Statistical Inference Overview

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Outline

Based on

"Understanding Statistics in the Behavioral Sciences" R. R. Pagano

Inferential Statistics (1)

- descriptive statistics summarize
  the characteristics of a data set
- inferential statistics help you come to conclusions
  and make predictions based on your data.

https://www.scribbr.com/statistics/inferential-statistics/

Inferential Statistics (2)

- When you have collected data from a sample,
  you can use inferential statistics
  to understand the larger population
  from which the sample is taken.

https://www.scribbr.com/statistics/inferential-statistics/

Inferential Statistics (3)

- Inferential statistics have two main uses:
  - making estimates about populations
    (eg., the mean SAT score of all 11th graders in the US).
  - testing hypotheses to draw conclusions about populations
    (eg., the relationship between SAT scores and family income).

https://www.scribbr.com/statistics/inferential-statistics/

Descriptive v.s. Inferential Statistics

- Descriptive statistics allow you
  to describe a data set
- Inferential statistics allow you
  to make inferences based on a data set.

https://www.scribbr.com/statistics/inferential-statistics/

Descriptive Statistics

- Using descriptive statistics, you can report characteristics of your data:
  - The distribution concerns the frequency of each value.
  - The central tendency concerns the averages of the values.
  - The variability concerns how spread out the values are.
- In descriptive statistics, there is no uncertainty - the statistics precisely describe the data that you collected. If you collect data from an entire population, you can directly compare these descriptive statistics to those from other populations.

https://www.scribbr.com/statistics/inferential-statistics/

Populations and Samples (1)

- population: everything in the group that we want to learn about.
- sample: a part of the population.

- Examples of populations and a sample from those populations:

Population	Sample
All of the people in Germany	500 Germans
All of the customers of Netflix	300 Netflix customers
Every car manufacturer	Tesla, Toyota, BMW, Ford

https://www.w3schools.com/statistics/statistics_statistical_inference.php

Populations and Samples (2)

- For good statistical analysis,
  the sample needs to be as *similar*
  as possible to the population.
- If they are *similar enough*, we say
  that the sample is representative of the population.
- The sample is used to make conclusions
  about the whole population.

https://www.w3schools.com/statistics/statistics_statistical_inference.php

Populations and Samples (3)

- If the sample is not *similar enough* to the whole population, the conclusions could be useless.

- Many words have specific meanings in statistics.
- The word population normally refers to a group of people.
- In statistics, it is any specific group that we are interested in learning about.

https://www.w3schools.com/statistics/statistics_statistical_inference.php

Statistical Inference

- Using data analysis and statistics to make conclusions about a population is called statistical inference.
- The main types of statistical inference are:
  - Estimation
  - Hypothesis testing

https://www.w3schools.com/statistics/statistics_statistical_inference.php

Estimation (1)

- Statistics from a sample are used
  to estimate population parameters.
- *The most likely* value is called a point estimate.
- There is always uncertainty when estimating.

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Estimation (2)

- The uncertainty is often expressed as confidence intervals defined by a *likely* lowest and highest value for the parameter.
- An example could be a confidence interval for the number of bicycles a Dutch person owns:
  - The average number of bikes a Dutch person owns is between 3.5 and 6.

https://www.w3schools.com/statistics/statistics_statistical_inference.php

Hypothesis Testing (1)

- a method to check if a claim about a population is true.
- checks how likely it is that a hypothesis is true is based on the sample data.
- there are different types of hypothesis testing.

https://www.w3schools.com/statistics/statistics_statistical_inference.php

Hypothesis Testing (2)

- the steps of the test depends on:
  - Type of data (categorical or numerical)
  - If you are looking at:
    - a single group
    - comparing one group to another
    - comparing the same group before and after a change

https://www.w3schools.com/statistics/statistics_statistical_inference.php

Hypothesis Testing (3)

