

# Tapped Delay

---

Copyright (c) 2024 - 2019 Young W. Lim.

Permission is granted to copy, distribute and/or modify this document under the terms of the GNU Free Documentation License, Version 1.2 or any later version published by the Free Software Foundation; with no Invariant Sections, no Front-Cover Texts, and no Back-Cover Texts. A copy of the license is included in the section entitled "GNU Free Documentation License".

Please send corrections (or suggestions) to [youngwlim@hotmail.com](mailto:youngwlim@hotmail.com).

This document was produced by using LibreOffice.

# Based on

---

Introduction to Signal Processing

S. J. Ofranidis

The necessities in DSP C Programming

FIR Filter (A.pdf) 20191114

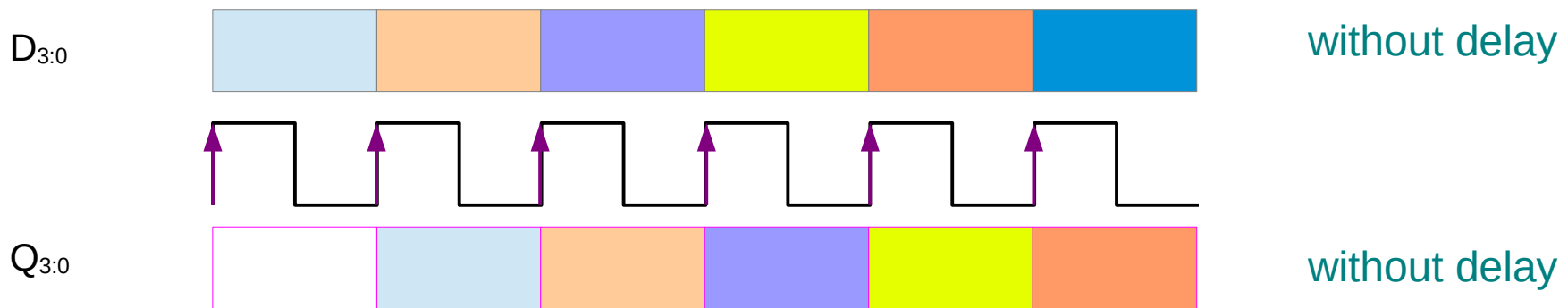
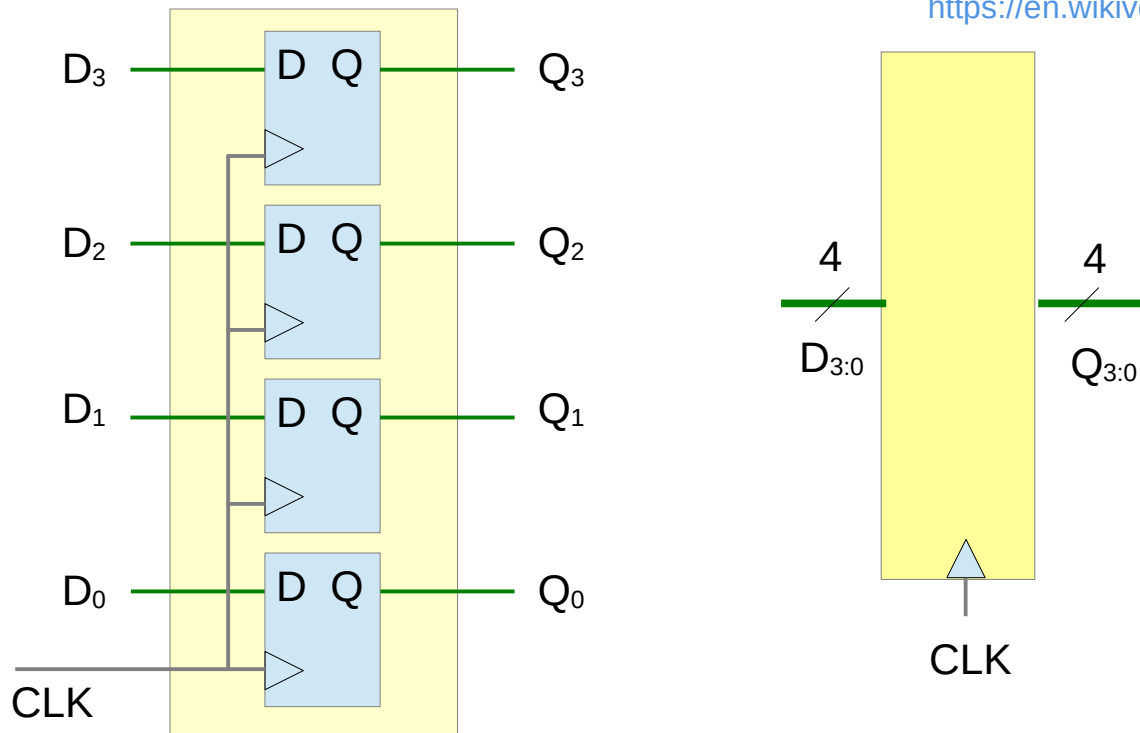
# D Flip Flop

---

Considering the widely used  
Edge triggered  
D-type Flip Flops

# Register

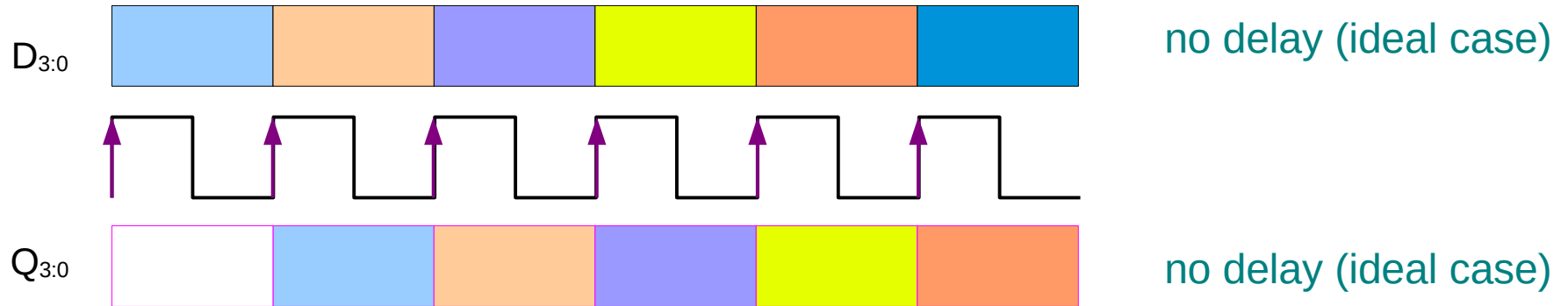
[https://en.wikiversity.org/wiki/The\\_necessities\\_in\\_Digital\\_Design](https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design)



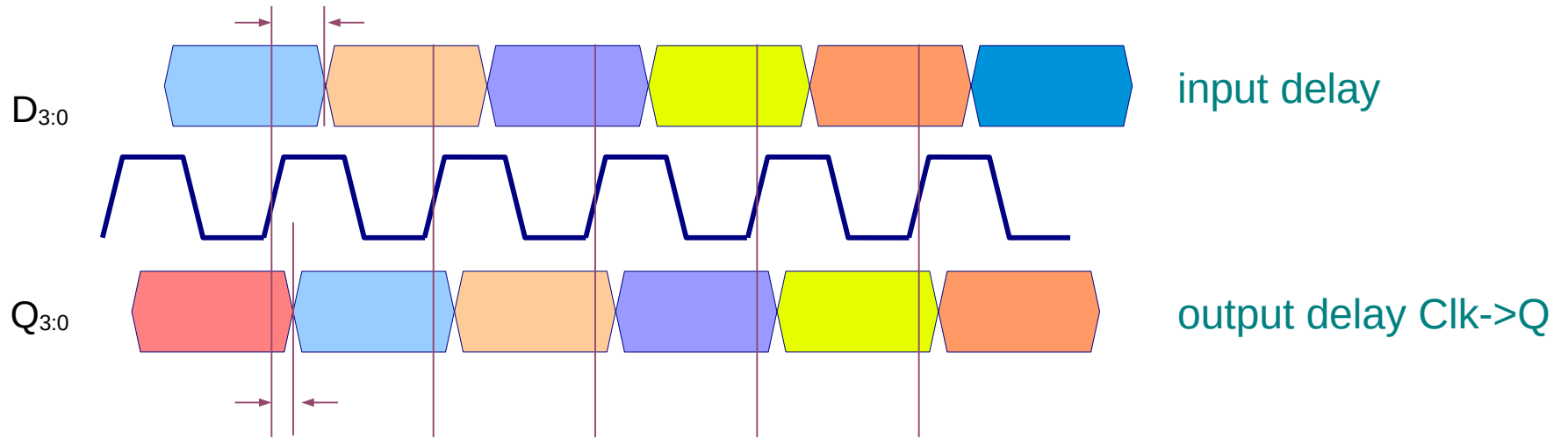
# Types of Timing Diagrams

## a timing diagram without delays

[https://en.wikiversity.org/wiki/The\\_necessities\\_in\\_Digital\\_Design](https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design)

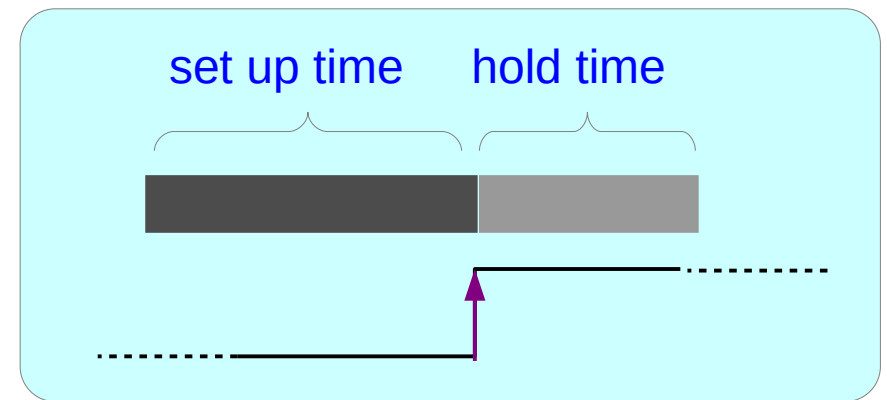
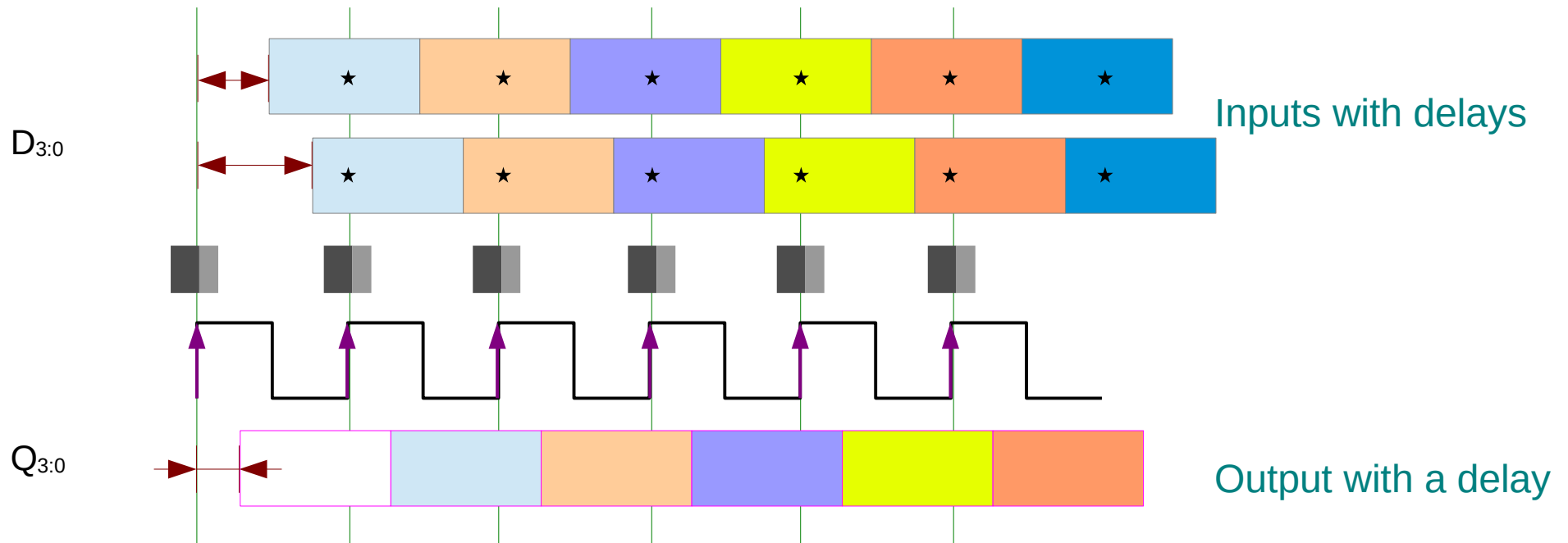


## a timing diagram with delays



# Setup & Hold Time (1)

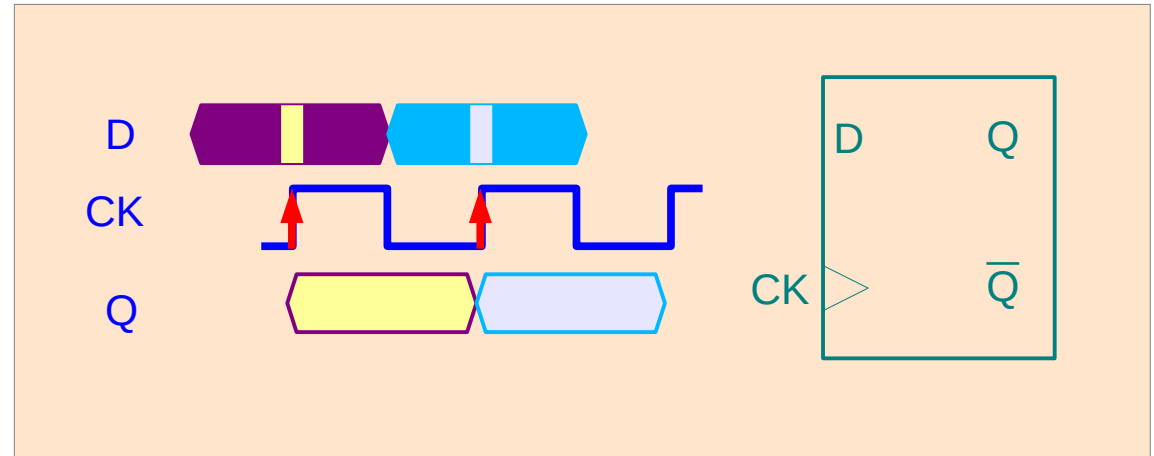
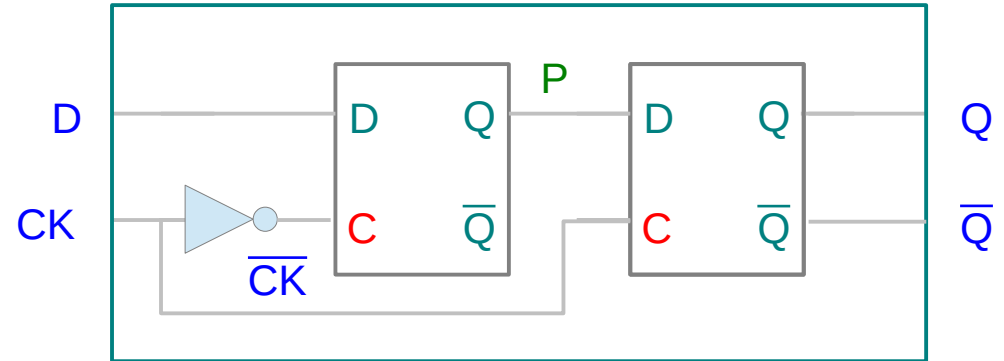
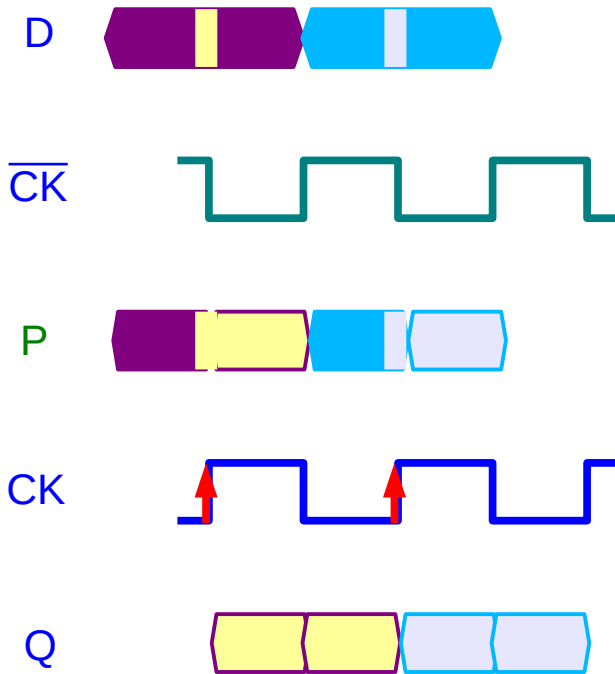
[https://en.wikiversity.org/wiki/The\\_necessities\\_in\\_Digital\\_Design](https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design)



