

# Logic Circuit Design

## NAND-1

20171009

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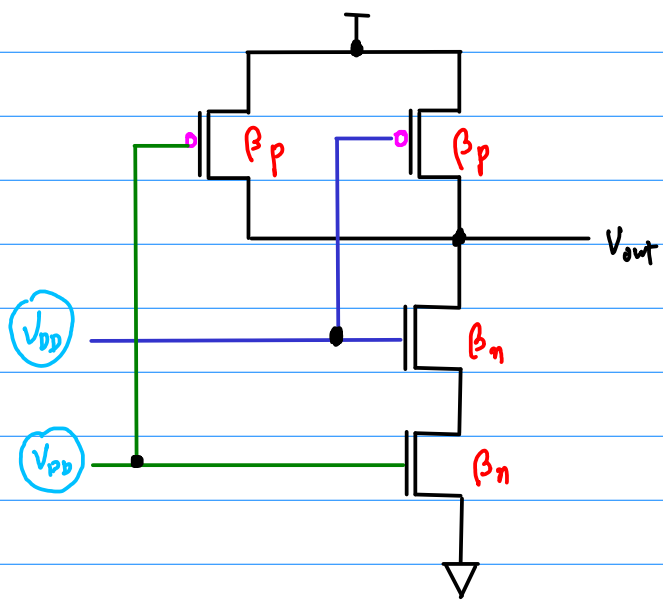
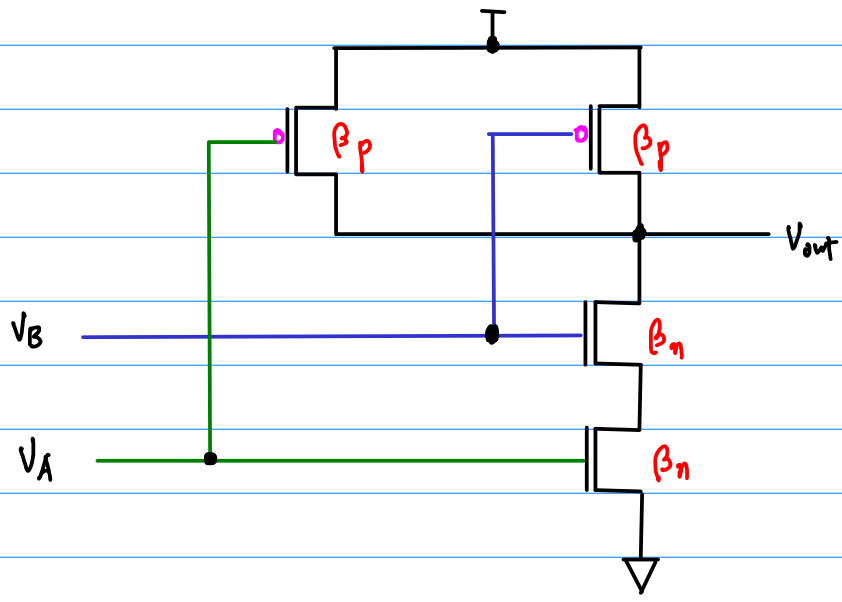
# References

Some Figures from the following sites

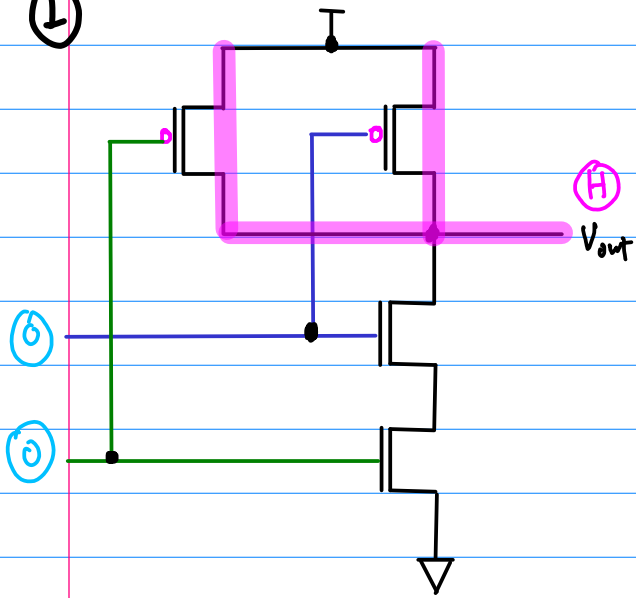
[1] <http://pages.hmc.edu/harris/cmosvlsi/4e/index.html>  
Weste & Harris Book Site

[2] Introduction to VLSI Circuits and Systems, Uyemura

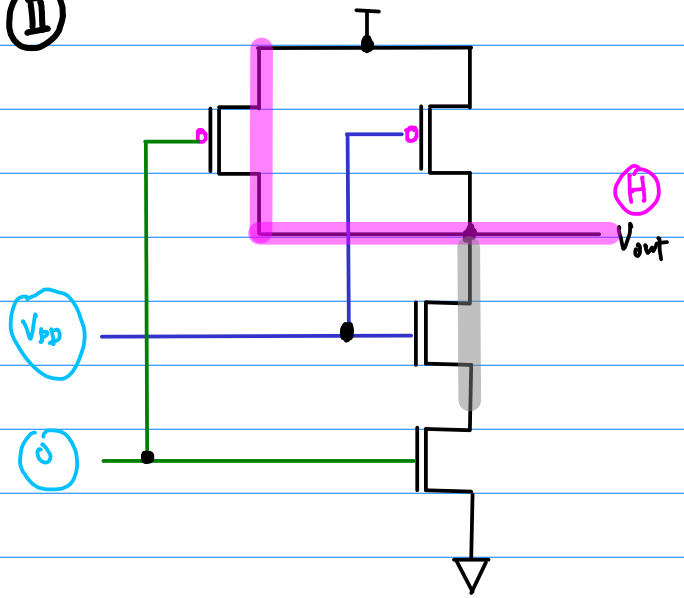
[2] [en.wikipedia.org](http://en.wikipedia.org)



