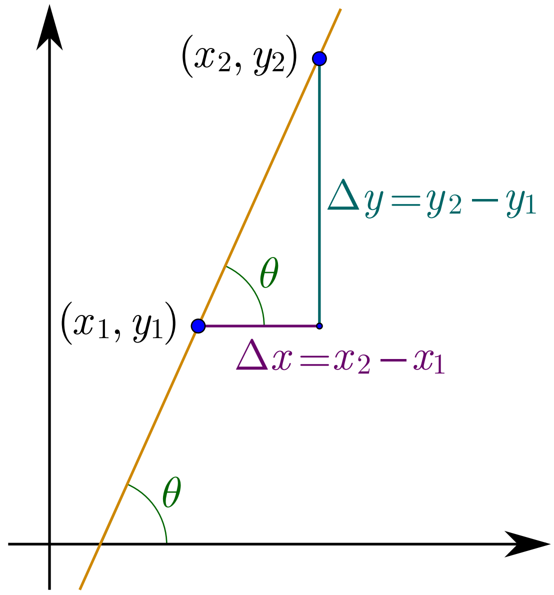
Slope

Contents

Chapter 1

Slope

This article is about the mathematical term. For other uses, see Slope (disambiguation).
For the grade (incline or gradient or pitch or slope) of any physical feature, see Grade (slope).
In mathematics, the **slope** or **gradient** of a line is a number that describes both the *direction* and the *steepness* of the line.[1] Slope is often denoted by the letter m; there is no clear answer to the question why the letter m is used for slope, but it might be from the "m for multiple" in the equation of a straight line "y = mx + b" or "y = mx + c".[2]



Slope: $m = \frac{\Delta y}{\Delta x} = \tan(\theta)$

- The *direction* of a line is either increasing, decreasing, horizontal or vertical.
    - A line is **increasing** if it goes **up** from left to right. The slope is **positive**, i.e. $m > 0$.
    - A line is **decreasing** if it goes **down** from left to right. The slope is **negative**, i.e. $m < 0$.
    - If a line is horizontal the slope is **zero**. This is a constant function.
    - If a line is vertical the slope is *undefined* (see below).
- The *steepness*, incline, or grade of a line is measured by the absolute value of the slope. A slope with a greater absolute value indicates a steeper line

Slope is calculated by finding the ratio of the "vertical change" to the "horizontal change" between (any) two distinct points on a line. Sometimes the ratio is expressed as a quotient ("rise over run"), giving the same number for every two distinct points on the same line. A line that is decreasing has a negative "rise". The line may be practical - as set by a road surveyor, or in a diagram that models a road or a roof either as a description or as a plan.

The rise of a road between two points is the difference between the altitude of the road at those two points, say y_1 and y_2, or in other words, the rise is $(y_2 - y_1) = \Delta y$. For relatively short distances - where the earth's curvature may be neglected, the run is the difference in distance from a fixed point measured along a level, horizontal line, or in other words, the run is $(x_2 - x_1) = \Delta x$. Here the slope of the road between the two points is simply described as the ratio of the altitude change to the horizontal distance between any two points on the line.

In mathematical language, the slope m of the line is

$$m = \frac{y_2 - y_1}{x_2 - x_1}.$$

The concept of slope applies directly to grades or gradients in geography and civil engineering. Through trigonometry, the grade m of a road is related to its angle of incline θ by the tangent function

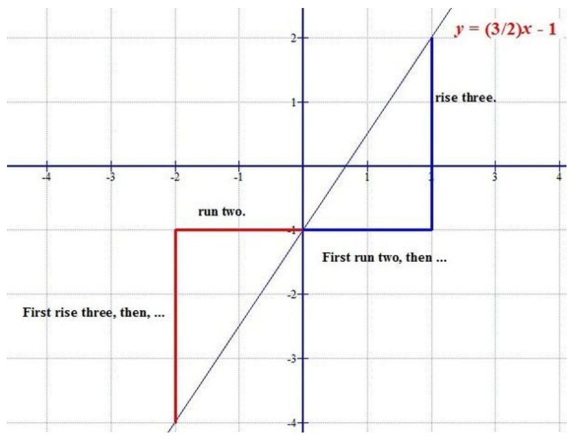
$$m = \tan(\theta)$$

Thus, a 45° rising line has a slope of +1 and a 45° falling line has a slope of −1.

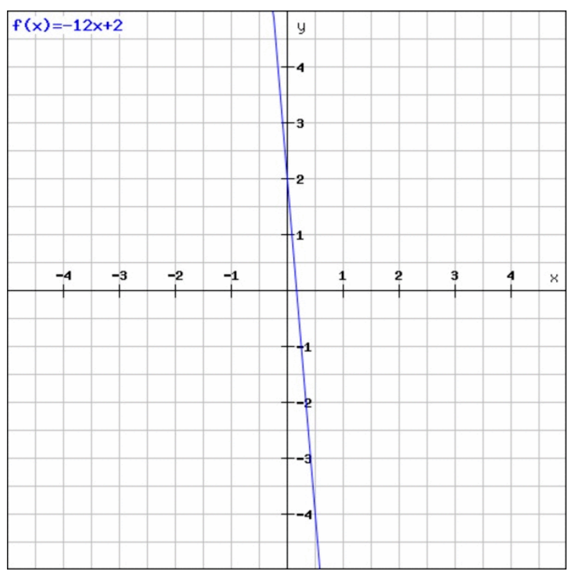
As a generalization of this practical description, the mathematics of differential calculus defines the slope of a curve at a point as the slope of the tangent line at that point. When the curve given by a series of points in a diagram or in a list of the coordinates of points, the slope may be calculated not at a point but between any two given points. When the curve is given as a continuous function, perhaps as an algebraic formula, then the differential calculus provides rules giving a formula for the slope of the curve at any point in the middle of the curve.