# Master-Slave D FlipFlop – Rising Edge

[https://en.wikiversity.org/wiki/The\\_necessities\\_in\\_Digital\\_Design](https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design)

## Master D Latch

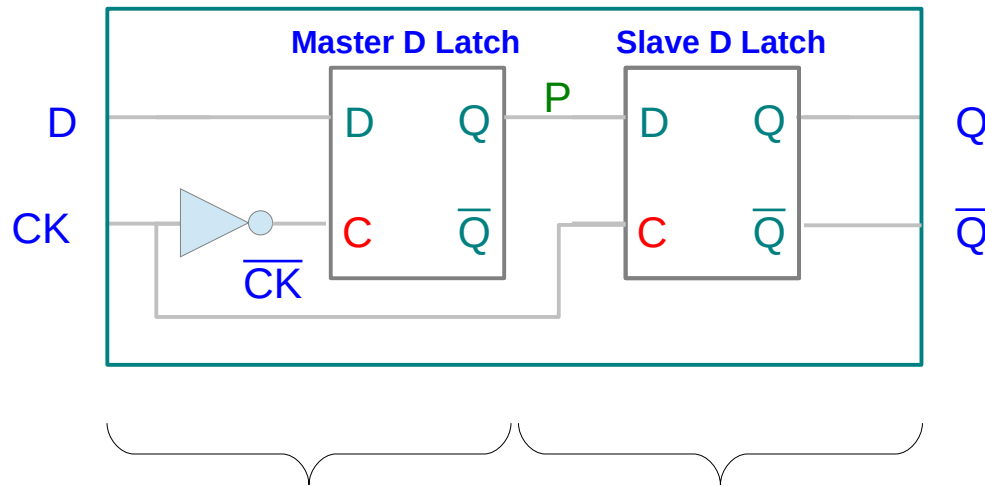


## Slave D Latch



# Master-Slave D FlipFlop – Rising Edge

[https://en.wikiversity.org/wiki/The\\_necessities\\_in\\_Digital\\_Design](https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design)

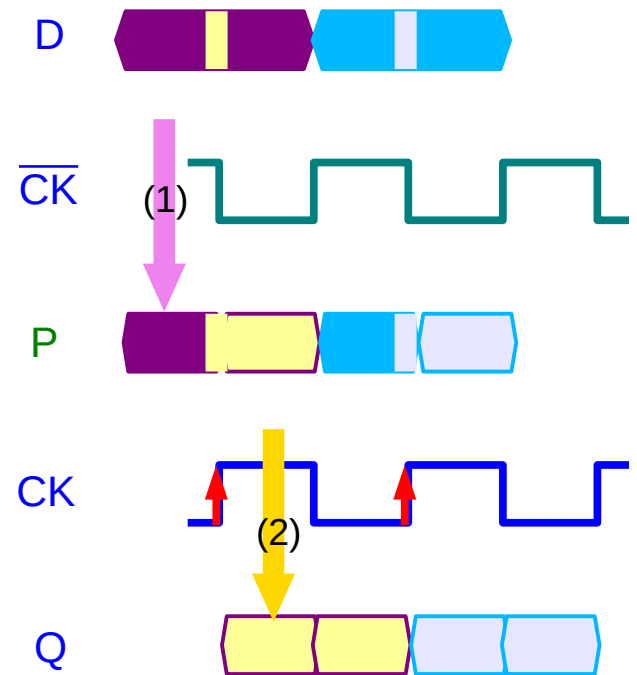


(1) the current input D gets stored in the master latch

(2) the current content P is clocked out to the output Q

Using **inverted clocks enable** (1) and (2) to be executed sequentially

Master D Latch

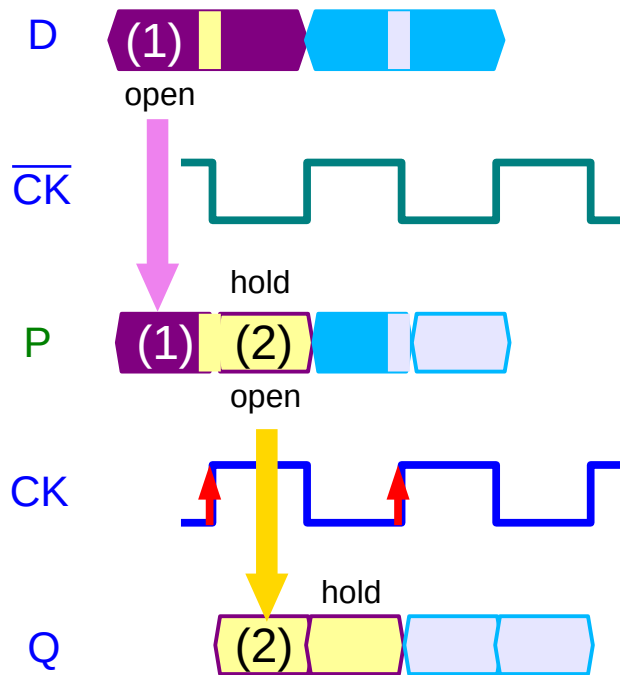


Slave D Latch

# Master-Slave D FlipFlop – open and hold

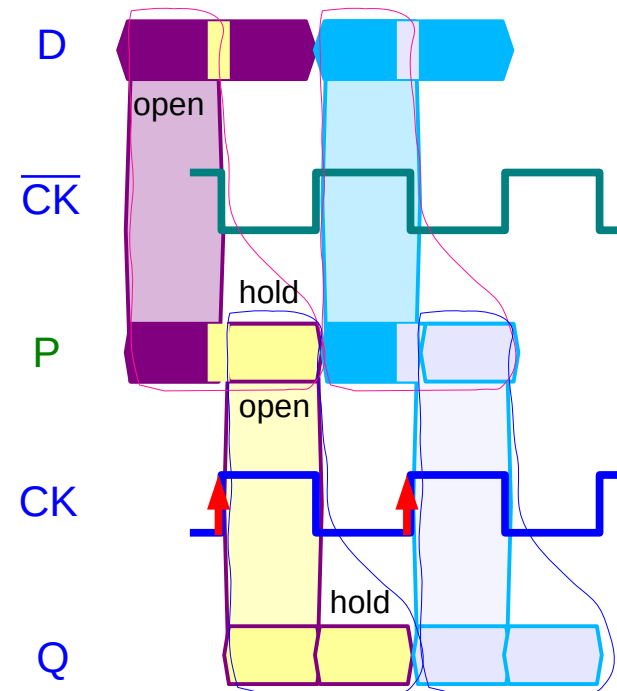
[https://en.wikiversity.org/wiki/The\\_necessities\\_in\\_Digital\\_Design](https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design)