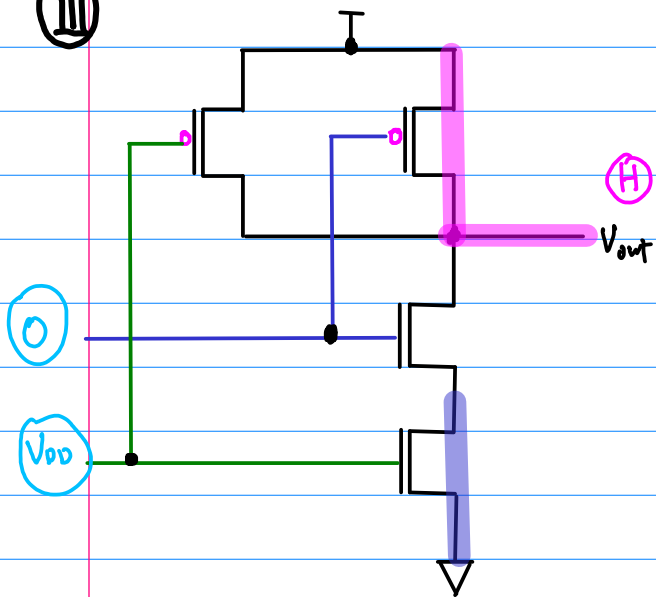
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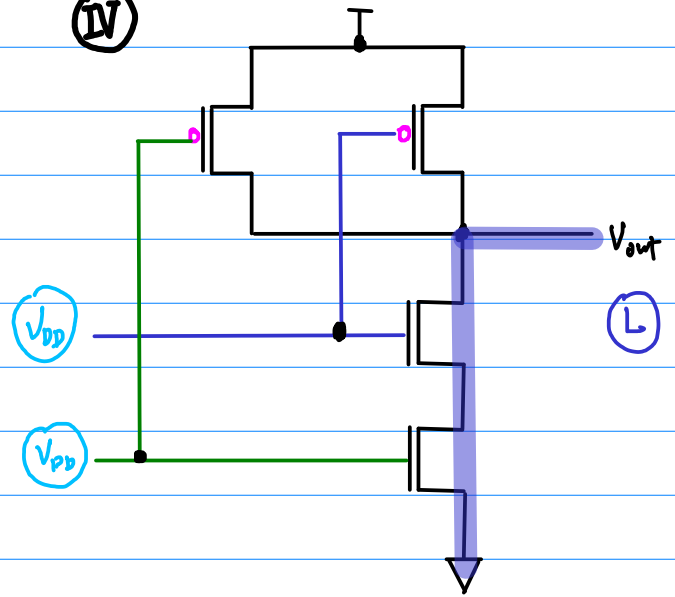
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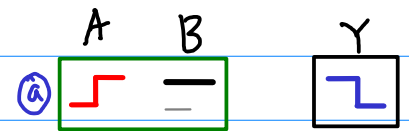
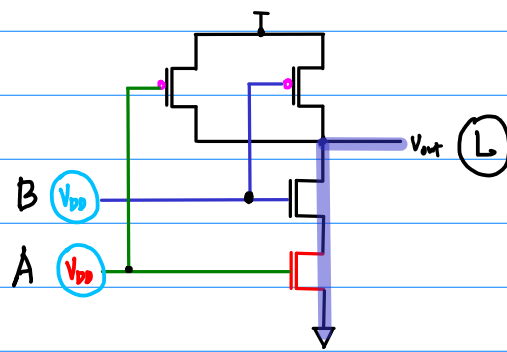
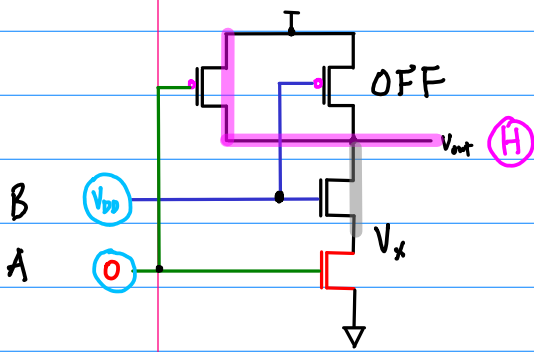


III

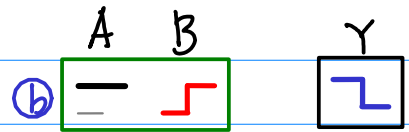
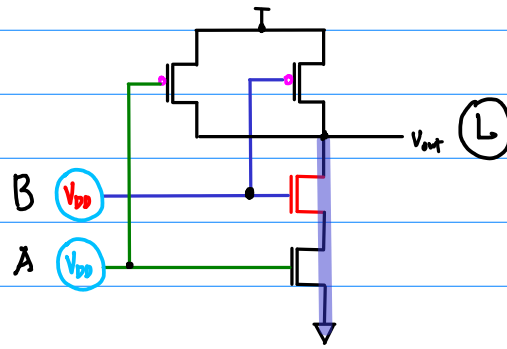
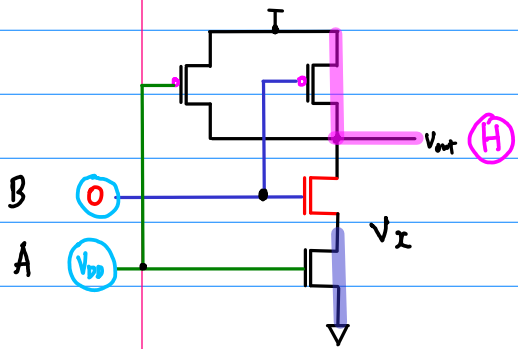


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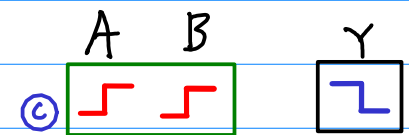
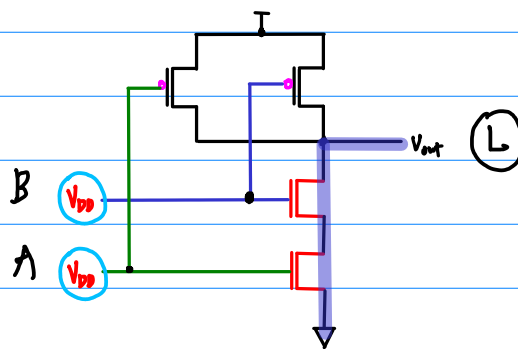
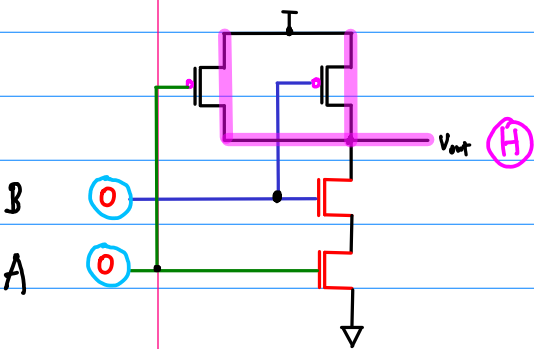




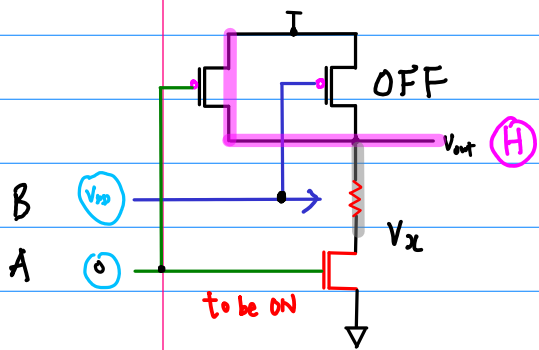
Weak PUN  
 nMB: Resistor  
 nMA: ~~Body Effect~~