- a hypothesis is a claim about a population parameter
- a hypothesis test is a formal procedure
  to check if a hypothesis is true or not.
- examples of claims that can be checked:
  - the average height of people in Denmark is more than 170 cm.
  - the share of left handed people in Australia is not 10%.
  - The average income of dentists is less the average income of lawyers.

https://www.w3schools.com/statistics/statistics_hypothesis_testing.php

Steps of Hypothesis Testing

1. *State* your research hypothesis as a null hypothesis (H_0) and alternate hypothesis and (H_a or H_1).
2. *Collect data* in a way designed to test the hypothesis.
3. *Perform* an appropriate statistical test
4. *Decide* whether to reject or fail to reject your null hypothesis
5. *Present* the findings in your results and discussion section.

https://www.scribbr.com/statistics/hypothesis-testing/

Statistical tests (1)

- Statistical tests are used in hypothesis testing.
  They can be used to:
  - determine whether a predictor variable has
    a statistically significant relationship
    with an outcome variable.
  - estimate the difference between two or more groups.

https://www.scribbr.com/statistics/statistical-tests/

Statistical tests (2)

- Statistical tests assume a null hypothesis of no relationship or no difference between groups.
  Then they determine whether the observed data fall outside of the range of values predicted by the null hypothesis.

https://www.scribbr.com/statistics/statistical-tests/

Statistical tests (3)

- Statistical tests work by calculating a test statistic
  - a number that describes how much
    the relationship between variables in your test differs
    from the null hypothesis of no relationship
- then calculates a p value (probability value).
  the p-value estimates how likely it is
  that you would see the difference described by the test statistic
  if the null hypothesis of no relationship were true.

https://www.scribbr.com/statistics/statistical-tests/

Statistical tests (4)

- If the value of the test statistic is more extreme than the statistic calculated from the null hypothesis, then you can infer a statistically significant relationship between the predictor and outcome variables
- If the value of the test statistic is less extreme than the one calculated from the null hypothesis, then you can infer no statistically significant relationship between the predictor and outcome variables.

https://www.scribbr.com/statistics/statistical-tests/

Statistical tests (5)

- You can perform statistical tests on data
  that have been collected in a statistically valid manner
  - either through an experiment,
  - or through observations made using probability sampling methods.
- For a statistical test to be valid,
  your sample size needs to be large
  enough to approximate the true distribution
  of the population being studied.
- To determine which statistical test to use,
  you need to know:
  - whether your data meets certain assumptions.
  - the types of variables that you're dealing with.

https://www.scribbr.com/statistics/statistical-tests/

Statistical assumptions (1)

- Statistical tests make some common assumptions about the data they are testing:
  - Independence of observations (a.k.a. no autocorrelation):
  - Homogeneity of variance:
  - Normality of data:

https://www.scribbr.com/statistics/statistical-tests/

Statistical assumptions (2)

- Independence of observations (a.k.a. no autocorrelation):
  - the observations / variables you include in your test are not related
  - for example, multiple measurements of a single test subject are not independent, while measurements of multiple different test subjects are independent

https://www.scribbr.com/statistics/statistical-tests/

Statistical assumptions (3)

- Homogeneity of variance:
  - the variance within each group being compared is similar among all groups.
  - If one group has much more variation than others, it will limit the test's effectiveness.

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Statistical assumptions (4)

- Normality of data:
  - the data follows a normal distribution (a.k.a. a bell curve).
  - This assumption applies only to quantitative data.

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Types of data (1)

- Data is a specific measurement of a variable
  it is the value you record in your data sheet.
  Data is generally divided into two categories:
  1. Quantitative data represents amounts
  2. Categorical data represents groupings

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Types of data (2)

- a variable that contains quantitative data is a quantitative variable;
- a variable that contains categorical data is a categorical variable.
- Each of these types of variables can be broken down into further types.

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Quantitative variables

- when you collect quantitative data,
  the numbers you record represent real amounts
  that can be added, subtracted, divided, etc.
- there are two types of quantitative variables:
  - discrete
  - continuous

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Catagorical variables

- Categorical variables represent groupings of some kind.
- sometimes recorded as numbers,
  but the numbers represent categories
  rather than actual amounts of things.
- There are three types of categorical variables:
  - binary
  - nominal
  - ordinal variables.