This generalization of the concept of slope allows very complex constructions to be planned and built that go well beyond static structures that are either horizontals or verticals, but can change in time, move in curves, and change depending on the rate of change of other factors. Thereby, the simple idea of slope becomes one of the main basis of the modern world in terms of both technology and the built environment.

1.1 Definition



Slope illustrated for y = *(3/2)*x *− 1. Click on to enlarge*



Slope of a line in coordinates system, from f(x)=−12x+2 to f(x)=12x+2

The slope of a line in the plane containing the *x* and *y* axes is generally represented by the letter *m*, and is defined as the change in the *y* coordinate divided by the corresponding change in the *x* coordinate, between two distinct points on the line. This is described by the following equation:

$$m = \frac{\Delta y}{\Delta x} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{\text{rise}}{\text{run}}.$$

(The Greek letter *delta*, Δ, is commonly used in mathematics to mean "difference" or "change".)

Given two points (x_1,y_1) and (x_2,y_2), the change in *x* from one to the other is $x_2 - x_1$ (*run*), while the change in *y* is $y_2 - y_1$ (*rise*). Substituting both quantities into the above equation generates the formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}.$$

The formula fails for a vertical line, parallel to the *y* axis (see Division by zero), where the slope can be taken as infinite, so the slope of a vertical line is considered undefined.

1.1.1 Examples

Suppose a line runs through two points: *P* = (1, 2) and *Q* = (13, 8). By dividing the difference in *y*-coordinates by the difference in *x*-coordinates, one can obtain the slope of the line:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8-2}{13-1} = \frac{6}{12} = \frac{1}{2}.$$

Since the slope is positive, the direction of the line is increasing. Since |m|<1, the incline is not very steep (incline <30°).

As another example, consider a line which runs through the points (4, 15) and (3, 21). Then, the slope of the line is

$$m = \frac{21-15}{3-4} = \frac{6}{-1} = -6.$$

Since the slope is negative, the direction of the line is decreasing. Since |m|>1, this decline is fairly steep (decline >45°).

1.2 Algebra and geometry

- If *y* is a linear function of *x*, then the coefficient of *x* is the slope of the line created by plotting the function. Therefore, if the equation of the line is given in the form

$$y = mx + k$$

then m is the slope. This form of a line's equation is called the *slope-intercept form*, because k can be interpreted as the y-intercept of the line, that is, the y-coordinate where the line intersects the y-axis.

- If the slope m of a line and a point (x_1,y_1) on the line are both known, then the equation of the line can be found using the point-slope formula:

  $$y - y_1 = m(x - x_1).$$

- The slope of the line defined by the linear equation

  $$ax + by + c = 0$$

  is

  $$-\frac{a}{b}$$

- Two lines are parallel if and only if they are not the same line (coincident) and either their slopes are equal or they both are vertical and therefore both have undefined slopes. Two lines are perpendicular if the product of their slopes is −1 or one has a slope of 0 (a horizontal line) and the other has an undefined slope (a vertical line).

- The angle θ between −90° and 90° that a line makes with the x-axis is related to the slope m as follows:

  $$m = \tan(\theta)$$

  and

  $\theta = \arctan(m)$ (this is the inverse function of tangent; see trigonometry).

1.2.1 Examples

For example, consider a line running through the points (2,8) and (3,20). This line has a slope, m, of

$$\frac{(20-8)}{(3-2)} = 12.$$

One can then write the line's equation, in point-slope form:

$$y - 8 = 12(x - 2) = 12x - 24$$

or:

$$y = 12x - 16.$$

The angle θ between −90° and 90° that this line makes with the x axis is

$$\theta = \arctan(12) \approx 85.2^\circ .$$

Consider the two lines: $y = -3x + 1$ and $y = -3x - 2$. Both lines have slope $m = -3$. They are not the same line. So they are parallel lines.

Consider the two lines $y = -3x + 1$ and $y = x/3 - 2$. The slope of the first line is $m_1 = -3$. The slope of the second line is $m_2 = 1/3$. The product of these two slopes is −1. So these two lines are perpendicular.

1.3 Statistics

In statistical mathematics, the gradient of the line of best fit for a given distribution of data which is linear, numerical, and free of outliers, is usually written as $b = \frac{rs_y}{s_x}$, where b is defined as the gradient (in statistics), r is Pearson's correlation coefficient, s_y is the standard deviation of the y-values and s_x is the standard deviation of the x-values.[3]

In this equation $y = a + bx$ for the least-squares regression line, b is the slope and a is the intercept.

1.4 Slope of a road or railway

Main articles: Grade (slope), Grade separation

There are two common ways to describe the steepness of a road or railroad. One is by the angle between 0° and 90° (in degrees), and the other is by the slope in a percentage. See also steep grade railway and rack railway.

The formulae for converting a slope given as a percentage into an angle in degrees and vice versa are:

$\text{angle} = \arctan\left(\frac{\text{slope}}{100\%}\right)$, (this is the inverse function of tangent; see trigonometry)

and

$$\text{slope} = 100\% \cdot \tan(\text{angle}),$$

where *angle* is in degrees and the trigonometric functions operate in degrees. For example, a slope of 100% or 1000‰ is an angle of 45°.

