Laurent Series and z-Transform - Geometric Series Causality B

20180612

2 formulas of z

① $\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$

$\updownarrow z^{-1}$

② $\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right)$

$$\boxed{\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} \xleftrightarrow{z^{-1}} \frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}}$$

$$\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \frac{3}{2}\,\frac{2}{3}\left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$$= \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$$\Big\downarrow z^{-1}$$

$$\frac{3}{2}\frac{-1}{(z^{-1}-0.5)(z^{-1}-2)} = \frac{3}{2}\,\frac{2}{3}\left(\frac{1}{z^{-1}-0.5} - \frac{1}{z^{-1}-2}\right)$$

$$= \left(\frac{2}{2z^{-1}-1} - \frac{0.5}{0.5z^{-1}-1}\right)$$

$$= \left(\frac{2z}{2-z} - \frac{0.5z}{0.5-z}\right)$$

$$= \left(\frac{-2z}{z-2} + \frac{0.5z}{z-0.5}\right)$$

$$= z\left(\frac{-2}{z-2} + \frac{0.5}{z-0.5}\right)$$

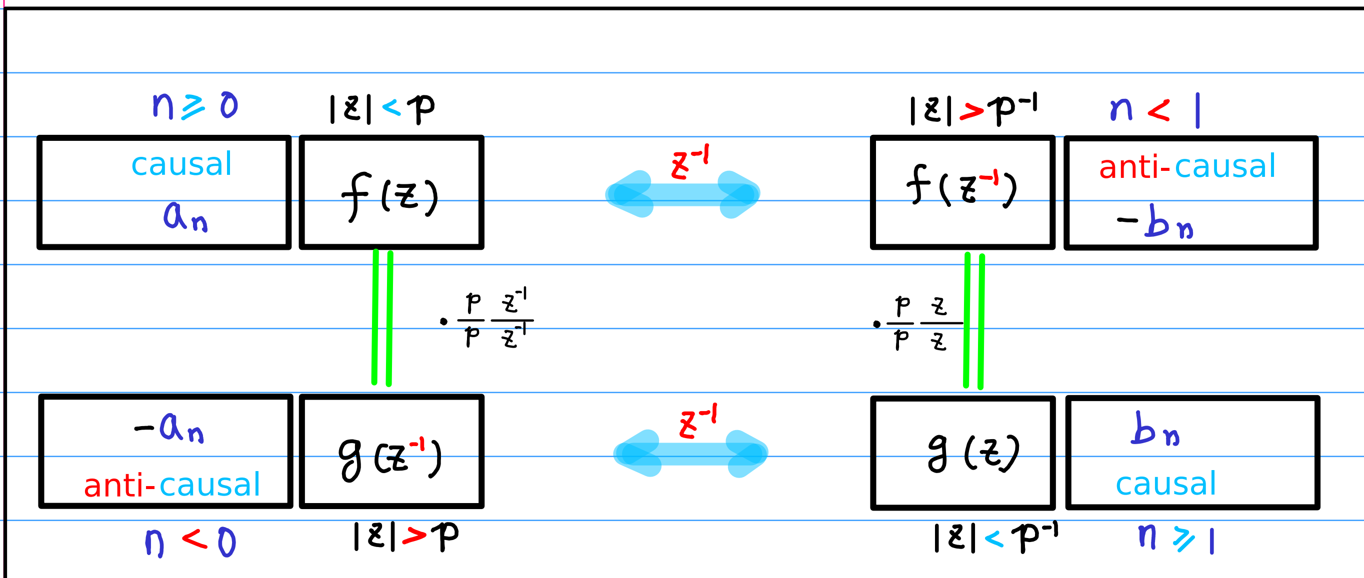
$$= z\left(\frac{-\frac{3}{2}z}{(z-2)(z-0.5)}\right)$$

$$= \frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$$

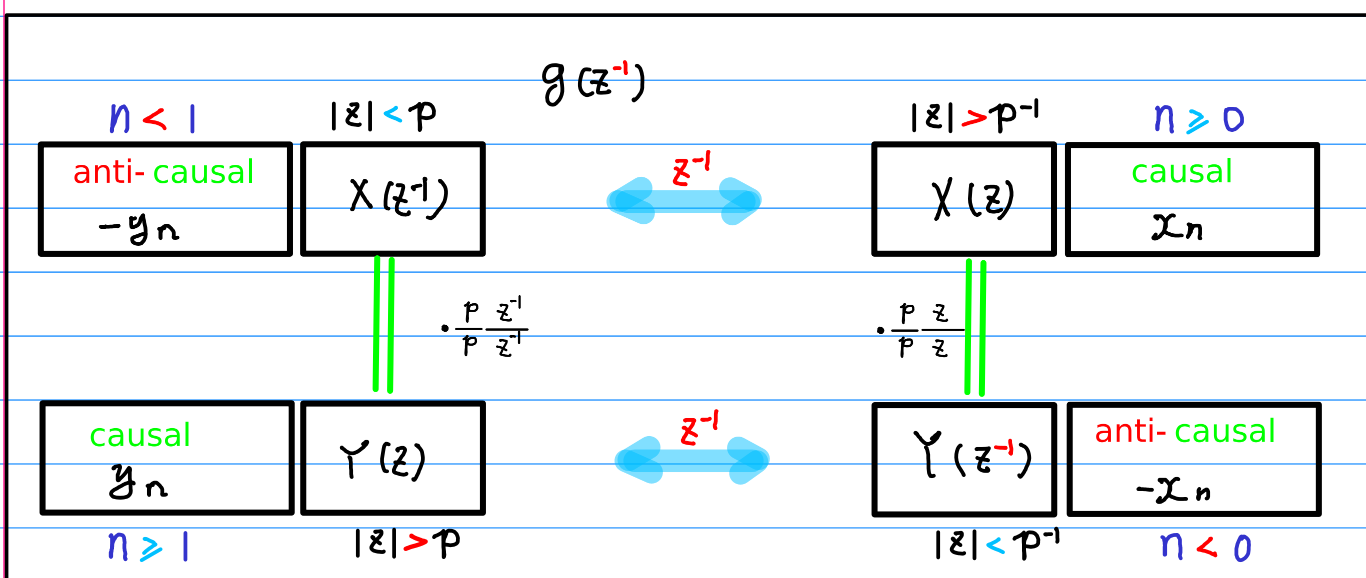
$$\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)} = \frac{3}{2}\,\frac{2}{3}\left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right)$$

Laurent Series & Z-Transform (1)

