]]**.

https://www.haskell.org/tutorial/goodies.html

Polymorphic Types

```
length              :: [a] -> Integer
length []            =  0
length (x:xs)        =  1 + length xs
```

The left-hand sides of the equations contain patterns such as [] and x:xs.

In a function application these patterns are matched against actual parameters in a fairly intuitive way

https://www.haskell.org/tutorial/goodies.html

Polymorphic Types

```
length            :: [a] -> Integer
length []          =  0
length (x:xs)      =  1 + length xs
```

[] only matches the empty list,

x:xs will successfully match any list with at least one element,

binding x to the first element and xs to the rest of the list

If the match succeeds,

the right-hand side is evaluated

and returned as the result of the application.

If it fails, the next equation is tried,

and if all equations fail, an error results.

https://www.haskell.org/tutorial/goodies.html

Polymorphic Types

Function head returns the first element of a list,
function tail returns all but the first.

```
head          :: [a] -> a
head (x:xs)   =  x

tail          :: [a] -> [a]
tail (x:xs)   =  xs
```

Unlike length, these functions are not defined for all possible values of their argument. A runtime error occurs when these functions are applied to an empty list.

https://www.haskell.org/tutorial/goodies.html

Polymorphic Types

With polymorphic types, we find that
some types are in a sense strictly more general than others
in the sense that the set of values they define is larger.

For example, the type **[a]** is more general than **[Char]**.
In other words, the latter type can be derived from the former
by a suitable substitution for **a**.

https://www.haskell.org/tutorial/goodies.html

Polymorphic Types

With regard to this **generalization ordering**,
Haskell's type system possesses two important properties:

First, every well-typed expression is guaranteed
to have a **unique principal type** (explained below),

and second, the **principal type** can be inferred automatically.

In comparison to a monomorphically typed language such as C,
the reader will find that polymorphism improves expressiveness,
and **type inference** lessens the burden of types on the programmer.

https://www.haskell.org/tutorial/goodies.html

Polymorphic Types

An expression's or function's **principal type** is
the least general type that, intuitively,
"contains all instances of the expression".

For example, the principal type of head is **[a]->a**; **[b]->a**, **a->a**,
or even **a** are correct types, but too general,
whereas something like **[Integer]->Integer** is too specific.
The existence of unique **principal types** is the hallmark feature
of the **Hindley-Milner type system**,
which forms the basis of the type systems of Haskell,
ML, Miranda, ("Miranda" is a trademark of Research Software, Ltd.)
and several other (mostly functional) languages.

https://www.haskell.org/tutorial/goodies.html

Explicitly Quantifying Type Variables

to explicitly bring fresh **type variables** into **scope**.

Example: **Explicitly quantifying** the **type variables**

map :: forall a b. (a -> b) -> [a] -> [b]

for any combination of types **a** and **b**

choose **a** = **Int** and **b** = **String**

then it's valid to say that map has the type

(Int -> String) -> [Int] -> [String]

Here we are **instantiating** the general type of **map**

to a more specific type.

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

Implicit **forall**

any introduction of a **lowercase type parameter**
implicitly begins with a **forall** keyword,

Example: Two equivalent type statements

```
id :: a -> a
id :: forall a . a -> a
```

We can apply additional **constraints**
on the quantified **type variables**

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

Existential Types

Normally when creating a new type

using **type**, **newtype**, **data**, etc.,

every **type variable** that appears on the right-hand side

must also appear on the left-hand side.

```
newtype ST s a = ST (State# s -> (# State# s, a #))
```

Existential types are a way of escaping

Existential types can be used for several different purposes.

But what they do is to **hide** a **type variable** on the right-hand side.

https://wiki.haskell.org/Existential_type

Type Variable Example – (1) error

Normally, any type variable appearing on the right must also appear on the left:

data Worker x y = Worker {buffer :: b, input :: x, output :: y}

This is an **error**, since the **type** of the **buffer** isn't specified on the right (it's a type variable rather than a type) but also isn't specified on the left (there's no '**b**' in the left part).

In Haskell98, you would have to write

data Worker b x y = Worker {buffer :: b, input :: x, output :: y}

https://wiki.haskell.org/Existential_type

Type Variable Example – (2) explicit type signature

However, suppose that a **Worker** can use any type '**b**'

so long as it belongs to some particular class.