A third way is to give one unit of rise in say 10, 20, 50 or 100 horizontal units, e.g. 1:10. 1:20, 1:50 or 1:100 (or *"1 in 10"*, *"1 in 20"* etc.) Note that 1:10 is steeper than 1:20.

For example, steepness of 20% means 1:5 or an incline with angle 11,3°.

Roads and railways have both longitudinal slopes and cross slopes.

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Slope warning sign in the Netherlands

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Slope warning sign in Poland

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A 1371-meter distance of a railroad with a 20‰ slope. Czech Republic

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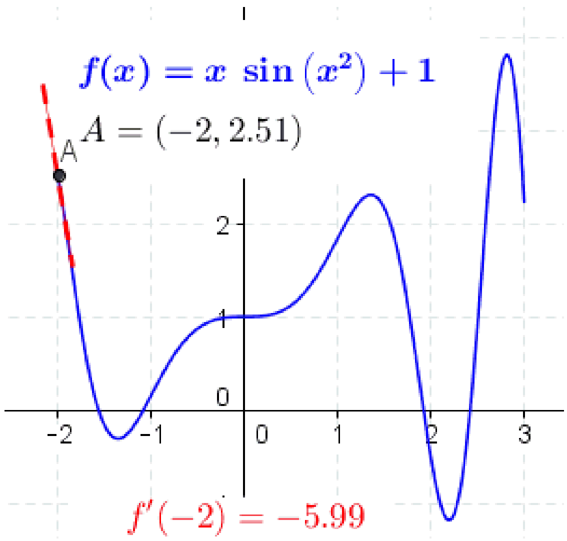
Steam-age railway gradient post indicating a slope in both directions at Meols railway station, United Kingdom

1.5 Calculus

The concept of a slope is central to differential calculus. For non-linear functions, the rate of change varies along the curve. The derivative of the function at a point is the slope of the line tangent to the curve at the point, and is thus equal to the rate of change of the function at that point.

If we let Δx and Δy be the distances (along the x and y axes, respectively) between two points on a curve, then the slope given by the above definition,

$$m = \frac{\Delta y}{\Delta x}$$



At each point, the derivative is the slope of a line that is tangent to the curve at that point. Note: the derivative at the point A is positive where green and dash-dot, negative where red and dashed, and zero where black and solid.

is the slope of a secant line to the curve. For a line, the secant between any two points is the line itself, but this is not the case for any other type of curve.

For example, the slope of the secant intersecting $y = x^2$ at (0,0) and (3,9) is 3. (The slope of the tangent at x = $^3/_2$ is also 3—*a* consequence of the mean value theorem.)

By moving the two points closer together so that Δy and Δx decrease, the secant line more closely approximates a tangent line to the curve, and as such the slope of the secant approaches that of the tangent. Using differential calculus, we can determine the limit, or the value that $\Delta y/\Delta x$ approaches as Δy and Δx get closer to zero; it follows that this limit is the exact slope of the tangent. If y is dependent on x, then it is sufficient to take the limit where only Δx approaches zero. Therefore, the slope of the tangent is the limit of $\Delta y/\Delta x$ as Δx approaches zero, or dy/dx. We call this limit the derivative.

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

Its value at a point on the function gives us the slope of the tangent at that point. For example, let $y=x^2$. A point on this function is (−2,4). The derivative of this function is $^{dy}/_{d}x=2x$. So the slope of the line tangent to y at (−2,4) is 2·(−2) = −4. The equation of this tangent line is: y−4=(−4)(x-(−2)) or y = −4x - 4.

1.6 Other generalizations

The concept of slope can be generalized to functions of more than one variable and is more often referred to as gradient.

1.7 See also

- Euclidean distance
- Inclined plane
- Linear function
- Slope definitions
- Theil–Sen estimator, a line with the median slope among a set of sample points

1.8 References

[1] Clapham, C.; Nicholson, J. (2009). "Oxford Concise Dictionary of Mathematics, Gradient" (PDF). Addison-Wesley. p. 348. Retrieved 1 September 2013.

[2] Weisstein, Eric W. "Slope". MathWorld--A Wolfram Web Resource. Retrieved 30 October 2016.

[3] *Further Mathematics Units 3&4 VCE (Revised)*. Cambridge Senior Mathematics. 2016. ISBN 9781316616222 – via Physical Copy.

1.9 External links

- "Slope of a Line (Coordinate Geometry)". Math Open Reference. 2009. Retrieved 30 October 2016. interactive

Chapter 2

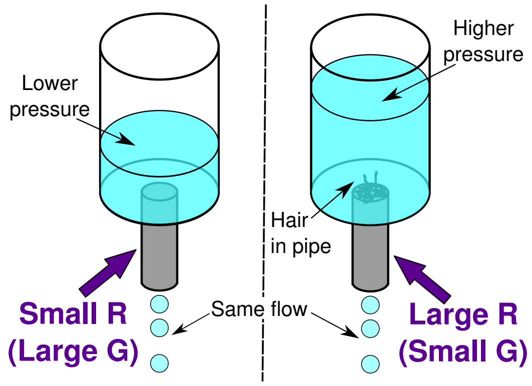
Electrical resistance and conductance

"Resistive" redirects here. For the term used when referring to touchscreens, see resistive touchscreen.

