

# Angle Recoding 2. Wu

## 3. MVR

20180919 Wed

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## ② MVR (Modified Vector Rotational)

two modifications

① **repetition** of elementary angles

each micro-rotation of elementary angle  
can be performed repeatedly

- more possible combinations
- smaller  $\xi_m$

② **confinement** of total micro-rotation number

confine the iteration number

in the micro-rotation phase  
to  **$R_m$**  ( $R_m \ll W$ )

The role of  **$R_m$**  is quite similar  
to the **number of non-zero digit**  
 **$N_D$**  in CSD recoding scheme

MV2

\* repetition of elementary angles

$$\theta = a(2) * 2 = \tan^{-1}(2^2) * 2$$

AR

$$+ a(1) + a(5) - a(8) - a(10) - a(14)$$

k= 0 theta= 0.4899573 ik= 1 uik=+1 a[ 1]= 0.4636476, new theta= 0.0263097  
k= 1 theta= 0.0263097 ik= 5 uik=+1 a[ 5]= 0.0312398, new theta=-0.0049301  
k= 2 theta=-0.0049301 ik= 8 uik=-1 a[ 8]= 0.0039062, new theta=-0.0010239  
k= 3 theta=-0.0010239 ik=10 uik=-1 a[10]= 0.0009766, new theta=-0.0000473  
k= 4 theta=-0.0000473 ik=14 uik=-1 a[14]= 0.0000610, new theta= 0.0000137

Conventional  
CORDIC

$$+ a(0) - a(1) + a(2) - a(3) + a(4) - a(5) + a(6) + a(7) \\ - a(8) - a(9) - a(10) - a(11) + a(12) + a(13) - a(14) - a(15)$$

k= 0 theta= 0.4899573 u[ 0]=+1 a[ 0]= 0.7853982, new theta=-0.2954408  
k= 1 theta=-0.2954408 u[ 1]=-1 a[ 1]= 0.4636476, new theta= 0.1682068  
k= 2 theta= 0.1682068 u[ 2]=+1 a[ 2]= 0.2449787, new theta=-0.0767719  
k= 3 theta=-0.0767719 u[ 3]=-1 a[ 3]= 0.1243550, new theta= 0.0475831  
k= 4 theta= 0.0475831 u[ 4]=+1 a[ 4]= 0.0624188, new theta=-0.0148357  
k= 5 theta=-0.0148357 u[ 5]=-1 a[ 5]= 0.0312398, new theta= 0.0164041  
k= 6 theta= 0.0164041 u[ 6]=+1 a[ 6]= 0.0156237, new theta= 0.0007804  
k= 7 theta= 0.0007804 u[ 7]=+1 a[ 7]= 0.0078123, new theta=-0.0070319  
k= 8 theta=-0.0070319 u[ 8]=-1 a[ 8]= 0.0039062, new theta=-0.0031257  
k= 9 theta=-0.0031257 u[ 9]=-1 a[ 9]= 0.0019531, new theta=-0.0011726  
k=10 theta=-0.0011726 u[10]=-1 a[10]= 0.0009766, new theta=-0.0001960  
k=11 theta=-0.0001960 u[11]=-1 a[11]= 0.0004883, new theta= 0.0002923  
k=12 theta= 0.0002923 u[12]=+1 a[12]= 0.0002441, new theta= 0.0000481  
k=13 theta= 0.0000481 u[13]=+1 a[13]= 0.0001221, new theta=-0.0000740  
k=14 theta=-0.0000740 u[14]=-1 a[14]= 0.0000610, new theta=-0.0000129  
k=15 theta=-0.0000129 u[15]=-1 a[15]= 0.0000305, new theta= 0.0000176