Weak PUN  
 nMB: Body Effect  
 nMA: resistor

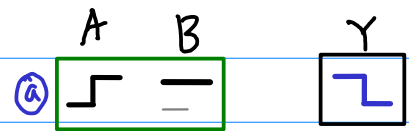


Strong PUN



{ small  $R$   
 large  $C$  (load)  
 { small voltage drop  
 slow change

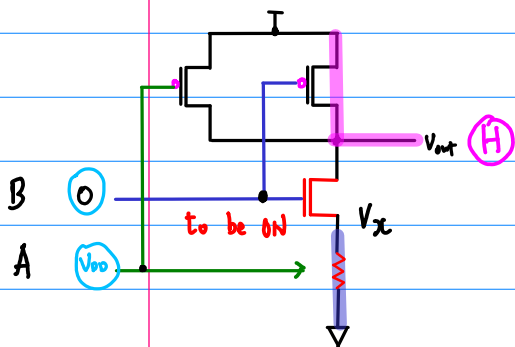
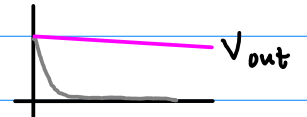
small effect on VTC



Weak PUN

nMB: Resistor

nMA: ~~Body Effect~~

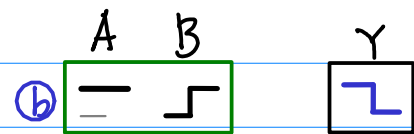


{ small  $R$   
 small  $C$  (load)  
 { large voltage drop  
 immediate change

increase  $V_T$

→ hard to turn ON nMB

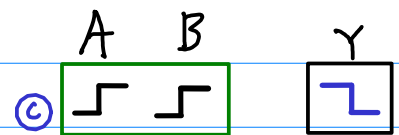
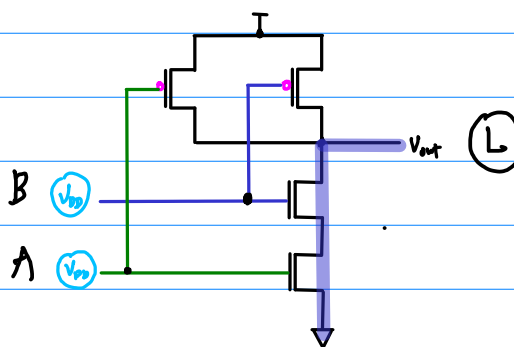
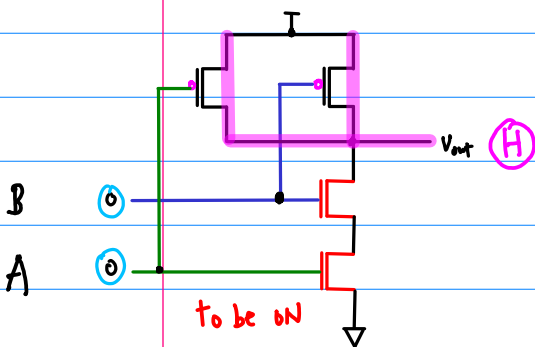
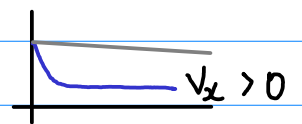
→ VTC shifted to the right



Weak PUN

nMB: Body Effect

nMA: resistor

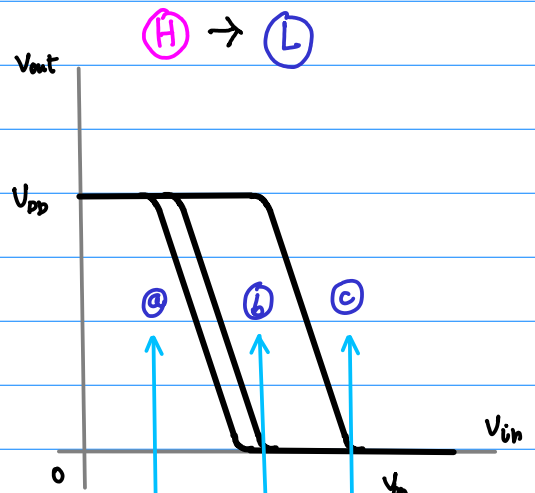
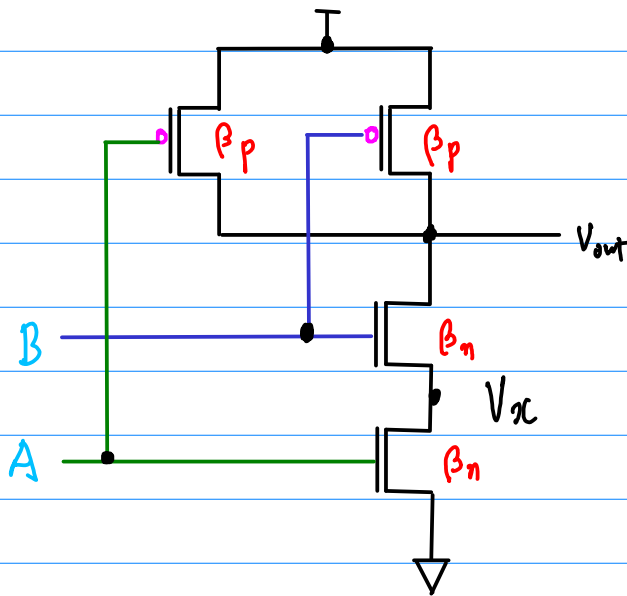
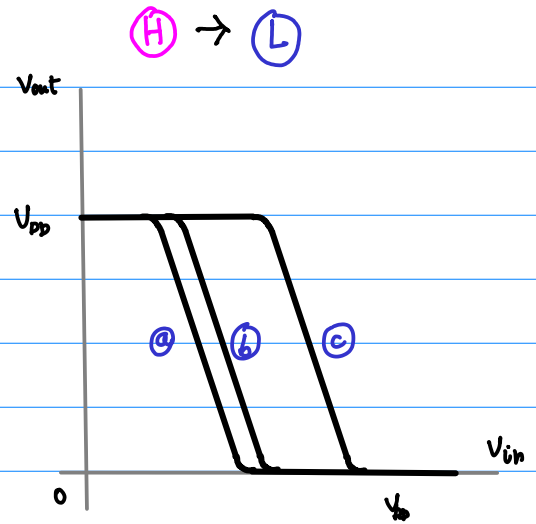


