

# Noise Definition

Young W Lim

November 21, 2019

Copyright (c) 2018 Young W. Lim. Permission is granted to copy, distribute and/or modify this document under the terms of the GNU Free Documentation License, Version 1.2 or any later version published by the Free Software Foundation; with no Invariant Sections, no Front-Cover Texts, and no Back-Cover Texts. A copy of the license is included in the section entitled "GNU Free Documentation License".

This work is licensed under a Creative Commons "Attribution-NonCommercial-ShareAlike 3.0 Unported" license.



Based on  
Probability, Random Variables and Random Signal Principles,  
P.Z. Peebles,Jr. and B. Shi



# White Noise

$N$  Gaussian random variables

## Definition

$$S_{NN}(\omega) = \frac{N_0}{2}$$

$$R_{NN}(\tau) = \frac{N_0}{2} \delta(\tau)$$

# Thermal Noise

$N$  Gaussian random variables

## Definition

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} S_{NN}(\omega) d\omega = \infty$$

$$S_{NN}(\omega) = \frac{(N_0/2)(\alpha|\omega|/T)}{e^{\alpha|\omega|/T} - 1} \delta(\tau)$$

# Discrete Time Noise

$N$  Gaussian random variables

## Definition

$$R_{XX}[k] = \sigma_X^2 \delta[k]$$

$$S_{YY}(e^{j\Omega}) = \sigma_X^2$$

# Band Limited Noise

$N$  Gaussian random variables

## Definition

$$S_{NN}(\omega) = \begin{cases} \frac{P\pi}{W} & -W < \omega < +W \\ 0 & \text{otherwise} \end{cases}$$

$$R_{NN}(\tau) = P \frac{\sin(W\tau)}{W\tau}$$

# Band Pass Noise

$N$  Gaussian random variables

## Definition

$$S_{NN}(\omega) = \begin{cases} \frac{P\pi}{W} & \omega_0 - (W/2) < |\omega| < \omega_0 + (W/2) \\ 0 & \text{otherwise} \end{cases}$$

$$R_{NN}(\tau) = P \frac{\sin(W\tau/2)}{W\tau/2} \cos(\omega_0\tau)$$



## Definition

$$Y(t) = X(t)A_0 \cos(\omega_0 t)$$

$$R_{YY}(t, t + \tau) = E[Y(t)Y(t + \tau)]$$

$$= E[A_0^2 X(t)X(t + \tau) \cos(\omega_0 t) \cos(\omega_0 t + \omega_0 \tau)]$$

$$= \frac{A_0^2}{2} R_{XX}(t, t + \tau) [\cos(\omega_0 \tau) + \cos(2\omega_0 t + \omega_0 \tau)]$$

## Definition

$$Y(t) = X(t)A_0 \cos(\omega_0 t)$$

$$R_{YY}(t, t + \tau) = E[Y(t)Y(t + \tau)]$$

$$= \frac{A_0^2}{2} R_{XX}(t, t + \tau) [\cos(\omega_0 \tau) + \cos(2\omega_0 t + \omega_0 \tau)]$$

$$A[R_{YY}(t, t + \tau)] = \frac{A_0^2}{2} R_{XX}(t, t + \tau) \cos(\omega_0 \tau)$$

$$S_{YY}(\omega) = \frac{A_0^2}{4} [S_{XX}(\omega - \omega_0) + S_{XX}(\omega + \omega_0)]$$