The **electrical resistance** of an electrical conductor is a measure of the difficulty to pass an electric current through that conductor. The inverse quantity is **electrical conductance**, and is the ease with which an electric current passes. Electrical resistance shares some conceptual parallels with the notion of mechanical friction. The SI unit of electrical resistance is the ohm (Ω), while electrical conductance is measured in siemens (S).

An object of uniform cross section has a resistance proportional to its resistivity and length and inversely proportional to its cross-sectional area. All materials show some resistance, except for superconductors, which have a resistance of zero.

The resistance (R) of an object is defined as the ratio of voltage across it (V) to current through it (I), while the conductance (G) is the inverse:



The hydraulic analogy compares electric current flowing through circuits to water flowing through pipes. When a pipe (left) is filled with hair (right), it takes a larger pressure to achieve the same flow of water. Pushing electric current through a large resistance is like pushing water through a pipe clogged with hair: It requires a larger push (electromotive force) to drive the same flow (electric current).

$$R = \frac{V}{I}, \qquad G = \frac{I}{V} = \frac{1}{R}$$

For a wide variety of materials and conditions, V and I are directly proportional to each other, and therefore R and G are constant (although they can depend on other factors like temperature or strain). This proportionality is called Ohm's law, and materials that satisfy it are called *ohmic* materials.

In other cases, such as a diode or battery, V and I are *not* directly proportional. The ratio V/I is sometimes still useful, and is referred to as a "chordal resistance" or "static resistance",[1][2] since it corresponds to the inverse slope of a chord between the origin and an *I*–*V* curve. In other situations, the derivative $\frac{dV}{dI}$ may be most useful; this is called the "differential resistance".

2.1 Introduction

In the hydraulic analogy, current flowing through a wire (or resistor) is like water flowing through a pipe, and the voltage drop across the wire is like the pressure drop that pushes water through the pipe. Conductance is proportional to how much flow occurs for a given pressure, and resistance is proportional to how much pressure is required to achieve a given flow. (Conductance and resistance are reciprocals.)

The voltage *drop* (i.e., difference between voltages on one side of the resistor and the other), not the voltage itself, provides the driving force pushing current through a resistor. In hydraulics, it is similar: The pressure *difference* between two sides of a pipe, not the pressure itself, determines the flow through it. For example, there may be a large water pressure above the pipe, which tries to push water down through the pipe. But there may be an equally large water pressure below the pipe, which tries to push water back up through the pipe. If these pressures are equal, no water flows. (In the image at right, the water pressure below the pipe is zero.)

The resistance and conductance of a wire, resistor, or other element is mostly determined by two properties:

- geometry (shape), and
- material

Geometry is important because it is more difficult to push water through a long, narrow pipe than a wide, short pipe. In the same way, a long, thin copper wire has higher resistance (lower conductance) than a short, thick copper wire.

Materials are important as well. A pipe filled with hair restricts the flow of water more than a clean pipe of the same shape and size. Similarly, electrons can flow freely and easily through a copper wire, but cannot flow as easily through a steel wire of the same shape and size, and they essentially cannot flow at all through an insulator like rubber, regardless of its shape. The difference between copper, steel, and rubber is related to their microscopic structure and electron configuration, and is quantified by a property called resistivity.

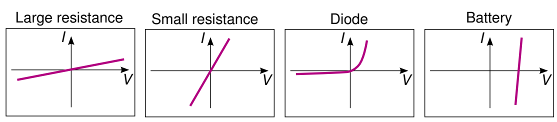
In addition to geometry and material, there are various other factors that influence resistance and conductance, such as temperature; see below.

2.2 Conductors and resistors

A 6.5 MΩ resistor, as identified by its electronic color code (blue–green–black-yellow-red). An ohmmeter could be used to verify this value.

Substances in which electricity can flow are called conductors. A piece of conducting material of a particular resistance meant for use in a circuit is called a resistor. Conductors are made of high-conductivity materials such as metals, in particular copper and aluminium. Resistors, on the other hand, are made of a wide variety of materials depending on factors such as the desired resistance, amount of energy that it needs to dissipate, precision, and costs.

2.3 Ohm's law



The current-voltage characteristics of four devices: Two resistors, a diode, and a battery. The horizontal axis is voltage drop, the vertical axis is current. Ohm's law is satisfied when the graph is a straight line through the origin. Therefore, the two resistors are ohmic, *but the diode and battery are not.*

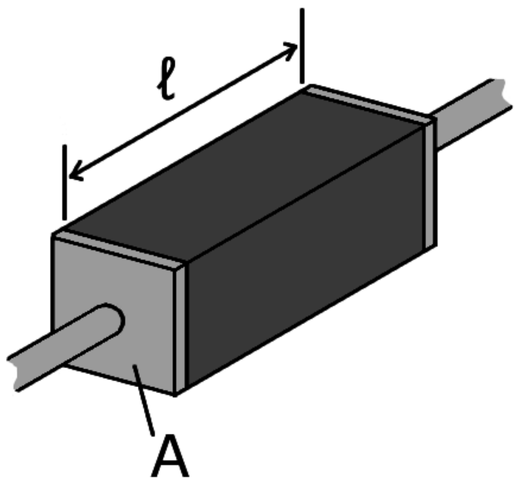
Main article: Ohm's law

Ohm's law is an empirical law relating the voltage V across an element to the current I through it:

$$I \propto V$$

(I is directly proportional to V). This law is not always true: For example, it is false for diodes, batteries, and other devices whose conductance is not constant. However, it is true to a very good approximation for wires and resistors (assuming that other conditions, including temperature, are held constant). Materials or objects where Ohm's law is true are called *ohmic*, whereas objects that do not obey Ohm's law are *non-ohmic*.