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Independant and dependant variables (1)

- experiments are usually designed to find out what effect one variable has on another
  - eg. the effect of salt addition on plant growth.
- manipulate the independent variable (the one you think might be the cause) and then
- measure the *dependent variable * (the one you think might be the effect) to find out what this effect might be.

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Independant and dependant variables (2)

- You will probably also have variables
  that you hold constant (control variables)
  in order to focus on your experimental treatment.

https://www.scribbr.com/statistics/statistical-tests/

The Null and Alternative Hypothesis (1)

- hypothesis testing is based on
  making two different claims about a population parameter.
- the claims.
  - the null hypothesis (H_0)
  - the alternative hypothesis (H_1)

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The Null and Alternative Hypothesis (2)

- the two claims needs to be mutually exclusive, meaning only one of them can be true.
- te *lternative hypothesis*is typically what we are trying to prove.
- for example, we want to check the following claim:
    - "The average height of people in Denmark is more than 170 cm."

https://www.w3schools.com/statistics/statistics_hypothesis_testing.php

Summary (1) comparing means

tests	
• one-sample test	comparing sample mean, population mean
• two-sample test	comparing two independent sample means
• paired test	comparing two related sample means

tests	test conditions
• t-test	1. when the population variance is *known*
	2. when the sample size is *large*
• z-test	1, when the population variance is *unknown*
	2. the sample size is *small*

https://www.qualitygurus.com/common-types-of-hypothesis-tests/

Summary (2) comparing means

one sample z-test	sample mean, population mean known population var / large sample size
one sample t-test	sample mean, population mean unknown population var / small sample size
two sample z-test	two *independent* sample means known population var / large sample size
two sample t-test	two *independent* sample means unknown population var / small sample size
paired t-test	two *related* sample means unknown population var / small sample size

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Summary (3) comparing proportions

one sample propotion test	sample proportion, population proportion when $np \geq 10$ and $n(1-p) \geq 10$
two sample proportion test	two *independent* sample proportions when $np \geq 10$ and $n(1-p) \geq 10$

test conditions
the normal approximation is used when both $np \geq 10$ and $n(1-p) \geq 10$ (data should have at least 10 "successes" and at least 10 "failures")

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Summary (4)

compare variances between	
sample variance, known population variance	Chi-square test
two independent sample variances	F-test
observed frequencies, expected frequencies	goodness of fit test
observed frequencies, expected frequencies	contingency tables
means of three or more independent samples	ANOVA (Analysis of Varianc

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Tests for Comparing Means (1)

- One-sample z-test:
  - used to compare the mean of a sample to a known population mean
  - used when the population variance is known, or the sample size is *large* ($n > 30$).
- Two-sample z-test:
  - used to compare the means of two independent samples.
  - used when the population variances are known, or the sample sizes are *large* ($n > 30$).

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Tests for Comparing Means (2)

- One-sample t-test:
  - used to compare the mean of a sample to a known population mean.
  - used when the population variance is unknown, and the sample size is *small* ($n < 30$).
- Two-sample t-test:
  - used to compare the means of two independent samples.
  - used when the population variances are unknown, and the sample sizes are *small* ($n < 30$).

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Tests for Comparing Means (3)

- Paired t-test:
  - used to compare the means of two *related* samples, such as the before and after measurements of the same group of subjects.
  - used when the population variances are unknown, and the sample size is *small* ($n < 30$).