data :  $W$ -bit word length

the iteration number :  $N$   $N \leq W$

the restricted iteration number :  $R_m$   $R_m \ll W$

$$i \in [0, R_m - 1]$$

$$R_m < W$$

the angle quantization error

$$\xi_{m, \text{MVR}} \triangleq \theta - \sum_{i=0}^{R_m-1} \alpha(i) a(s(i))$$

the rotational sequence

$$s(i) \in \{0, 1, \dots, W-1\} \quad * \text{repetition allowed}$$

the micro-rotation angle  
in the  $i$ -th iteration

the directional sequence

$$\alpha(i) \in \{-1, 0, +1\}$$

the direction of the  $i$ -th  
micro-rotation of  $a(s(i))$

$$\alpha(i) a(s(i)) = \tilde{\theta}(i)$$

the rotational sequence

$$s(i) \in \{0, 1, \dots, W-1\} \quad [0, 3, 6, 7]$$

the directional sequence

$$\alpha(i) \in \{-1, 0, +1\} \quad [1, -1, -1, 1]$$

$$\text{atan}(2^0) - \text{atan}(2^{-3}) - \text{atan}(2^{-6}) + \text{atan}(2^{-7})$$

$$\alpha(i) a(s(i)) = \tilde{\theta}(j)$$

MVR-CORDIC Algorithm with $R_n = 4$	Greedy Algorithm	3	$\vec{\alpha} = [1 \ -1 \ -1 \ -1]$ $\vec{s} = [0 \ 3 \ 6 \ 7]$	$5.2891 \cdot 10^{-4}$
	Semi-greedy Algorithm ( $D = 2$ )	4	$\vec{\alpha} = [1 \ -1 \ -1 \ 1]$ $\vec{s} = [0 \ 3 \ 5 \ 7]$	$5.2033 \cdot 10^{-4}$
	TBS Algorithm	5	$\vec{\alpha} = [1 \ 1 \ -1 \ -1]$ $\vec{s} = [1 \ 2 \ 4 \ 7]$	$2.5911 \cdot 10^{-4}$

```
>> s = [0, 3, 6, 7]
>> alpha = [1, -1, -1, -1]
>> sum(atan(2.^(-s) .* alpha))
ans = 0.63761
>>
```

```
>> s = [0, 3, 5, 7]
>> alpha = [1, -1, -1, 1]
>> sum(atan(2.^(-s) .* alpha))
ans = 0.63762
```

```
>> alpha = [1, 1, -1, -1]
>> s = [1, 2, 4, 7]
>> sum(atan(2.^(-s) .* alpha))
ans = 0.63840
```

```
0 7.85398163397448e-01
1 4.63647609000806e-01
2 2.44978663126864e-01
3 1.24354994546761e-01
4 6.24188099959574e-02
5 3.12398334302683e-02
6 1.56237286204768e-02
7 7.81234106010111e-03
8 3.90623013196697e-03
9 1.95312251647882e-03
10 9.76562189559319e-04
11 4.88281211194898e-04
12 2.44140620149362e-04
13 1.22070311893670e-04
14 6.10351561742088e-05
15 3.05175781155261e-05
```

$w=16$

0	7.85398163397448e-01	s(0)
1	4.63647609000806e-01	.
2	2.44978663126864e-01	.
3	1.24354994546761e-01	s(1)
4	6.24188099959574e-02	.
5	3.12398334302683e-02	.
6	1.56237286204768e-02	s(2)
7	7.81234106010111e-03	s(3)
8	3.90623013196697e-03	.
9	1.95312251647882e-03	.
10	9.76562189559319e-04	.
11	4.88281211194898e-04	.
12	2.44140620149362e-04	.
13	1.22070311893670e-04	.
14	6.10351561742088e-05	.
15	3.05175781155261e-05	.

0	7.85398163397448e-01	s(0)
3	1.24354994546761e-01	s(1)
6	1.56237286204768e-02	s(2)
7	7.81234106010111e-03	s(3)

$R_m = 4$

$s(0)=0$   
 $s(1)=3$   
 $s(2)=6$   
 $s(3)=7$

0	7.85398163397448e-01
1	4.63647609000806e-01
2	2.44978663126864e-01
3	1.24354994546761e-01
4	6.24188099959574e-02
5	3.12398334302683e-02
6	1.56237286204768e-02
7	7.81234106010111e-03
8	3.90623013196697e-03
9	1.95312251647882e-03
10	9.76562189559319e-04
11	4.88281211194898e-04
12	2.44140620149362e-04
13	1.22070311893670e-04
14	6.10351561742088e-05
15	3.05175781155261e-05