2.4 Relation to resistivity and conductivity



A piece of resistive material with electrical contacts on both ends.

Main article: Electrical resistivity and conductivity

The resistance of a given object depends primarily on two factors: What material it is made of, and its shape. For a given material, the resistance is inversely proportional to the cross-sectional area; for example, a thick copper wire has lower resistance than an otherwise-identical thin copper wire. Also, for a given material, the resistance is proportional to the length; for example, a long copper wire has higher resistance than an otherwise-identical short copper wire. The resistance R and conductance G of a conductor of uniform cross section, therefore, can be computed as

$$R = \rho \frac{\ell}{A},$$
$$G = \sigma \frac{A}{\ell}.$$

where ℓ is the length of the conductor, measured in metres [m], A is the cross-sectional area of the conductor measured in square metres [m^2], σ (sigma) is the electrical conductivity measured in siemens per meter ($S \cdot m^{-1}$), and ρ (rho) is the electrical resistivity (also called *specific electrical resistance*) of the material, measured in ohm-metres (Ω·m). The resistivity and conductivity are proportionality constants, and therefore depend only on the material the wire is made of, not the geometry of the wire. Resistivity and conductivity are reciprocals: $\rho = 1/\sigma$. Resistivity is a measure of the material's ability to oppose electric current.

This formula is not exact, as it assumes the current density is totally uniform in the conductor, which is not always true in practical situations. However, this formula still provides a good approximation for long thin conductors such as wires.

Another situation for which this formula is not exact is with alternating current (AC), because the skin effect inhibits current flow near the center of the conductor. For this reason, the *geometrical* cross-section is different from the *effective* cross-section in which current actually flows, so resistance is higher than expected. Similarly, if two conductors near each other carry AC current, their resistances increase due to the proximity effect. At commercial power frequency, these effects are significant for large conductors carrying large currents, such as busbars in an electrical substation,[3] or large power cables carrying more than a few hundred amperes.

2.4.1 What determines resistivity?

Main article: Electrical resistivity and conductivity

The resistivity of different materials varies by an enormous amount: For example, the conductivity of teflon is about 10^{30} times lower than the conductivity of copper. Why is there such a difference? Loosely speaking, a metal has large numbers of "delocalized" electrons that are not stuck in any one place, but free to move across large distances, whereas in an insulator (like teflon), each electron is tightly bound to a single molecule, and a great force is required to pull it away. Semiconductors lie between these two extremes. More details can be found in the article: Electrical resistivity and conductivity. For the case of electrolyte solutions, see the article: Conductivity (electrolytic).

Resistivity varies with temperature. In semiconductors, resistivity also changes when exposed to light. See below.

2.5 Measuring resistance

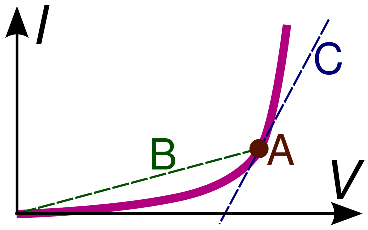
Main article: ohmmeter

An instrument for measuring resistance is called an ohmmeter. Simple ohmmeters cannot measure low resistances accurately because the resistance of their measuring leads causes a voltage drop that interferes with the measurement, so more accurate devices use four-terminal sensing.

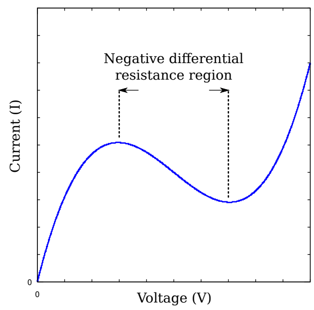
2.6 Typical resistances

See also: Electrical resistivities of the elements (data page) and Electrical resistivity and conductivity

2.7 Static and differential resistance



The IV curve of a non-ohmic device (purple). The **static resistance** at point *A* is the inverse slope of line *B* through the origin. The **differential resistance** at *A* is the inverse slope of tangent line *C*.



The IV curve of a component with negative differential resistance, an unusual phenomenon where the IV curve is non-monotonic. See also: Small-signal model

Many electrical elements, such as diodes and batteries do *not* satisfy Ohm's law. These are called *non-ohmic* or *non-linear*, and their *I–V* curves are *not* straight lines through the origin.

Resistance and conductance can still be defined for non-ohmic elements. However, unlike ohmic resistance, non-linear resistance is not constant but varies with the voltage or current through the device; i.e., its operating point. There are two types of resistance:[1][2]

- **Static resistance** (also called *chordal* or *DC resistance*) - This corresponds to the usual definition of resistance; the voltage divided by the current

$R_{\text{static}} = \frac{V}{I}$.

It is the slope of the line (chord) from the origin through the point on the curve. Static resistance determines the power dissipation in an electrical component. Points on the *IV* curve located in the 2nd or 4th quadrants, for which the slope of the chordal line is negative, have *negative static resistance*. Passive devices, which have no source of energy, cannot have negative static resistance. However active devices such as transistors or op-amps can synthesize negative static resistance with feedback, and it is used in some circuits such as gyrators.

- **Differential resistance** (also called *dynamic*, *incremental* or *small signal resistance*) - Differential resistance is the derivative of the voltage with respect to the current; the slope of the *IV* curve at a point

  $R_{\text{diff}} = \frac{dV}{dI}$.

