Angle Recoding 2. Wu
3. MVR

20180917 Mon

② MVR (Modified Vector Rotational)

two modifications

① repeatition of elementary angles

each micro-rotation of elementary angle
can be performed repeatedly

- more possible combinations
- smaller ξ_m

② confinement of total micro-rotation number

confine the iteration number
in the micro-rotation phase
to R_m ($R_m \ll W$)

The role of R_m is quite similar
to the number of non-zero digit
N_D in CSD recoding scheme

data : W-bit word length

the iteration number : N $\qquad N \leqslant W$

the restricted iteration number : R_m $\qquad R_m \ll W$

$i \in [0, R_m - 1]$ $\quad R_m < W$

the angle quantization error

$$\boxed{\xi_{m,MVR} \triangleq \theta - \sum_{i=0}^{R_m-1} \alpha(i)\, a(s(i))}$$

the rotational sequence

$s(i) \in \{0, 1, \cdots, W-1\}$ * repetition allowed

the micro-rotation angle
in the i-th iteration

the directional sequence

$\alpha(i) \in \{-1, 0, +1\}$

the direction of the i-th
micro-rotation of $a(s(i))$

$$\alpha(i)\, a(s(i)) = \tilde{\theta}(i)$$

the rotational sequence

$s(i) \in \{0, 1, \cdots, w-1\}$ [0, 3, 6, 7]

the directional sequence

$\alpha(i) \in \{-1, 0, +1\}$ [1, -1, -1, 1]

atan(2^0) - atan(2^{-3}) - atan(2^{-6}) + atan(2^{-7})

$$\alpha(i)\, a(s(i)) = \tilde{\theta}(j)$$

MVR-CORDIC Algorithm with $R_m = 4$	Greedy Algorithm	3	$\overline{\alpha} = [1\ \ -1\ \ -1\ \ -1]$ $\overline{s} = [0\ \ 3\ \ 6\ \ 7]$	$5.2891 * 10^{-4}$
	Semi-greedy Algorithm ($D = 2$)	4	$\overline{\alpha} = [1\ \ -1\ \ -1\ \ 1]$ $\overline{s} = [0\ \ 3\ \ 5\ \ 7]$	$5.2033 * 10^{-4}$
	TBS Algorithm	5	$\overline{\alpha} = [1\ \ 1\ \ -1\ \ -1]$ $\overline{s} = [1\ \ 2\ \ 4\ \ 7]$	$2.5911 * 10^{-4}$

```
>> s = [0, 3, 6, 7]
>> alpha = [1, -1, -1, -1]
>> sum(atan(2.^(-s) .* alpha))
ans =  0.63761
>>
```

```
>> s = [0, 3, 5, 7]
>> alpha = [1, -1, -1, 1]
>> sum(atan(2.^(-s) .* alpha))
ans =  0.63762
```

```
>> alpha = [1, 1, -1, -1]
>> s = [1, 2, 4, 7]
>> sum(atan(2.^(-s) .* alpha))
ans =  0.63840
```

0	7.85398163397448e-01
1	4.63647609000806e-01
2	2.44978663126864e-01
3	1.24354994546761e-01
4	6.24188099959574e-02
5	3.12398334302683e-02
6	1.56237286204768e-02
7	7.81234106010111e-03
8	3.90623013196697e-03
9	1.95312251647882e-03
10	9.76562189559319e-04
11	4.88281211194898e-04
12	2.44140620149362e-04
13	1.22070311893670e-04
14	6.10351561742088e-05
15	3.05175781155261e-05

$W=16$

0	7.85398163397448e-01	s(0)
1	4.63647609000806e-01	.
2	2.44978663126864e-01	.
3	1.24354994546761e-01	s(1)
4	6.24188099959574e-02	.
5	3.12398334302683e-02	.
6	1.56237286204768e-02	s(2)
7	7.81234106010111e-03	s(3)
8	3.90623013196697e-03	.
9	1.95312251647882e-03	.
10	9.76562189559319e-04	.
11	4.88281211194898e-04	.
12	2.44140620149362e-04	.
13	1.22070311893670e-04	.
14	6.10351561742088e-05	.
15	3.05175781155261e-05	.