Strong PUN

→ hard to turn ON nMA & nMB

→ VTC shifted to the right

	$V_A$	$V_B$	$V_{out}$
©	0	0	$V_{DD}$
Ⓐ	0	$V_{DD}$	$V_{DD}$
Ⓑ	$V_{DD}$	0	$V_{DD}$
	$V_{DD}$	$V_{DD}$	0



A : gate input of the bottom nMOS

B : gate input of the top nMOS

A switched while B=0

A & B switched simultaneously strong PUN

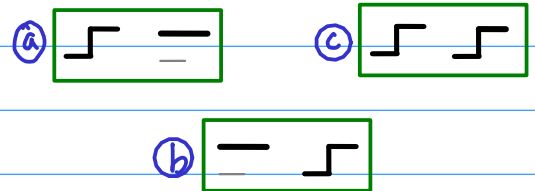
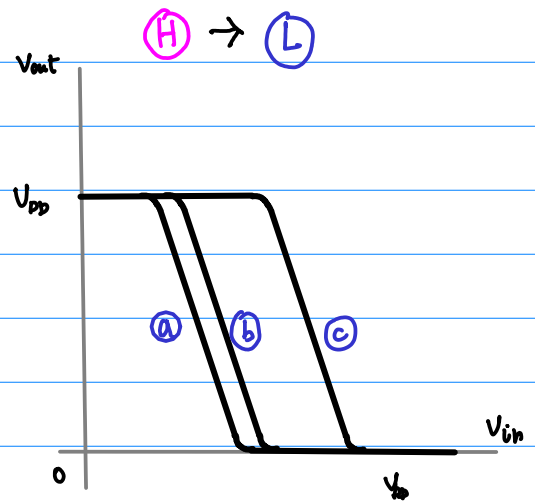
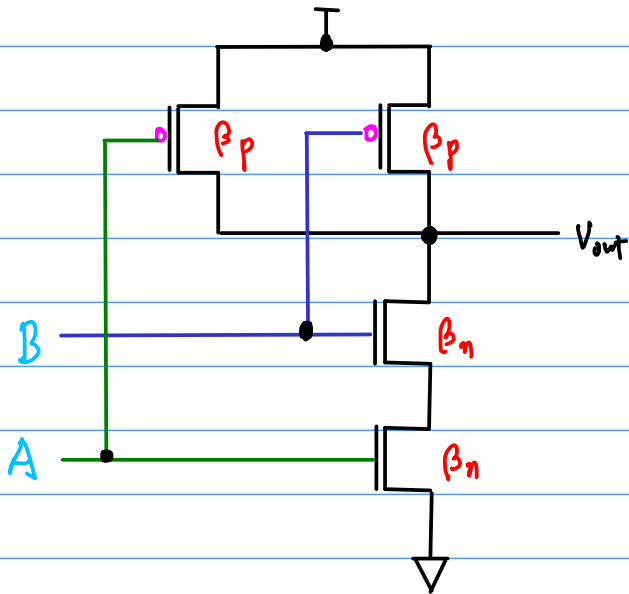
B switched while A=0  
Body Effect

Strong PUN } difficult to turn on nMOS  
Body Effect }

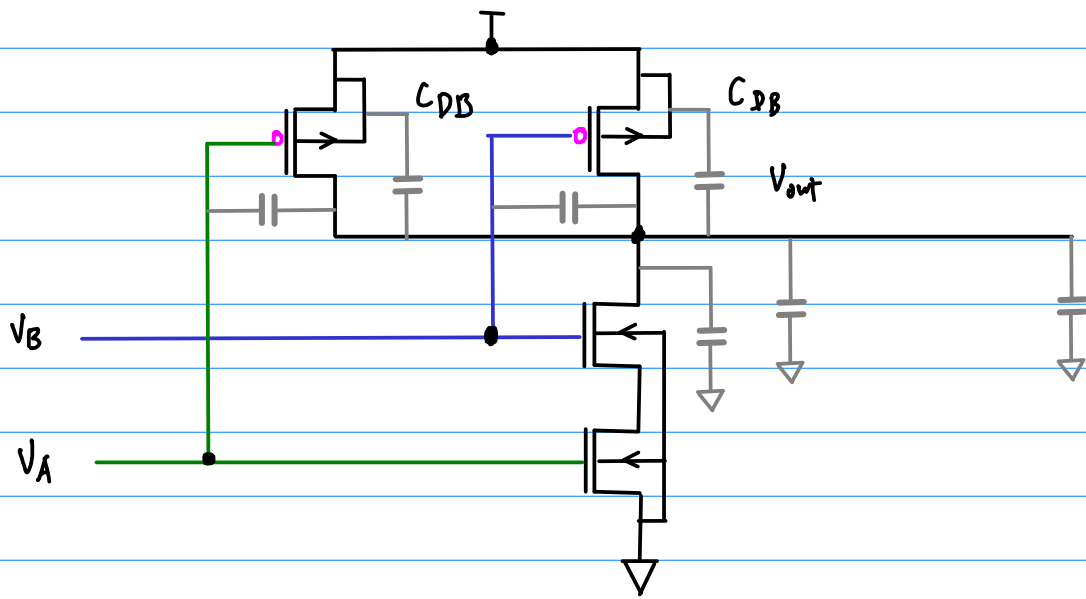
VTC shifted the right

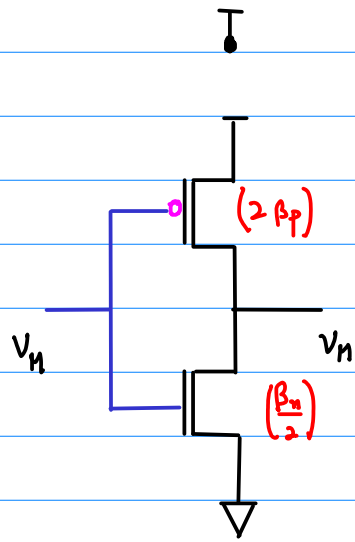
	$V_A$	$V_B$	$V_{out}$
Ⓒ	0	0	$V_{DD}$
Ⓐ	0	$V_{DD}$	$V_{DD}$
Ⓑ	$V_{DD}$	0	$V_{DD}$
	$V_{DD}$	$V_{DD}$	0