Then every **function** that uses a Worker will have a type like

foo :: (Buffer b) => Worker b Int Int

In particular, failing to write an **explicit type signature** **(Buffer b)**

will invoke the dreaded monomorphism restriction.

Using **existential types**, we can avoid this:

https://wiki.haskell.org/Existential_type

Type Variable Example – (3) existential type

```
data Worker x y = forall b. Buffer b =>
                     Worker {buffer :: b, input :: x, output :: y}

foo :: Worker Int Int
```

The **type** of the **buffer** (**Buffer**) now does not appear in the **Worker** type at all.

https://wiki.haskell.org/Existential_type

Type Variable Example – (4) characteristics

```
data Worker x y = forall b. Buffer b =>
                        Worker {buffer :: b, input :: x, output :: y}
foo :: Worker Int Int
```

- it is now impossible for a function to demand a **Worker** having a specific type of **buffer**.
- the **type** of **foo** can now be derived automatically without needing an explicit **type signature**. (No **monomorphism** restriction.)

https://wiki.haskell.org/Existential_type

Type Variable Example – (4) characteristics

```
data Worker x y = forall b. Buffer b =>
                        Worker {buffer :: b, input :: x, output :: y}
foo :: Worker Int Int
```

- since code now has <u>no</u> <u>idea</u>
  what **type** the buffer function <u>returns</u>,
  you are more <u>limited</u> in what you can do to it.

https://wiki.haskell.org/Existential_type

Hiding a type

In general, when you use a '**hidden**' type in this way,
you will usually want that **type** to belong to a **specific class**,
or you will want to **pass some functions** along
that can work on that type.

Otherwise you'll have some value belonging
to a **random unknown type**,
and you won't be able to do anything to it!

https://wiki.haskell.org/Existential_type

Conversion to less a specific type

Note: You can use **existential types**

to **convert** a **more specific type**

into a **less specific one**.

There is no way to perform the reverse conversion!

https://wiki.haskell.org/Existential_type

A heterogeneous list example

This illustrates **creating a heterogeneous list**,
all of whose members implement "**Show**",
and progressing through that list to show these items:

```
data Obj = forall a. (Show a) => Obj a

xs :: [Obj]
xs = [Obj 1, Obj "foo", Obj 'c']

doShow :: [Obj] -> String
doShow [] = ""
doShow ((Obj x):xs) = show x ++ doShow xs
```

With output: **doShow xs ==> "1\"foo\"'c'"**

https://wiki.haskell.org/Existential_type

Bottom

https://stackoverflow.com/questions/6872898/what-is-weak-head-normal-form

Bottom

bottom in Haskell specifically called **undefined**.

This is only one form of it

though technically **bottom** is also

a non-terminating computation, such as length [1..]

bottom is used to represent an expression which is

- not computable
- runs forever
- never returns a value
- throws an exception
- etc.

https://stackoverflow.com/questions/26428828/what-does-%E2%8A%A5-mean-in-the-strictness-monad-from-p-wadlers-paper

Bottom represents computations

The term **bottom** refers to

a **computation** which never completes successfully.

a **computation** that fails due to some kind of error,

a **computation** that just goes into an infinite loop

(without returning any data).

The mathematical symbol for bottom is '⊥'.

In plain ASCII, '_|_'.

https://wiki.haskell.org/Bottom

Bottom – a member of any type

Bottom is a **member** of any type,

even the trivial type **()** or

the equivalent simple type:

data Unary = Unary

https://wiki.haskell.org/Bottom

Bottom – definitions

Bottom can be expressed in Haskell thus:

```
bottom = bottom
bottom = error "Non-terminating computation!"
```

Indeed, the Prelude exports a function

```
undefined = error "Prelude.undefined"
```

Other implementations of Haskell, such as Gofer, defined bottom as:

```
undefined | False = undefined
```

The type of bottom is arbitrary, and defaults to the most general type:

```
undefined :: a
```

https://wiki.haskell.org/Bottom

Bottom – Usage

As **bottom** is an **inhabitant** of every **type** *a value of every type*

bottoms can be used wherever a value of that type would be.