0	7.85398163397448e-01	s(0)
3	1.24354994546761e-01	s(1)
6	1.56237286204768e-02	s(2)
7	7.81234106010111e-03	s(3)

$R_m = 4$

s(0)=0
s(1)=3
s(2)=6
s(3)=7

0	7.85398163397448e-01
1	4.63647609000806e-01
2	2.44978663126864e-01
3	1.24354994546761e-01
4	6.24188099959574e-02
5	3.12398334302683e-02
6	1.56237286204768e-02
7	7.81234106010111e-03
8	3.90623013196697e-03
9	1.95312251647882e-03
10	9.76562189559319e-04
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13	1.22070311893670e-04
14	6.10351561742088e-05
15	3.05175781155261e-05

AQ & MVR CORDIC

$$\xi_{m,MVR} \triangleq \theta - \left[\sum_{j=0}^{R_m-1} \alpha(j)\, a(s(j)) \right]$$

the rotational sequence $s(j)$

$$j = 0, 1, 2, \cdots, R_m - 1$$

$$\downarrow$$

$s(j) \in \{0, 1, \cdots, W-1\}$ rotational sequence

determines the micro-rotation angle $a(s(j))$ in the j-th iteration

the directional sequence $\alpha(j)$

$\alpha(j) \in \{-1, 0, +1\}$

controls the direction of the j-th micro-rotation of $a(s(j))$

$$\alpha(j)\, a(s(j)) = \tilde{\theta}(j)$$

$i =$	0,	1,	2,	3,	$\cdots$,	$W-1$		
$s(j) =$	0,	1,	2,	3,	$\cdots$,	$W-1$	rotational sequence	
$\alpha(j) =$	-1,	0,	0,	+1,	$\cdots$,	-1	directional sequence	
$j =$	0,	-,	-,	1,	$\cdots$,	R_m-1	effective iteration number	

$R_m \ll W$

i	j	$S(j)$	
0	0	$S(0) = 0$	
1			
2			
3			
4	1, 4	$S(1) = 4$, $S(4) = 4$	repetition allowed
5	2	$S(2) = 5$	
6			
7			
8	3	$S(3) = 8$	
9		$\boxed{\text{rotational sequence}}$	
10	effective iteration number		
11			
12			
13			
14			
$W-1 = 15$			

i	Conventional CORDIC	j	S(j)
0	S(0) = 0	0	S(0) = 0
1	S(1) = 1		
2	S(2) = 2		
3	S(3) ..3		
4	S(4) = 4	1	S(1) = 4
5	S(5) = 5	2	S(2) = 5
6	S(6) = 6		
7	S(7) = 7		
8	S(8) = 8	3	S(3) = 8
9	S(9) = 9		
10	S(10) = 10		
11	S(11) = 11		
12	S(12) = 12		
13	S(13) = 13		
14	S(14) = 14		
15	S(15) = 15		

effective iteration number

rotational sequence

W-1 =

sub-angle $\left(\alpha(j)\, a(s(j))\right) \sim \tilde{\theta}(j)$

$$\xi_{m,AR} = \theta - \left[\sum_{j=0}^{N'-1} \tilde{\theta}(j)\right], \quad \tilde{\theta}(j) = \tan^{-1}\left(\alpha(j)\cdot 2^{-s(j)}\right)$$

$$= \theta - \left[\sum_{j=0}^{N'-1} \tan^{-1}\left(\alpha(j)\cdot 2^{-s(j)}\right)\right]$$

$N' \triangleq \sum_{j=0}^{N-1} |\mu(j)|$ the effective iteration number

EAS formed by MVR-CORDIC
is the same as AR
also performs AQ

the EAS consists of all possible values of $\tilde{\theta}(j)$

the EAS S_1 in AR

$$S_1 = \left\{ \tan^{-1}\left(\alpha^* \cdot 2^{-s^*}\right) : \alpha^* \in \{-1, 0, +1\},\ s^* \in \{0, 1, \cdots, N-1\} \right\}$$