Laurent Series $a_n \leftrightarrow f(z)$ $\quad b_n \leftrightarrow g(z)$

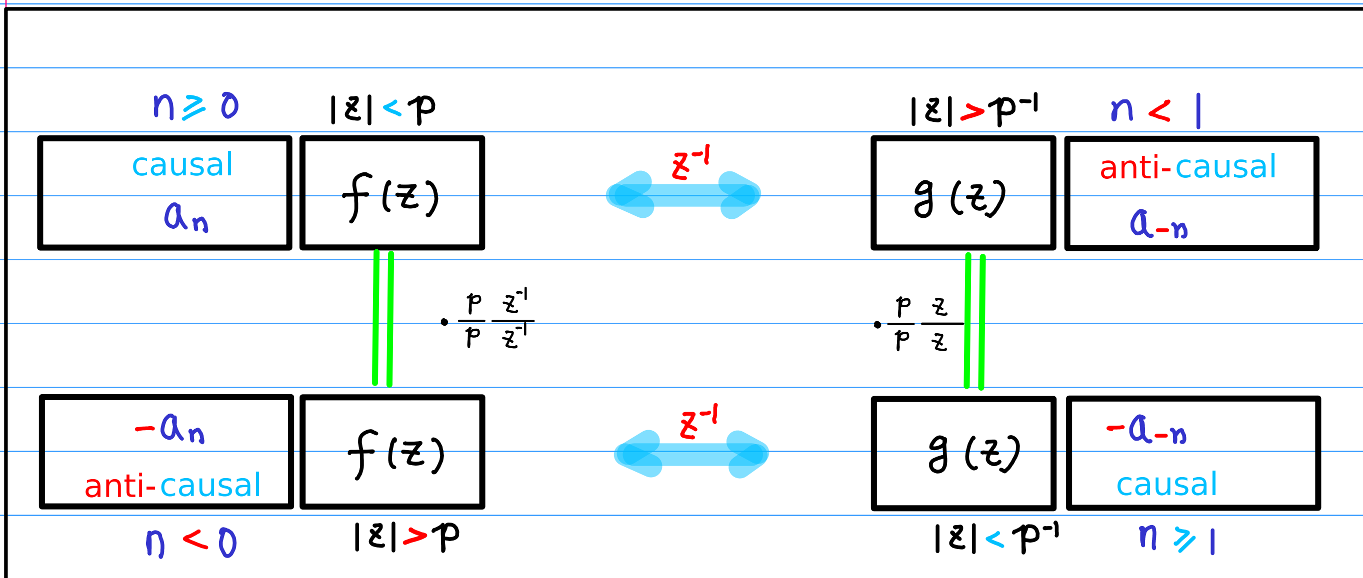


Z- Transform $X(z) \leftrightarrow x_n$ $\quad Y(z) \leftrightarrow y_n$

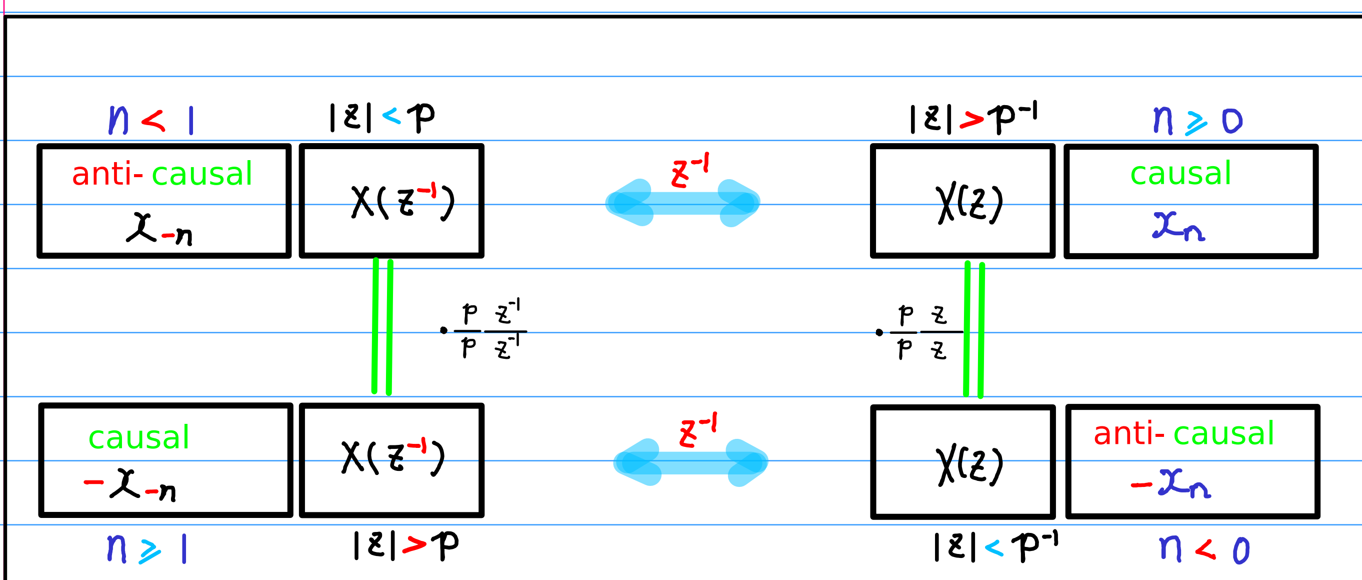


Laurent Series & Z-Transform ②

Laurent Series $a_n \leftrightarrow f(z)$



Z-Transform $X(z) \leftrightarrow x_n$



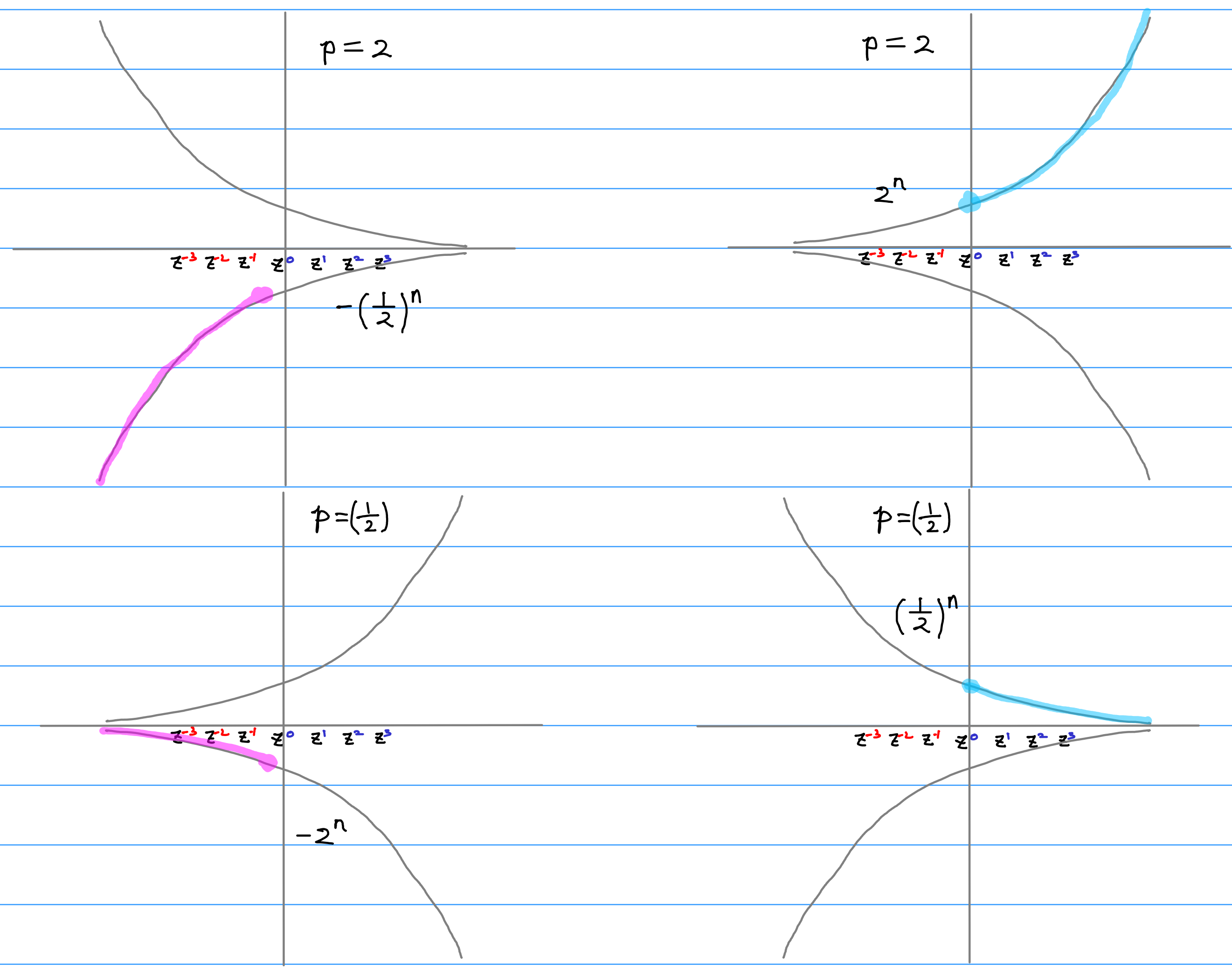
anti-causal $X(z^{-1})$ $(|z|<p)$ causal $Y(z)$ $(|z|>p)$

$X(z^{-1}) \leftrightarrow x_{-n}$ $(n<1)$ $Y(z) \leftrightarrow y_n$ $(n \geq 0)$

	$n<1$	$\lvert z\rvert<p$		
x_{-n}	anti-causal X(z): $-(p^{-1})^{-n+1}$	$-\dfrac{p^{-1}}{1-p^{-1}z}$	$-(p^{-1}+p^{-2}z^{1}+p^{-3}z^{2}+\cdots)=\sum_{n=0}^{-\infty}(p^{-1})^{-n+1}z^{-n}$	$n<0$
	$\cdot(-1)$	$\cdot\dfrac{p}{p}\dfrac{z^{-1}}{z^{-1}}$		
y_n	$(p)^{n-1}$ causal X(z)	$\dfrac{z^{-1}}{1-pz^{-1}}$	$p^0z^{-1}+p^1z^{-2}+p^2z^{-3}+\cdots=\sum_{n=1}^{\infty}(p)^{n-1}z^{-n}$	$n\geq 0$
	$n\geq 1$	$\lvert z\rvert>p$		

anti-causal $n=-1,-2,-3,\ldots.$ causal $n=0,+1,+2,+3,\ldots.$

$-(p^{-1},p^{-2},p^{-3},\cdots)$ $(p^0,p^1,p^2,\cdots)$



causal $f(z)$ $(|z|<p)$ anti-causal $f(z)$ $(|z|>p)$

$f(z) \leftrightarrow a_n$ $(n \geq 0)$ $g(z^{-1}) \leftrightarrow -a_n$ $(n<0)$

$n \geq 0$ $|z|<p$

causal f(z)	
$-(p^{-1})^{n+1}$	$-\dfrac{p^{-1}}{1-p^{-1}z}$

$$-(p^{-1}+p^{-2}z^1+p^{-3}z^2+\cdots)=\sum_{n=0}^{\infty}-(p^{-1})^{n+1}z^n \qquad n \geq 0$$