Master D Latch



Slave D Latch

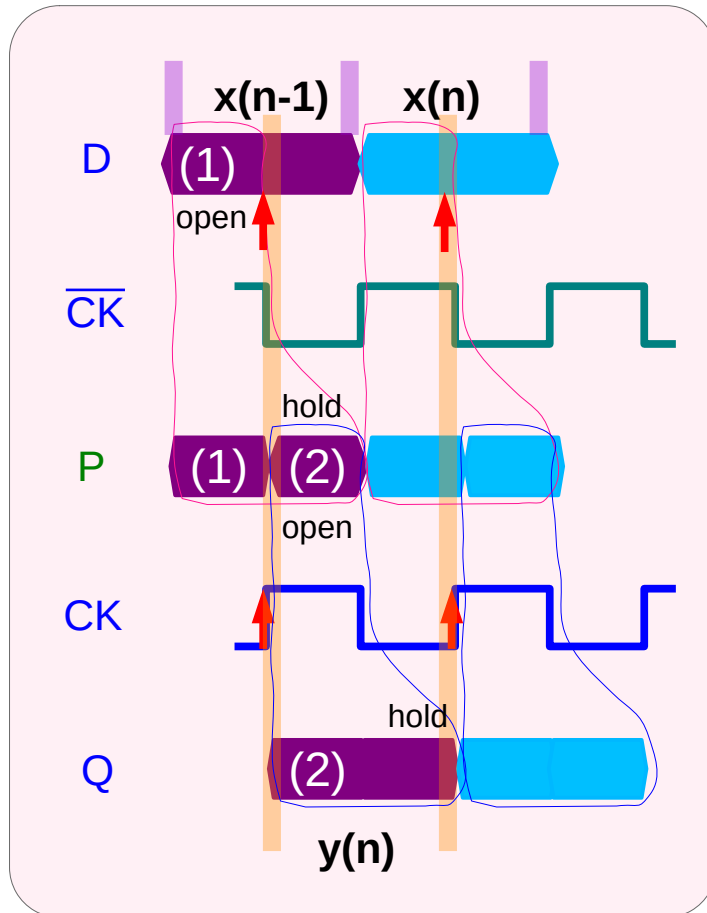
Master D Latch



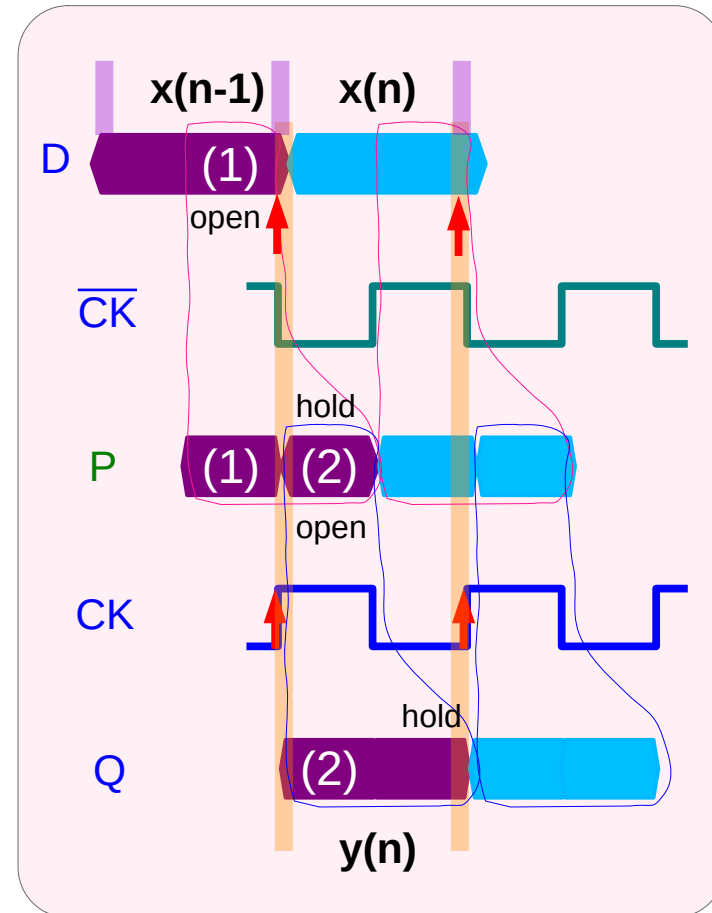
Slave D Latch

# Master-Slave D FlipFlop – typical vs. ideal timing (1)

## Typical Timing

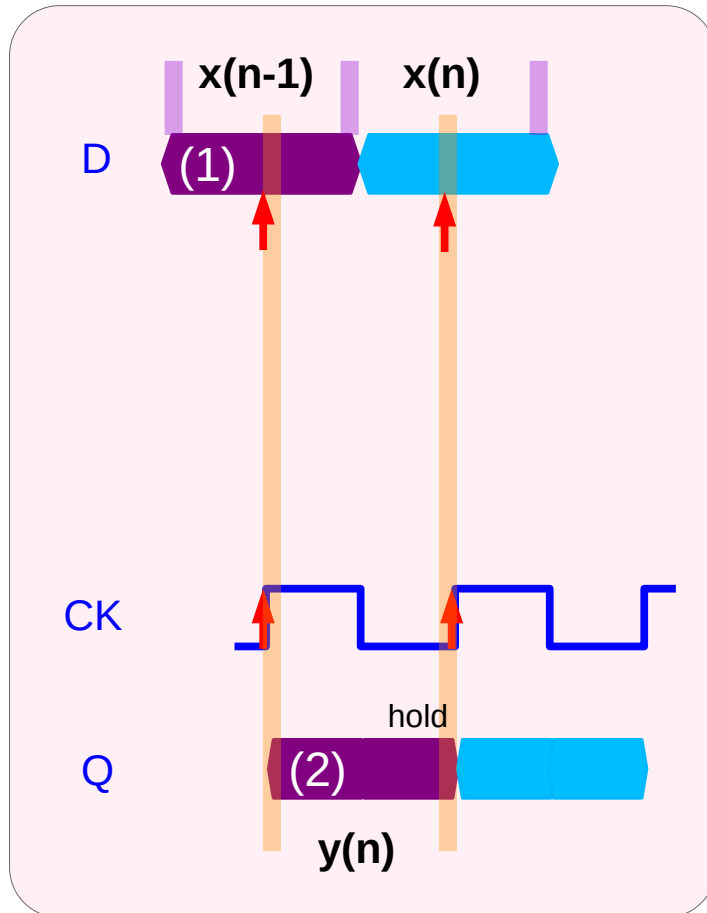


## Ideal Timing

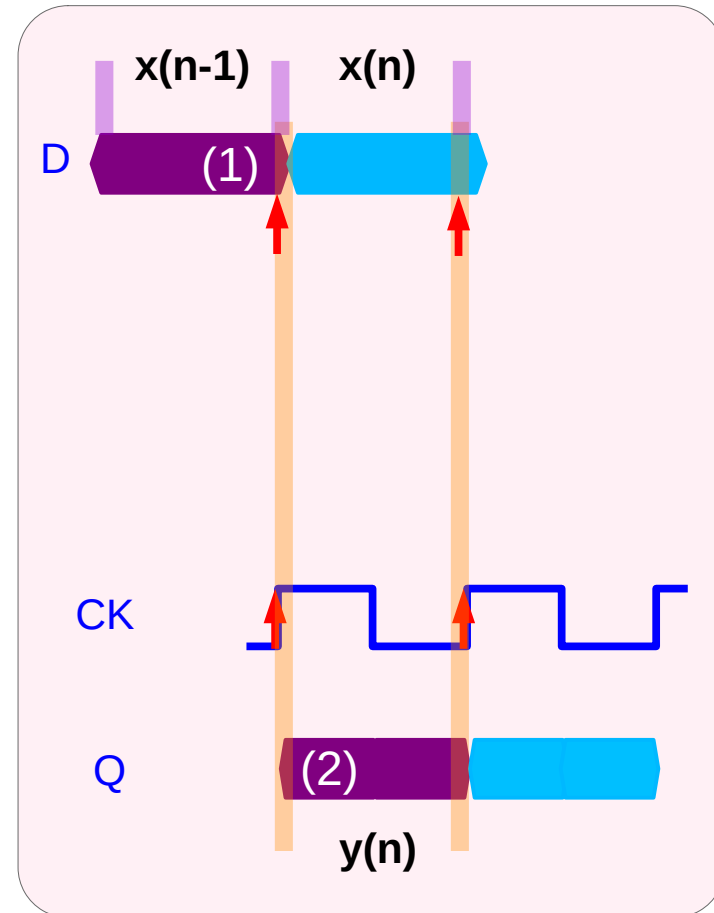


# Master-Slave D FlipFlop – typical vs. ideal timing (2)

## Typical Timing

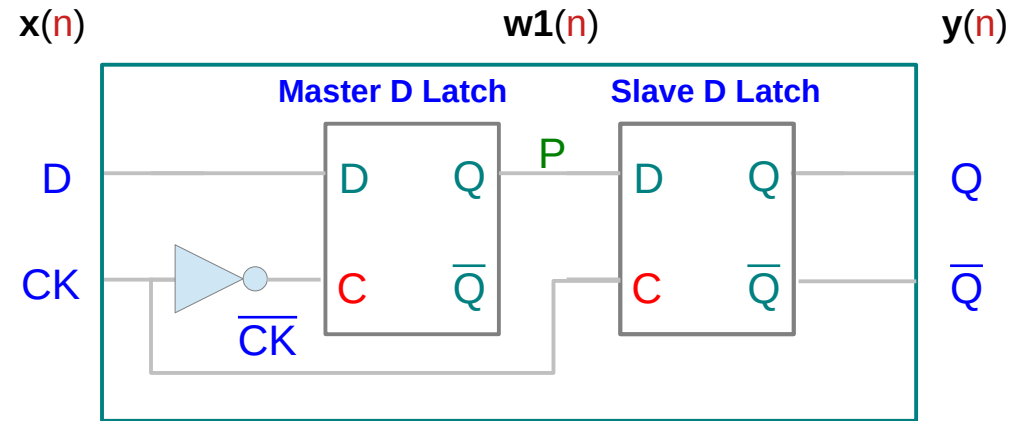
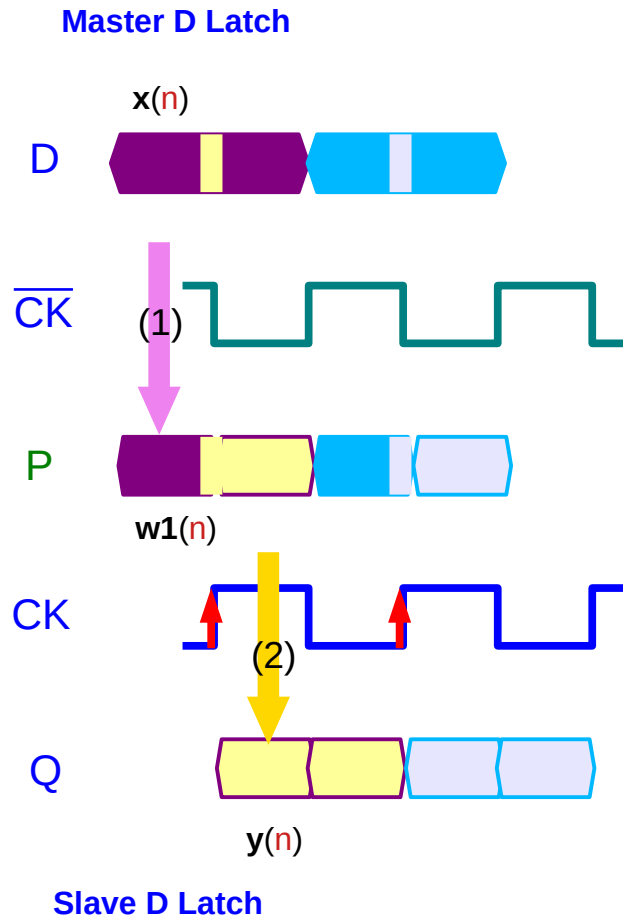


## Ideal Timing



# Master-Slave D FlipFlop – Rising Edge Sampling

[https://en.wikiversity.org/wiki/The\\_necessities\\_in\\_Digital\\_Design](https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design)



(1) the current input  $x(n)$  gets stored in the master latch

(2) the current content  $w1(n)$  is clocked out to the output  $y(n)$

# Fixed point representation

fractional numbers

**Floating Point Representation**

**Fixed Point Representation**

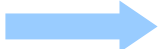
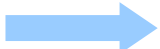
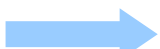
integer + implicit fixed scaling factor

a 2's complement number  $(11110101)_2 = -11$

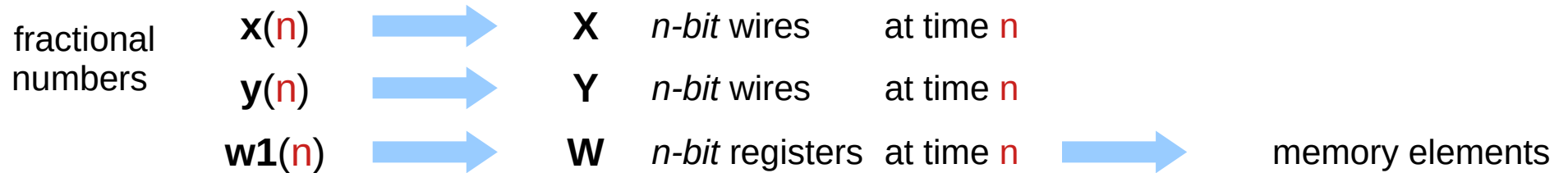
could represent  $-11 \cdot 2^{-3} = -88$ ,  
 $-11 \cdot 2^{-5} = -0.34375$   
 $-11 \cdot 2^{-12} = -0.002685546875$

with implied scaling factors -3, -5, -12

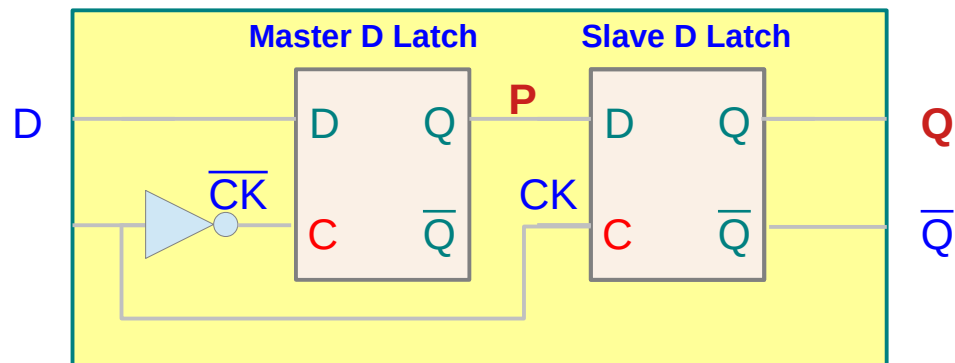
fractional numbers

$x(n)$		<b>X</b>	<i>n-bit</i> wires	at time <i>n</i>
$y(n)$		<b>Y</b>	<i>n-bit</i> wires	at time <i>n</i>
$w1(n)$		<b>W</b>	<i>n-bit</i> registers	at time <i>n</i>

# Memory Elements

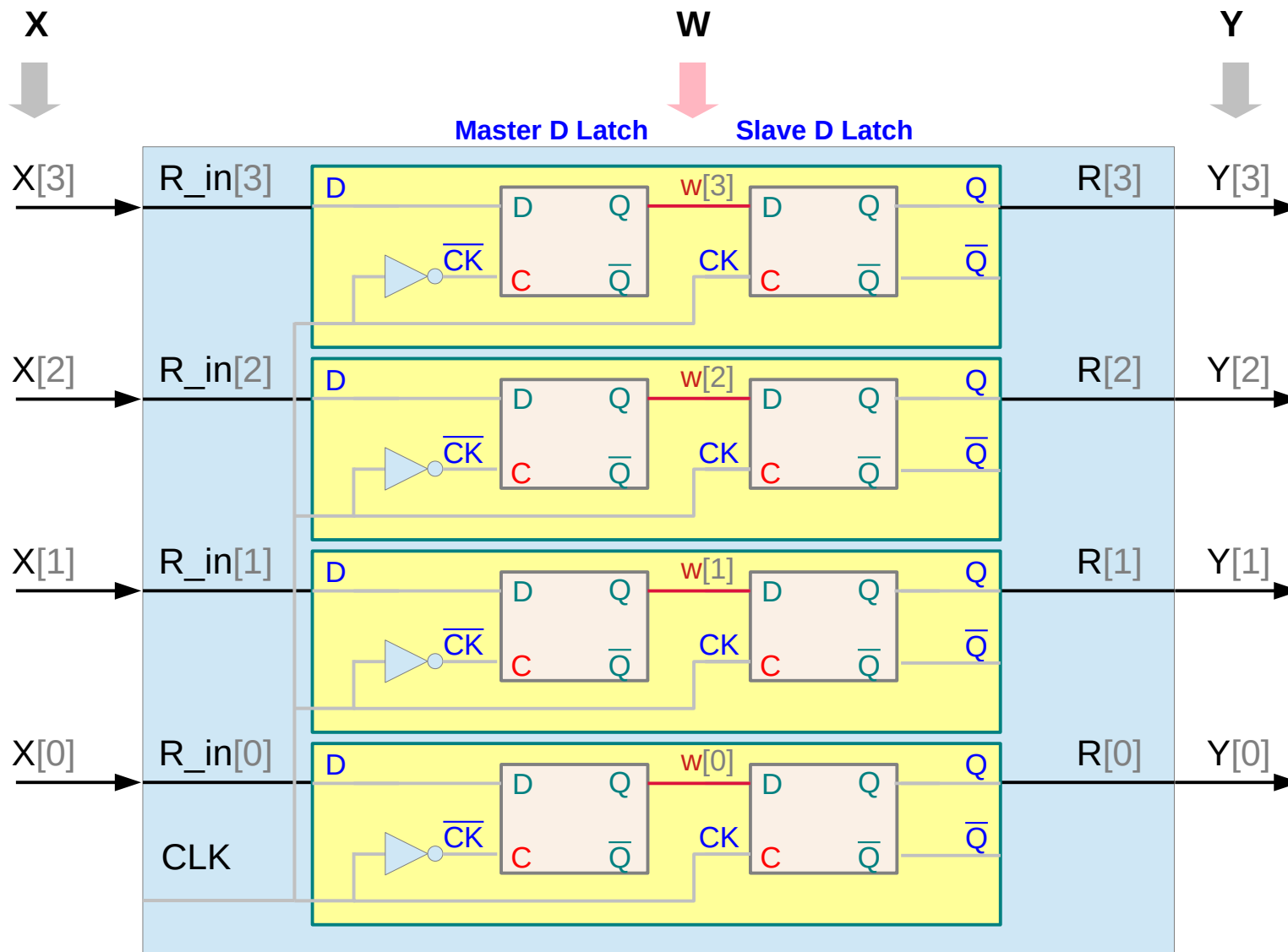


1. using the **P** outputs of **master latches** as the name for **memory elements**
2. using the **Q** outputs of **flipflops** as the name for **memory elements**



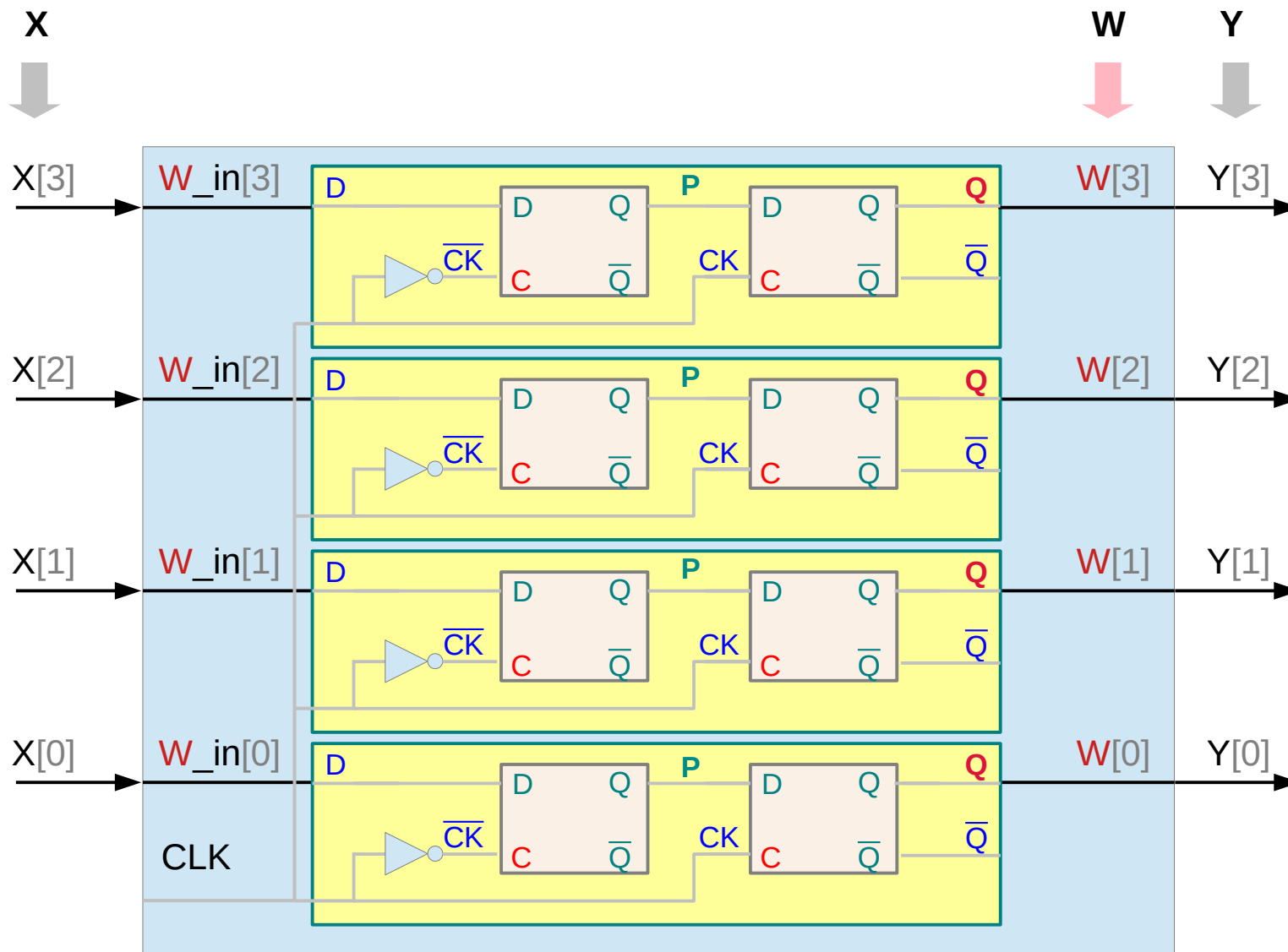
D FlipFlop using Master and Slave D Latches