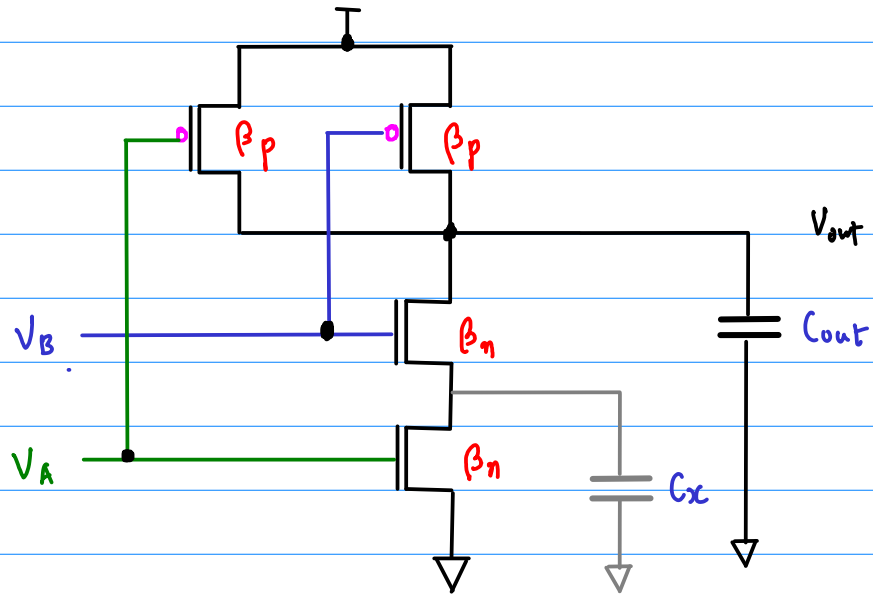
	$V_A$	$V_B$	$V_{out}$
Ⓒ	⌋	⌋	⌋
Ⓐ	⌋	—	⌋
Ⓑ	—	⌋	⌋

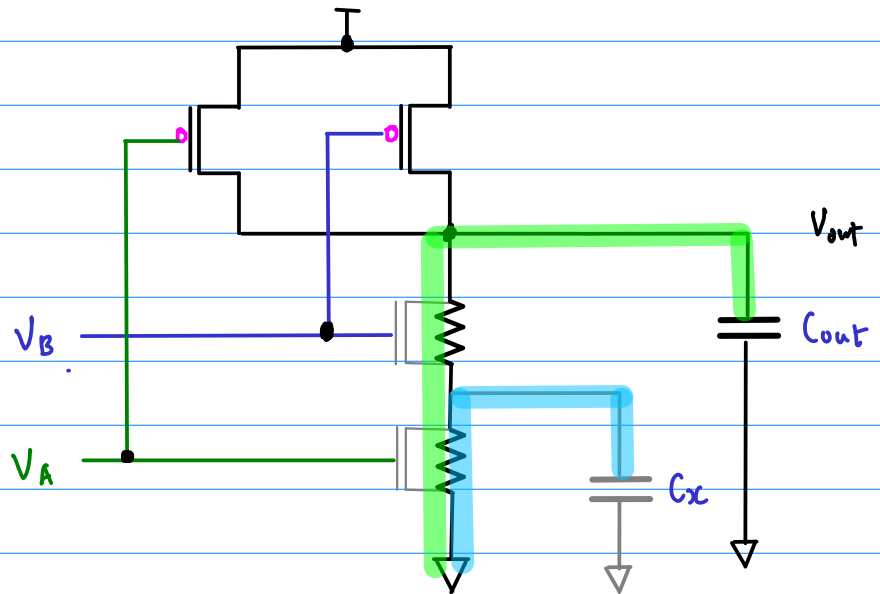
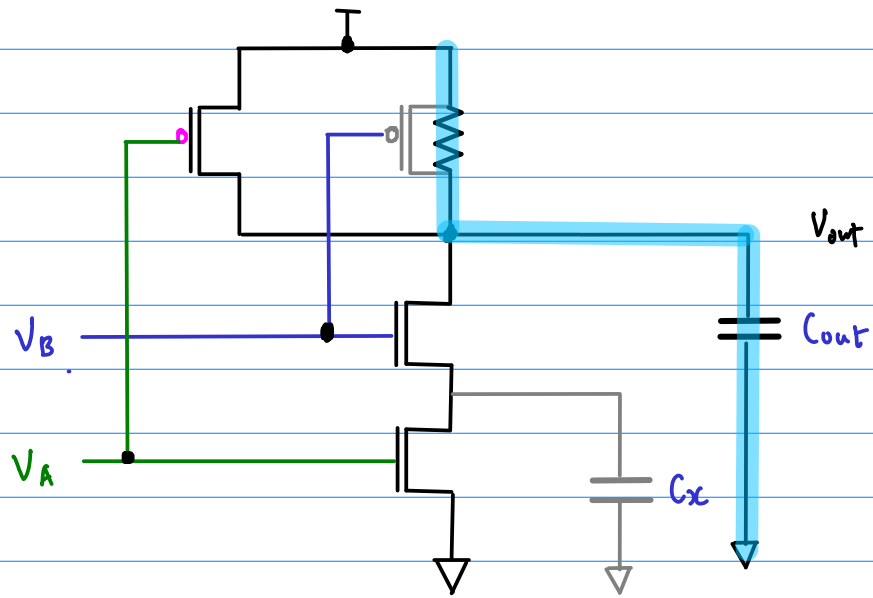