$\cdot(-1)$ $\cdot\dfrac{p}{p}\dfrac{z^{-1}}{z^{-1}}$

anti-causal f(z)	
$(p)^{-n-1}$	$\dfrac{z^{-1}}{1-pz^{-1}}$

$$p^0z^{-1}+p^1z^{-2}+p^2z^{-3}+\cdots=\sum_{n=-1}^{-\infty}(p)^{-n-1}z^n \qquad n<0$$

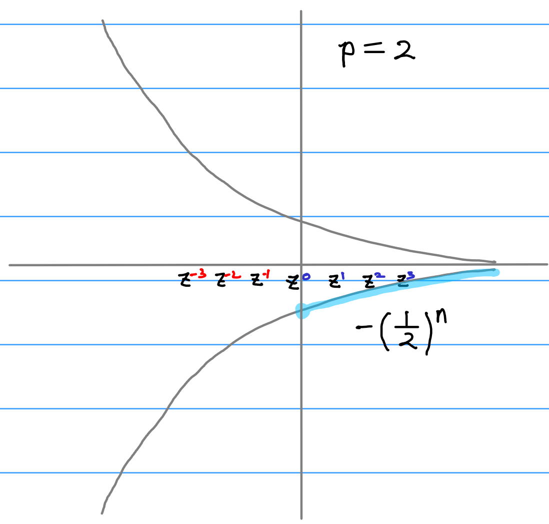
$n<0$ $|z|>p$

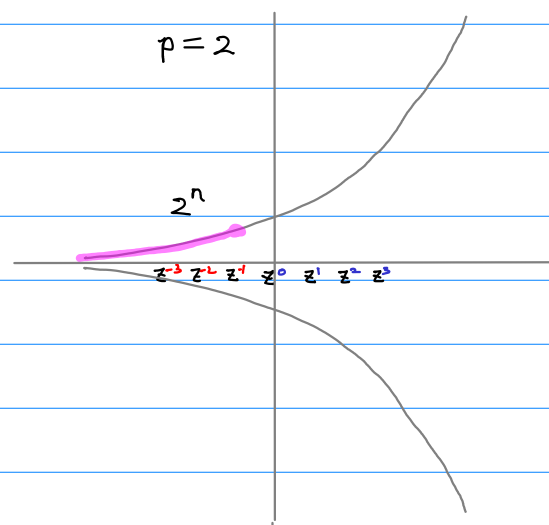
causal   n=0,+1,+2,+3,....

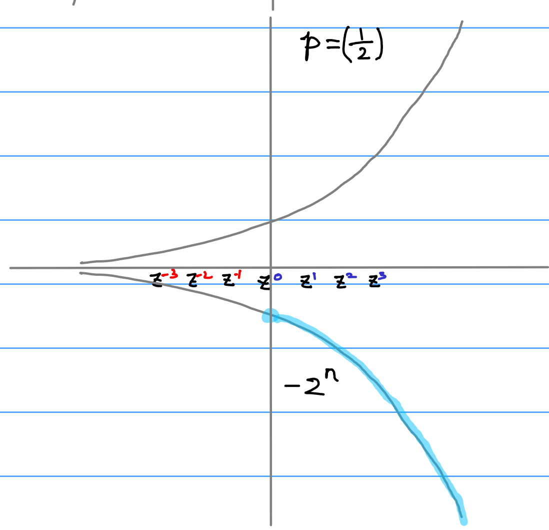
$-(p^{-1},p^{-2},p^{-3},\cdots)$

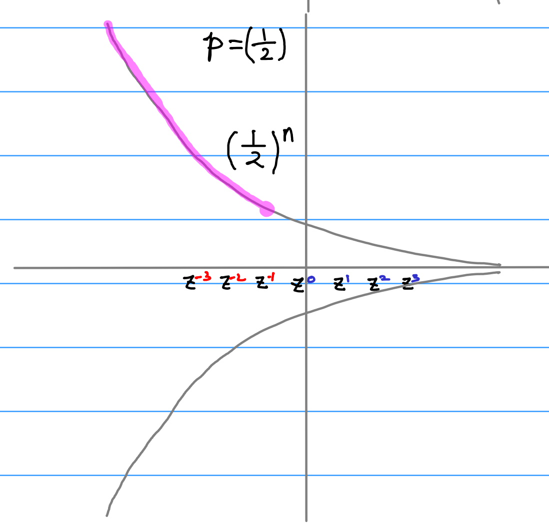
anti-causal   n=-1,-2,-3,....

$(p^0,p^1,p^2,\cdots)$









anti-causal $g(z)$ $(|z|>p^{-1})$ causal $g(z)$ $(|z|<p^{-1})$

$f(z^{-1}) \longleftrightarrow -b_n$ $(n<1)$ $g(z) \longleftrightarrow b_n$ $(n\geq 1)$

$|z|>p^{-1}$ $n<1$

$n<1$ $$-(p^{-1}+p^{-2}z^{-1}+p^{-3}z^{-2}+\cdots) = \sum_{n=0}^{-\infty} -(p)^{n-1} z^n$$ $\boxed{-\dfrac{p^{-1}}{1-p^{-1}z^{-1}}}$ $\boxed{\text{anti-causal } f(z) \quad -(p^{-1})^{-n+1}}$

$\cdot\dfrac{p}{p}\dfrac{z}{z}$ $\cdot(-1)$

$n\geq 1$ $$p^0z^1+p^1z^2+p^2z^3+\cdots = \sum_{n=1}^{\infty} (p)^{n-1} z^n$$ $\boxed{\dfrac{z}{1-pz}}$ $\boxed{(p)^{n-1} \quad \text{causal } f(z)}$