  If the *IV* curve is nonmonotonic (with peaks and troughs), the curve has a negative slope in some regions—so in these regions the device has *negative differential resistance*. Devices with negative differential resistance can amplify a signal applied to them, and are used to make amplifiers and oscillators. These include tunnel diodes, Gunn diodes, IMPATT diodes, magnetron tubes, and unijunction transistors.

2.8 AC circuits

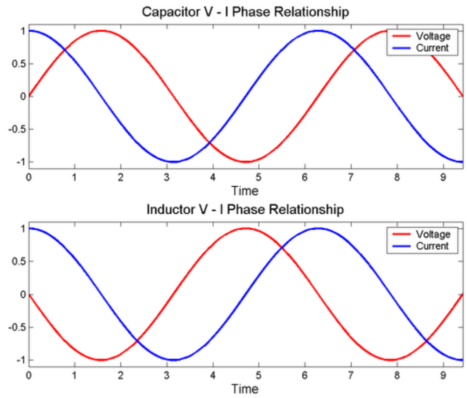
2.8.1 Impedance and admittance

Main articles: Electrical impedance and Admittance

When an alternating current flows through a circuit, the relation between current and voltage across a circuit element is characterized not only by the ratio of their magnitudes, but also the difference in their phases. For example, in an ideal resistor, the moment when the voltage reaches its maximum, the current also reaches its maximum (current and voltage are oscillating in phase). But for a capacitor or inductor, the maximum current flow occurs as the voltage passes through zero and vice versa (current and voltage are oscillating 90° out of phase, see image at right). Complex numbers are used to keep track of both the phase and magnitude of current and voltage:

$$V(t) = \text{Re}(V_0 e^{j\omega t}), \quad I(t) = \text{Re}(I_0 e^{j\omega t}), \quad Z = \frac{V_0}{I_0}, \quad Y = \frac{I_0}{V_0}$$

where:



The voltage (red) and current (blue) versus time (horizontal axis) for a capacitor (top) and inductor (bottom). Since the amplitude of the current and voltage sinusoids are the same, the absolute value of impedance is 1 for both the capacitor and the inductor (in whatever units the graph is using). On the other hand, the phase difference between current and voltage is −90° for the capacitor; therefore, the complex phase of the impedance of the capacitor is −90°. Similarly, the phase difference between current and voltage is +90° for the inductor; therefore, the complex phase of the impedance of the inductor is +90°.

- t is time,
- $V(t)$ and $I(t)$ are, respectively, voltage and current as a function of time,
- V_0, I_0, Z, and Y are complex numbers,
- Z is called impedance,
- Y is called admittance,
- Re indicates real part,
- ω is the angular frequency of the AC current,
- $j = \sqrt{-1}$ is the imaginary unit.

The impedance and admittance may be expressed as complex numbers that can be broken into real and imaginary parts:

$$Z = R + jX, \quad Y = G + jB$$

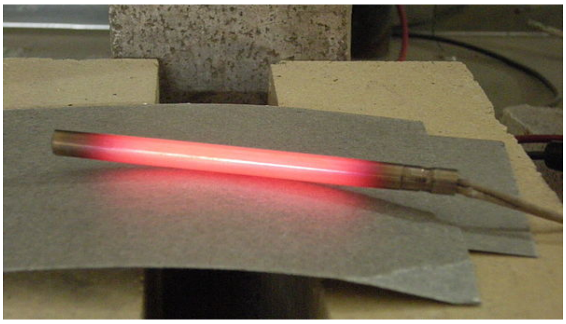
where R and G are resistance and conductance respectively, X is reactance, and B is susceptance. For ideal resistors, Z and Y reduce to R and G respectively, but for AC networks containing capacitors and inductors, X and B are nonzero.

$Z = 1/Y$ for AC circuits, just as $R = 1/G$ for DC circuits.

2.8.2 Frequency dependence of resistance

Another complication of AC circuits is that the resistance and conductance can be frequency-dependent. One reason, mentioned above is the skin effect (and the related proximity effect). Another reason is that the resistivity itself may depend on frequency (see Drude model, deep-level traps, resonant frequency, Kramers–Kronig relations, etc.)

2.9 Energy dissipation and Joule heating



Running current through a material with high resistance creates heat, in a phenomenon called Joule heating. In this picture, a cartridge heater, warmed by Joule heating, is glowing red hot.

Main article: Joule heating

Resistors (and other elements with resistance) oppose the flow of electric current; therefore, electrical energy is required to push current through the resistance. This electrical energy is dissipated, heating the resistor in the process. This is called *Joule heating* (after James Prescott Joule), also called *ohmic heating* or *resistive heating*.

The dissipation of electrical energy is often undesired, particularly in the case of transmission losses in power lines. High voltage transmission helps reduce the losses by reducing the current for a given power.

On the other hand, Joule heating is sometimes useful, for example in electric stoves and other electric heaters (also called *resistive heaters*). As another example, incandescent lamps rely on Joule heating: the filament is heated to such a high temperature that it glows "white hot" with thermal radiation (also called incandescence).

The formula for Joule heating is:

$$P = I^2 R$$

where P is the power (energy per unit time) converted from electrical energy to thermal energy, R is the resistance, and I is the current through the resistor.

2.10 Dependence of resistance on other conditions

2.10.1 Temperature dependence

Main article: Electrical resistivity and conductivity § Temperature dependence

Near room temperature, the resistivity of metals typically increases as temperature is increased, while the resistivity of semiconductors typically decreases as temperature is increased. The resistivity of insulators and electrolytes may increase or decrease depending on the system. For the detailed behavior and explanation, see Electrical resistivity and conductivity.