# AQ & MVR CORDIC

$$\xi_{m, MVR} \triangleq \theta - \left[ \sum_{j=0}^{R_m-1} \alpha(j) a(s(j)) \right]$$

the rotational sequence  $s(j)$

$$j = 0, 1, 2, \dots, R_m-1$$

$s(j) \in \{0, 1, \dots, W-1\}$  rotational sequence

determines the micro-rotation angle  $a(s(j))$   
in the  $j$ -th iteration

the directional sequence  $\alpha(j)$

$$\alpha(j) \in \{-1, 0, +1\}$$

controls the direction of the  $j$ -th  
micro-rotation of  $a(s(j))$

$$\alpha(j) a(s(j)) = \tilde{\theta}(j)$$

$$\begin{array}{ll} i = 0, 1, 2, 3, \dots, W-1 & \\ s(j) = 0, 1, 2, 3, \dots, W-1 & \text{rotational sequence} \\ \alpha(j) = -1, 0, 0, +1, \dots, -1 & \text{directional sequence} \\ j = 0, -, -, 1, \dots, R_m-1 & \text{effective iteration number} \\ R_m \ll W & \end{array}$$

$i$	$j$	$S(j)$
0	0	$S(0) = 0$

1

2

3

4	1, 4	$S(1) = 4, S(4) = 4$
---	------	----------------------

repetition allowed

5	2	$S(2) = 5$
---	---	------------

6

7

8	3	$S(3) = 8$
---	---	------------

9

10

11

12

13

14

$W-1 = 15$

effective  
iteration  
number

rotational  
sequence

$i$	Conventional	$j$	$S(j)$
①	$S(0) = 0$	0	$S(0) = 0$
1	$S(1) = 1$		
2	$S(2) = 2$		
3	$S(3) = 3$		
④	$S(4) = 4$	1	$S(1) = 4$
⑤	$S(5) = 5$	2	$S(2) = 5$
6	$S(6) = 6$		
7	$S(7) = 7$		
⑧	$S(8) = 8$	3	$S(3) = 8$
9	$S(9) = 9$		
10	$S(10) = 10$		
11	$S(11) = 11$		
12	$S(12) = 12$		
13	$S(13) = 13$		
14	$S(14) = 14$		
15	$S(15) = 15$		

effective  
iteration  
number

rotational  
sequence

$W-1 =$

$$\text{sub-angle } (\alpha(j) a(s(j))) \sim \tilde{\theta}(j)$$

$$\begin{aligned} \xi_{m,AR} &= \theta - \left[ \sum_{j=0}^{N'-1} \tilde{\theta}(j) \right], \quad \tilde{\theta}(j) = \tan^{-1}(\alpha(j) \cdot 2^{-s(j)}) \\ &= \theta - \left[ \sum_{j=0}^{N'-1} \tan^{-1}(\alpha(j) \cdot 2^{-s(j)}) \right] \end{aligned}$$

$$N' \triangleq \sum_{j=0}^{N-1} |\mu(j)| \quad \text{the effective iteration number}$$

EAS formed by MVR-CORDIC  
is the same as AR  
also performs AQ

the EAS consists of all possible values of  $\tilde{\theta}(j)$

the EAS  $S_1$  in AR

$$S_1 = \{ \tan^{-1}(\alpha^* \cdot 2^{-s^*}) : \alpha^* \in \{-1, 0, +1\}, s^* \in \{0, 1, \dots, N-1\} \}$$