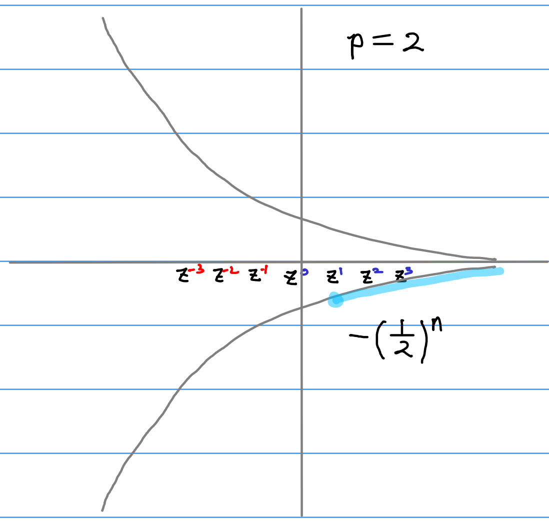
$|z|<p^{-1}$ $n\geq 1$

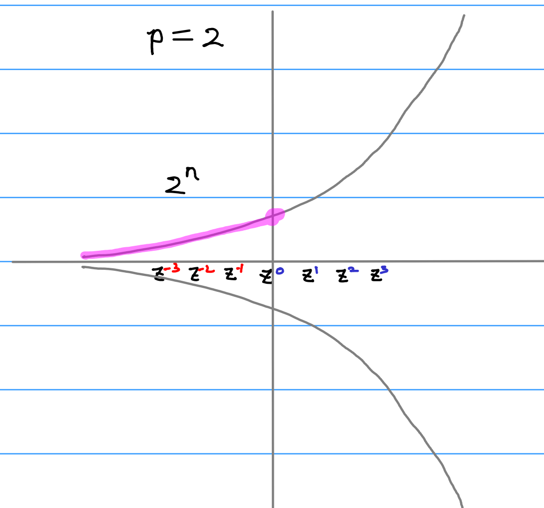
causal $n=+1,+2,+3,\ldots.$

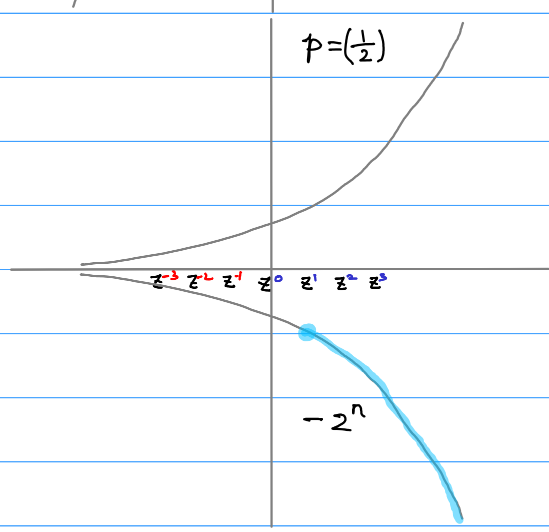
$-(p^0, p^1, p^2, \cdots)$

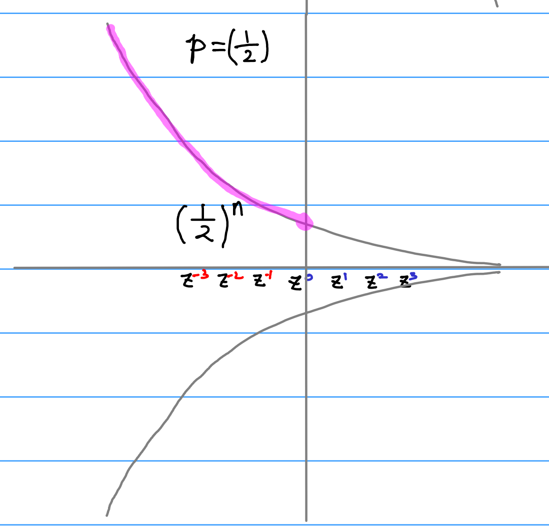
anti-causal $n=0,-1,-2,-3,\ldots.$

$(p^{-1}, p^{-2}, p^{-3}, \cdots)$



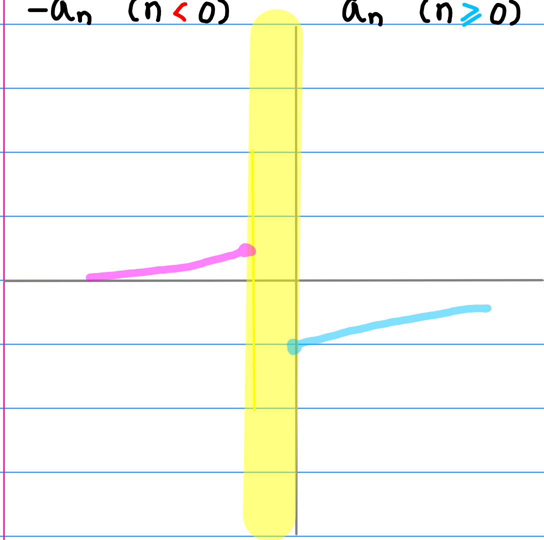






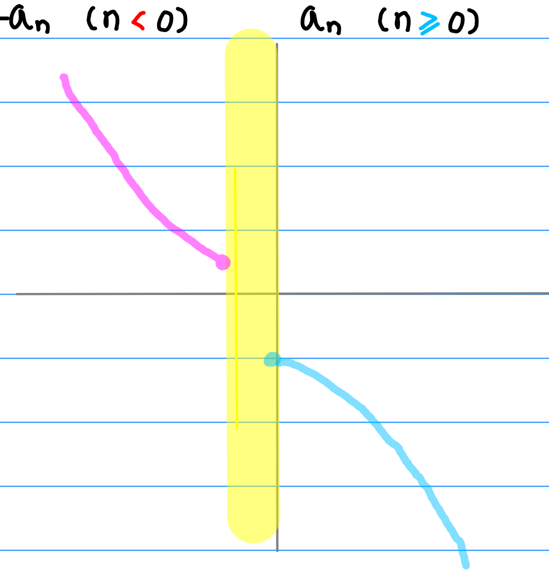
$f(z)$ $(|z|>p)$ $f(z)$ $(|z|<p)$

$-a_n$ $(n<0)$ a_n $(n\geq 0)$



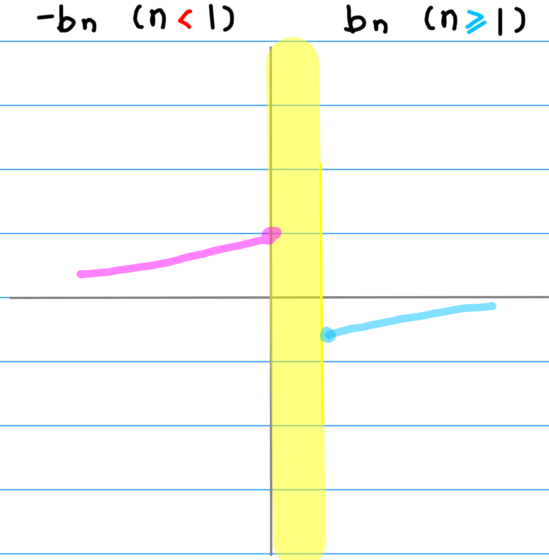
$f(z)$ $(|z|>p)$ $f(z)$ $(|z|<p)$

$-a_n$ $(n<0)$ a_n $(n\geq 0)$



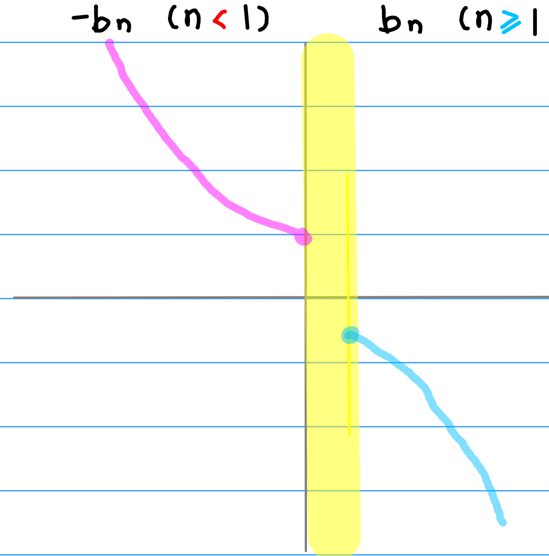
$g(z)$ $(|z|>p^{-1})$ $g(z)$ $(|z|<p^{-1})$

$-b_n$ $(n<1)$ b_n $(n\geq 1)$



$g(z)$ $(|z|>p^{-1})$ $g(z)$ $(|z|<p^{-1})$

$-b_n$ $(n<1)$ b_n $(n\geq 1)$



causal $f(z)$	anti-causal $f(z)$
$f(z) \leftrightarrow a_n \quad (n \geq 0)$	$f(z^{-1}) \leftrightarrow a_{-n} \quad (n < 1)$

$n \geq 0$ $\quad |z| < p$

causal f(z)
$-(p^{-1})^{n+1}$

$-\dfrac{p^{-1}}{1 - p^{-1} z}$

$$-(p^{-1} + p^{-2} z^{1} + p^{-3} z^{2} + \cdots) = \sum_{n=0}^{\infty} -(p^{-1})^{n+1} z^{n} \qquad n \geq 0$$

$|z| > p^{-1}$ $\quad n < 1$

$n < 1$

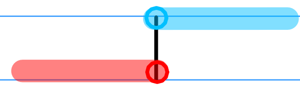
$$-(p^{-1} + p^{-2} z^{-1} + p^{-3} z^{-2} + \cdots) = \sum_{n=0}^{-\infty} -(p)^{n-1} z^{n}$$

$-\dfrac{p^{-1}}{1 - p^{-1} z^{-1}}$

anti-causal f(z)
$-(p^{-1})^{-n+1}$

$n \geq 0$

causal f(z)
$-p^{-n-1}$



$-n$

$n < 1$

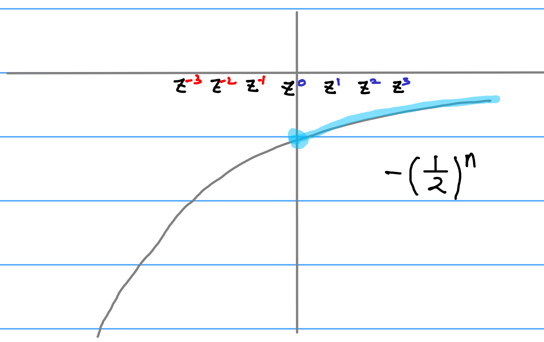
anti-causal f(z)
$-p^{+n-1}$

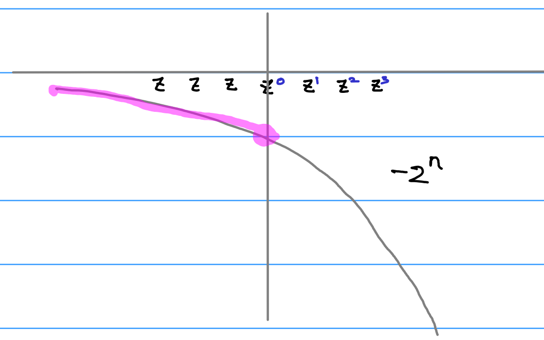
causal $\quad$ n=0,+1,+2,+3,....

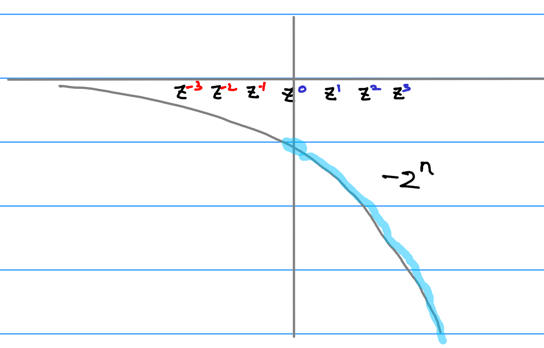
$-(p^{-1}, p^{-2}, p^{-3}, \cdots)$

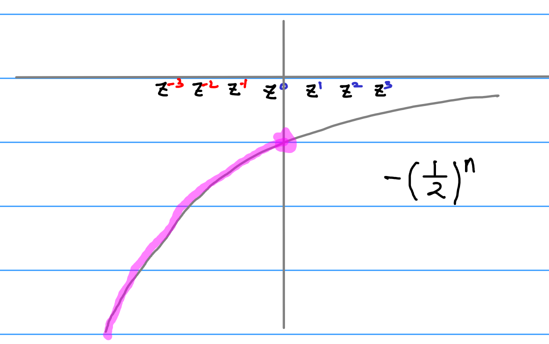
anti-causal $\quad$ n=0,-1,-2,-3,....