As a consequence, the resistance of wires, resistors, and other components often change with temperature. This effect may be undesired, causing an electronic circuit to malfunction at extreme temperatures. In some cases, however, the effect is put to good use. When temperature-dependent resistance of a component is used purposefully, the component is called a resistance thermometer or thermistor. (A resistance thermometer is made of metal, usually platinum, while a thermistor is made of ceramic or polymer.)

Resistance thermometers and thermistors are generally used in two ways. First, they can be used as thermometers: By measuring the resistance, the temperature of the environment can be inferred. Second, they can be used in conjunction with Joule heating (also called self-heating): If a large current is running through the resistor, the resistor's temperature rises and therefore its resistance changes. Therefore, these components can be used in a circuit-protection role similar to fuses, or for feedback in circuits, or for many other purposes. In general, self-heating can turn a resistor into a nonlinear and hysteretic circuit element. For more details see Thermistor#Self-heating effects.

If the temperature T does not vary too much, a linear approximation is typically used:

$$R(T) = R_0[1 + \alpha(T - T_0)]$$

where α is called the *temperature coefficient of resistance*, T_0 is a fixed reference temperature (usually room temperature), and R_0 is the resistance at temperature T_0 . The parameter α is an empirical parameter fitted from measurement data. Because the linear approximation is only an approximation, α is different for different reference temperatures. For this reason it is usual to specify the temperature that α was measured at with a suffix, such as α_{15} , and the relationship only holds in a range of temperatures around the reference.[9]

The temperature coefficient α is typically $+3\times10^{-3}$ K^{-1} to $+6\times10^{-3}$ K^{-1} for metals near room temperature. It is usually negative for semiconductors and insulators, with highly variable magnitude.[10]

2.10.2 Strain dependence

Main article: Strain gauge

Just as the resistance of a conductor depends upon temperature, the resistance of a conductor depends upon strain. By placing a conductor under tension (a form of stress that leads to strain in the form of stretching of the conductor), the length of the section of conductor under tension increases and its cross-sectional area decreases. Both these effects contribute to increasing the resistance of the strained section of conductor. Under compression (strain in the opposite direction), the resistance of the strained section of conductor decreases. See the discussion on strain gauges for details about devices constructed to take advantage of this effect.

2.10.3 Light illumination dependence

Main articles: Photoresistor and Photoconductivity

Some resistors, particularly those made from semiconductors, exhibit *photoconductivity*, meaning that their resistance changes when light is shining on them. Therefore, they are called *photoresistors* (or *light dependent resistors*). These are a common type of light detector.

2.11 Superconductivity

Main article: Superconductivity

Superconductors are materials that have exactly zero resistance and infinite conductance, because they can have V=0 and I≠0. This also means there is no joule heating, or in other words no dissipation of electrical energy. Therefore, if superconductive wire is made into a closed loop, current flows around the loop forever. Superconductors require cooling to temperatures near 4 K with liquid helium for most metallic superconductors like NbSn alloys, or cooling to temperatures near 77K with liquid nitrogen for the expensive, brittle and delicate ceramic high temperature superconductors. Nevertheless, there are many technological applications of superconductivity, including superconducting magnets.

2.12 See also

- Electrical measurements
- Resistor
- Electrical conduction for more information about the physical mechanisms for conduction in materials.
- Voltage divider
- Voltage drop
- Thermal resistance
- Sheet resistance
- SI electromagnetism units
- Quantum Hall effect, a standard for high-accuracy resistance measurements.
- Conductance quantum and its reciprocal, the Von Klitzing constant (under Von Klitzing)
- Series and parallel circuits
- Johnson–Nyquist noise
- Letter and digit code for resistance values

2.13 References

[1] Forbes T. Brown (2006). *Engineering System Dynamics*. CRC Press. p. 43. ISBN 978-0-8493-9648-9.

[2] Kenneth L. Kaiser (2004). *Electromagnetic Compatibility Handbook*. CRC Press. pp. 13–52. ISBN 978-0-8493-2087-3.

[3] Fink and Beaty, *Standard Handbook for Electrical Engineers 11th Edition*, page 17-19

[4] The resistivity of copper is about 1.7×10^{-8} Ωm. See .

[5] *Electric power substations engineering* by John Douglas McDonald, p 18-37, google books link

[6] For a fresh Energizer E91 AA alkaline battery, the internal resistance varies from 0.9 Ω at −40 °C, to 0.1 Ω at +40 °C.

[7] A 60 W light bulb in the USA (120 V mains electricity) draws RMS current 60 W/120 V=500 mA, so its resistance is 120 V/500 mA=240 Ω. The resistance of a 60 W light bulb in Europe (230 V mains) is 900 Ω. The resistance of a filament is temperature-dependent; these values are for when the filament is already heated up and the light is already glowing.

[8] 100,000 Ω for dry skin contact, 1000 Ω for wet or broken skin contact. High voltage breaks down the skin, lowering resistance to 500 Ω. Other factors and conditions are relevant as well. For more details, see the electric shock article, and: "Publication No. 98-131: Worker Deaths by Electrocution" (PDF). National Institute for Occupational Safety and Health. Retrieved 2014-11-02.

[9] Ward, MR, *Electrical Engineering Science*, pp36–40, McGraw-Hill, 1971.