This can be useful in a number of circumstances:

-- For leaving a todo in your program to come back to later:

foo = undefined

-- When dispatching to a type class instance:

print (sizeOf (undefined :: Int))

-- When using laziness:

print (head (1 : undefined))

https://wiki.haskell.org/Bottom

Bottom Rule

if **x** is computable,

then **strict f x** evaluates to **f x**,

but if **x** is not computable, *undefined*

then **strict f x** evaluates to "not computable". *undefined*

for example, **f x = 2 * x**.

consider **f (1 / 0)**

can't evaluate it because you can't evaluate (1 / 0)

(1 / 0) not computable

f (1 / 0) not computable

https://stackoverflow.com/questions/26428828/what-does-%E2%8A%A5-mean-in-the-strictness-monad-from-p-wadlers-paper

strict f x

Sometimes it is necessary

to control order of evaluation in a **lazy** functional program.

Use the computable function **strict**,

strict f x = if x ≠ ⊥ then f x else ⊥.

Operationally, **strict f x** is reduced by

first reducing **x** to **weak head normal form** (**WHNF**)

and then reducing the application **f x**.

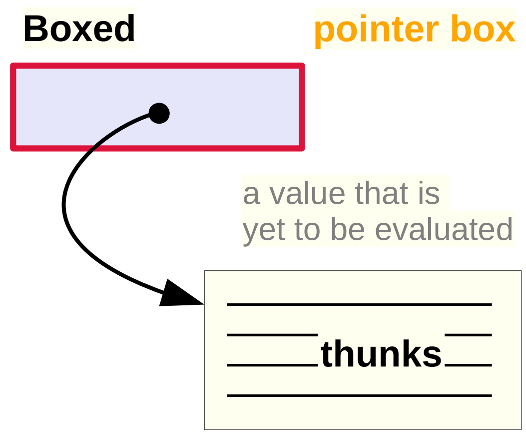
Alternatively, it is safe to reduce **x** and **f x** in parallel,

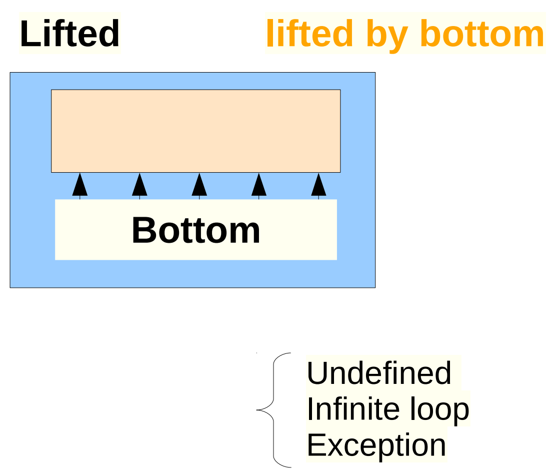
but not allow access to the result until **x** is in **WHNF**.

https://stackoverflow.com/questions/26428828/what-does-%E2%8A%A5-mean-in-the-strictness-monad-from-p-wadlers-paper

Classifying types – Summary

Boxed	a **pointer** to a **heap** object.
Unboxed	no **pointer**
Lifted	**bottom** as an element.
Unlifted	no **extra values**.
Algebraic	one or more **constructors**,
Primitive	a **built-in type**