$-(p^{-1}, p^{-2}, p^{-3}, \cdots)$









anti-causal $g(z)$

$g(z^{-1}) \leftrightarrow b_{-n}$ $(n<0)$

causal $g(z)$

$g(z) \leftrightarrow b_n$ $(n \geq 1)$

$n<0$ $\boxed{(p)^{-n-1}}$ anti-causal f(z)

$|z|>p$ $\boxed{\dfrac{z^{-1}}{1-pz^{-1}}}$

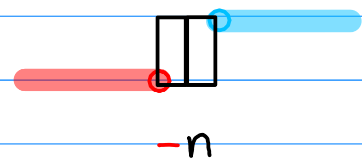
$$p^0 z^{-1} + p^1 z^{-2} + p^2 z^{-3} + \cdots = \sum_{n=-1}^{-\infty} (p)^{-n-1} z^n \qquad n<0$$

$$n \geq 1 \qquad p^0 z^1 + p^1 z^2 + p^2 z^3 + \cdots = \sum_{n=1}^{\infty} (p)^{n-1} z^n$$

$|z|<p^{-1}$ $\boxed{\dfrac{z}{1-pz}}$

$n \geq 1$ $\boxed{(p)^{n-1}}$ causal f(z)

$n<0$ $\boxed{p^{-n-1}}$ anti-causal f(z)



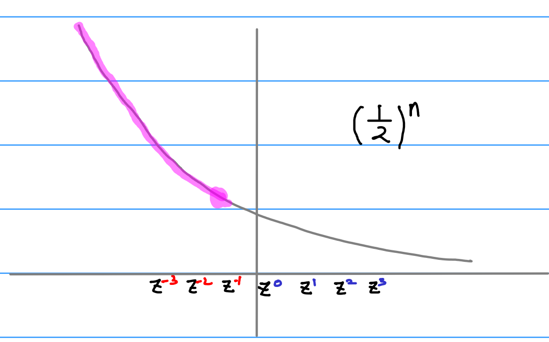
$n \geq 1$ $\boxed{p^{n-1}}$ causal f(z)

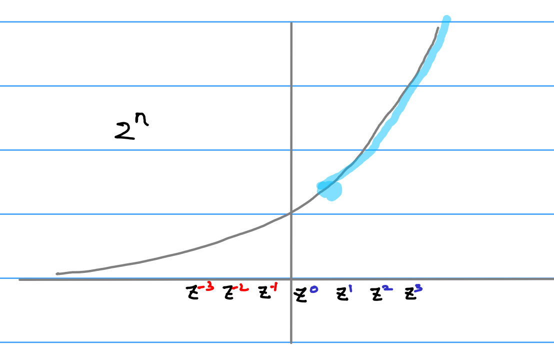
anti-causal $n=-1,-2,-3,\ldots.$

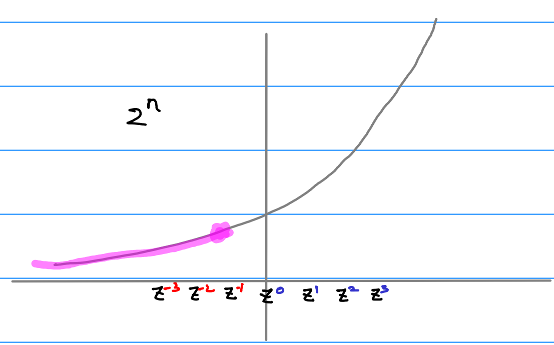
$(p^{-2}, p^{-3}, p^{-4}, \ldots)$

causal $n=+1,+2,+3,\ldots.$

$(p^{-2}, p^{-3}, p^{-4}, \ldots)$







$f(z^{-1})$ $(|z|>p^{-1})$ $\quad$ $f(z)$ $(|z|<p)$

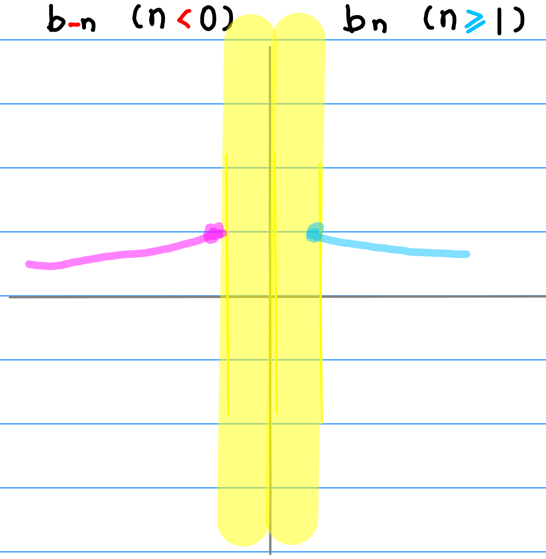
a_{-n} $(n<1)$ $\quad$ a_n $(n\geq 0)$

$f(z^{-1})$ $(|z|>p^{-1})$ $\quad$ $f(z)$ $(|z|<p)$

a_{-n} $(n<1)$ $\quad$ a_n $(n\geq 0)$

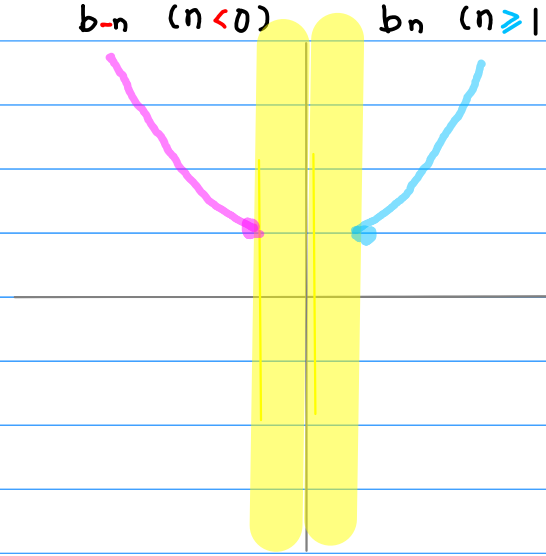
$g(z^{-1})$ $(|z|>p)$ $\quad$ $g(z)$ $(|z|<p^{-1})$

b_{-n} $(n<0)$ $\quad$ b_n $(n\geq 1)$



$g(z^{-1})$ $(|z|>p)$ $\quad$ $g(z)$ $(|z|<p^{-1})$

b_{-n} $(n<0)$ $\quad$ b_n $(n\geq 1)$



x_n		a_{-n}	
y_n		b_{-n}	
causal		anti-causal	
$n \geq 0$	$-(p^{-1}, p^{-2}, p^{-3}, \cdots)$	$n < 1$	
$n \geq 1$	$(p^{-2}, p^{-3}, p^{-4}, \cdots)$	$n < 0$	
anti-causal		causal	
$n < 1$	$-(p^{-1}, p^{-2}, p^{-3}, \cdots)$	$n \geq 0$	
$n < 0$	$(p^{-2}, p^{-3}, p^{-4}, \cdots)$	$n \geq 1$	

causal $f(z)$ $(|z|<p)$ anti-causal $f(z)$ $(|z|>p)$

$f(z) \leftrightarrow a_n$ $(n \geq 0)$ $g(z^{-1}) \leftrightarrow -a_n$ $(n<0)$

$n \geq 0$ $|z|<p$

$\boxed{\text{causal } f(z) \quad -(p^{-1})^{n+1}}$ $\boxed{-\dfrac{p^{-1}}{1-p^{-1}z}}$ $-(p^{-1}+p^{-2}z^{1}+p^{-3}z^{2}+\cdots) = \sum_{n=0}^{\infty} -(p^{-1})^{n+1} z^{n}$ $n \geq 0$