# (1) Using master latch outputs P

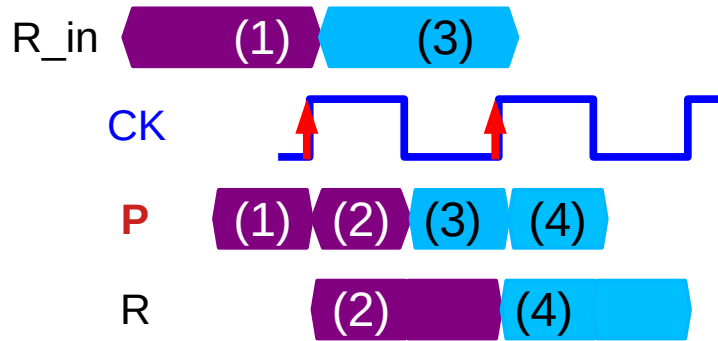




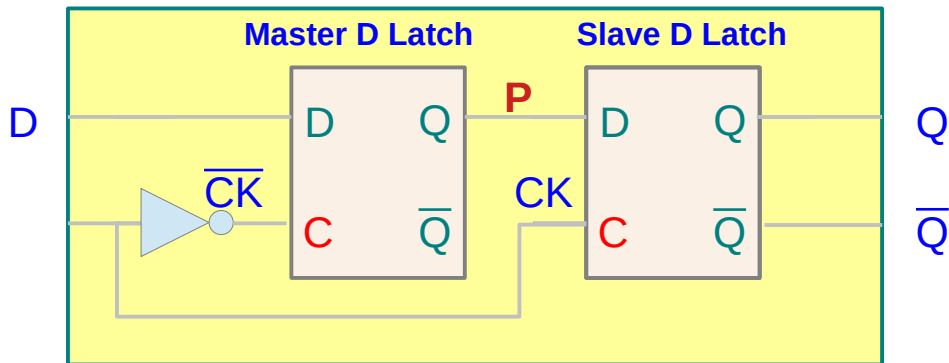
## (2) Using flipflop outputs Q



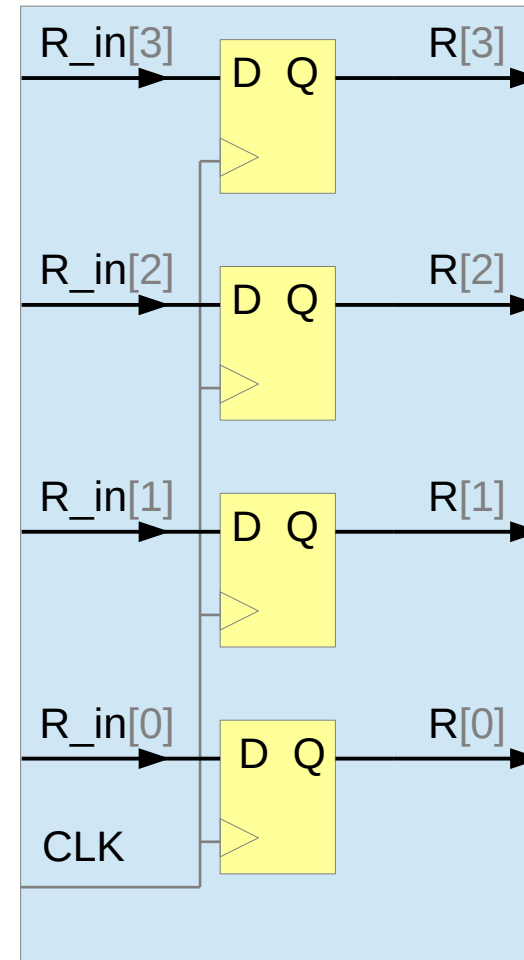
# (1) 4-bit Register Using master latch outputs **P**



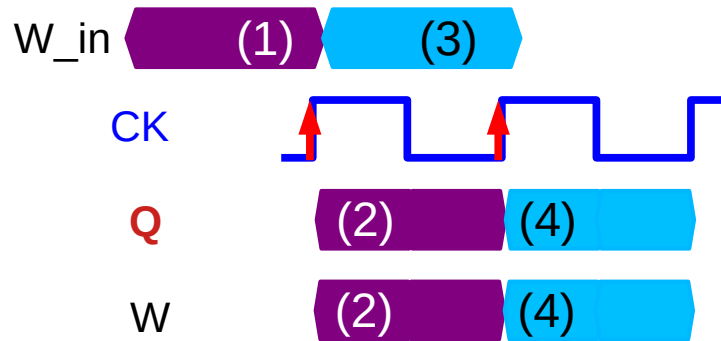
D FlipFlop using Master and Slave D Latches



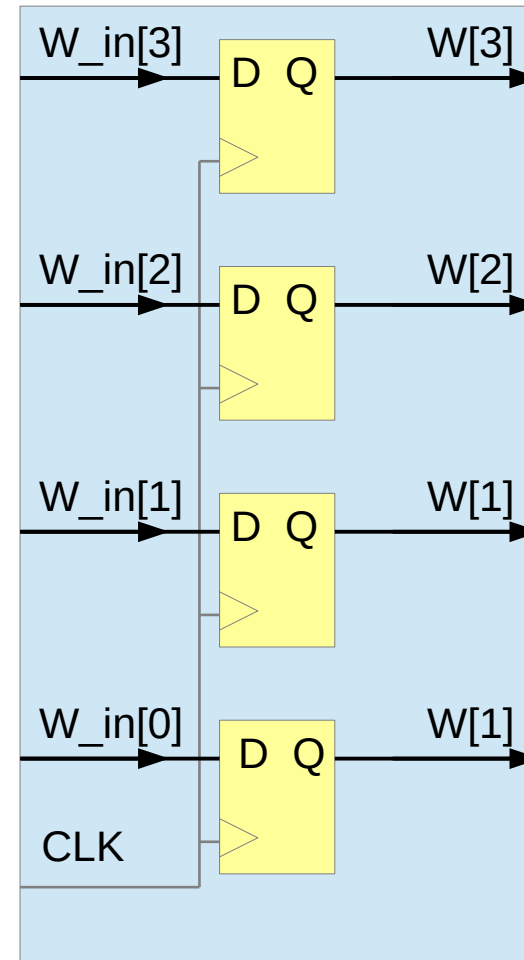
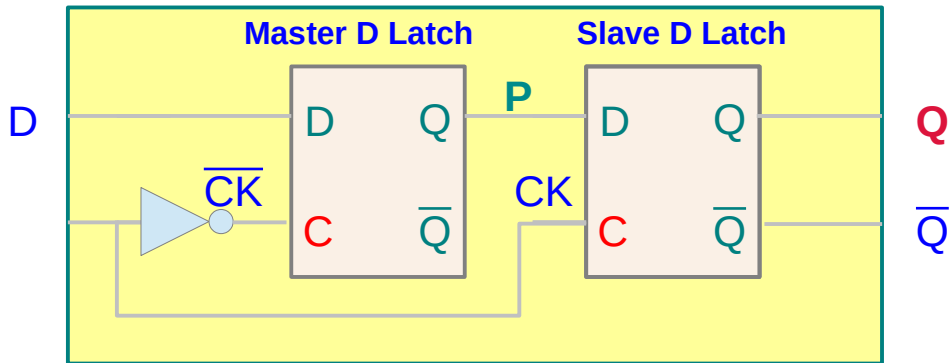
4-bit Register using 4 D flipflops



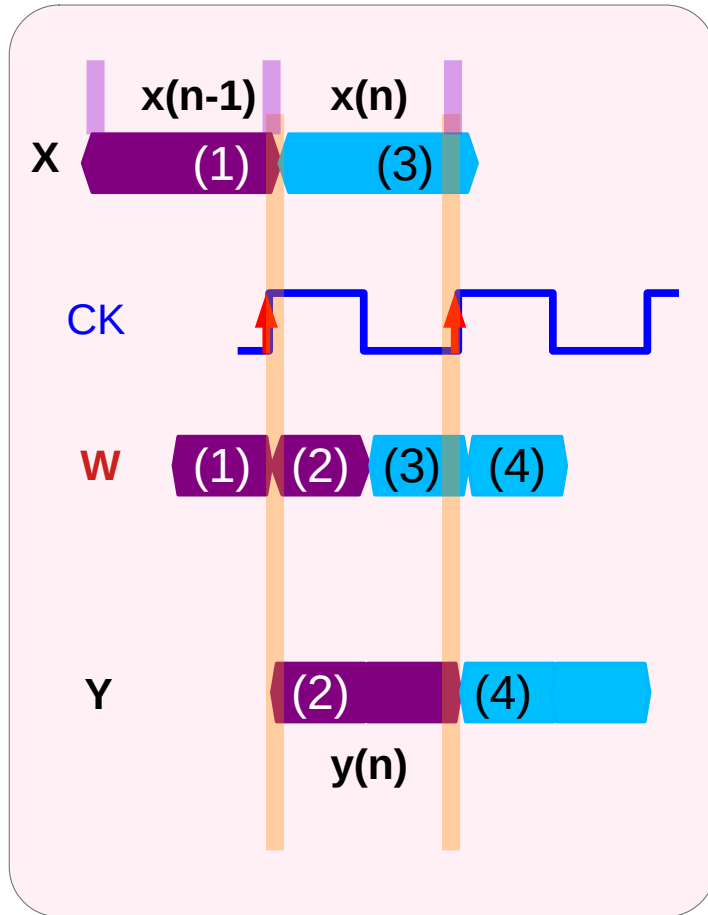
## (2) 4-bit Register Using flipflop outputs Q