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Tests for Comparing Proportions (1)

- Let us consider the parameter p of the population proportion
  - eg) we might want to know the proportion of males within a total population of adults when we conduct a survey.
- A test of proportion will assess whether or not a sample from a population represents the true proportion of the entire population

https://online.stat.psu.edu/statprogram/reviews/statistical-concepts/proportions

Examples of proportions

- an example
  - newborn babies are more likely to be boys than girls.
  - a random sample found 13,173 boys were born among 25,468 newborn children
  - the sample proportion of boys was 0.5172 ($= \frac{13173}{25468}$)
  - is this sample evidence that the birth of boys is more common than the birth of girls in the entire population? ($0.5172 > 0.4828$)

https://online.stat.psu.edu/statprogram/reviews/statistical-concepts/proportions

Tests for Comparing Proportions (2-1)

- examples involved testing
  whether a single population proportion p equals some value .
- different examples of testing
  whether one population proportion equals
  a second population proportion

https://online.stat.psu.edu/stat415/lesson/9/9.4

Tests for Comparing Proportions (2-2)

- Additionally, most of our examples thus far have involved
  - left-tailed tests in which the alternative hypothesis involved
  - right-tailed tests in which the alternative hypothesis involved
- Here, let's consider an example that tests
  the equality of two proportions
  against the alternative that they are not equal

https://online.stat.psu.edu/stat415/lesson/9/9.4

Tests for Comparing Proportions (2-3)

- Time magazine reported the result of a telephone poll of 800 adult Americans.
- the question posed of the Americans who were surveyed was: "Should the federal tax on cigarettes be raised to pay for health care reform?"
- the results of the survey were:

Non-smokers	Smokers
$n_1 = 605$	$n_2 = 195$
$y_1 = 351$ said yes	$y_2 = 41$ said yes
$\hat{p}_1 = \frac{351}{605} = 0.58$	$\hat{p}_2 = \frac{41}{195} = 0.21$

https://online.stat.psu.edu/stat415/lesson/9/9.4

Tests for Comparing Proportions (2-4)

- If $p_1 =$ the proportion of the non-smoker population who reply "yes"
- and $p_2 =$ the proportion of the smoker population who reply "yes,"
- then we are interested in testing the null hypothesis:
  $H_0 : p_1 = p_2$
  against the alternative hypothesis:
  $H_A : p_1 \neq p_2$
- Before we can actually conduct the hypothesis test, we'll have to derive the appropriate test statistic.

https://online.stat.psu.edu/stat415/lesson/9/9.4

Tests for Comparing Proportions (2-5)

- The overall sample proportion is:
  $\hat{p} = \frac{41+351}{195+605} = \frac{392}{800} = 0.49$
- that implies then that the test statistic for testing:
  $H_0 : p_1 = p_2$ versus $H_A : p_1 \neq p_2$
  is:
  $Z = \frac{(0.58-0.21)-0}{\sqrt{0.49(0.51)(\frac{1}{195}+\frac{1}{605})}} = 8.9$

https://online.stat.psu.edu/stat415/lesson/9/9.4

Tests for Comparing Proportions (3)

- One-sample proportion test :
  - used to compare the proportion of a sample
    to a known population proportion.
  - the normal approximation is used
    when both $np \geq 10$ and $n(1-p) \geq 10$
    (data should have at least 10 "successes" and at least 10 "failures")
    (in some books, it is 5)

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Tests for Comparing Proportions (4)

- Two-sample proportion test :
  - used to compare the proportions of two independent samples.
  - the normal approximation is used
    when both $np \geq 10$ and $n(1-p) \geq 10$
    (data should have at least 10 "successes" and at least 10 "failures")
    (in some books, it is 5)

https://www.qualitygurus.com/common-types-of-hypothesis-tests/

Tests for Comparing Proportions (5)

- Time magazine reported the result of a telephone poll of 800 adult Americans.
- The question posed of the Americans who were surveyed was: "Should the federal tax on cigarettes be raised to pay for health care reform?"
  The results of the survey were:

https://online.stat.psu.edu/stat415/lesson/9/9.4

Tests for Comparing Proportions (6)

Non-Smokers	Smokers
$n_1 = 605$	$n_2 = 195$
$y_1 = 351$ said "yes"	$y_2 = 41$ said "yes"
$\hat{p}_1 = \frac{351}{605} = 0.58$	$\hat{p}_2 = \frac{41}{195} = 0.21$