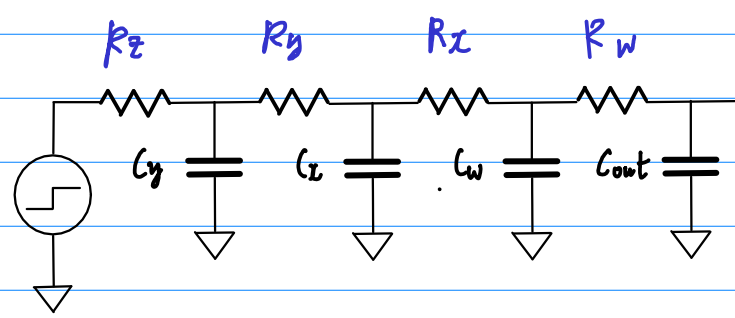
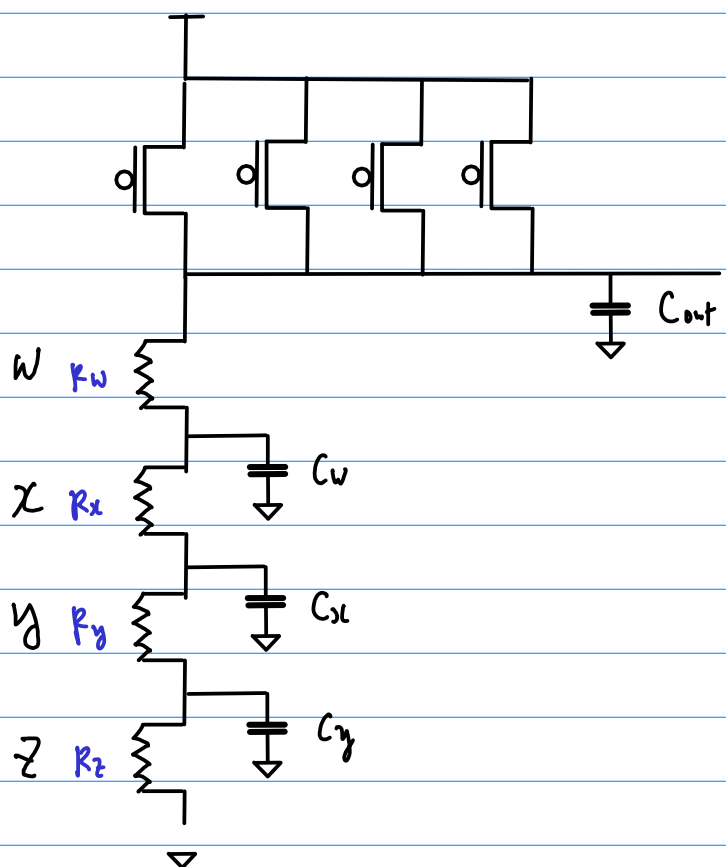
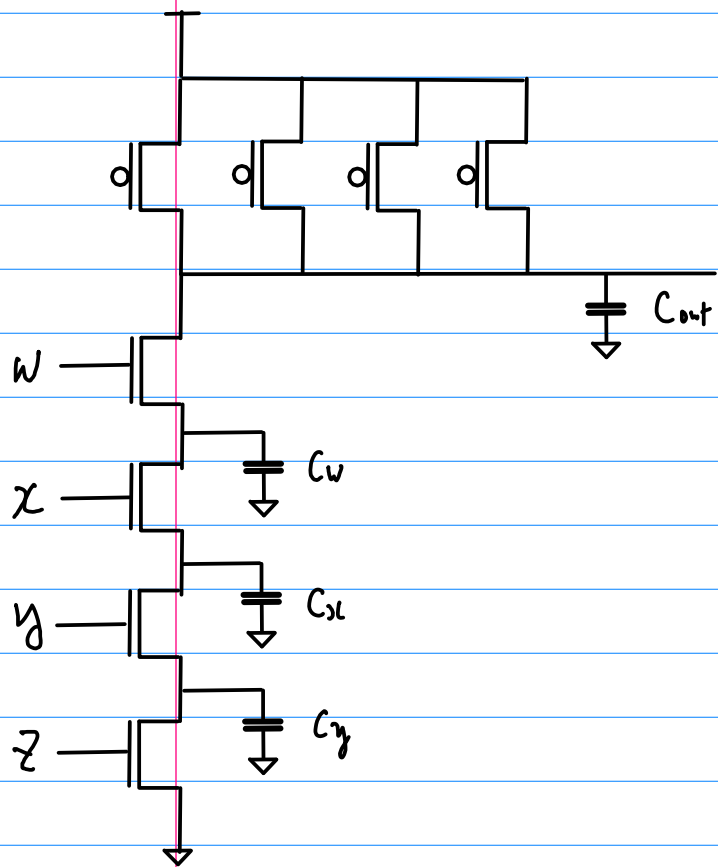




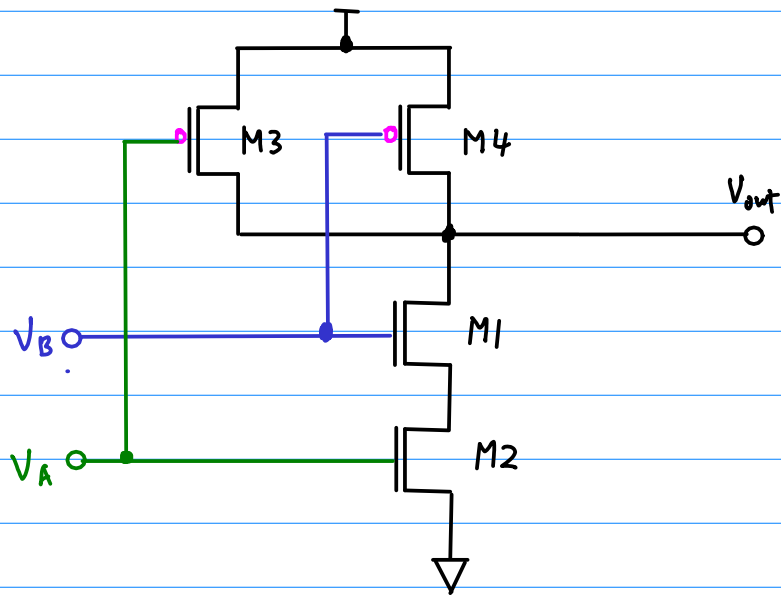
$$C_{out} = C_{FET} + C_L$$

$$C_{FG1} = C_{Dn} + 2 C_{Dp}$$

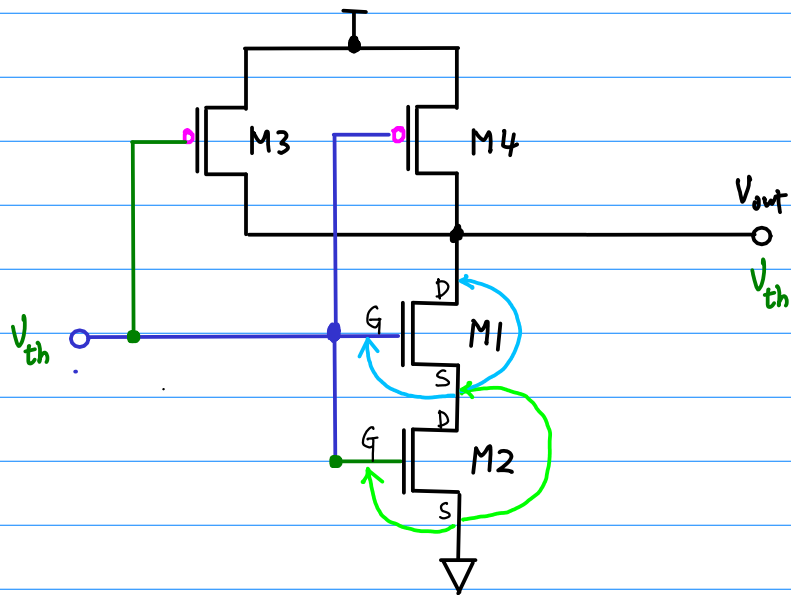
$$R_p = \frac{1}{\beta_p (v_{DD} - |V_{Tp}|)} \quad R_n = \frac{1}{\beta_n (v_{DD} - V_{Tn})}$$



