DTFS (3A)

- CTFS and DFT
- CTFT and DFT
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Young Won Lim
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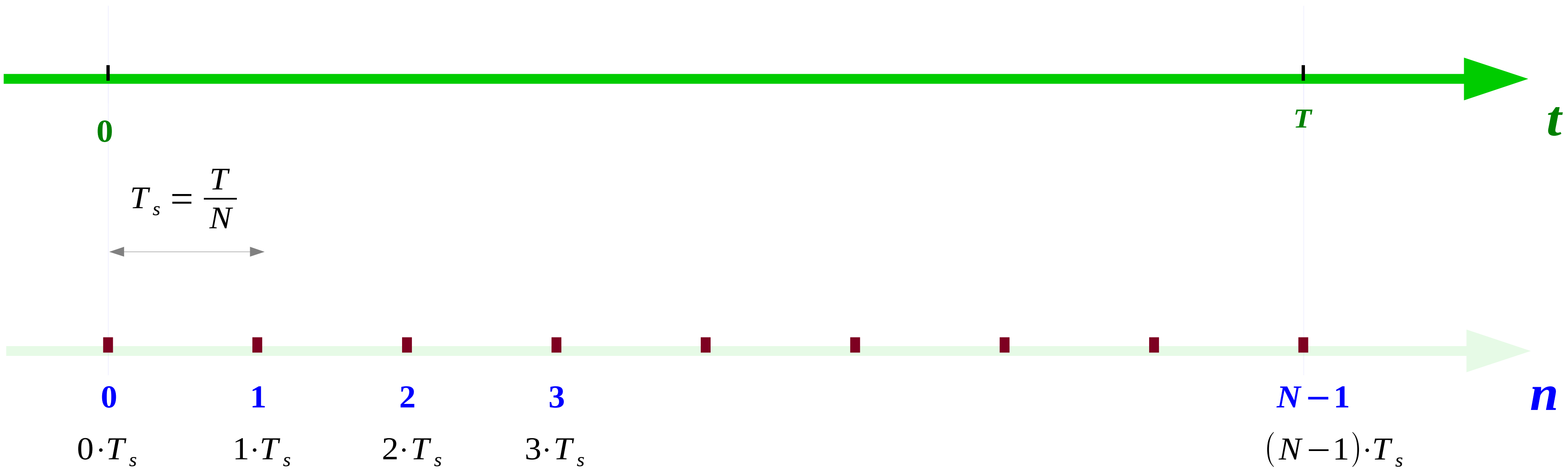
Continuous Time vs. Discrete Time