- Is there sufficient evidence at the , say, to conclude that the two populations - smokers and non-smokers - differ significantly with respect to their opinions?

https://online.stat.psu.edu/stat415/lesson/9/9.4

Tests for Comparing Proportions (7)

- Errr.... that Z-value is off the charts, so to speak. Let's go through the formalities anyway making the decision first using the rejection region approach, and then using the P-value approach. Putting half of the rejection region in each tail, we have:
- That is, we reject the null hypothesis if or if . We clearly reject , since 8.99 falls in the "red zone," that is, 8.99 is (much) greater than 1.96. There is sufficient evidence at the 0.05 level to conclude that the two populations differ with respect to their opinions concerning imposing a federal tax to help pay for health care reform.

https://online.stat.psu.edu/stat415/lesson/9/9.4

Tests for Comparing Proportions (8)

- That is, the P-value is less than 0.0001. Because, we reject the null hypothesis. Again, there is sufficient evidence at the 0.05 level to conclude that the two populations differ with respect to their opinions concerning imposing a federal tax to help pay for health care reform.
- Thankfully, as should always be the case, the two approaches. . . . the critical value approach and the P-value approach. . . lead to the same conclusion

https://online.stat.psu.edu/stat415/lesson/9/9.4

Tests for Comparing Variance

- Chi-square test for variance :
    - used to compare the variance of a sample to a known population variance
- F-test for variance :
    - used to compare the variances of two *independent* samples

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Other Common Tests (1)

- Goodness of fit test :
- used to determine whether a sample fits a *specific* distribution.
- used to compare the observed frequencies of a *categorical variable* to the expected frequencies under a *particular* distribution.

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Other Common Tests (2)

- Testing for independence of two attributes (Contingency Tables) :
- used to determine whether there is a relationship between two *categorical variables*.
- often used in the form of a chi-square test, which compares the observed frequencies in a contingency table to the expected frequencies under the assumption of independence.

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Other Common Tests (3)

- ANOVA (Analysis of Variance) :
- used to compare the means of three or more independent samples.
- used to determine whether there is a significant difference between the means of the groups.

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One-sample z-test

- used to test a hypothesis about the *population* mean
- based on the assumption that the sample is drawn from a normally distributed population.
  - the null hypothesis
    the *population* mean is equal to a specific value
  - the alternative hypothesis
    the *population* mean is not equal to that value

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Two-sample z-test

- based on the assumption that both samples are drawn from normally distributed populations with equal variances.
- the two-sample z-test requires that the population standard deviations be known or that the sample sizes be *large* (30 or more),
  - the null hypothesis
    the means of the two samples are equal
  - the alternative hypothesis
    the means are not equal

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One-sample t-test

- used to test a hypothesis about the *population* mean
- based on the assumption that the sample is drawn from a normally distributed population
  - the null hypothesis
    the *population* mean is equal to a specific value
  - the alternative hypothesis
    the *population* mean is not equal to that value

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Two-sample t-test

- based on the assumption that the samples are drawn from populations with normal distributions.
- the two-sample t-test
  that the population standard deviations need not be known or
  that the sample sizes need not be *large* (30 or more),
  - the null hypothesis
    the means of the two samples are equal
  - the alternative hypothesis
    the means are not equal

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Paired t-test

- used to test a hypothesis about the difference between the means of the two samples
- based on the assumption that the differences between the pairs are normally distributed
- In a dependent two-sample t-test (a paired t-test), the samples in the two groups being compared are *related* in some way.
  - the null hypothesis
    there is no difference between the means of the two samples
  - the alternative hypothesis
    there is a difference between the means

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Two proportions z-test

- used to test a hypothesis about the difference between the proportions of the two samples and
- based on the assumption that the samples are drawn from populations with a normal distribution
  - the null hypothesis :
    there is no difference between the proportions of the two samples
  - the alternative hypothesis :
    there is a difference between the proportion

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OOO