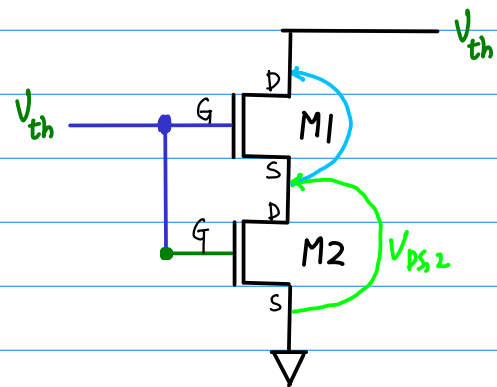
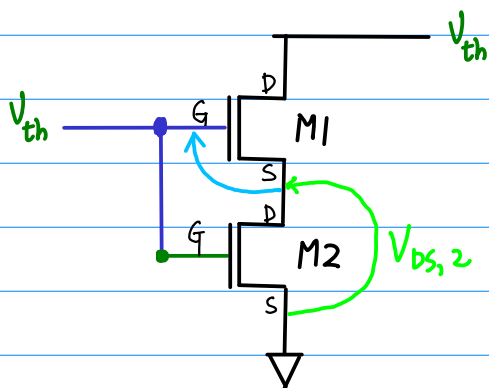
$$t_{pd} = C_y R_Z + C_x (R_Y + R_Z) + C_w (R_X + R_Y + R_Z) + C_{out} (R_W + R_X + R_Y + R_Z)$$



$V_A$	$V_B$	$V_{out}$		$V_A$	$V_B$	$V_{out}$
0	0	$V_{DD}$	⇒	$V_{th}$	$V_{th}$	$V_{th}$
$V_{DD}$	0	$V_{DD}$		$V_{DD}$	$V_{th}$	$V_{th}$
0	$V_{DD}$	$V_{DD}$		$V_{th}$	$V_{DD}$	$V_{th}$



$V_A$	$V_B$	$V_{out}$
0	0	$V_{DD}$
$\downarrow$	$\downarrow$	$\downarrow$
$V_{th}$	$V_{th}$	$V_{th}$



$$V_{GS,1} = V_{th} - V_{DS,2}$$

$$V_{th} = V_{DS,1} + V_{DS,2}$$

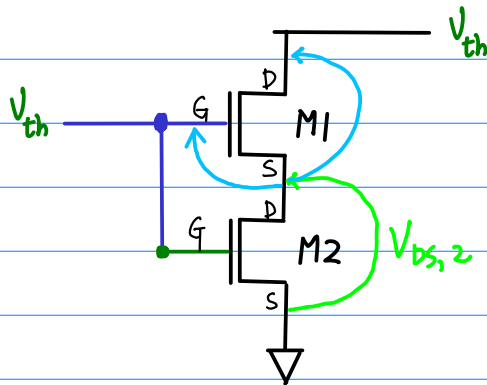
$$V_{th} - V_{DS,2} = V_{DS,1}$$

$\therefore$

$$V_{GS,1} = V_{DS,1}$$

M1 saturated.





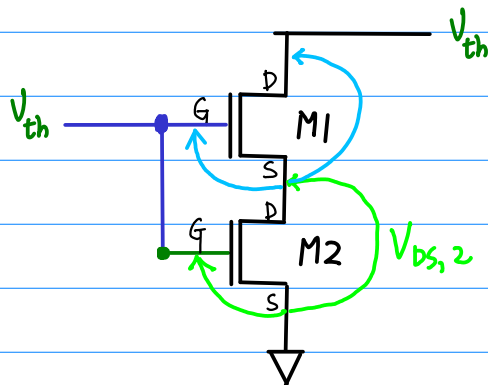
$$I_D = \frac{\beta_n}{2} (V_{GS,1} - V_{T0n})^2$$

$$V_{GS,1} = V_{th} - V_{DS,2}$$

$$I_D = \frac{\beta_n}{2} (V_{th} - V_{DS,2} - V_{T0n})^2$$

$$I_D = \frac{\beta_n}{2} (V_{th} - V_{T0n} - V_{DS,2})^2$$

$\therefore$   $V_{GS,1} = V_{DS,1}$  M1 saturated.



$$V_{th} = V_{GS1} + V_{DS2}$$

$$= V_{GS2}$$

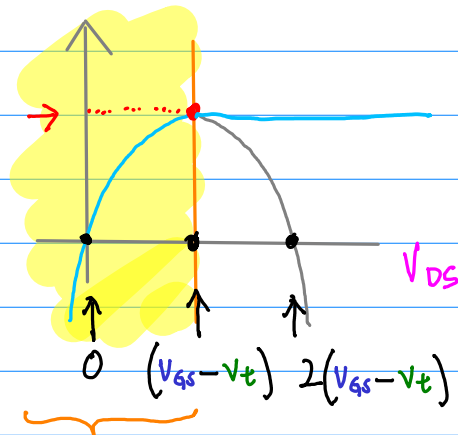
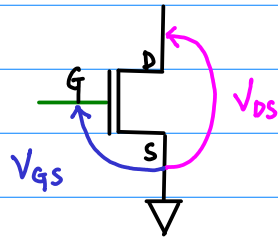
$$V_{GS1} < V_{GS2}$$

$$V_{GS2} - V_{DS2} = V_{GS1} > 0$$

$V_{GS,2} > V_{GS,1}$  Linear

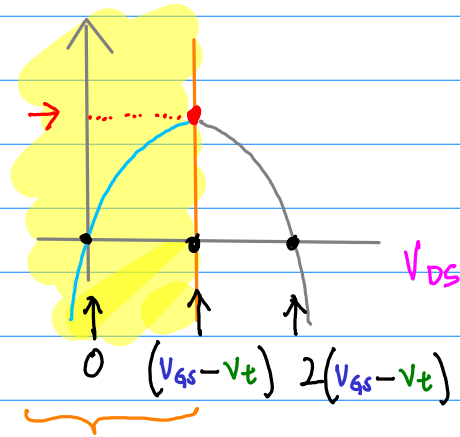
$$I_{ds} = \begin{cases} 0 & V_{gs} < V_t & \text{cutoff} \\ \beta \left( V_{gs} - V_t - \frac{V_{ds}}{2} \right) V_{ds} & V_{ds} < V_{dsat} & \text{linear} \\ \frac{\beta}{2} (V_{gs} - V_t)^2 & V_{ds} > V_{dsat} & \text{saturation} \end{cases}$$

$$I_{ds} = f(V_{ds}) = \frac{\beta}{2} \left( 2(V_{gs} - V_t) - V_{ds} \right) V_{ds}$$



