Complex Series (3B)

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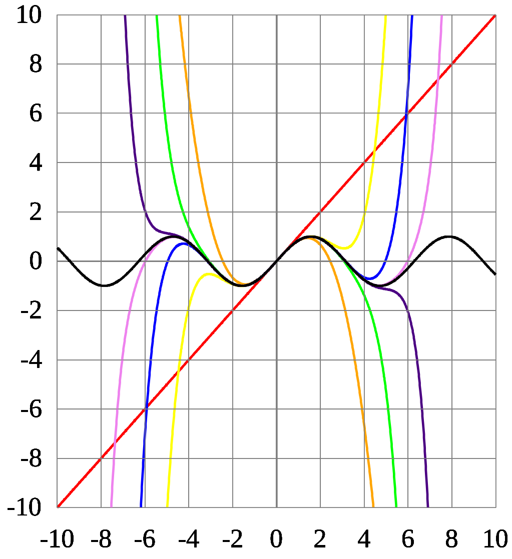
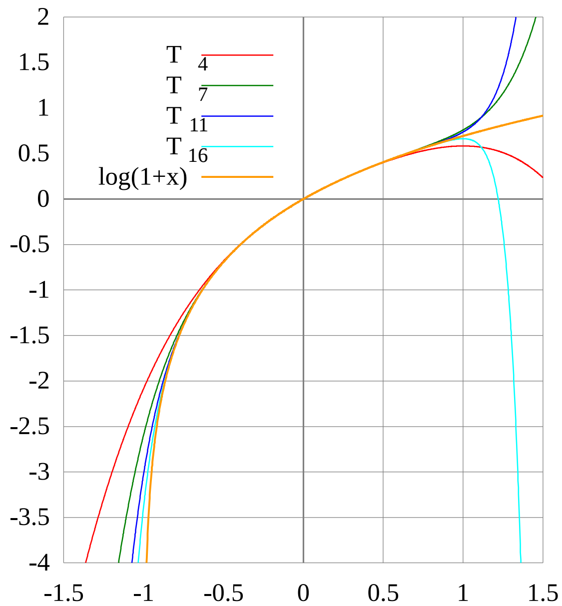
Taylor Series – real





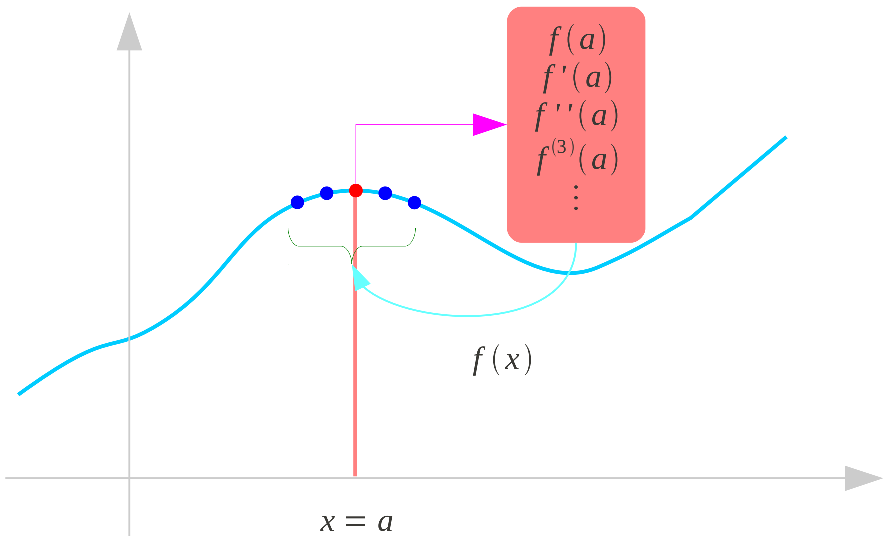
As the degree of the Taylor polynomial rises, it approaches the correct function. This image shows sin(x) and its Taylor approximations, polynomials of degree 1, 3, 5, 7, 9, 11 and 13.

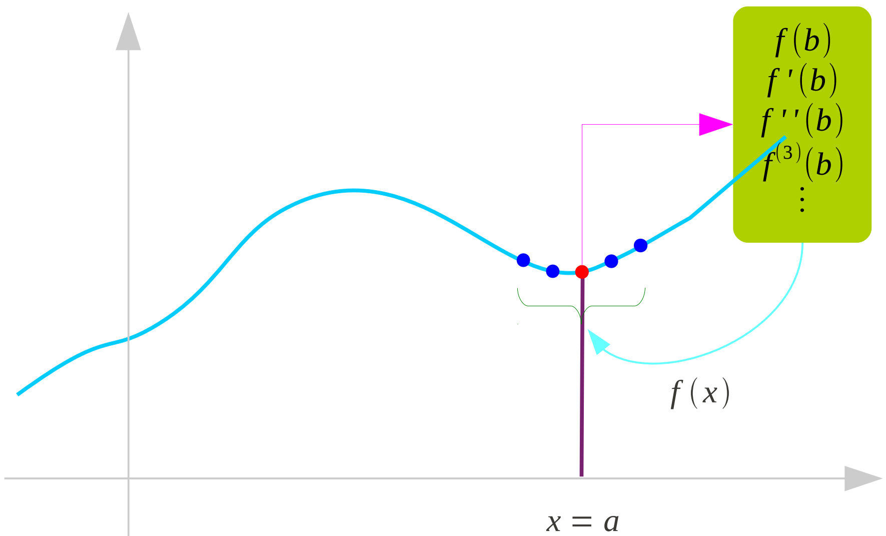
The Taylor polynomials for log(1+x) only provide accurate approximations in the range $-1 < x \leq 1$. Note that, for $x > 1$, the Taylor polynomials of higher degree are worse approximations.