D FlipFlop using Master and Slave D Latches



# (1) Timing diagrams with master latch outputs **P**



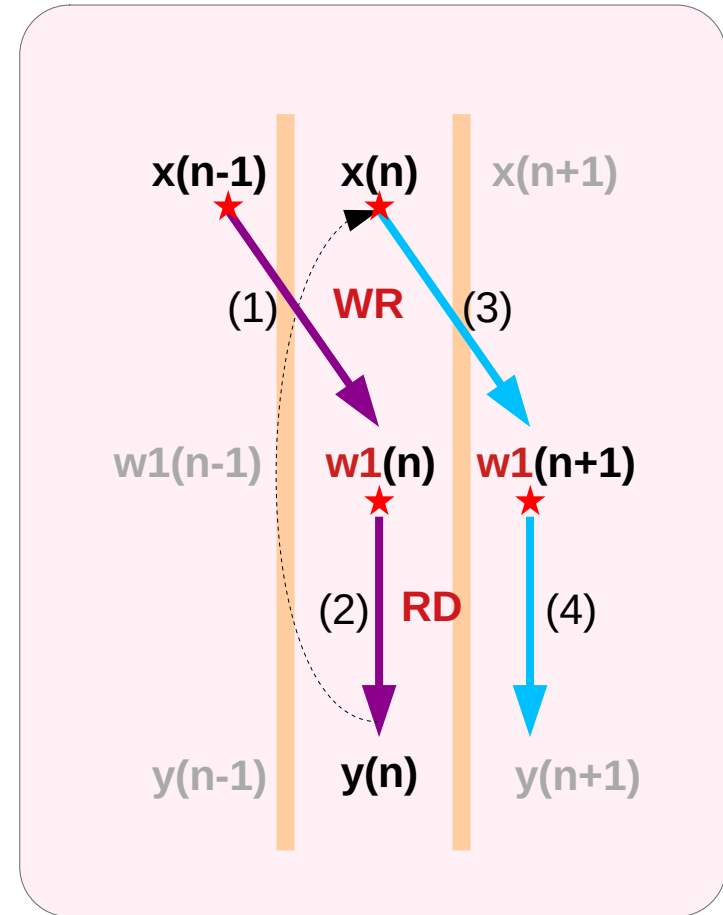
Hardware model

## data centered view

Input

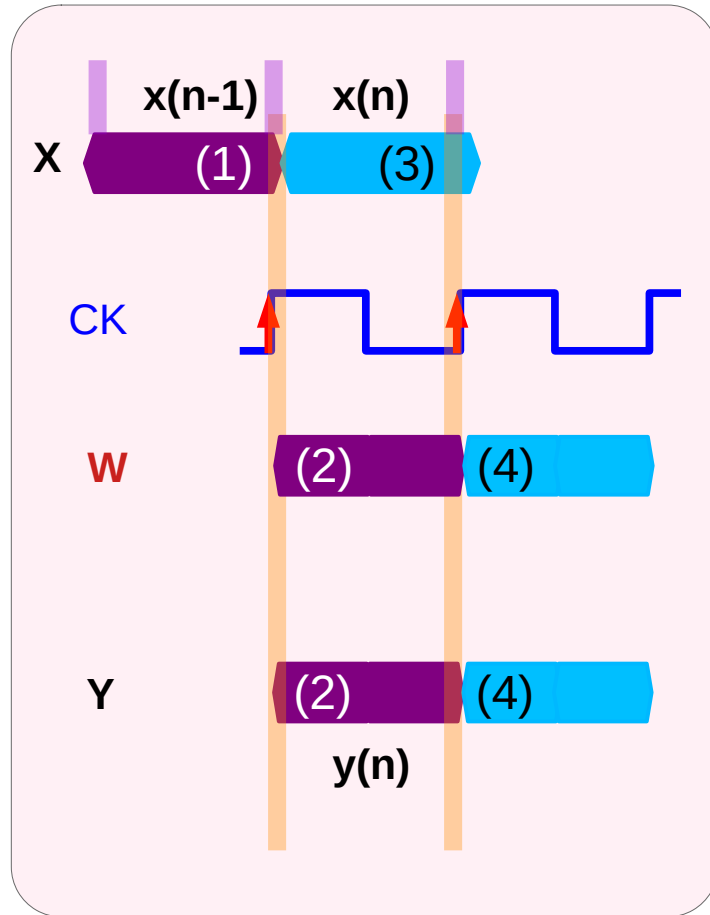
Internal State

Output



The same **DSP C** model

## (2) Timing diagrams with flipflop outputs Q



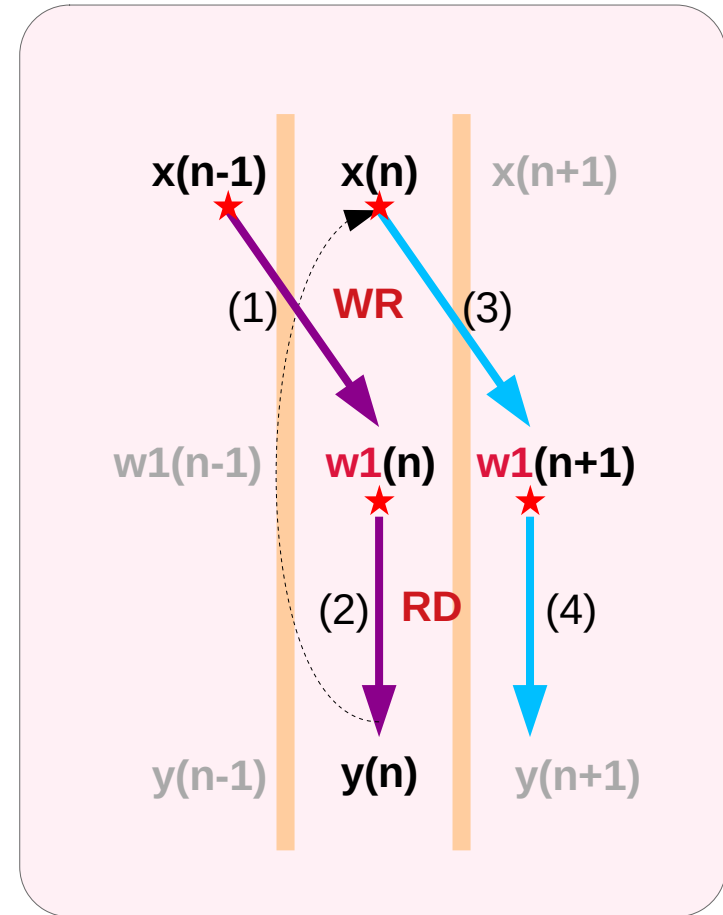
Hardware model

data centered view

Input

Simplified Internal State

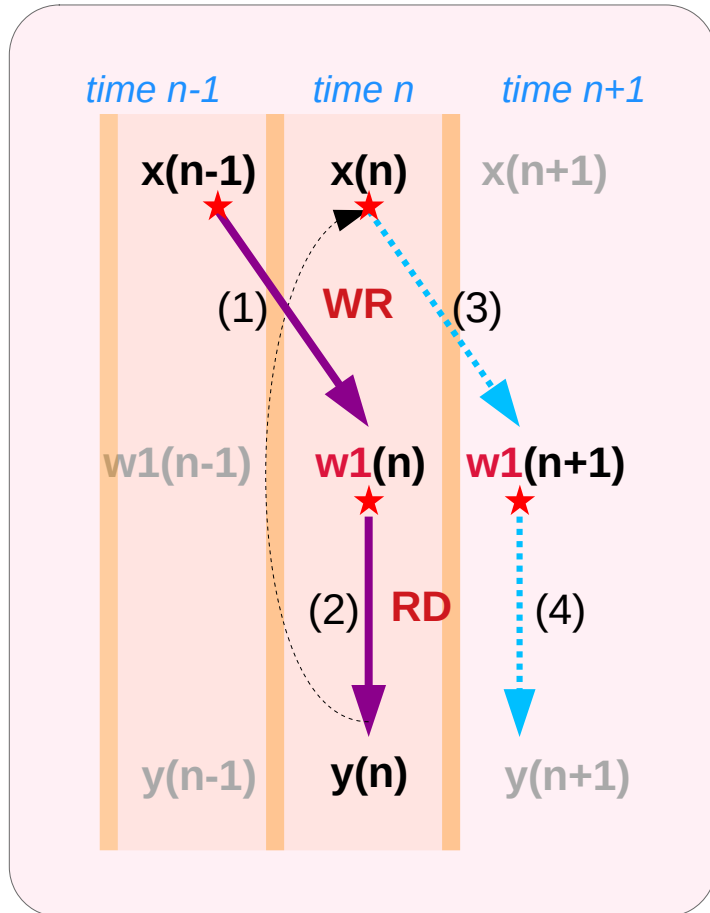
Output



The same DSP C model

# Data centered operations

## data centered operations



## DSP C model

## operation spans two time steps (n-1, n)

- (1)  $w1(n) = x(n-1)$       *WR*  $w1(n)$
- (2)  $y(n) = w1(n)$       *RD*  $w1(n)$

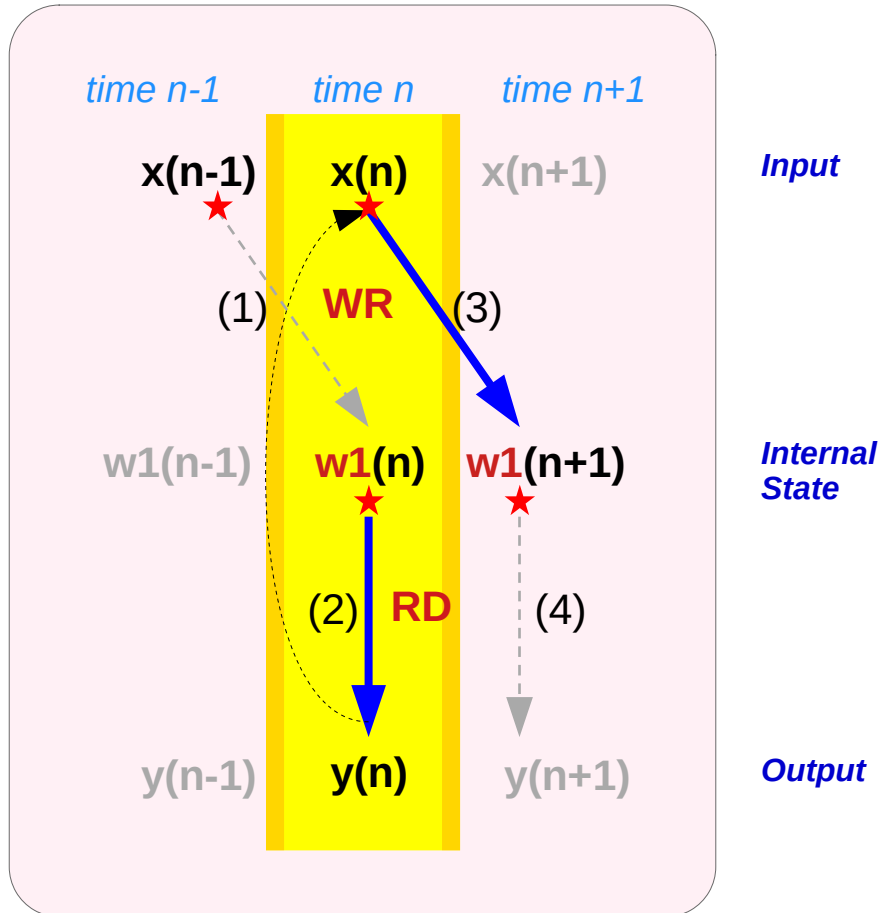
first **WR**, then **RD**  
must avoid **RAW (read after write)** hazards

here, because the written data is read out,  
there is no **RAW (read after write)** hazards

$$\begin{array}{l}
 \underline{y(n-1) = w1(n-1)} \quad \underline{w1(n) = w1(n-1)} \\
 \underline{w1(n) = x(n-1)} \\
 \underline{y(n) = w1(n)} \quad \underline{w1(n+1) = w1(n)} \\
 \underline{w1(n+1) = x(n)} \\
 \underline{y(n+1) = w1(n+1)} \quad \underline{w1(n+2) = w1(n+1)} \\
 \underline{w1(n+2) = x(n+1)} \\
 \underline{y(n+2) = w1(n+2)}
 \end{array}$$

# Time centered operations

## time centered operations



## DSP C model

## operation spans only one time step (n)

- (2)  $y(n) = w1(n)$       *RD*  $w1(n)$   
 (3)  $w1(n+1) = x(n)$       *WR*  $w1(n)$

first **RD**, then **WR**  
 no **RAW (read after write)** hazards

$y(n-1) = w1(n-1)$	$w1(n) = w1(n-1)$
$w1(n) = x(n-1)$	
$y(n) = w1(n)$	$w1(n+1) = w1(n)$
$w1(n+1) = x(n)$	
$y(n+1) = w1(n+1)$	$w1(n+2) = w1(n+1)$
$w1(n+2) = x(n+1)$	
$y(n+2) = w1(n+2)$	

# Data vs. time centered operations (1)

## data centered operations

## time centered operations

$y(n-1) = w1(n-1)$ <i>output from <math>w1(n-1)</math></i> <hr/> $w1(n) = x(n-1)$ <i>input to <math>w1(n)</math></i>	$w1(n) = w1(n-1)$	$y(n-1) = w1(n-1)$ <i>output from <math>w1(n-1)</math></i> <hr/> $w1(n) = x(n-1)$ <i>input to <math>w1(n)</math></i>	$w1(n) = w1(n-1)$	} <i>time n-1</i> } <i>time n</i> } <i>time n+1</i> } <i>time n+2</i>
$y(n) = w1(n)$ <i>output from <math>w1(n)</math></i> <hr/> $w1(n+1) = x(n)$ <i>input to <math>w1(n+1)</math></i>	$w1(n+1) = w1(n)$	$y(n) = w1(n)$ <i>output from <math>w1(n)</math></i> <hr/> $w1(n+1) = x(n)$ <i>input to <math>w1(n+1)</math></i>	$w1(n+1) = w1(n)$	
$y(n+1) = w1(n+1)$ <i>output from <math>w1(n+1)</math></i> <hr/> $w1(n+2) = x(n+1)$ <i>input to <math>w1(n+2)</math></i>	$w1(n+2) = w1(n+1)$	$y(n+1) = w1(n+1)$ <i>output from <math>w1(n+1)</math></i> <hr/> $w1(n+2) = x(n+1)$ <i>input to <math>w1(n+2)</math></i>	$w1(n+2) = w1(n+1)$	
$y(n+2) = w1(n+2)$ <i>output from <math>w1(n+2)</math></i>		$y(n+2) = w1(n+2)$ <i>output from <math>w1(n+2)</math></i>		

- (1)  $w1(n) = x(n-1)$       *WR  $w1(n)$*   
 (2)  $y(n) = w1(n)$       *RD  $w1(n)$*

- (2)  $y(n) = w1(n)$       *RD  $w1(n)$*   
 (1)  $w1(n) = x(n)$       *WR  $w1(n)$*

$W1 = X$	<i>WR <math>W1</math></i>
$Y = W1$	<i>RD <math>W1</math></i>

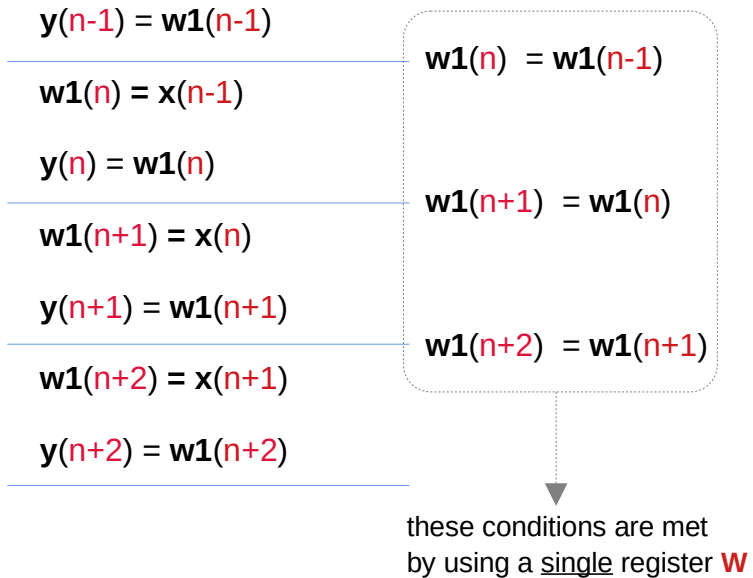
$Y = W1$	<i>RD <math>W1</math></i>
$W1 = X$	<i>WR <math>W1</math></i>

← *Register Transfer Level*

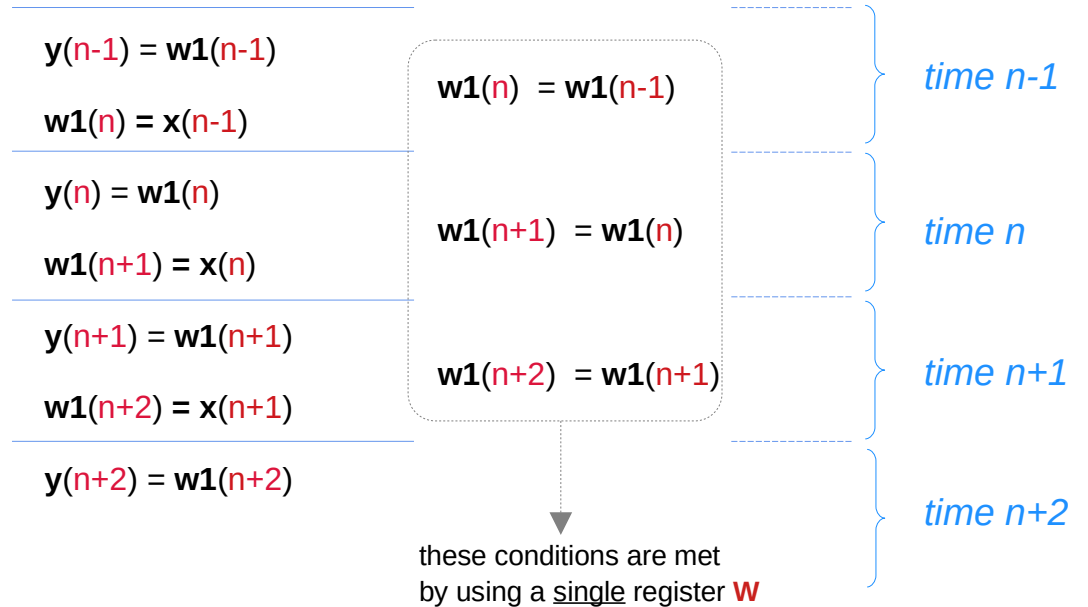


# Data vs. time centered operations (2)

## data centered operations



## time centered operations



# Data centered operations

## data centered operations

## in the Register Transfer Level

$y(n-1) = w1(n-1)$ <small>output <math>y(n-1)</math> from <math>w1(n-1)</math></small> $w1(n) = x(n-1)$ <small>input <math>x(n-1)</math> to <math>w1(n)</math></small>	$w1(n) = w1(n-1)$	$Y = W1$ <small>output <math>y(n-1)</math> from <math>W1</math></small> $W1 = X$ <small>input <math>x(n-1)</math> to <math>W1</math></small>	} time n-1
$y(n) = w1(n)$ <small>output <math>y(n)</math> from <math>w1(n)</math></small> $w1(n+1) = x(n)$ <small>input <math>x(n)</math> to <math>w1(n+1)</math></small>	$w1(n+1) = w1(n)$	$Y = W1$ <small>output <math>y(n)</math> from <math>W1</math></small> $W1 = X$ <small>input <math>x(n)</math> to <math>W1</math></small>	
$y(n+1) = w1(n+1)$ <small>output <math>y(n+1)</math> from <math>w1(n+1)</math></small> $w1(n+2) = x(n+1)$ <small>input <math>x(n+1)</math> to <math>w1(n+2)</math></small>	$w1(n+2) = w1(n+1)$	$Y = W1$ <small>output <math>y(n+1)</math> from <math>W1</math></small> $W1 = X$ <small>input <math>x(n+1)</math> to <math>W1</math></small>	} time n+1
$y(n+2) = w1(n+2)$ <small>output <math>y(n+2)</math> from <math>w1(n+2)</math></small>		$Y = W1$ <small>output <math>y(n+2)</math> from <math>W1</math></small>	
(1) $w1(n) = x(n-1)$ $WR\ w1(n)$ (2) $y(n) = w1(n)$ $RD\ w1(n)$	$W1 = X$ $WR\ W1$ $Y = W1$ $RD\ W1$		