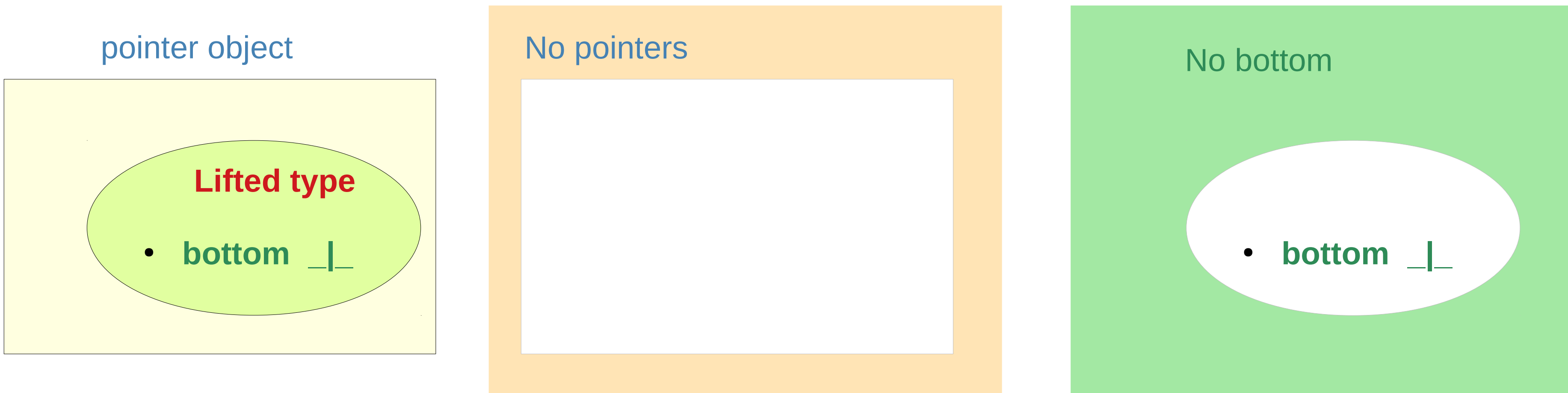




https://gitlab.haskell.org/ghc/ghc/-/wikis/commentary/compiler/type-type

(Un)Lifted and (Un)Boxed types

Boxed type | **Unboxed type** | **Unlifted type**



Lifted type → **Boxed type**

kind *

Unboxed type → **Unlifted type**

kind #

https://stackoverflow.com/questions/39985296/what-are-lifted-and-unlifted-product-types-in-haskell

Bottom in a programming language

programming language :

bottom refers to a value that is less defined than any other.

It's common to assign the **bottom value** to every computation

that either produces an **error** or **fails** to **terminate**,

because trying to distinguish these conditions

which greatly weakens

the mathematics and

complicates program analysis.

https://stackoverflow.com/questions/26428828/what-does-%E2%8A%A5-mean-in-the-strictness-monad-from-p-wadlers-paper

Bottom in an order theory

order theory (particularly **lattice theory**) :

The **bottom** element of a partially ordered set,

if one exists, is the one that precedes all others.

https://stackoverflow.com/questions/26428828/what-does-%E2%8A%A5-mean-in-the-strictness-monad-from-p-wadlers-paper

Bottom in a lattice theory

Lattice theory

the logical **false** value

is the **bottom element** of a **lattice** of **truth values**,

and **true** is the **top element**

classical logic

these are the only two – **true** and **false**

but one can also consider logics

with infinitely many truthfulness values,

such as **intuitionism** and various forms of **constructivism**.

These take the notions in a rather different direction.

https://stackoverflow.com/questions/26428828/what-does-%E2%8A%A5-mean-in-the-strictness-monad-from-p-wadlers-paper

Bottom in a standard Boolean logic

standard Boolean logic

the symbol ⊥ read **falsum** or **bottom**,
is simply a statement which is always false,
the equivalent of the false constant in programming languages.

The form is an inverted (upside-down) version of the symbol ⊤
(**verum** or **top**), which is the equivalent of true -
and there's mnemonic value in the fact that the symbol looks
like a capital letter T.

https://stackoverflow.com/questions/26428828/what-does-%E2%8A%A5-mean-in-the-strictness-monad-from-p-wadlers-paper

Bottom – verum an falsum

The names **verum** and **falsum** are Latin for "**true**" and "**false**";
the names "**top**" and "**bottom**" come from
the use of the symbols in the **theory** of **ordered sets**,
where they were chosen based on
the location of the horizontal crossbar

https://stackoverflow.com/questions/26428828/what-does-%E2%8A%A5-mean-in-the-strictness-monad-from-p-wadlers-paper

Bottom – computability theory

computability theory, ⊥ is also
the value of an **uncomputable computation**,
so you can also think of it as the **undefined value**.