$$V_{ds} \leq V_{gs} - V_t$$

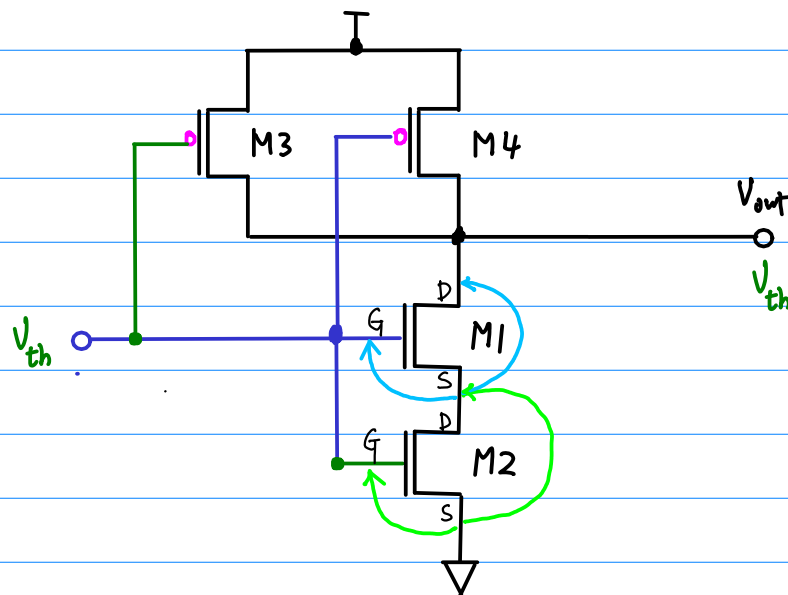
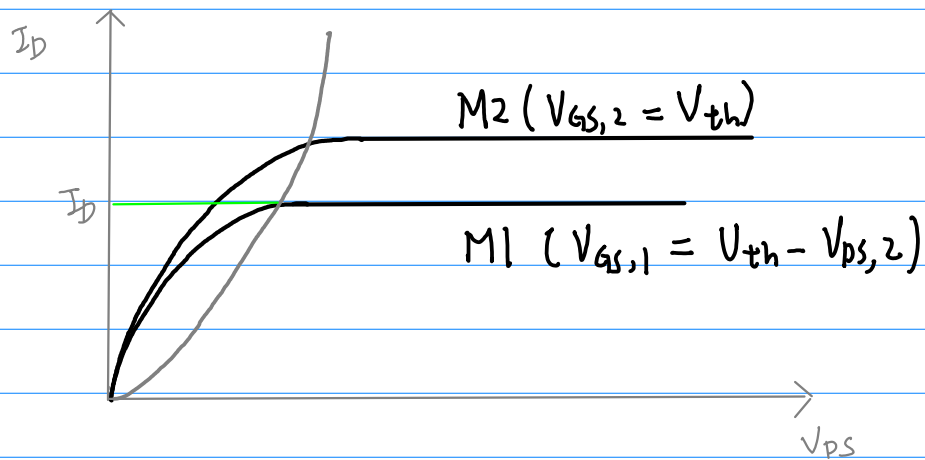
$$f(V_{DS}) = \frac{\mu}{2} \left( 2(V_{GS} - V_t) - V_{DS} \right) V_{DS}$$



$$V_{DS} \leq V_{GS} - V_t$$

$$\begin{aligned} f(V_{GS} - V_t) &= \frac{\mu}{2} \left( 2(V_{GS} - V_t) - (V_{GS} - V_t) \right) (V_{GS} - V_t) \\ &= \frac{\mu}{2} (V_{GS} - V_t)^2 \end{aligned}$$

$$I_{DS} = \frac{\mu}{2} (V_{GS} - V_t)^2 \quad : \text{Saturated current}$$



M1 SAT  $I_D = \frac{\beta_n}{2} (V_{th} - V_{T0n} - V_{DS,2})^2$

$V_{GS,2} > V_{GS,1}$  Linear

$$I_D = \frac{\beta_n}{2} (2(V_{th} - V_{T0n})V_{DS,2} - V_{DS,2}^2)$$

# Simultaneous switching

$$V_{GS,1} = V_{th} - V_{DS,2} \quad V_{GS,2} = V_{th}$$

$$V_{th} = V_{DS,1} - V_{DS,2}$$

$$V_{GS,1} = V_{DS,1}$$

$$I_D = \frac{\beta_n}{2} (V_{th} - V_{T0n} - V_{DS,2})^2$$

$$I_D = \frac{\beta_n}{2} (2(V_{th} - V_{T0n})V_{DS,2} - V_{DS,2}^2)$$

$$V_{DS,2} = (V_{th} - V_{T0n}) - \sqrt{\frac{2I_D}{\beta_n}}$$

$$V_{th} - V_{T0n} = (V_{DD} - V_{th})$$

$$V_{SG,3} = V_{SG,4} = (V_{DD} - V_{th})$$

$$V_{SD,3} = V_{SD,4} = (V_{DD} - V_{th})$$

$$I_D = I_{D,3} + I_{D,4} = \beta_p (V_{DD} - V_{th} - |V_{T0p}|)^2$$

$$V_{th} = \frac{V_{T0n} + 2\sqrt{\frac{\beta_p}{\beta_n}} (V_{DD} - |V_{T0p}|)}{1 + 2\sqrt{\frac{\beta_p}{\beta_n}}}$$

$$V_{th} = \frac{2V_{DD} - V_{T0}}{1 + 2\sqrt{\frac{\beta_p}{\beta_n}}}$$

# Single Input Switching

$$V_{GS,1} = V_{th} - V_{DS,2} \quad V_{GS,2} = V_{DD}$$

$$\begin{aligned} I_D &= \frac{\beta_n}{2} (V_{th} - V_{T0n} - V_{DS,2})^2 \\ &= \frac{\beta_n}{2} [2(V_{DD} - V_{T0n})V_{DS,2} - V_{DS,2}^2] \end{aligned}$$

$$V_{DS,2} = (V_{th} - V_{T0n}) - \sqrt{\frac{2I_D}{\beta_n}}$$

$$\frac{4I_D}{\beta_n} = 2(V_{DD} - V_{T0n})(V_{th} - V_{T0n}) + 2\sqrt{\frac{2I_D}{\beta_n}}(V_{th} - V_{DD}) - (V_{th} - V_{T0n})^2$$

$$\sqrt{\frac{2I_D}{\beta_n}} = \sqrt{\frac{\beta_p}{\beta_n}} (V_{DD} - |V_{T0p}| - V_{th})$$

$$\begin{aligned} & \left[ 1 + 2 \left( \sqrt{\frac{\beta_p}{\beta_n}} + \frac{\beta_p}{\beta_n} \right) \right] V_{th}^2 \\ & - \left\{ \left[ 4 \left( \frac{\beta_p}{\beta_n} \right) + 2 + 2\sqrt{\frac{\beta_p}{\beta_n}} \right] (V_{DD} - V_{T0}) + 2V_{T0} + 2V_{DD}\sqrt{\frac{\beta_p}{\beta_n}} \right\} V_{th} \\ & + 2 \left[ 2 \left( \frac{\beta_p}{\beta_n} \right) (V_{DD} - V_{T0})^2 + 2(V_{T0} + \sqrt{\frac{\beta_p}{\beta_n}}) (V_{DD} - V_{T0}) + V_{T0}^2 \right] = 0 \end{aligned}$$

$$V_{th} = (V_{DD} - 0.6V_{T0}) - \frac{1}{5} \sqrt{5V_{DD}^2 - 10V_{DD}V_{T0} + 4V_{T0}^2}$$