# Time centered operations

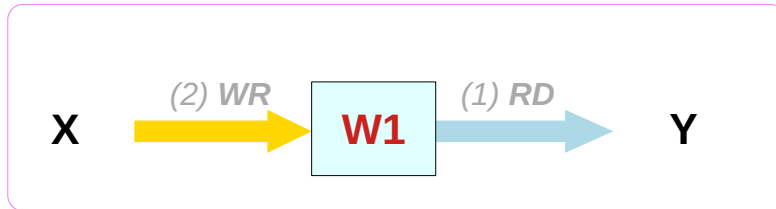
## time centered operations

## in the Register Transfer Level

$y(n-1) = w1(n-1)$ <small>output from <math>w1(n-1)</math></small> $w1(n) = x(n-1)$ <small>input to <math>w1(n)</math></small> $w1(n) = w1(n-1)$	$Y = W1$ <small>output <math>y(n-1)</math> from <math>W1</math></small> $W1 = X$ <small>input <math>x(n-1)</math> to <math>W1</math></small>	} time n-1 } time n } time n+1 } time n+2
$y(n) = w1(n)$ <small>output from <math>w1(n)</math></small> $w1(n+1) = x(n)$ <small>input to <math>w1(n+1)</math></small> $w1(n+1) = w1(n)$	$Y = W1$ <small>output <math>y(n)</math> from <math>W1</math></small> $W1 = X$ <small>input <math>x(n)</math> to <math>W1</math></small>	
$y(n+1) = w1(n+1)$ <small>output from <math>w1(n+1)</math></small> $w1(n+2) = x(n+1)$ <small>input to <math>w1(n+2)</math></small> $w1(n+2) = w1(n+1)$	$Y = W1$ <small>output <math>y(n+1)</math> from <math>W1</math></small> $W1 = X$ <small>input <math>x(n+1)</math> to <math>W1</math></small>	
$y(n+2) = w1(n+2)$ <small>output from <math>w1(n+2)</math></small>	$Y = W1$ <small>output <math>y(n+2)</math> from <math>W1</math></small>	
(2) $y(n) = w1(n)$ <span style="margin-left: 100px;"><math>RD\ w1(n)</math></span>	$Y = W1$ <span style="margin-left: 100px;"><math>RD\ W1</math></span>	
(1) $w1(n) = x(n)$ <span style="margin-left: 100px;"><math>WR\ w1(n)</math></span>	$W1 = X$ <span style="margin-left: 100px;"><math>WR\ W1</math></span>	

# Simultaneous RD and WR actions

## Register Transfer Level



at time  $n$

at time  $n+1$

$$Y = W1$$

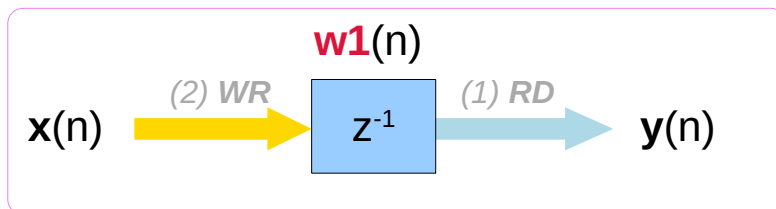
$$W1 = X$$

$$Y = W1$$

$$W1 = X$$

(1) RD  
(2) WR

## DSP C Model



at time  $n$

at time  $n+1$

$$y(n) = w1(n)$$

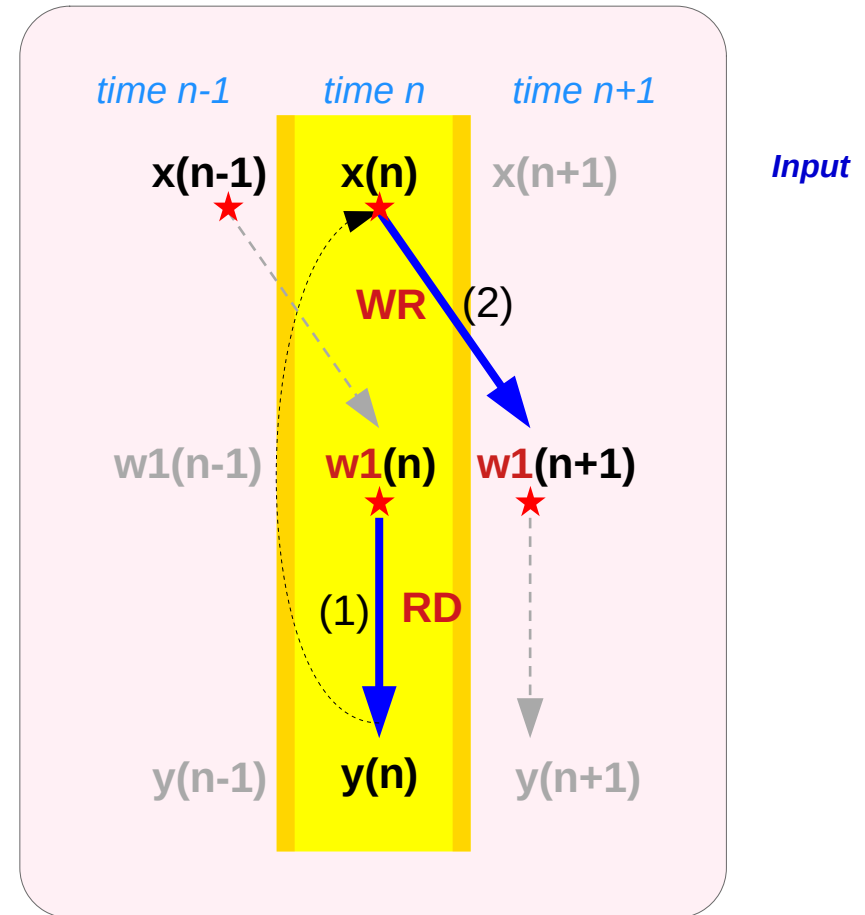
$$w1(n+1) = x(n)$$

$$y(n+1) = w1(n+1)$$

$$w1(n+2) = x(n+1)$$

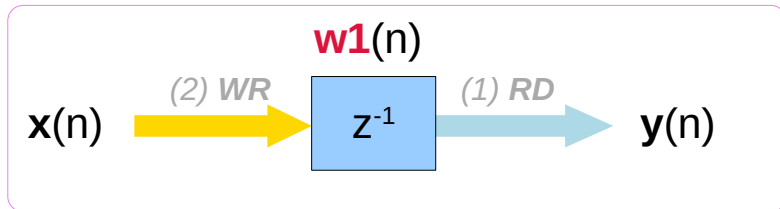
(1) RD  
(2) WR

## time centered operations



## DSP C model

# Simultaneous RD and WR actions



current content	$w1(n)$	$= x(n-1)$
next content	$w1(n+1)$	$= x(n)$

at time  $n$

$$y(n) = w1(n)$$

$$w1(n+1) = x(n)$$

at time  $n+1$

$$y(n+1) = w1(n+1)$$

$$w1(n+2) = x(n+1)$$

- (1) **RD**    *read internal state*  
 (2) **WR**    *update internal state*

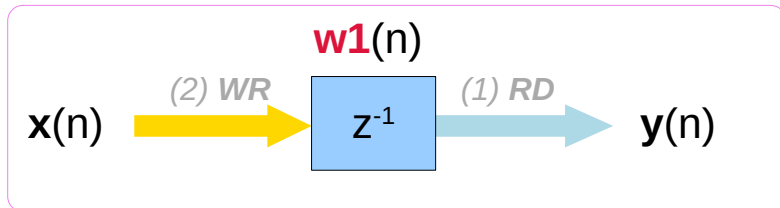
at time  $n$ ,

- 1) the content of the register  $w1(n)$   
becomes the output  $y(n)$
- 2) the input  $x(n)$  is saved and  
becomes the new content  $w1(n+1)$

**RD** access of  $w1(n) = x(n-1)$

**WR** access of  $w1(n+1) = x(n)$

# Current content $w1(n)$ and current input $x(n)$

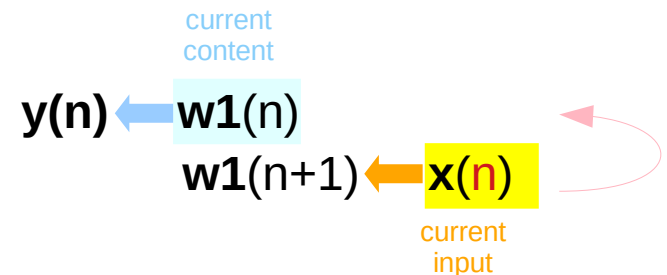


current content	$w1(n)$	$= x(n-1)$
next content	$w1(n+1)$	$= x(n)$

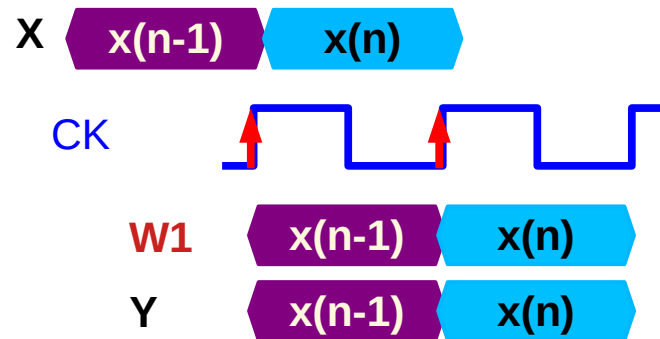
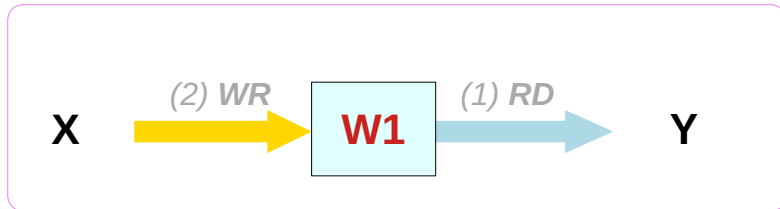
a register holding the previous input sample  $x(n-1)$

- (1) the **current content**  $x(n-1)$  is clocked out to the output
- (2) the **current input**  $x(n)$  gets stored in the register

It will be held for one sampling instant and become the output at the next time  $n+1$



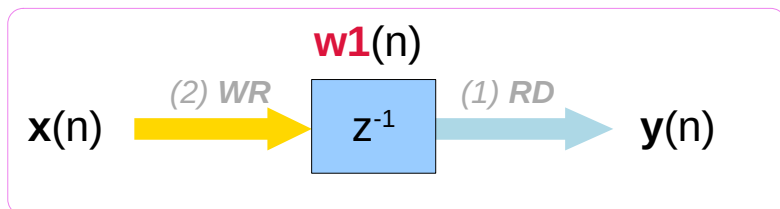
# Current content $w1(n)$ and current input $x(n)$



*simulate a clocked hardware  
ignoring delay constraints in hardware*

*zero-delay simulation*

# Delay element modeling



$w1(n) \longrightarrow y(n)$	$y(n) = w1(n)$ (1) RD old w1
$x(n) \longrightarrow w1(n+1)$	$w1(n+1) = x(n)$ (2) WR new w1

The content of the delay register at time  $n$  as the **internal state** of the filter by

internal state at time  $n$

$$w1(n) = x(n-1)$$

internal state at time  $n+1$

$$w1(n+1) = x(n)$$

output at time  $n$

$$y(n) = w1(n)$$

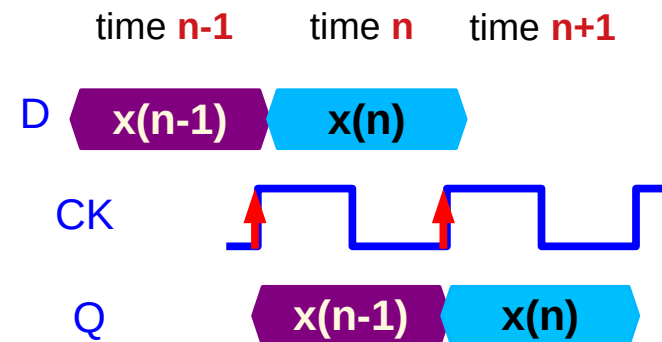
RD before WR

WAR (Write after Read) Access

WR at time  $n-1$

WR at time  $n$

RD at time  $n$



*simulate a clocked hardware  
ignoring delay constraints in hardware*

*zero-delay simulation*

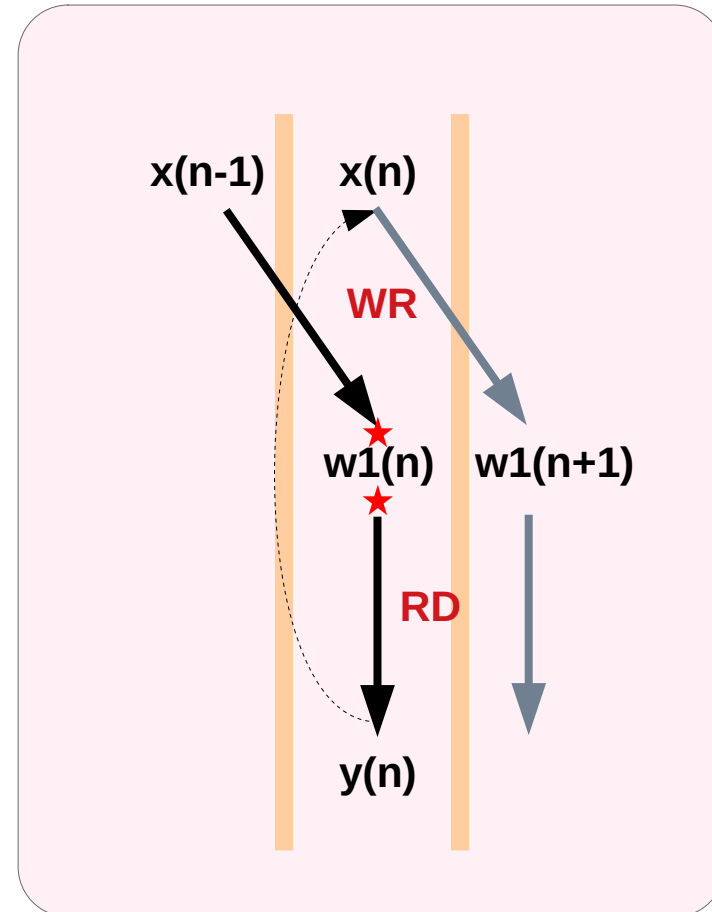


# WAR (Write after Read)

$y(n) = w1(n)$	(1) RD	old w1
$w1(n+1) = x(n)$	(2) WR	new w1
$y(n+1) = w1(n+1)$	(1) RD	old w1
$w1(n+2) = x(n+1)$	(2) WR	new w1