The major difference

1) the total number of sub-angles N_A

the total iteration number

in the micro-rotation phase

is kept fixed to a pre-defined value of R_m

$N_A = R_m$

2) the sub-angle θ_i corresponds to $\alpha(j)\, a(s(j))$

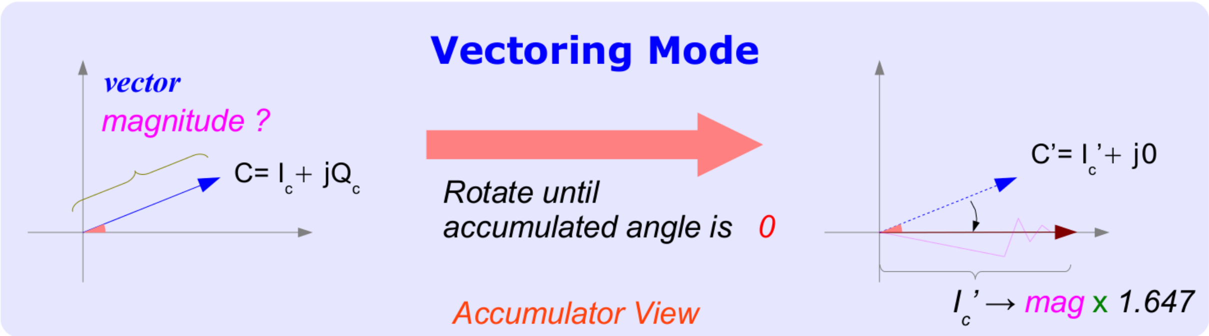
$$\theta_j = \alpha(j)\, a(s(j)) = \tilde{\theta}_j$$

MVR (Modified Vector Rotation)

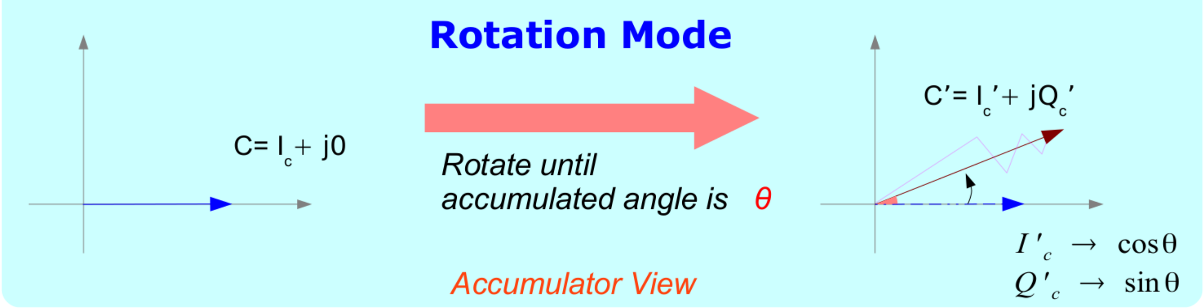
1) Repeat of Elementary Angles θ_i, θ_i

2) fixed total micro-rotation Number R_m

* Vector Rotation Mode
* and the rotation angles are known in advance



vector rotation mode



Modified Vector Rotational MVR CORDIC

- reduce the iteration number
- maintaining the SQNR performance
- modifying the basic microrotation procedure

Three Searching Algorithm

1. the selective prerotation
2. the selective scaling
3. iteration-tradeoff scheme

Optimization Problem

EAS point of view

Given θ, find the combination of R_m elementary angles from EAS S_1, such that the angle quantization error $|\xi_{m,MUR}|$ is minimized.

semi-greedy algorithm
trade offs between computational complexities and performance

key issue in the MVR-CORDIC
is to find the best sequences of
$s(i)$ and $\alpha(i)$ to minimize $|\xi_m|$
subject to the constraint that
the total iteration number is confined to R_m