It doesn't matter why the computation is uncomputable -
whether because it has **undefined inputs**,
or **never terminates**, or whatever.

it defines **strict** as a **function**
that makes any computation (another function) **undefined**
whenever its inputs (arguments) are **undefined**.

https://stackoverflow.com/questions/26428828/what-does-%E2%8A%A5-mean-in-the-strictness-monad-from-p-wadlers-paper

WHNF (Weak Head Normal Form)

https://stackoverflow.com/questions/6872898/what-is-weak-head-normal-form

Normal Form

An **expression** in **normal form**

is fully evaluated,

contains no un-evaluated thunks

no **sub-expression** could be evaluated any further

https://stackoverflow.com/questions/6872898/what-is-weak-head-normal-form

Normal Form Examples

in normal form:

42

(2, "hello")

\x -> (x + 1)

not in normal form:

1 + 2 -- we could evaluate this to 3

(\x -> x + 1) 2 -- we could apply the function

"he" ++ "llo" -- we could apply the (++)

(1 + 1, 2 + 2) -- we could evaluate 1 + 1 and 2 + 2

https://stackoverflow.com/questions/6872898/what-is-weak-head-normal-form

Head – outermost function application

The **head** in **WHNF** (Weak Head Normal Form)

does not refer to the **head** of a **list**,

but to the outermost **function application**.

thunks

generally refer to **unevaluated expressions**

HNF (Head normal form) is irrelevant for Haskell.

It differs from **WHNF** in that

the **bodies** of lambda expressions

are also evaluated *to some extent*.

https://stackoverflow.com/questions/6872898/what-is-weak-head-normal-form

NF is WHNF

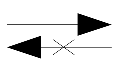
An **expression** in **WHNF** (weak head normal form)

has been evaluated to the outermost

data constructor or **lambda abstraction** (the **head**).

sub-expressions may or may not have been evaluated.

Normal form (NF)



Weak Head Normal Form (WHNF)

No unevaluated subexpressions

No unevaluated head expression

https://stackoverflow.com/questions/6872898/what-is-weak-head-normal-form

Weak Head Normal Form Test

To determine whether an expression is in weak head normal form,
we only have to look at the outermost part of the expression.

If the **outermost** part of the expression

 is a **data constructor** or a **lambda**,

  then it is in **weak head normal form**.

 is a **function application**,

  then it is not in **weak head normal form**.

https://stackoverflow.com/questions/6872898/what-is-weak-head-normal-form

Evaluation Example

<u>outermost</u> application

from left to right;

lazy evaluation.

Example:

take 1 (1:2:3:[]) => { apply **take** }

1 : take (1-1) (2:3:[]) => { apply (-) }

1 : take 0 (2:3:[]) => { apply **take** }

1 : []

https://stackoverflow.com/questions/6872898/what-is-weak-head-normal-form

Reduced Normal Form

evaluation stops when there are

no more function applications left to replace.

the result is in **normal form**

(or reduced normal form, **RNF**).

no unevaluated subexpressions

https://stackoverflow.com/questions/6872898/what-is-weak-head-normal-form

Lazy Evaluation

No matter in which **order** you evaluate an expression,

you will always end up with the same **normal form**

(but only if the evaluation terminates).

There is a slightly different description for **lazy evaluation**.

Namely, it says that you should evaluate everything

to weak head normal form (**WHNF**) only.

https://stackoverflow.com/questions/6872898/what-is-weak-head-normal-form

The head of the expression

There are precisely **three cases** for an expression to be in **WHNF**:

A **constructor**: constructor expression_1 expression_2 ...

A **built-in function** with too few arguments, like (+) 2 or sqrt

A **lambda-expression**: \x -> expression

In other words, the **head** of the **expression**

(i.e. the **outermost function application**)

cannot be evaluated any further,

but the function argument may contain

unevaluated expressions.

https://stackoverflow.com/questions/6872898/what-is-weak-head-normal-form

Weak Head Normal Form Test

in weak head normal form:

(1 + 1, 2 + 2) -- the outermost part is the data constructor (,)

\x -> 2 + 2 -- the outermost part is a lambda abstraction

'h' : ("e" ++ "llo") -- the outermost part is the data constructor (:)

As mentioned, all the normal form expressions listed above are also in weak head normal form.

not in weak head normal form:

1 + 2 -- the outermost part here is an application of (+)

(\x -> x + 1) 2 -- the outermost part is an application of (\x -> x + 1)

"he" ++ "llo" -- the outermost part is an application of (++)

in normal form:
42
(2, "hello")
\x -> (x + 1)

https://stackoverflow.com/questions/6872898/what-is-weak-head-normal-form

References

[1] ftp://ftp.geoinfo.tuwien.ac.at/navratil/HaskellTutorial.pdf

[2] https://www.umiacs.umd.edu/~hal/docs/daume02yaht.pdf