Taylor Series – real

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n + \cdots$$

around $x = a$





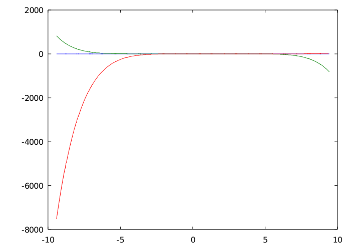
$$f(x) = f(b) + f'(b)(x-b) + \frac{f''(b)}{2!}(x-b)^2 + \cdots + \frac{f^{(n)}(b)}{n!}(x-b)^n + \cdots$$

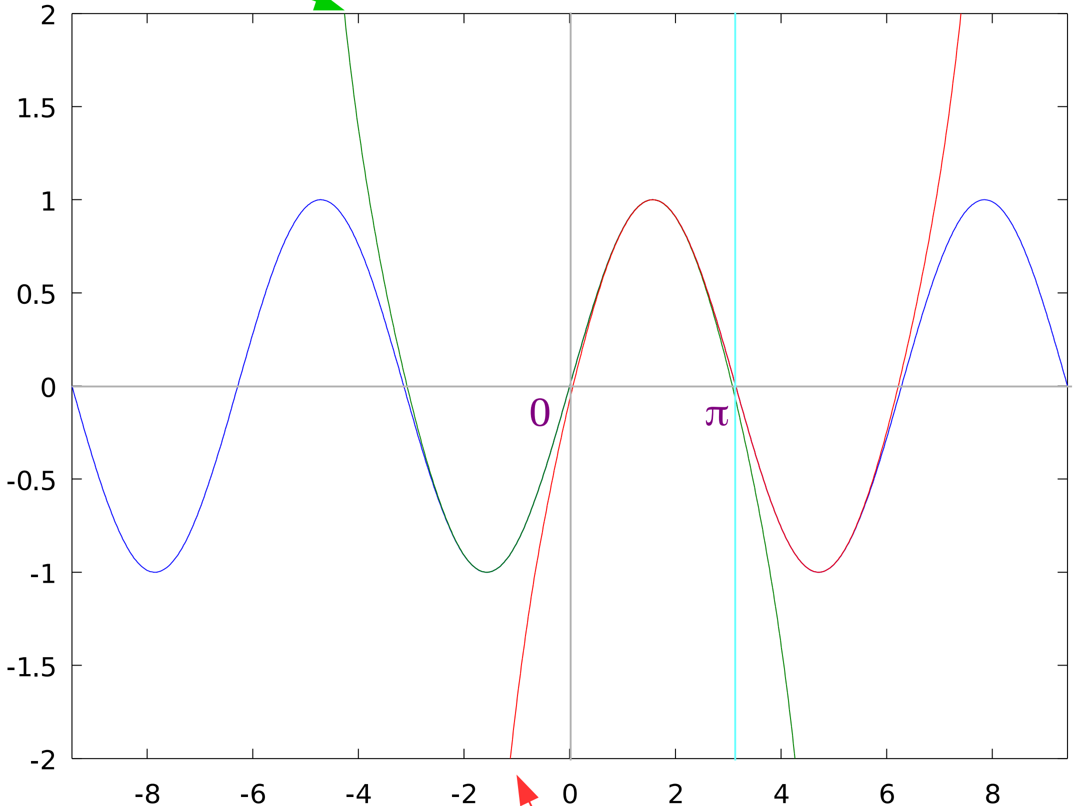
around $x = b$

Taylor Series Expansion – sin(x)

$$f(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}$$

Taylor series expansion at $x = 0$





for $-1 < x < +1$

$f(x) = \sin(x)$

for $\pi - 1 < x < \pi + 1$

$g(x) = \sin(x)$

Taylor series expansion at $x = \pi$

$$g(x) = -(x-\pi) + \frac{(x-\pi)^3}{3!} - \frac{(x-\pi)^5}{5!} + \frac{(x-\pi)^7}{7!}$$

Taylor Series – real

References

[1] http://en.wikipedia.org/
[2] http://planetmath.org/
[3] M.L. Boas, “Mathematical Methods in the Physical Sciences”
[4] E. Kreyszig, “Advanced Engineering Mathematics”
[5] D. G. Zill, W. S. Wright, “Advanced Engineering Mathematics”
[6] T. J. Cavicchi, “Digital Signal Processing”