1) Greedy Algorithm
2) Exhaustive Algorithm
3) Semigreedy Algorithm

data : W-bit word length

the iteration number : N $\quad N \leqslant W$

the restricted iteration number : R_m $\quad R_m \ll W$

Hu's greedy algorithm

$\theta(0) = \theta$, $\{\mu(i) = 0,\ 0 \leqslant i \leqslant N-1\}$, $k=0$.

repeat until $|\theta(k)| < a(N-1)$ Do

Choose i_k, $0 \leqslant i_k \leqslant N-1$

$$\big|\,|\theta(k)| - a(i_k)\big| = \min_{0 \leqslant i \leqslant N-1} \big|\,|\theta(k)| - a(i_k)\big|$$

$$\theta(k+1) = \theta(k) - \mu(i_k)\, a(i_k)$$

$$\mu(i_k) = \operatorname{Sign}(\theta(k))$$

$J(i) = |\theta(i) - \alpha(i)\, a(s(i))|$ is minimited

1) Greedy Algorithm

given θ, W, R_m

try to approach the target rotation angle, θ, step by step
in each step, decisions are made on $\alpha(i)$ and $s(i)$
by choosing the best combination of $\alpha(i)\, a(s(i))$
so as to minimize $|\xi_m|$

$\alpha(i)$ and $s(i)$ are determined such that
the error function $J(i) = |\theta(i) - \alpha(i)\, a(s(i))|$ is minimited

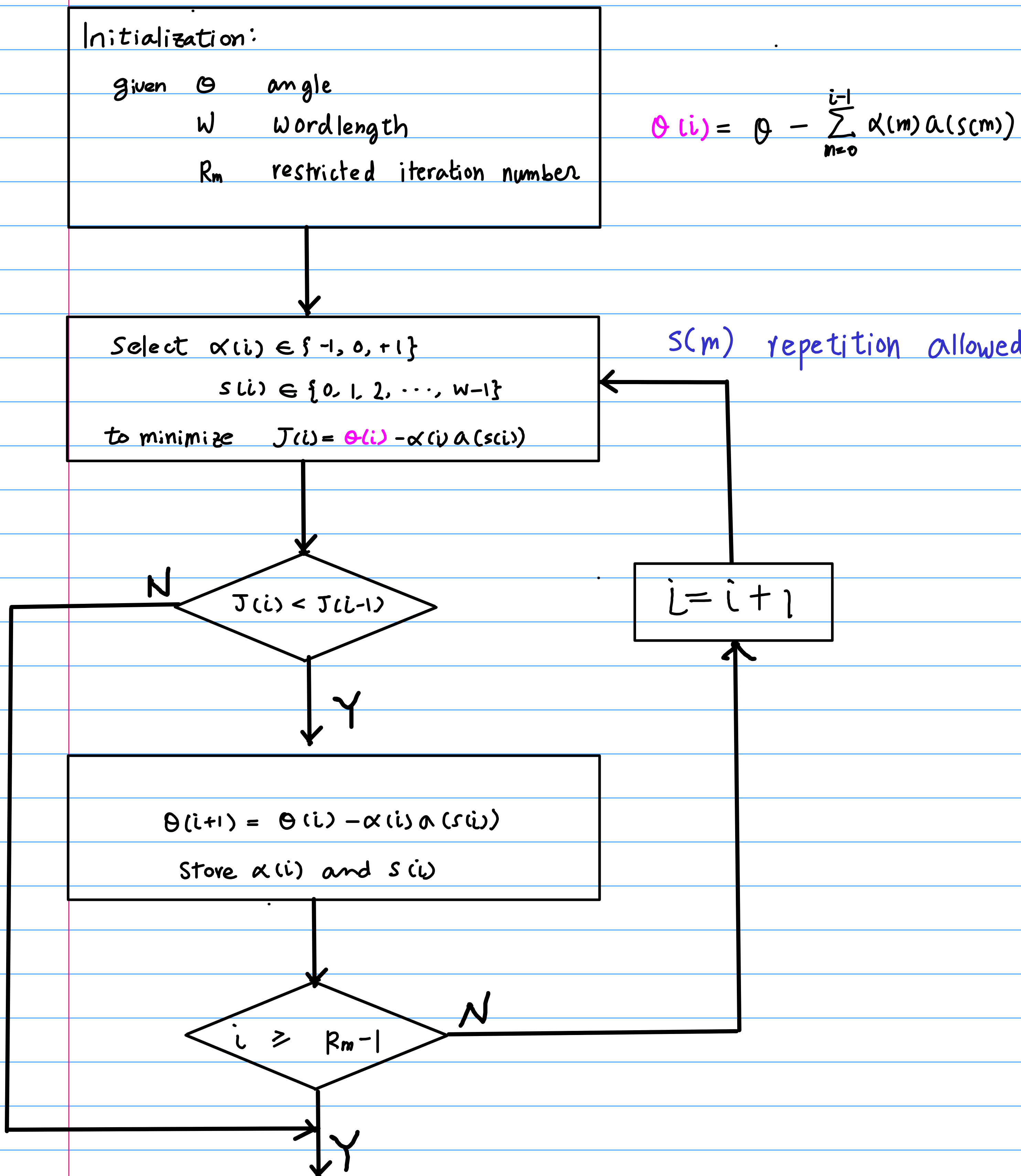
$\theta(i)$: the residue angle in the i-th step