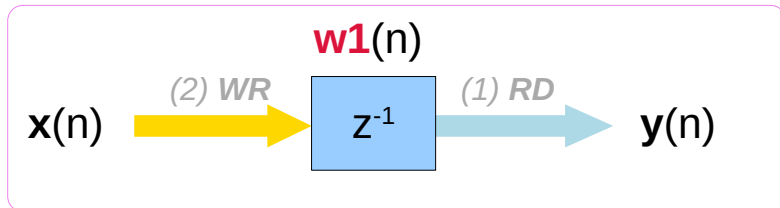
$w1(n+1) = x(n)$	(2) WR	new w1
$y(n) = w1(n)$	(1) RD	old w1
$w1(n+2) = x(n+1)$	(2) WR	new w1
$y(n+1) = w1(n+1)$	(1) RD	old w1

WAR (Write after Read) Violation



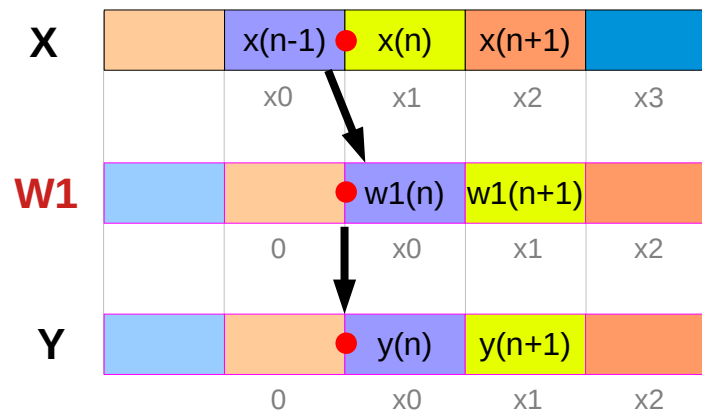
DSP C model

# Single Delay

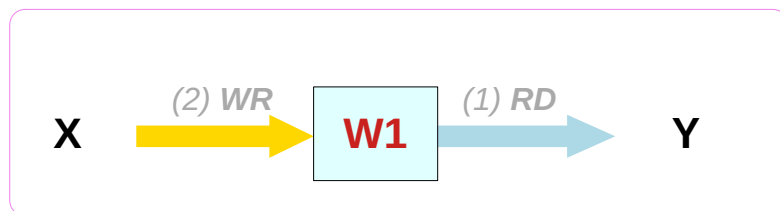


$$y(n) = w1(n) \quad (1) \text{ RD } \rightarrow$$

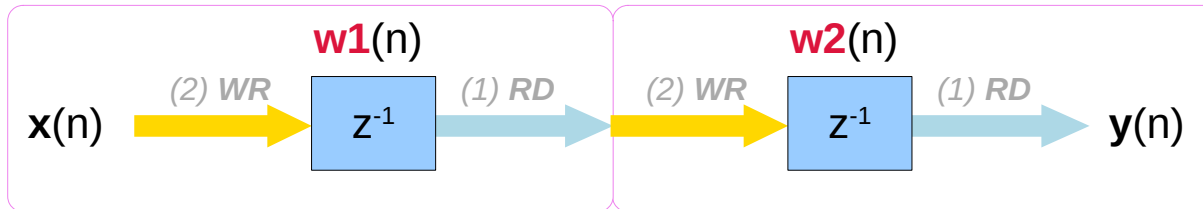
$$w1(n+1) = x(n) \quad (2) \text{ WR } \rightarrow$$



n	x(n)	w1(n)	y(n)
0	x0	0	0
1	x1	x0	x0
2	x2	x1	x1
3	x3	x2	x2
4	x4	x3	x3



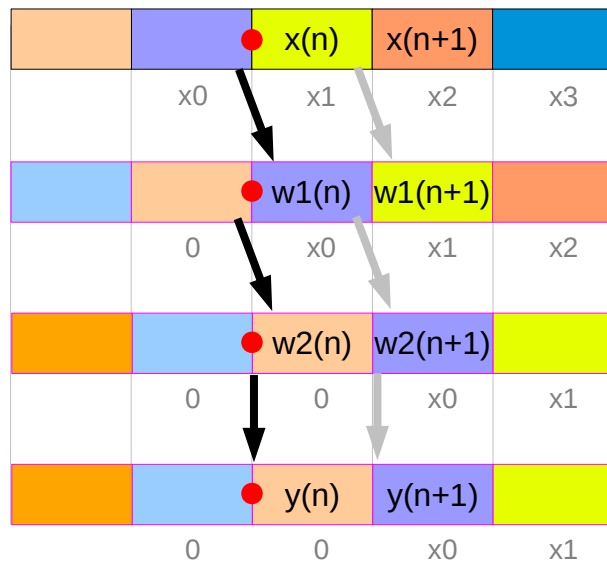
# Double Delay



$$y(n) = w2(n)$$

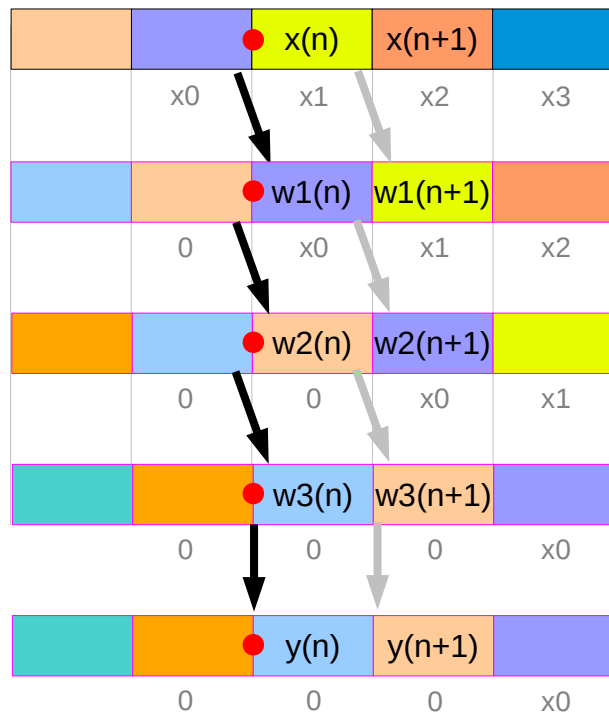
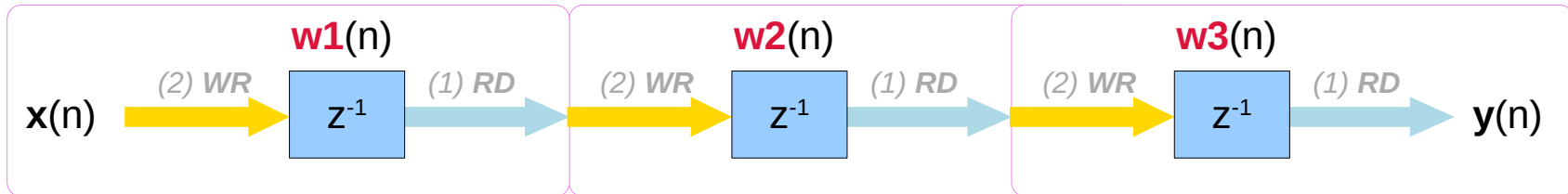
$$w2(n+1) = w1(n)$$

$$w1(n+1) = x(n)$$



$n$	$x(n)$	$w1(n)$	$w2(n)$	$y(n)$
0	$x_0$	0	0	0
1	$x_1$	$x_0$	0	0
2	$x_2$	$x_1$	$x_0$	$x_0$
3	$x_3$	$x_2$	$x_1$	$x_1$
4	$x_4$	$x_3$	$x_2$	$x_2$

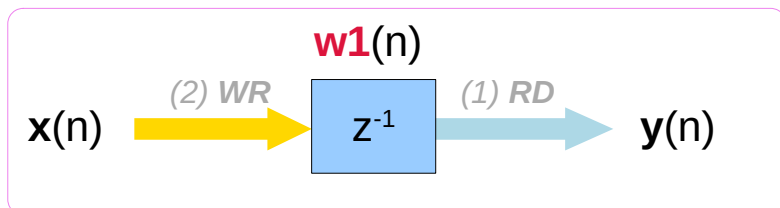
# Triple Delay



$n$	$x(n)$	$w1(n)$	$w2(n)$	$w3(n)$	$y(n)$
0	$x_0$	0	0	0	0
1	$x_1$	$x_0$	0	0	0
2	$x_2$	$x_1$	$x_0$	0	0
3	$x_3$	$x_2$	$x_1$	$x_0$	$x_0$
4	$x_4$	$x_3$	$x_2$	$x_1$	$x_1$

$$\begin{aligned}
 y(n) &= w3(n) \\
 w3(n+1) &= w2(n) \\
 w2(n+1) &= w1(n) \\
 w1(n+1) &= x(n)
 \end{aligned}$$

# Single Delay – IO Equations



## single delay

$y(n) = w1(n)$	output
$w1(n+1) = x(n)$	input

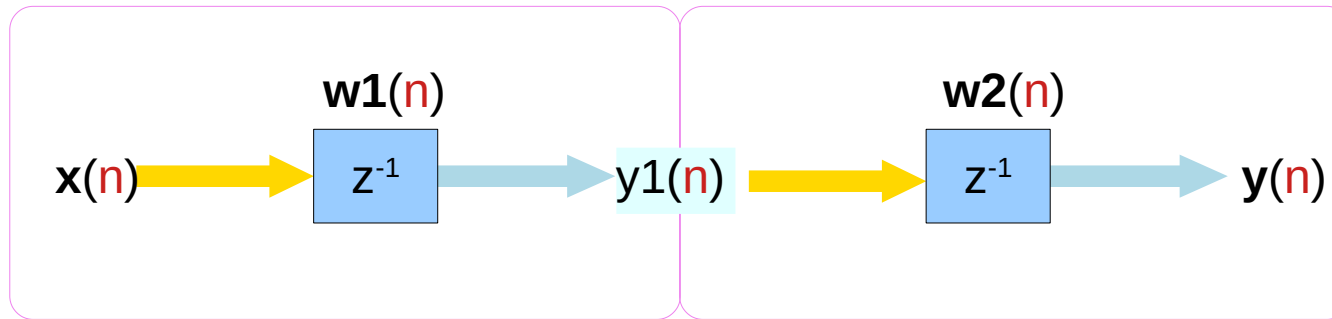
## double delay

$y(n) = w2(n)$	output
$w2(n+1) = w1(n)$	
$w1(n+1) = x(n)$	input

## triple delay

$y(n) = w3(n)$	output
$w3(n+1) = w2(n)$	
$w2(n+1) = w1(n)$	
$w1(n+1) = x(n)$	input

# Double Delay – IO Equations



$$y1(n) = w1(n)$$

$$w1(n+1) = x(n)$$

$$y(n) = w2(n)$$

$$w2(n+1) = y1(n)$$

$$y1(n) = w1(n)$$

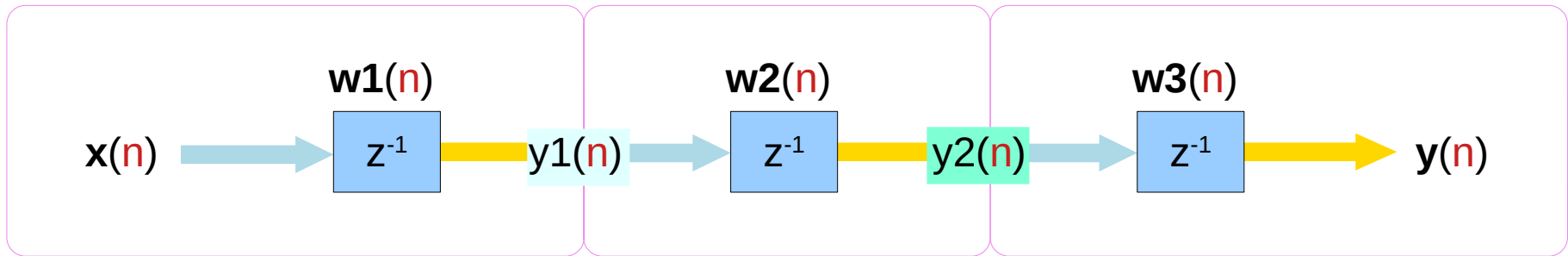
$$w2(n+1) = y1(n)$$


---


$$w2(n+1) = w1(n)$$

$y(n) = w2(n)$	output
$w2(n+1) = w1(n)$	
$w1(n+1) = x(n)$	input

# Triple Delay – IO Equations



$$y1(n) = w1(n)$$

$$w1(n+1) = x(n)$$

$$y2(n) = w2(n)$$

$$w2(n+1) = y1(n)$$

$$y(n) = w3(n)$$

$$w3(n+1) = y2(n)$$

$$y1(n) = w1(n)$$

$$w2(n+1) = y1(n)$$


---


$$w2(n+1) = w1(n)$$

$$y2(n) = w2(n)$$

$$w3(n+1) = y2(n)$$

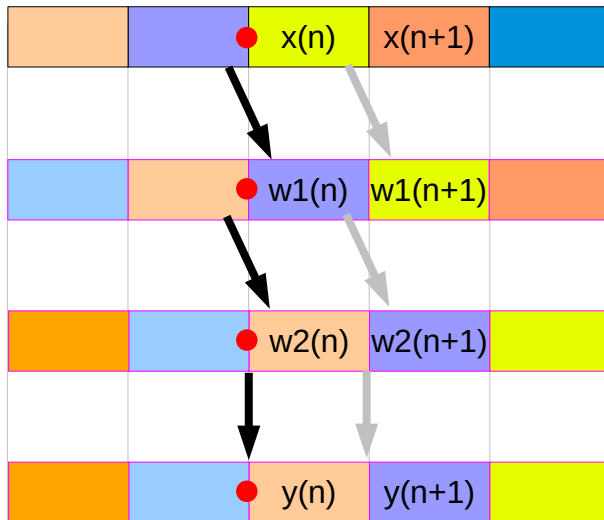

---


$$w3(n+1) = w2(n)$$

$y(n) = w3(n)$	output
$w3(n+1) = w2(n)$	
$w2(n+1) = w1(n)$	
$w1(n+1) = x(n)$	input

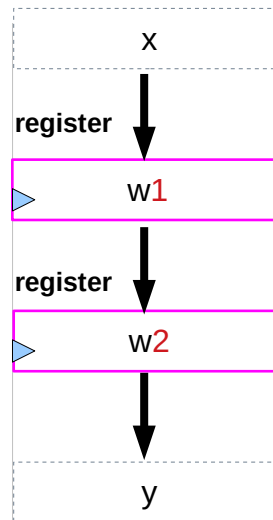
# Delay C Model

Timing Chart



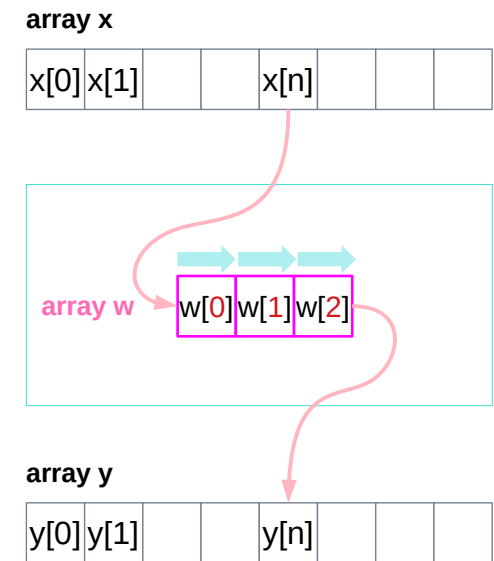
$$\begin{aligned}
 y(n) &= w2(n) \\
 w2(n+1) &= w1(n) \\
 w1(n+1) &= x(n)
 \end{aligned}$$