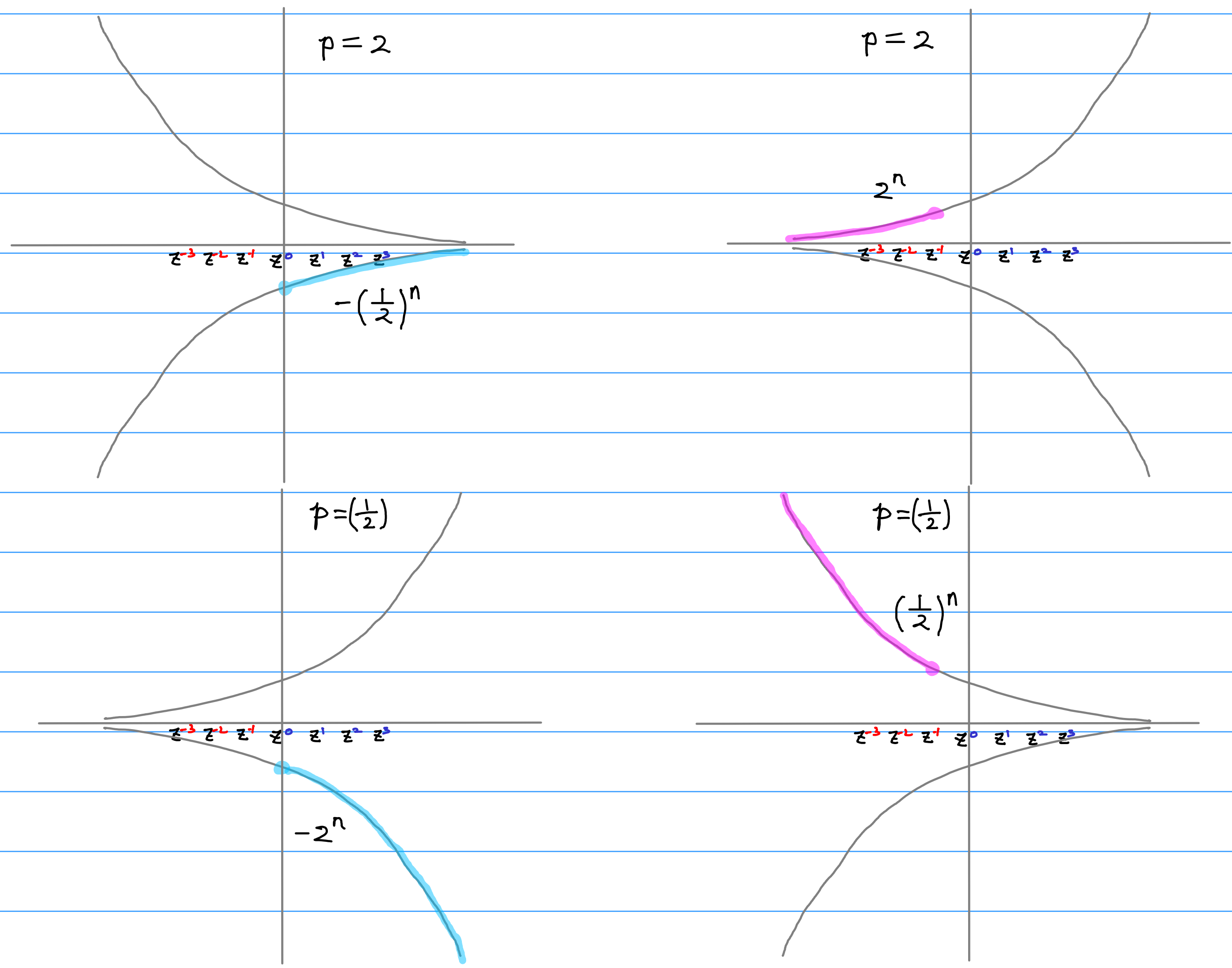
$\cdot(-1)$ $\cdot \dfrac{p}{p}\dfrac{z^{-1}}{z^{-1}}$

$\boxed{(p)^{-n-1} \quad \text{anti-causal } f(z)}$ $\boxed{\dfrac{z^{-1}}{1-pz^{-1}}}$ $p^{0}z^{-1}+p^{1}z^{-2}+p^{2}z^{-3}+\cdots = \sum_{n=-1}^{-\infty} (p)^{-n-1} z^{n}$ $n<0$

$n<0$ $|z|>p$

causal $n=0,+1,+2,+3,\ldots.$ anti-causal $n=-1,-2,-3,\ldots.$

$-(p^{-1}, p^{-2}, p^{-3}, \cdots)$ $(p^{0}, p^{1}, p^{2}, \cdots)$



Getting causal sequence

$$
\begin{tikzcd}
\boxed{\dfrac{1}{z - p}} \arrow[r, leftrightarrow, "\frac{p^{-1}}{p^{-1}}"] \arrow[d, leftrightarrow, "\frac{z^{-1}}{z^{-1}}"'] & \boxed{-\dfrac{p^{-1}}{1 - p^{-1} z}} = f(z) \arrow[r, leftrightarrow] & \boxed{?} \\
\boxed{\dfrac{z^{-1}}{1 - p z^{-1}}} & &
\end{tikzcd}
$$

$$
\| \\
Y(z) \leftrightarrow \boxed{?}
$$

$$
\begin{tikzcd}
\boxed{\dfrac{1}{z^{-1} - p}} \arrow[r, leftrightarrow, "\frac{p^{-1}}{p^{-1}}"] \arrow[d, leftrightarrow, "\frac{z^{-1}}{z^{-1}}"'] & \boxed{-\dfrac{p^{-1}}{1 - p^{-1} z^{-1}}} = X(z) \arrow[r, leftrightarrow] & \boxed{?} \\
\boxed{\dfrac{z}{1 - p z}} & &
\end{tikzcd}
$$

$$
\| \\
g(z) \leftrightarrow \boxed{?}
$$

getting causal sequence w/o memorizing

$$\boxed{-\frac{p^{-1}}{1-p^{-1}z}} = f(z) \longleftrightarrow \boxed{-(p^{-1})^{n+1}}$$

$$\boxed{\frac{z}{1-pz}} = g(z) \longleftrightarrow \boxed{(p)^{n-1}}$$

Left shift

$$\boxed{\frac{z^{-1}}{1-pz^{-1}}} = Y(z) \longleftrightarrow \boxed{(p)^{n-1}}$$

Left shift

$$\boxed{-\frac{p^{-1}}{1-p^{-1}z^{-1}}} = X(z) \longleftrightarrow \boxed{-(p^{-1})^{n+1}}$$

Getting anti-causal sequence

$$
\begin{tikzcd}
\boxed{\dfrac{1}{z-p}} \arrow[r, leftrightarrow, "\frac{p^{-1}}{p^{-1}}"] \arrow[d, leftrightarrow, "\frac{z^{-1}}{z^{-1}}"'] & \boxed{-\dfrac{p^{-1}}{1-p^{-1}z}} \\
\boxed{\dfrac{z^{-1}}{1-pz^{-1}}} &
\end{tikzcd}
$$

$$-\frac{p^{-1}}{1-p^{-1}z} = f(z) \leftrightarrow -(p^{-1})^{n+1}$$

$$X(z^{-1}) \leftrightarrow \boxed{?}$$

$$\frac{z^{-1}}{1-pz^{-1}} = g(z^{-1}) \leftrightarrow \boxed{?}$$

$$Y(z) \leftrightarrow (p)^{n-1}$$

$$
\begin{tikzcd}
\boxed{\dfrac{1}{z^{-1}-p}} \arrow[r, leftrightarrow, "\frac{p^{-1}}{p^{-1}}"] \arrow[d, leftrightarrow, "\frac{z^{-1}}{z^{-1}}"'] & \boxed{-\dfrac{p^{-1}}{1-p^{-1}z^{-1}}} \\
\boxed{\dfrac{z}{1-pz}} &
\end{tikzcd}
$$