Period T

Number of Samples N

Sample Instant $t = n\dfrac{T}{N}$

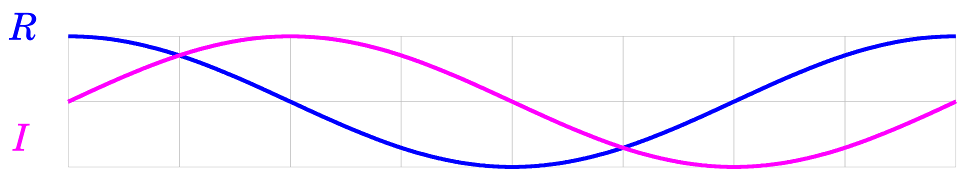


Two Types of Orthogonal Sinusoids

$$e^{+j\left(\frac{2\pi}{T}\right)k\,t}$$

$$k\,\omega_0\,t = \left(\frac{2\pi}{T}\right)k\,t$$

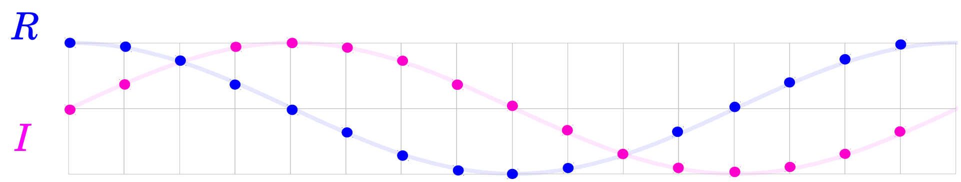
$$\left(\frac{2\pi}{T}\right)$$



$$e^{+j\left(\frac{2\pi}{T}\right)(1)\,t}$$

$$0 \le t \le T$$

$$k = -2,\ -1,\ 0,\ +1,\ +2,\ \ldots$$



$$e^{+j\left(\frac{2\pi}{N}\right)(1)\,n}$$

$$n = 0,\ 1,\ 2,\ \ldots,\ N-1$$

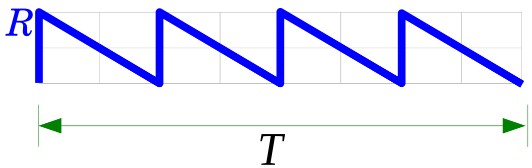
$$k = 0,\ 1,\ 2,\ \ldots,\ N-1$$

$$e^{+j\left(\frac{2\pi}{N}\right)k\,n}$$

$$k\left(\frac{2\pi}{T}\right)n\left(\frac{T}{N}\right) = \left(\frac{2\pi}{N}\right)k\,n$$

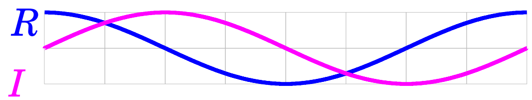
$$\left(\frac{2\pi}{N}\right)$$

Two Types of Inner Products



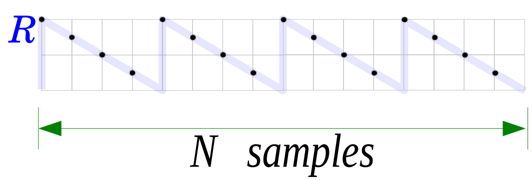
$$f_0 = 1/T$$

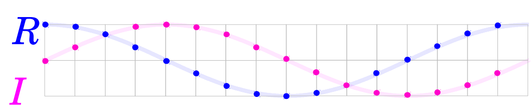
$$\omega_0 = 2\pi/T$$



$$\frac{1}{T}\langle x(t),\ e^{+j(1)\omega_0 t}\rangle = C_1$$

$$\langle e^{+j(1)\omega_0 t},\ e^{+j(1)\omega_0 t}\rangle = T$$





$$\frac{1}{N}\langle x[n],\ e^{+j\left(\frac{2\pi}{N}\right)n(1)}\rangle = \gamma_1$$

$$\langle e^{+j\left(\frac{2\pi}{N}\right)n(1)},\ e^{+j\left(\frac{2\pi}{N}\right)n(1)}\rangle = N$$

Inner Product Representations

$x(t)$ ⟷ invertible ⟷ C_k

$$\frac{1}{T}\int_0^T x(t)\, e^{-jk\omega_0 t}\, dt = C_k$$

$$\frac{\langle x(t) \;,\; e^{+j(2\pi/T)kt} \rangle}{\langle e^{+j(2\pi/T)kt} ,\; e^{+j(2\pi/T)kt} \rangle} = C_k$$

$x[n]$ ⟷ invertible ⟷ γ_k

$$\frac{1}{N}\sum_{n=0}^{N-1} x[n]\, e^{-j\left(\frac{2\pi}{N}\right)nk} = \gamma_k$$

$$\frac{\langle x(t) \;,\; e^{+j(2\pi/N)kn} \rangle}{\langle e^{+j(2\pi/N)kn} ,\; e^{+j(2\pi/N)kn} \rangle} = \gamma_k$$

CTFS and DTFS

Continuous Time $x(t)$

$$x(t) = \sum_{k=-\infty}^{+\infty} C_k \, e^{+jk\omega_0 t}$$

$$0 \le t \le T$$

$$C_k = \frac{1}{T}\int_0^T x(t)\, e^{-jk\omega_0 t}\, dt$$

$$k = -2, -1, 0, +1, +2, \ldots$$

CTFS

Discrete Time $x[n]$

$$x[n] = \sum_{k=-M}^{+M} \gamma_k \, e^{+j\left(\frac{2\pi}{N}\right)kn}$$

$$n = 0, 1, 2, \ldots, N-1,$$

$$\gamma_k = \frac{1}{N}\sum_{n=0}^{N-1} x[n]\, e^{-j\left(\frac{2\pi}{N}\right)kn}$$