Register Transfer



$$\begin{aligned}
 y &= w2 \\
 w2 &= w1 \\
 w1 &= x
 \end{aligned}$$

DSP C Model for simulation



$$\begin{aligned}
 y[n] &= w[2] \\
 w[0] &= x[n] \\
 w[2] &= w[1] \\
 w[1] &= w[0]
 \end{aligned}$$

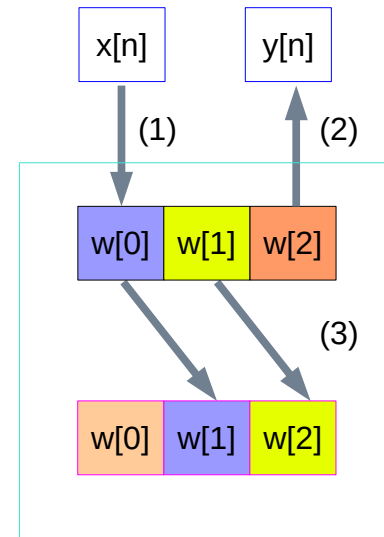


# IO Equations for the Triple Delay (1)

$$\begin{aligned}y(n) &= w_2(n) \\w_0(n) &= x(n) \\w_2(n+1) &= w_1(n) \\w_1(n+1) &= w_2(n)\end{aligned}$$

$$D = 2, 1$$

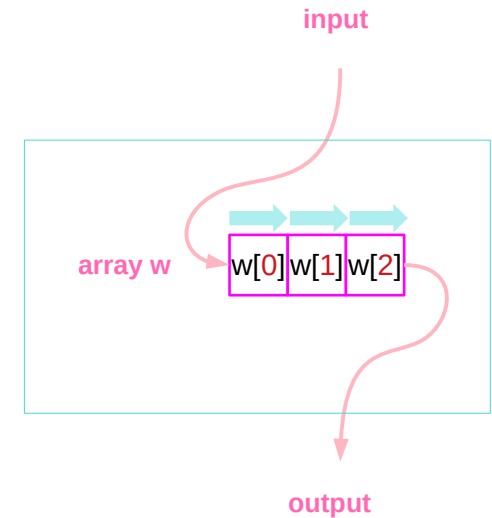
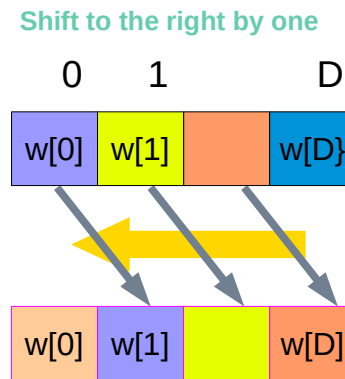
```
y[n] = w[2]           // get the output
w[0] = x[n]           // put the input
w[2] = w[1]           // shift
w[1] = w[0]           // shift
```



# delay.c

```
/* delay.c - delay by D time samples */  
/* w[0] = input, w[D] = output */
```

```
void delay(int D, double *w)  
{  
    int i;  
  
    for (i=D; i>=1; i--)  
        w[i] = w[i-1];  
  
    // reverse-order updating  
}
```



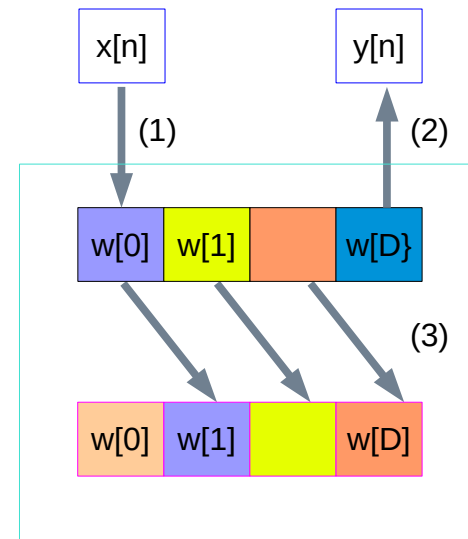
order of execution

$$\begin{aligned} w[D] &= w[D-1] \\ \dots & \quad \dots \\ w[2] &= w[1] \\ w[1] &= w[0] \end{aligned}$$

# Using the delay function

```
double *w;  
w = (double *) calloc(D+1, sizeof(double)); // (D+1)-dimensional
```

```
for (n = 0; n < Ntot; n++) {  
    y[n] = w[D]; // (1) write output  
    w[0] = x[n]; // (2) read input  
    delay(D, w); // (3) update delay line  
}
```



# Delay Functions

$$y(n) = w_1(n)$$
$$w_1(n+1) = x(n)$$

$$y(n) = w_2(n)$$
$$w_2(n+1) = w_1(n)$$
$$w_1(n+1) = x(n)$$

$$y(n) = w_3(n)$$
$$w_3(n+1) = w_2(n)$$
$$w_2(n+1) = w_1(n)$$
$$w_1(n+1) = x(n)$$

$$y(n) = w_D(n)$$
$$w_0(n) = x(n)$$
$$w_i(n+1) = w_{i-1}(n),$$
$$i = D, D-1, \dots, 2, 1$$

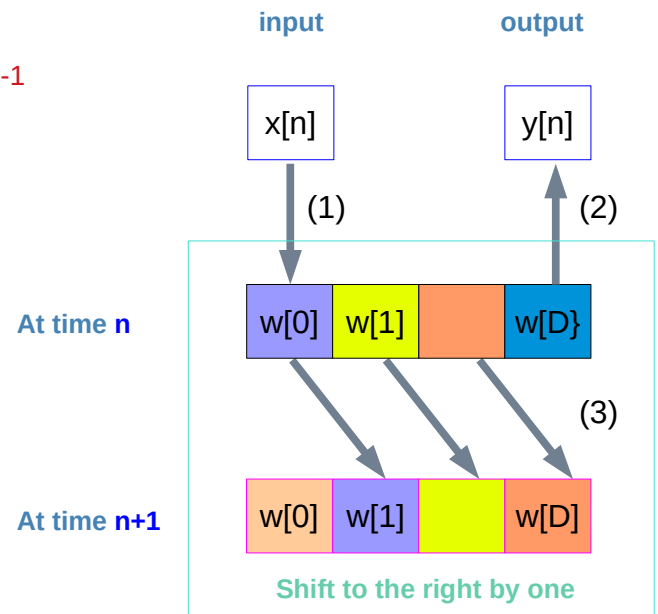
time index :  $n$

memory location :  $W_i$

memory index :  $i$

$$w_i(n+1) = w_{i-1}(n)$$

the current value at  $w_{i-1}$   
will become  
the next value at  $w_i$



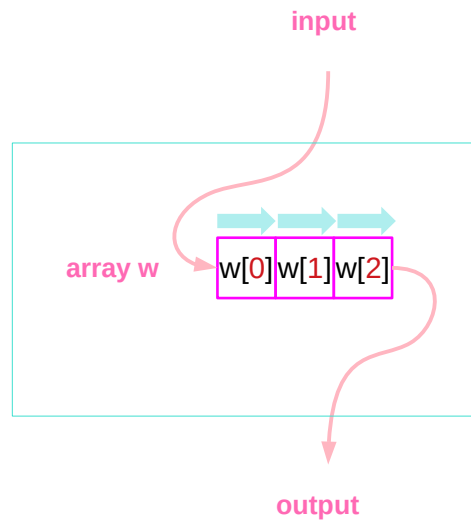
# Holding a delayed input sequence

$$w_0(n) = x(n)$$

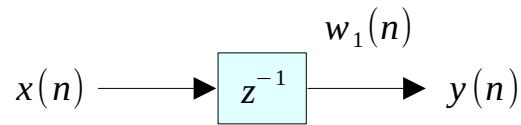
$$w_1(n) = x(n-1) = w_0(n-1)$$

$$w_2(n) = x(n-2) = w_1(n-1)$$

$$w_3(n) = x(n-3) = w_2(n-1)$$



# Single Delay (1)



$$w_1(n) = x(n-1) \quad (\text{internal state at time } n)$$

$$w_1(n+1) = x(n) \quad (\text{internal state at time } n+1)$$

$$y(n) = w_1(n)$$

$$w_1(n+1) = x(n)$$

$$1 \quad x_1 \quad x_0 \quad x_0$$

$$y(n+1) = w_1(n+1)$$

$$w_1(n+2) = x(n+1)$$

$$4 \quad x_4 \quad x_3 \quad x_3$$

$$w_1(0) = 0$$

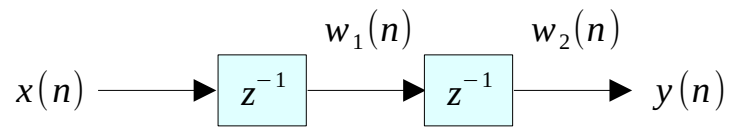
$$[x_0, x_1, x_2, x_3, \dots] \rightarrow [0, x_0, x_1, x_2, x_3, \dots]$$

for each input sample  $x$  do:

$$y := w_1$$

$$w_1 := x$$

# Double Delay (1)



$$w_2(n) = w_1(n-1) = x((n-1)-1) = x(n-2)$$

$$w_1(n) = x(n-1)$$

$$w_2(n+1) = w_1(n) \quad n \quad x(n) \quad w_1(n) \quad w_2(n) \quad y(n)$$

$$w_1(n+1) = x(n) \quad 0 \quad x_0 \quad 0 \quad 0 \quad 0$$

$$1 \quad x_1 \quad x_0 \quad 0 \quad 0$$

$$y(n) = w_2(n) \quad 2 \quad x_2 \quad x_1 \quad x_0 \quad x_0$$

$$w_2(n+1) = w_1(n) \quad 3 \quad x_3 \quad x_2 \quad x_1 \quad x_1$$

$$w_1(n+1) = x(n) \quad 4 \quad x_4 \quad x_3 \quad x_2 \quad x_2$$

$$w_1(0) = 0$$

$$[x_0, x_1, x_2, x_3, \dots] \rightarrow [0, x_0, x_1, x_2, x_3, \dots]$$

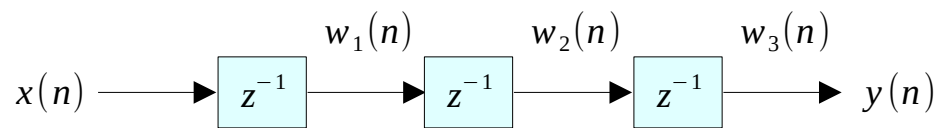
for each input sample  $x$  do:

$$y := w_2$$

$$w_2 := w_1$$

$$w_1 := x$$

# Triple Delay (1)



$$w_3(n) = w_2(n-1) = w_1(n-2) = x(n-3)$$

$$w_2(n) = w_1(n-1)$$

$$w_1(n) = x(n-1)$$

$$[x_0, x_1, x_2, x_3, \dots] \rightarrow [0, x_0, x_1, x_2, x_3, \dots]$$

for each input sample  $x$  do:

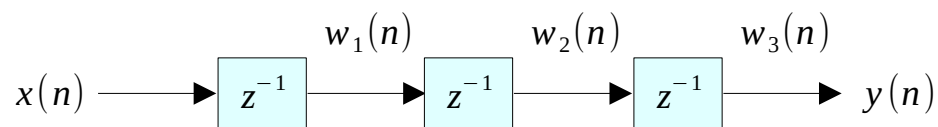
```

y := w_3
w_3 := w_2
w_2 := w_1
w_1 := x
    
```

$w_3(n+1) = w_2(n)$	$n$	$x(n)$	$w_1(n)$	$w_2(n)$	$y(n)$
$w_2(n+1) = w_1(n)$	0	$x_0$	0	0	0
$w_1(n+1) = x(n)$	1	$x_1$	$x_0$	0	0
$y(n) = w_3(n)$	2	$x_2$	$x_1$	$x_0$	$x_0$
$w_3(n+1) = w_2(n)$	3	$x_3$	$x_2$	$x_1$	$x_1$
$w_2(n+1) = w_1(n)$	4	$x_4$	$x_3$	$x_2$	$x_2$
$w_1(n+1) = x(n)$					



# D Unit Delay (1)



$$w_i(n) = w_{i-1}(n-1) \quad \text{for } i = 1, 2, \dots, D$$

$$w_3(n+1) = w_2(n) \quad n \quad x(n) \quad w_1(n) \quad w_2(n) \quad y(n)$$

$$w_2(n+1) = w_1(n) \quad 0 \quad x_0 \quad 0 \quad 0 \quad 0$$

$$w_1(n+1) = x(n) \quad 1 \quad x_1 \quad x_0 \quad 0 \quad 0$$

$$2 \quad x_2 \quad x_1 \quad x_0 \quad x_0$$

$$y(n) = w_D(n) \quad 3 \quad x_3 \quad x_2 \quad x_1 \quad x_1$$

$$w_0(n) = x(n) \quad 4 \quad x_4 \quad x_3 \quad x_2 \quad x_2$$

$$w_i(n+1) = w_{i-1}(n)$$

$$i = D, D-1, \dots, 2, 1$$

$$[x_0, x_1, x_2, x_3, \dots] \rightarrow [0, x_0, x_1, x_2, x_3, \dots]$$

for each input sample  $x$  do:

$$y := w_D$$

$$w_0 := x$$

$$w_0 := x$$

for  $i = D, D-1, \dots, 1$  do:

$$w_i := w_{i-1}$$

for each input sample  $w_0$  do:

for  $i = D, D-1, \dots, 1$  do:

$$w_i := w_{i-1}$$

# D Unit Delay (1)

```
/* delay.c - delay by D time samples */
void delay(int D, double *w)      w[0] = input, w[D] = output
{
    int i;

    for (i=D; i>=1; i--)          reverse-order updating
        w[i] = w[i-1];

}
```

# dot

```
/* dot.c - dot product of two length-(M+1) vectors */
```

```
double dot(int M, double *h, double *w)
```

```
{
```

```
    int i;
```

```
    double y;
```

```
    for (y=0, i=0; i<=M; i++)
```

```
        y += h[i] * w[i];
```

```
    return y;
```

```
}
```

*Usage: y = dot(M, h, w);*

*h = filter vector, w = state vector*

*M = filter order*

compute dot product

$$y = h_0 w_0 + h_1 w_1 + \dots + h_M w_M = [h_0, h_1, \dots, h_M] \begin{bmatrix} w_0 \\ w_1 \\ \vdots \\ w_M \end{bmatrix} = \mathbf{h}^T \mathbf{w}$$

# Direct Form

Considering the widely used  
Edge triggered  
D-type Flip Flops

$$H(z) = \frac{N(z)}{D(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

$$y_n = -a_1 y_{n-1} - a_2 y_{n-2} + b_0 x_n + b_1 x_{n-1} + b_2 x_{n-2}$$

---

## References

- [1] S. J. Ofranidis , Introduction to Signal Processing