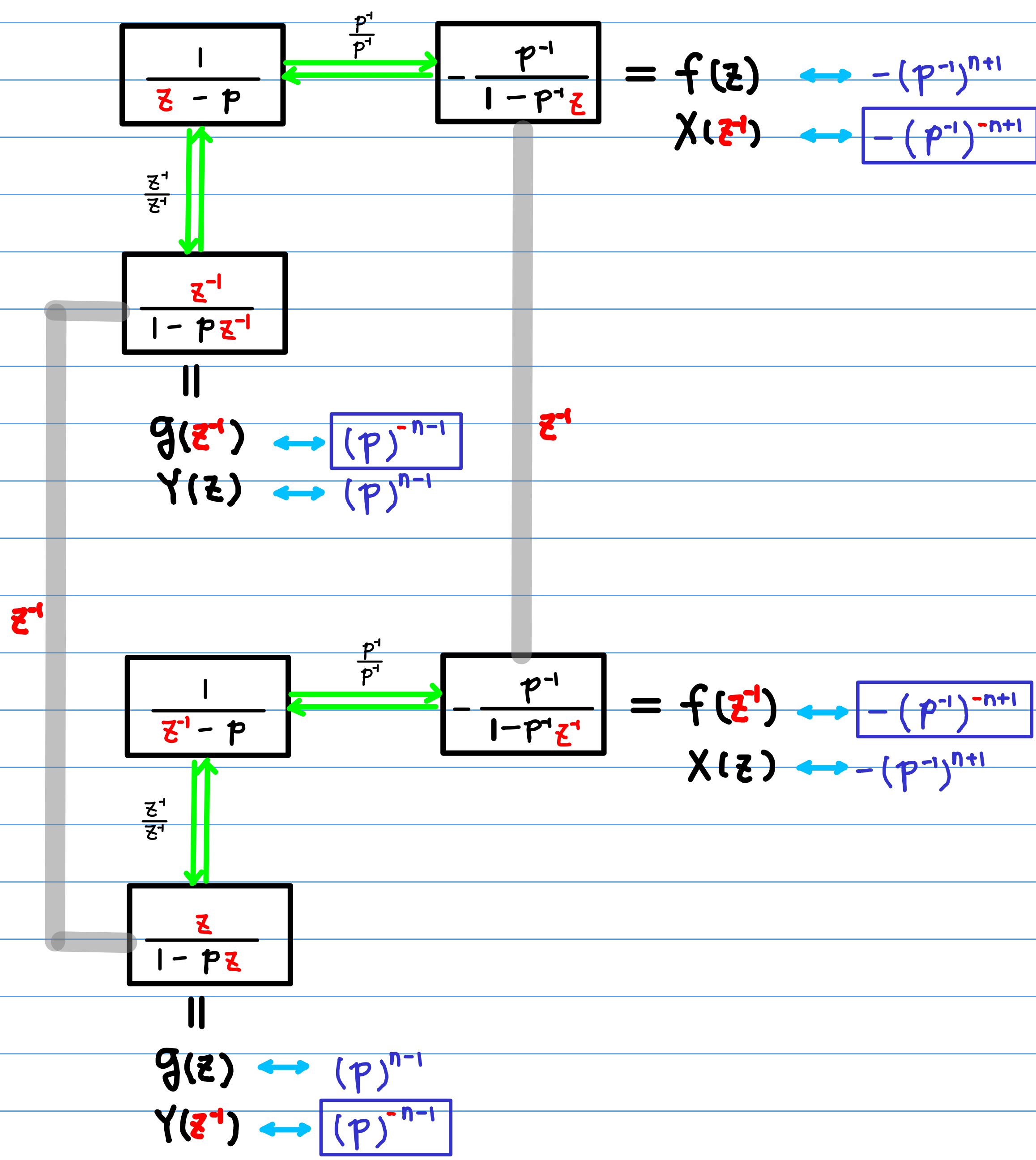
$$-\frac{p^{-1}}{1-p^{-1}z^{-1}} = f(z^{-1}) \leftrightarrow \boxed{?}$$

$$X(z) \leftrightarrow -(p^{-1})^{n+1}$$

$$\frac{z}{1-pz} = g(z) \leftrightarrow (p)^{n-1}$$

$$Y(z^{-1}) \leftrightarrow \boxed{?}$$

① $z \leftarrow z^{-1}$ ② $a_n \leftarrow a_{-n}$



Getting anti-causal sequence w/o memorizing

$$\begin{tikzcd}
f(z^{-1}) = \boxed{-\dfrac{p^{-1}}{1-p^{-1}z^{-1}}} \arrow[r, "z^{-1}"] \arrow[d, leftrightarrow] & \boxed{-\dfrac{p^{-1}}{1-p^{-1}z}} = f(z) \arrow[d, leftrightarrow] \\
a_{-n} = \boxed{-(p^{-1})^{-n+1}} & \boxed{-(p^{-1})^{n+1}} = a_n \arrow[l, "-n"']
\end{tikzcd}$$

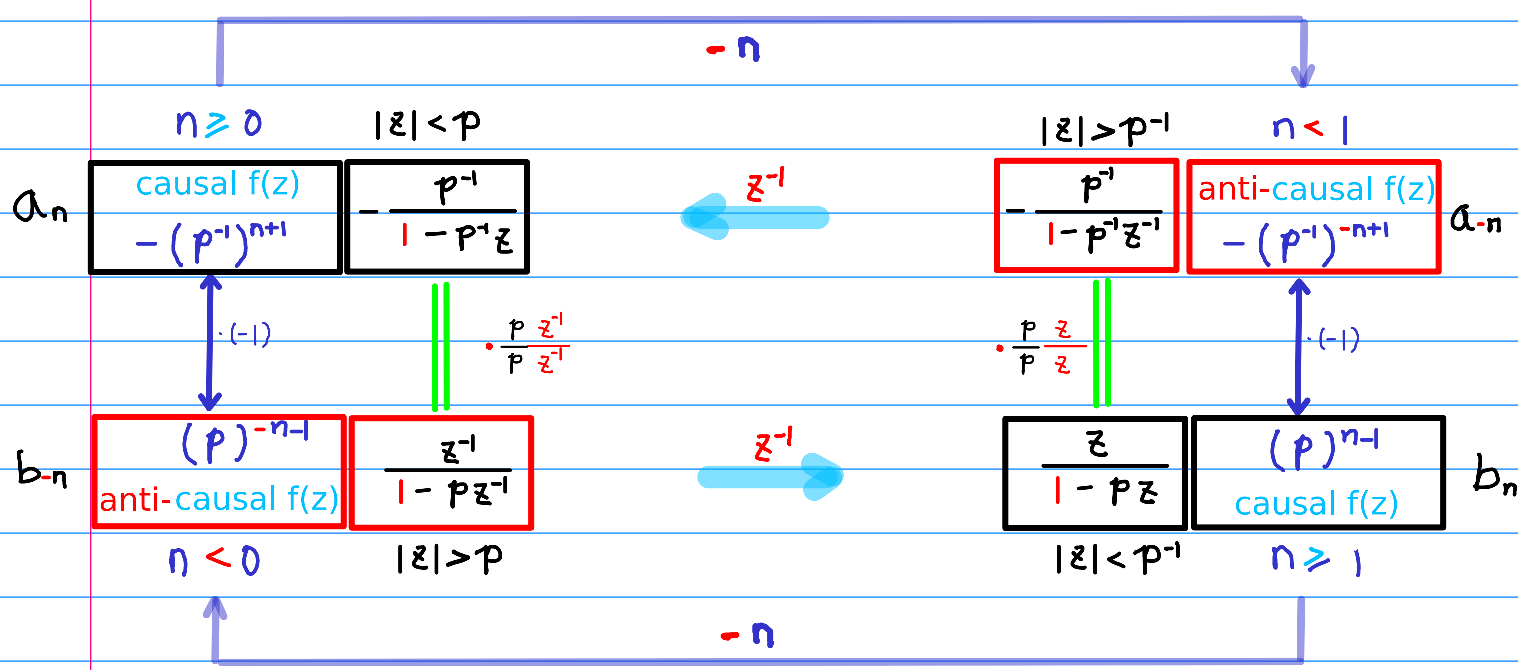
$$\begin{tikzcd}
g(z^{-1}) = \boxed{\dfrac{z^{-1}}{1-pz^{-1}}} \arrow[r, "z^{-1}"] \arrow[d, leftrightarrow] & \boxed{\dfrac{z}{1-pz}} = g(z) \arrow[d, leftrightarrow] \\
b_{-n} = \boxed{(p)^{-n-1}} & \boxed{(p)^{n-1}} = b_n \arrow[l, "-n"']
\end{tikzcd}$$

$$\begin{tikzcd}
Y(z^{-1}) = \boxed{\dfrac{z}{1-pz}} \arrow[r, "z^{-1}"] \arrow[d, leftrightarrow] & \boxed{\dfrac{z^{-1}}{1-pz^{-1}}} = Y(z) \arrow[d, leftrightarrow] \\
y_{-n} = \boxed{(p)^{-n-1}} & \boxed{(p)^{n-1}} = y_n \arrow[l, "-n"']
\end{tikzcd}$$