$$\theta(i) = \theta - \sum_{m=0}^{i-1} \alpha(m)\, a(s(m))$$

the searching is terminated
- if no further improvements can be found
  $J(i) \geq J(i-1)$
- if the iteration i reaches $R_m - 1$

$\alpha(R_m-1)$ and $s(R_m-1)$ are determined
at the end of the searching

the greedy algorithm terminates
Only when the residue angle error
cannot be further reduced.


Initialization:
given θ angle
W wordlength
R_m restricted iteration number
$\theta(i) = \theta - \sum_{m=0}^{i-1} \alpha(m)\, a(s(m))$
Select $\alpha(i) \in \{-1, 0, +1\}$
$s(i) \in \{0, 1, 2, \cdots, W-1\}$
to minimize $J(i) = \theta(i) - \alpha(i)\, a(s(i))$
s(m) repetition allowed
$J(i) < J(i-1)$
N
Y
$i = i + 1$
$\theta(i+1) = \theta(i) - \alpha(i)\, a(s(i))$
Store $\alpha(i)$ and $s(i)$
$i \geq R_m - 1$
N
Y

2) Exhaustive Algorithm

search for the entire solution space

$$\underbrace{\{-1, 0, +1\}}_{3}^{\alpha(i)} \times \underbrace{\{s(0), s(1), \cdots, s(W-1)\}}_{W}^{a(s(i))} \times \underbrace{\{0, 1, \cdots, R_m - 1\}}_{R_m}^{i} \Rightarrow (3W)^{R_m}$$

all possible combinations of

$$\sum_{i=0}^{R_m-1} \alpha(i)\, a(s(i))$$

in a single step

decisions for $\alpha(i)$ and $s(i)$, $0 \leq i \leq R_m - 1$

by minimizing the error function

$$J = \left| \theta - \sum_{i=0}^{R_m-1} \alpha(i)\, a(s(i)) \right|$$

global optimal solution

Initialization:

given Θ, W, R_m

let $\Theta(0) = \Theta$,

$i = 0$

$J(-1) = \infty$

Select $\alpha(i) \in \{-1, 0, +1\}$

$s(i) \in \{0, 1, 2, \cdots, W-1\}$

for $0 \leq i \leq R_m - 1$

to minimize $J(i) = \Theta - \sum_{i=0}^{R_m-1} \alpha(i)\, a(s(i))$

Store $\alpha(i)$ and $s(i)$

for $0 \leq i \leq R_m - 1$

$(3 \cdot W) \cdot (3 \cdot W) \cdots (3 \cdot W)$

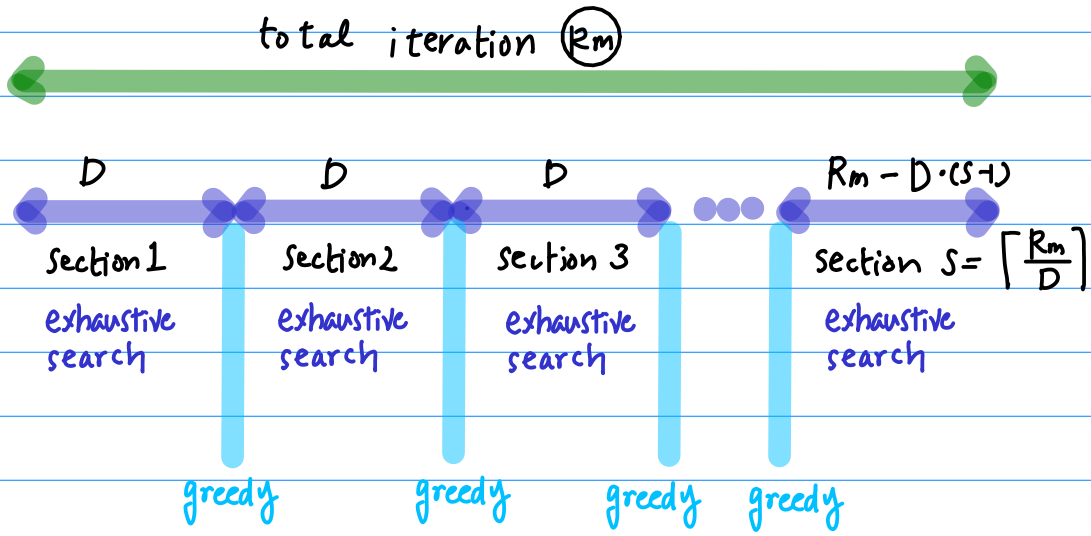
$= 3^{R_m} \cdot W^{R_m}$

3) Semi-greedy Algorithm

a combination of greedy and exhaustive algorithm

the search space of $\alpha(i)$ and $s(i)$ for $0 \le i \le R_m - 1$
are divided into several sections
with D iterations as a segment
↓ block length ↓ block