The major difference

1) the total number of sub-angles  $N_A$

the total iteration number

in the micro-rotation phase

is kept fixed to a pre-defined value of  $R_m$

$$N_A = R_m$$

2) the sub-angle  $\theta_i$  corresponds to  $\alpha(j) \alpha(s(j))$

$$\theta_j = \alpha(j) \alpha(s(j)) = \tilde{\theta}_j$$

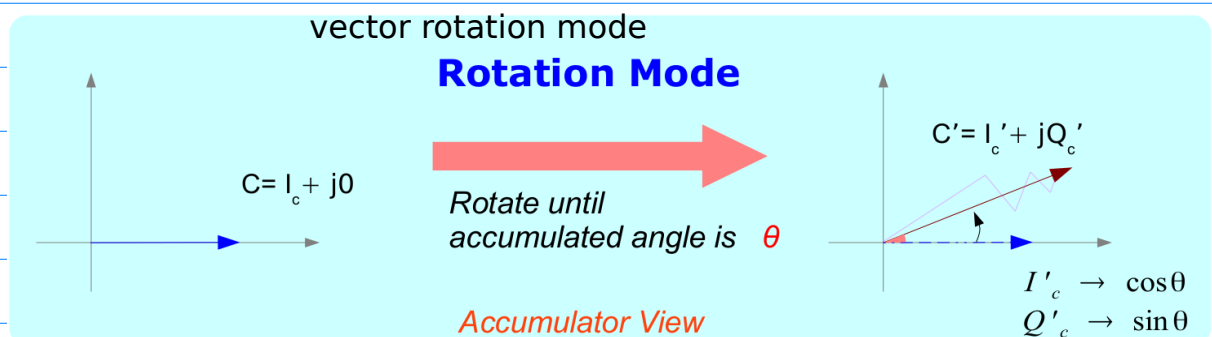
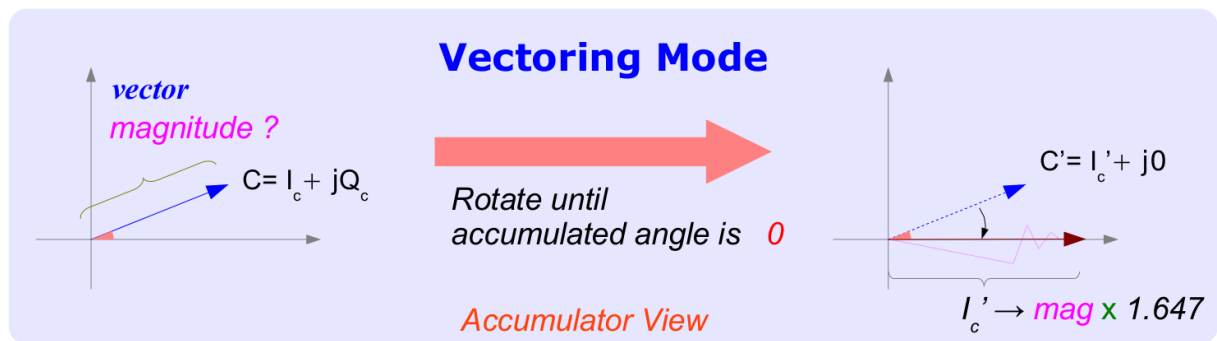
# MVR (Modified Vector Rotation)

1) Repeat of Elementary Angles  $\theta_i, \theta_i$

2) fixed total micro-rotation Number  $R_m$

\* Vector Rotation Mode

\* and the rotation angles are known in advance



# Modified Vector Rotational MVR CORDIC

- reduce the iteration number
- maintaining the SQNR performance
- modifying the basic microrotation procedure

## Three Searching Algorithm

- ① the selective prerotation
- ② the selective scaling
- ③ iteration-tradeoff scheme

# Optimization Problem

EAS point of view

Given  $\theta$ , find the combination of  $R_m$  elementary angles from EAS  $S_i$ , such that the angle quantization error  $|\xi_{m,nur}|$  is minimized.

semi-greedy algorithm  
trade offs between computational complexities  
and performance



key issue in the MVR-CORDIC  
is to find the best sequences of  
 $s(i)$  and  $\alpha(i)$  to minimize  $|\xi_m|$   
subject to the constraint that  
the total iteration number is confined to  $R_m$