$$k = -M, \ldots, 0, \ldots, +M$$

DTFS

$$\sum_{k=-M}^{+M} = \sum_{k=0}^{N-1} = \sum_{k=k_0}^{k_0+N-1} = \sum_{k=\langle N\rangle}$$

$$N = 2M+1$$

C_k **Infinite set of** ***k*****'s**

γ_k **Finite set of** ***k*****'s**

Truncate CTFS Coefficients

$$x(t) = \sum_{k=-\infty}^{+\infty} C_k \, e^{+jk\omega_0 t}$$

$$0 \le t \le T$$

$$C_k = \frac{1}{T} \int_0^T x(t) \, e^{-jk\omega_0 t} \, dt$$

$$k = -2, \; -1, \; 0, \; +1, \; +2, \; \ldots$$

Infinite set of *k*'s **CTFS**

$$x(t) \approx \sum_{k=-M}^{+M} C_k \, e^{+jk\omega_0 t} \qquad N = 2M+1$$

synthesis with truncated coefficients

Use a finite subset of N coefficients

Finite set of *k*'s

Approximated Coefficients

CTFS DTFS DFT

$$C_k \approx \gamma_k = \frac{X[k]}{N}$$

$$x[n] = \sum_{k=-M}^{+M} \gamma_k \, e^{+j\left(\frac{2\pi}{N}\right)k n}$$

$$n = 0, \; 1, \; 2, \; \ldots \; , \; N-1,$$

$$\gamma_k = \frac{1}{N} \sum_{n=0}^{N-1} x[n] \, e^{-j\left(\frac{2\pi}{N}\right)k n}$$

$$k = -M, \; \ldots \; , \; 0, \; \ldots \; , \; +M$$

DTFS

Approximated Synthesis

$$x(t) = \sum_{k=-\infty}^{+\infty} C_k \, e^{+jk\omega_0 t}$$

$$x_{FS}(t) = \sum_{k=-M}^{+M} \gamma_k \, e^{+jk\omega_0 t}$$

$$x[n] = \sum_{k=-M}^{+M} \gamma_k \, e^{+j\left(\frac{2\pi}{N}\right)kn}$$

$$n = 0,\ 1,\ 2,\ \ldots,\ N-1,$$

$$\gamma_k = \frac{1}{N} \sum_{n=0}^{N-1} x[n] \, e^{-j\left(\frac{2\pi}{N}\right)kn}$$

$$k = -M,\ \ldots,\ 0,\ \ldots,\ +M$$

DTFS

CTFS, DTFS, and DFT

$$x(t) = \sum_{k=-\infty}^{+\infty} C_k \, e^{+jk\omega_0 t}$$

$$0 \le t \le T$$

$$C_k = \frac{1}{T} \int_0^T x(t) \, e^{-jk\omega_0 t} \, dt$$

$$k = -2, -1, 0, +1, +2, \ldots$$

CTFS

$$x[n] = \sum_{k=0}^{N} \gamma_k \, e^{+j\left(\frac{2\pi}{N}\right)kn}$$

$$n = 0, 1, 2, \ldots, N-1$$

$$\gamma_k = \frac{1}{N} \sum_{n=0}^{N-1} x[n] \, e^{-j\left(\frac{2\pi}{N}\right)kn}$$

$$k = 0, 1, 2, \ldots, N-1$$

DTFS

$$x(t) \approx \sum_{k=-M}^{+M} \gamma_k \, e^{+jk\omega_0 t}$$

Approximated Synthesis

$$0 \le t \le T$$

$$C_k \approx \gamma_k = \frac{X[k]}{N}$$

Approximated Coefficients

$$k = -2, -1, 0, +1, +2, \ldots$$

$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] \, e^{+j\left(\frac{2\pi}{N}\right)kn}$$

$$n = 0, 1, 2, \ldots, N-1$$

$$X[k] = \sum_{n=0}^{N-1} x[n] \, e^{-j\left(\frac{2\pi}{N}\right)kn}$$

$$k = 0, 1, 2, \ldots, N-1$$

DFT

CTFT of a Sampled Signal

References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003