the segmentation scheme



in the i-th block

decision of $\alpha(k)$ and $S(k)$ for $iD \leq k \leq (i+1)D-1$

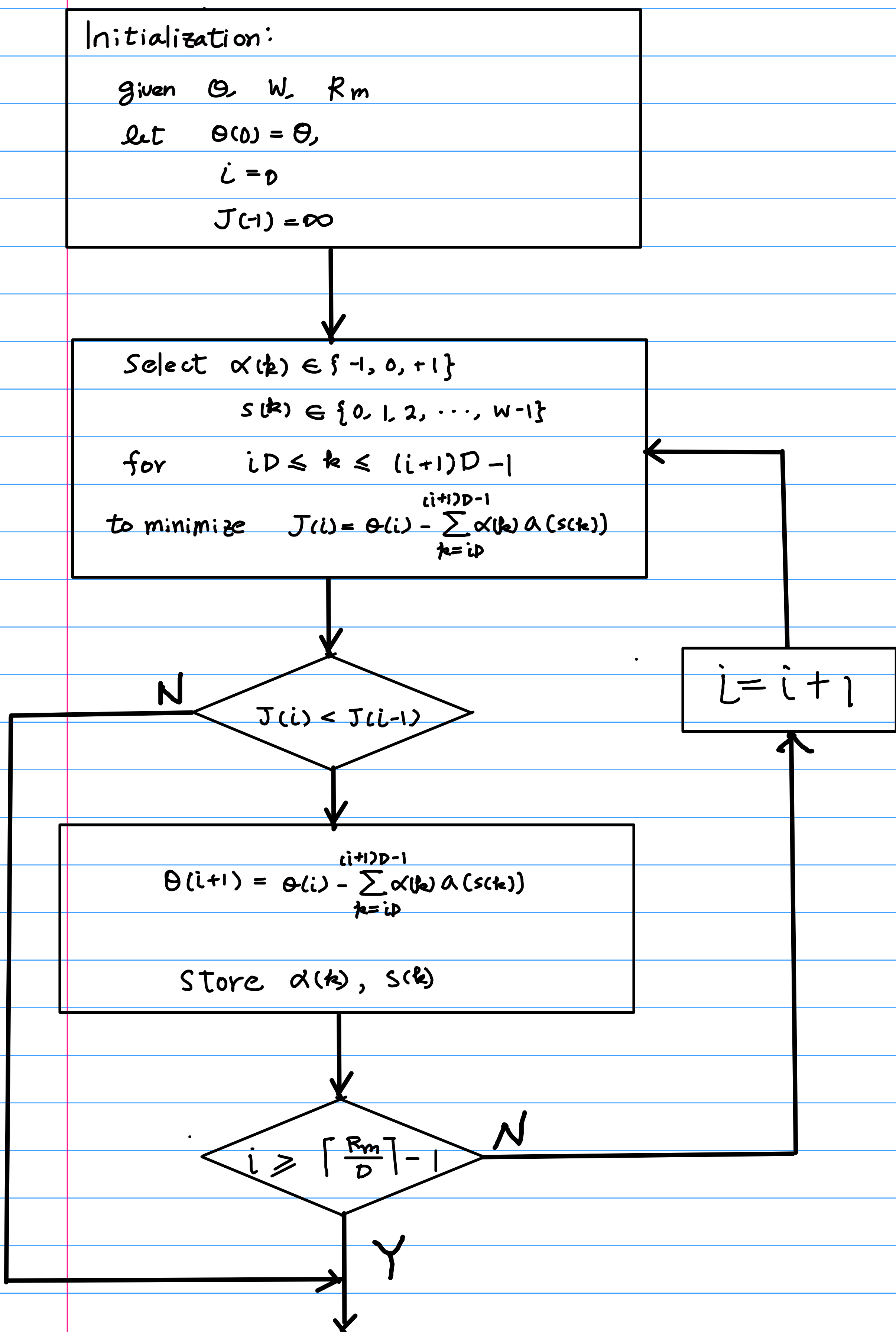
minimizes $J = \left| \theta(i) - \sum_{k=iD}^{(i+1)D-1} \alpha(k)\, a(S(k)) \right|$

where $\theta(i) = \theta - \sum_{m=0}^{i-1} \left[\sum_{k=mD}^{(m+1)D-1} \alpha(k)\, a(S(k)) \right]$

the residue angle in the i-th step

$S = \left\lceil \frac{R_m}{D} \right\rceil$

$$\theta(i) = \theta - \left[\sum_{k=0 \cdot D}^{1 \cdot D-1} \alpha(k)\, a(S(k)) + \sum_{k=1D}^{2 \cdot D-1} \alpha(k)\, a(S(k)) + \cdots + \sum_{k=2D}^{S \cdot D-1} \alpha(k)\, a(S(k)) \right]$$


Initialization:
given Θ, W, R_m
let $\Theta(0) = \Theta$, $i = 0$, $J(-1) = \infty$
Select $\alpha(k) \in \{-1, 0, +1\}$, $s(k) \in \{0, 1, 2, \cdots, W-1\}$
for $iD \leq k \leq (i+1)D-1$
to minimize $J(i) = \Theta(i) - \sum_{k=iD}^{(i+1)D-1} \alpha(k)\, a(s(k))$
$J(i) < J(i-1)$
N
$\Theta(i+1) = \Theta(i) - \sum_{k=iD}^{(i+1)D-1} \alpha(k)\, a(s(k))$
Store $\alpha(k)$, $s(k)$
$i \geq \lceil \frac{R_m}{D} \rceil - 1$
N
Y
$i = i+1$