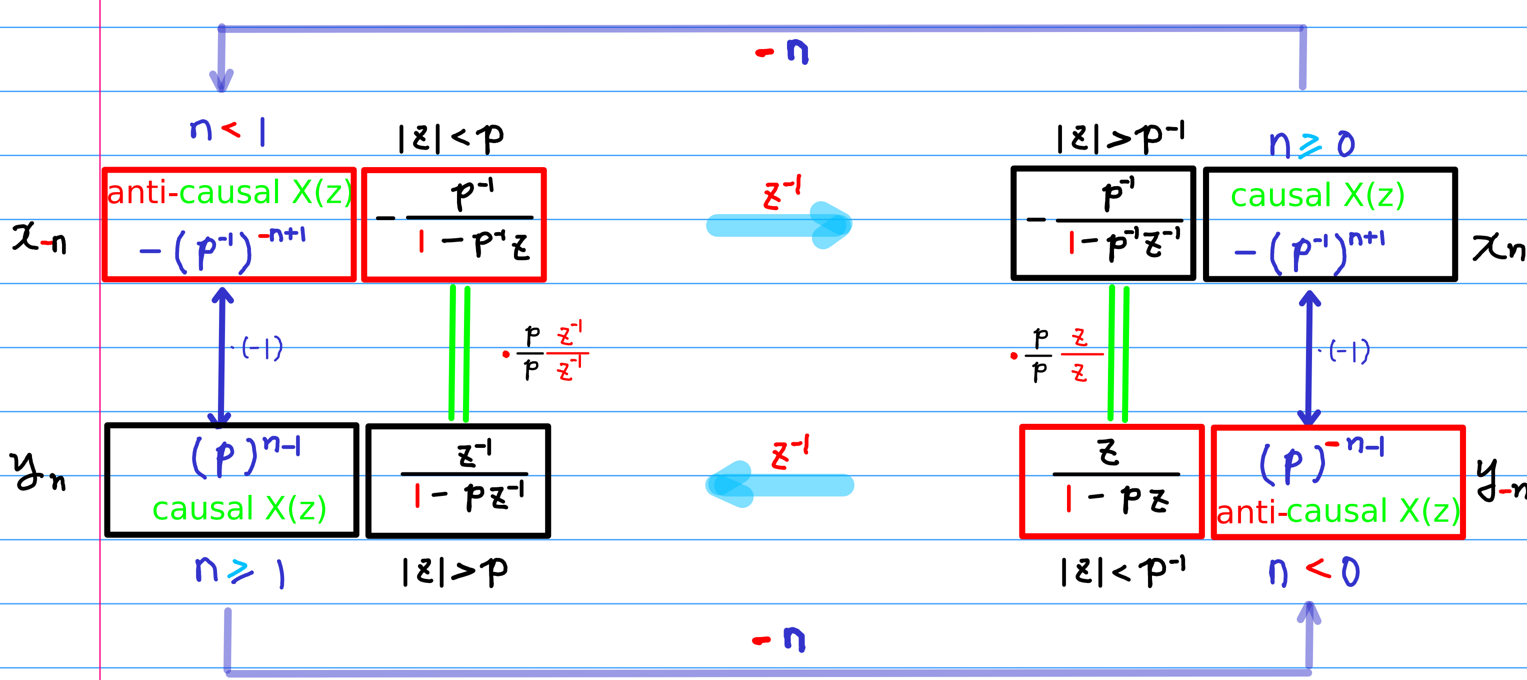
$$\begin{tikzcd}
X(z^{-1}) = \boxed{-\dfrac{p^{-1}}{1-p^{-1}z}} \arrow[r, "z^{-1}"] \arrow[d, leftrightarrow] & \boxed{-\dfrac{p^{-1}}{1-p^{-1}z^{-1}}} = X(z) \arrow[d, leftrightarrow] \\
x_{-n} = \boxed{-(p^{-1})^{-n+1}} & \boxed{-(p^{-1})^{n+1}} = x_n \arrow[l, "-n"']
\end{tikzcd}$$

Getting anti-causal sequence

Laurent Series



Z-Transform



$f(z^{-1})$ $g(z^{-1})$

① $\boxed{z^{-1} \to z}$ $f(z)$, $g(z)$

② $f(z) \leftrightarrow a_n$ $g(z) \leftrightarrow b_n$

③ $\boxed{n \to -n}$ a_{-n}, b_{-n}

$X(z^{-1})$ $Y(z^{-1})$

① $\boxed{z^{-1} \to z}$ $X(z)$, $Y(z)$

② $X(z) \leftrightarrow x_n$ $Y(z) \leftrightarrow y_n$

③ $\boxed{n \to -n}$ x_{-n}, x_{-n}

$$f(z^{-1}) = \boxed{-\frac{p^{-1}}{1-p^{-1}z^{-1}}} \qquad g(z^{-1}) = \boxed{\frac{z^{-1}}{1-pz^{-1}}}$$

anti-causal

$$f(z) = \boxed{-\frac{p^{-1}}{1-p^{-1}z}} \qquad g(z) = \boxed{\frac{z}{1-pz}}$$

$$Y(z^{-1}) = \boxed{\frac{z}{1-pz}} \qquad X(z^{-1}) = \boxed{-\frac{p^{-1}}{1-p^{-1}z}}$$

anti-causal

$$Y(z) = \boxed{\frac{z^{-1}}{1-pz^{-1}}} \qquad X(z) = \boxed{-\frac{p^{-1}}{1-p^{-1}z^{-1}}}$$

$f(z^{-1})$ $g(z^{-1})$

① $\boxed{z^{-1} \to z}$ $f(z)$, $g(z)$

② $f(z) \leftrightarrow a_n$ $g(z) \leftrightarrow b_n$

③ $\boxed{n \to -n}$ a_{-n}, b_{-n}

$$f(z^{-1}) = \boxed{-\frac{p^{-1}}{1-p^{-1}z^{-1}}} \qquad g(z^{-1}) = \boxed{\frac{z^{-1}}{1-pz^{-1}}}$$

① $$f(z) = \boxed{-\frac{p^{-1}}{1-p^{-1}z}} \qquad g(z) = \boxed{\frac{z}{1-pz}}$$

② $$a_n = \boxed{-(p^{-1})^{n+1}} \qquad b_n = \boxed{(p)^{n-1}}$$

③ $$a_{-n} = \boxed{-(p^{-1})^{-n+1}} \qquad b_{-n} = \boxed{(p)^{-n-1}}$$

$X(z^{-1})$ $Y(z^{-1})$

1. $\boxed{z^{-1} \to z}$ $X(z)$, $Y(z)$

2. $X(z) \leftrightarrow x_n$ $Y(z) \leftrightarrow y_n$

3. $\boxed{n \to -n}$ x_{-n}, x_{-n}

$$Y(z^{-1}) = \boxed{\frac{z}{1-pz}} \qquad X(z^{-1}) = \boxed{-\frac{p^{-1}}{1-p^{-1}z}}$$

1. $$Y(z) = \boxed{\frac{z^{-1}}{1-pz^{-1}}} \qquad X(z) = \boxed{-\frac{p^{-1}}{1-p^{-1}z^{-1}}}$$

2. $$y_n = \boxed{(p)^{n-1}} \qquad x_n = \boxed{-(p^{-1})^{n+1}}$$

3. $$y_{-n} = \boxed{-(p^{-1})^{-n+1}} \qquad x_{-n} = \boxed{(p)^{-n-1}}$$

① $\dfrac{-1}{(z-1)(z-2)}$

② $\dfrac{-0.5z^2}{(z-1)(z-0.5)}$

$-\dfrac{1}{1-z}+\dfrac{0.5}{1-0.5z}$

$f(z)$ $|z|<1$ causal

$X(z)$ $|z|<1$ anti-causal

$+\dfrac{z}{1-z}-\dfrac{z}{1-2z}$

$f(z)$ $|z|<0.5$ causal

$X(z)$ $|z|<0.5$ anti-causal

$+\dfrac{z^{-1}}{1-z^{-1}}-\dfrac{z^{-1}}{1-2z^{-1}}$

$f(z)$ $|z|>1$ anti-causal

$X(z)$ $|z|>1$ causal

$-\dfrac{1}{1-z^{-1}}+\dfrac{1}{1-0.5z^{-1}}$

$f(z)$ $|z|>2$ anti-causal

$X(z)$ $|z|>2$ causal

$$-\frac{1}{1-z}+\frac{0.5}{1-0.5z}$$

$f(z) \quad |z|<1$

$$+\frac{z}{1-z}-\frac{z}{1-2z}$$

$f(z) \quad |z|<0.5$

$\boxed{\cdot z \quad n-1}$

$$+\frac{1}{1-z}-\frac{1}{1-2z}$$

$g(z) \quad |z|<0.5$

$$+\frac{z^{-1}}{1-z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}$$

$X(z) \quad |z|>1$

$\boxed{\cdot z^{-1} \quad n-1}$

$$+\frac{1}{1-z^{-1}}-\frac{1}{1-2z^{-1}}$$

$V(z) \quad |z|>1$

$$-\frac{1}{1-z^{-1}}+\frac{1}{1-0.5z^{-1}}$$

$X(z) \quad |z|>2$

$$-\frac{1}{1-z}+\frac{0.5}{1-0.5z}$$

$X(z) \quad |z|<1$

$z^{-1} \quad -n$

$$-\frac{1}{1-z^{-1}}+\frac{1}{1-0.5z^{-1}}$$

$V(z) \quad |z|>2$

$$+\frac{z}{1-z}-\frac{z}{1-2z}$$

$X(z) \quad |z|<0.5$

$z^{-1} \quad -n$

$$+\frac{z^{-1}}{1-z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}$$

$V(z) \quad |z|>1$

$\cdot z^{-1} \quad n-1$

$$+\frac{1}{1-z^{-1}}-\frac{1}{1-2z^{-1}}$$

$W(z) \quad |z|>1$

$$+\frac{z^{-1}}{1-z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}$$

$f(z) \quad |z|>1$

$z^{-1} \quad -n$

$$+\frac{z}{1-z}-\frac{z}{1-2z}$$

$g(z) \quad |z|<0.5$

$\cdot z \quad n-1$

$$+\frac{1}{1-z}-\frac{1}{1-2z}$$

$h(z) \quad |z|<0.5$

$$-\frac{1}{1-z^{-1}}+\frac{1}{1-0.5z^{-1}}$$

$f(z) \quad |z|>2$

$z^{-1} \quad -n$

$$-\frac{1}{1-z}+\frac{0.5}{1-0.5z}$$

$g(z) \quad |z|<1$