- 1) Greedy Algorithm
- 2) Exhaustive Algorithm
- 3) Semigreedy Algorithm

data :  $W$ -bit word length

the iteration number :  $N$   $N \leq W$

the restricted iteration number :  $R_m$   $R_m \ll W$

# Hu's greedy algorithm

$$\theta(0) = \theta, \{\mu(i) = 0, 0 \leq i \leq N-1\}, k=0$$

repeat until  $|\theta(k)| < \alpha(N-1)$  do

choose  $i_k, 0 \leq i_k \leq N-1$

$$|\theta(k) - a(i_k)| = \min_{0 \leq i \leq N-1} |\theta(k) - a(i)|$$

$$\theta(k+1) = \theta(k) - \mu(i_k) a(i_k)$$

$$\mu(i_k) = \text{sign}(\theta(k))$$

$$J(i) = |\theta(i) - \alpha(i) a(i)| \text{ is minimized}$$

# 1) Greedy Algorithm

given  $\theta$ ,  $W$ ,  $R_m$

try to approach the target rotation angle,  $\theta$ , step by step  
in each step, decisions are made on  $\alpha(i)$  and  $s(i)$   
by choosing the best combination of  $\alpha(i)$  and  $s(i)$   
so as to minimize  $|\xi_m|$

$\alpha(i)$  and  $s(i)$  are determined such that

the error function  $J(i) = |\theta(i) - \alpha(i)\alpha(s(i))|$  is minimized

$\theta(i)$  : the residue angle in the  $i$ -th step

$$\theta(i) = \theta - \sum_{m=0}^{i-1} \alpha(m)\alpha(s(m))$$

the searching is terminated

$\left\{ \begin{array}{l} \text{if no further improvements can be found} \\ J(i) \geq J(i-1) \\ \text{if the iteration } (i) \text{ reaches } R_m - 1 \end{array} \right.$

$\alpha(R_m - 1)$  and  $s(R_m - 1)$  are determined

at the end of the searching

the greedy algorithm terminates

Only when the residue angle error  
cannot be further reduced.

Initialization:

given  $\theta$  angle

$W$  wordlength

$R_m$  restricted iteration number

$$\theta(i) = \theta - \sum_{n=0}^{i-1} \alpha(n) a(s(n))$$

Select  $\alpha(i) \in \{-1, 0, +1\}$

$s(i) \in \{0, 1, 2, \dots, W-1\}$

to minimize  $J(i) = \theta(i) - \alpha(i) a(s(i))$

$s(m)$  repetition allowed

N

$J(i) < J(i-1)$

Y

$\theta(i+1) = \theta(i) - \alpha(i) a(s(i))$

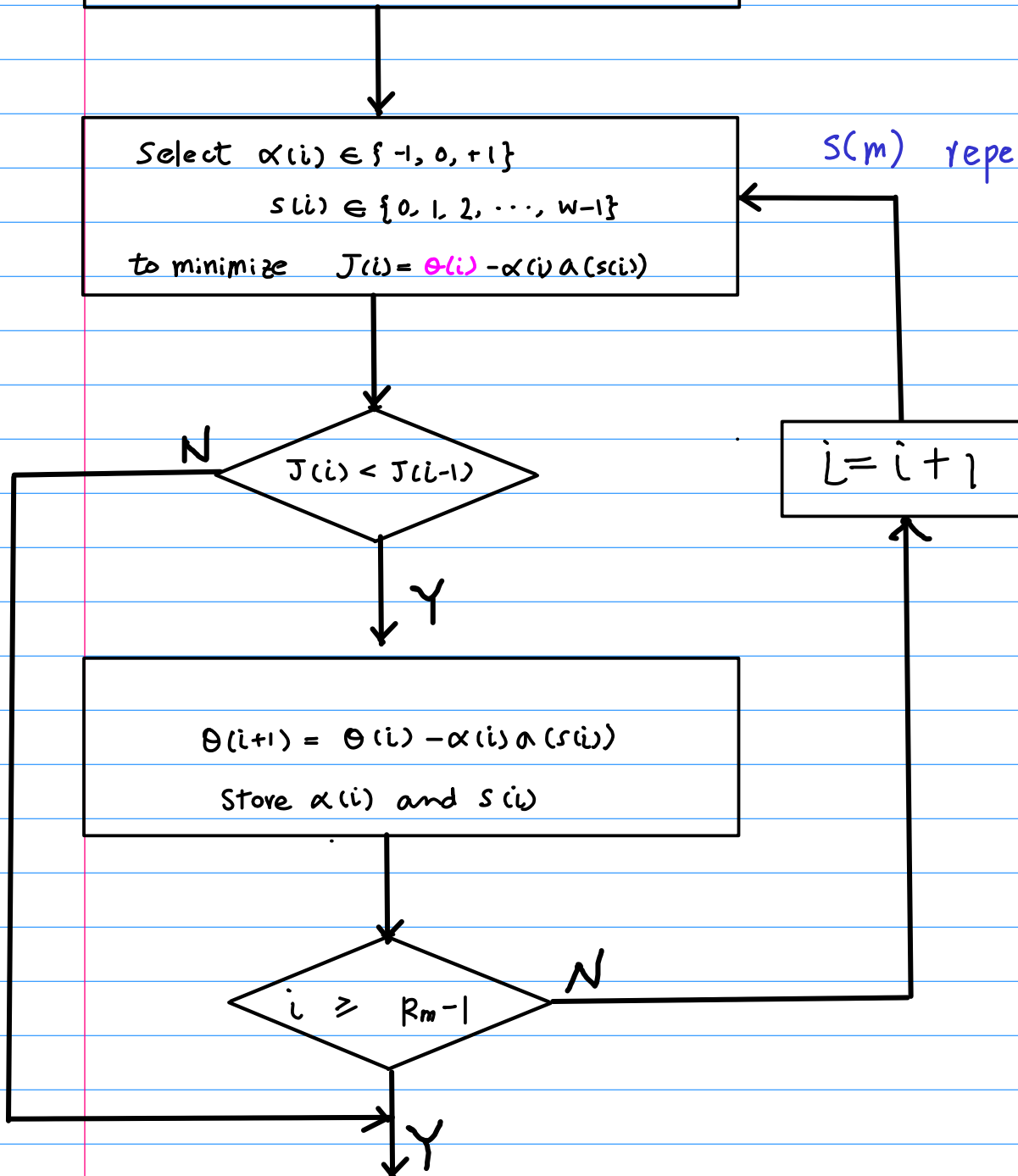
Store  $\alpha(i)$  and  $s(i)$

$i = i + 1$

$i \geq R_m - 1$

N

Y



## 2) Exhaustive Algorithm

search for the entire solution space

$$\begin{array}{ccc} \alpha(i) & a(s(i)) & i \\ \{-1, 0, +1\} & \{s(0), s(1), \dots, s(W-1)\} & \{0, 1, \dots, R_m-1\} \\ 3 & W & R_m \end{array} \Rightarrow (3W)^{R_m}$$

all possible combinations of

$$\sum_{i=0}^{R_m-1} \alpha(i) a(s(i))$$

in a single step

decisions for  $\alpha(i)$  and  $s(i)$ ,  $0 \leq i \leq R_m-1$   
by minimizing the error function

$$J = \left| 0 - \sum_{i=0}^{R_m-1} \alpha(i) a(s(i)) \right|$$

global optimal solution

Initialization:

given  $\theta, W, R_m$

let  $\theta(0) = \theta,$

$i = 0$

$J(-1) = \infty$



Select  $\alpha(i) \in \{-1, 0, +1\}$

$s(i) \in \{0, 1, 2, \dots, W-1\}$

for  $0 \leq i \leq R_m - 1$

to minimize  $J(i) = \theta - \sum_{i=0}^{R_m-1} \alpha(i) a(s(i))$



Store  $\alpha(i)$  and  $s(i)$

for  $0 \leq i \leq R_m - 1$

$$(3 \cdot W) \cdot (3 \cdot W) \cdots (3 \cdot W) \\ = 3^{R_m} \cdot W^{R_m}$$

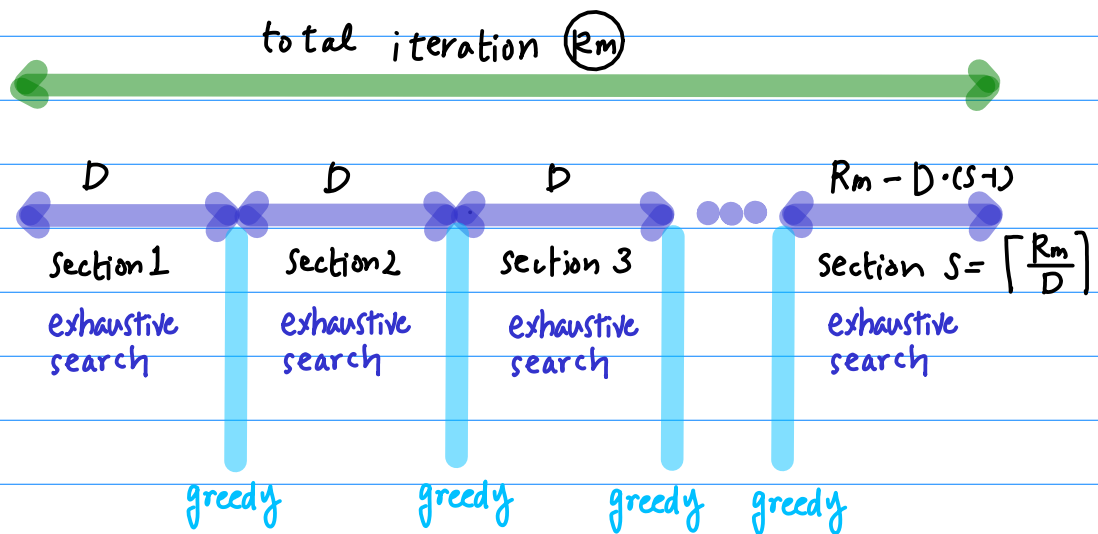
### 3) Semi-greedy Algorithm

a combination of greedy and exhaustive algorithm

the search space of  $\alpha(i)$  and  $s(i)$  for  $0 \leq i \leq R_m - 1$   
are divided into several sections

with  $D$  iterations as a segment  
 $\downarrow$  block length                       $\downarrow$  block

the segmentation scheme



in the  $i$ -th block

decision of  $\alpha(k)$  and  $s(k)$  for  $iD \leq k \leq (i+1)D-1$

$$\text{minimizes } J = \left| \theta(i) - \sum_{k=iD}^{(i+1)D-1} \alpha(k) a(s(k)) \right|$$

$$\text{where } \theta(i) = \theta - \sum_{m=0}^{i-1} \left[ \sum_{k=mD}^{(m+1)D-1} \alpha(k) a(s(k)) \right]$$

the residue angle in the  $i$ -th step

$$s = \left\lceil \frac{R_m}{D} \right\rceil$$

$$\theta(i) = \theta - \left[ \sum_{k=0D}^{1D-1} \alpha(k) a(s(k)) + \sum_{k=1D}^{2D-1} \alpha(k) a(s(k)) + \dots + \sum_{k=(i-1)D}^{iD-1} \alpha(k) a(s(k)) \right]$$



Initialization:

given  $\Theta$ ,  $W$ ,  $R_m$

let  $\Theta(0) = \Theta$ ,

$i = 0$

$J(-1) = \infty$

Select  $\alpha(k) \in \{-1, 0, +1\}$

$s(k) \in \{0, 1, 2, \dots, W-1\}$

for  $iD \leq k \leq (i+1)D - 1$

to minimize  $J(i) = \Theta(i) - \sum_{k=iD}^{(i+1)D-1} \alpha(k) a(s(k))$

N

$J(i) < J(i-1)$

$\Theta(i+1) = \Theta(i) - \sum_{k=iD}^{(i+1)D-1} \alpha(k) a(s(k))$

Store  $\alpha(k)$ ,  $s(k)$

$i \geq \lceil \frac{R_m}{D} \rceil - 1$

N

Y

$i = i + 1$