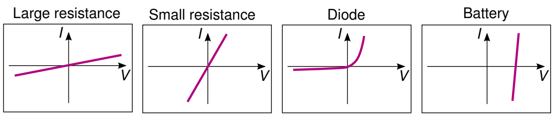
[10] See Electrical resistivity and conductivity for a table. The temperature coefficient of resistivity is similar but not identical to the temperature coefficient of resistance. The small difference is due to thermal expansion changing the dimensions of the resistor.

2.14 External links

- *Clemson Vehicular Electronics Laboratory: Resistance Calculator*
- Electron Conductance Models Using Maximal Entropy Random Walks Wolfram Demonstrantions Project

Chapter 3

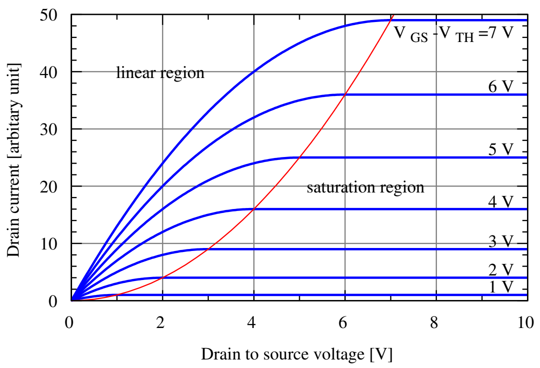
Current–voltage characteristic



The current–voltage characteristics of four devices: a resistor with large resistance, a resistor with small resistance, a P–N junction diode, and a battery with nonzero internal resistance. The horizontal axis represents the voltage drop, the vertical axis the current. All four plots use the passive sign convention.

A **current–voltage characteristic** or **I–V curve** (current–voltage curve) is a relationship, typically represented as a chart or graph, between the electric current through a circuit, device, or material, and the corresponding voltage, or potential difference across it.

3.1 In electronics



MOSFET drain current vs. drain-to-source voltage for several values of the overdrive voltage, $V_{GS} - V_{th}$ *; the boundary between* ***linear (Ohmic)*** *and* ***saturation (active)*** *modes is indicated by the upward curving parabola.*

In electronics, the relationship between the direct current (DC) through an electronic device and the DC voltage across its terminals is called a current–voltage characteristic of the device. Electronic engineers use these charts to determine basic parameters of a device and to model its behavior in an electrical circuit. These characteristics are also known as IV curves, referring to the standard symbols for current and voltage.

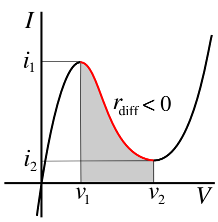
In electronic components with more than two terminals, such as vacuum tubes and transistors, the current-voltage relationship at one pair of terminals may depend on the current or voltage on a third terminal. This is usually displayed on a more complex current–voltage graph with multiple curves, each one representing the current-voltage relationship at a different value of current or voltage on the third terminal.[1]

For example the diagram at right shows a family of IV curves for a MOSFET as a function of drain voltage with overvoltage (*VGS* − *Vth*) as a parameter.

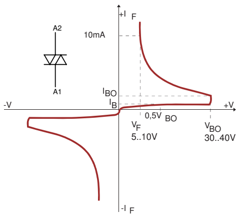
The simplest IV characteristic involves a resistor, which according to Ohm's Law exhibits a linear relationship between the applied voltage and the resulting electric current. However, even in this case, environmental factors such as temperature or material characteristics of the resistor can produce a non-linear curve.

The transconductance and Early voltage of a transistor are examples of parameters traditionally measured with the assistance of an I–V chart, or laboratory equipment that traces the charts in real time on an oscilloscope.

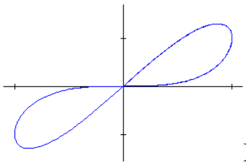
3.1.1 Special I-V curves characteristics

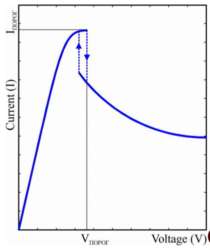
- IV curve may be non-monotonic: for example a tunnel diode has a negative resistance region.
- IV curve isn't necessarily a mathematical function: for example a DIAC or a spark gap starts to conduct electrical current only after its breakover voltage is exceeded
- IV curve may be directional or may have hysteresis: for example Memristor and Gunn diode
- 

IV curve similar to a tunnel diode characteristic curve. It has negative resistance in the shaded voltage region, between v_1 and v_2

- 

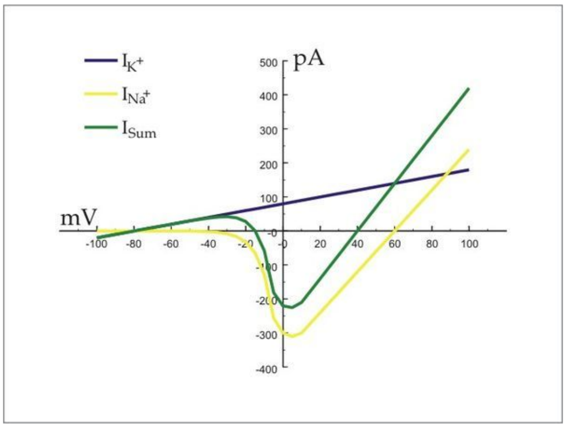
DIAC IV curve. VBO is the *breakover voltage*.

- 
Memristor IV curve, showing a pinched hysteresis

- 

Gunn diode IV curve, showing negative differential resistance with hysteresis (notice arrows)

3.2 In electrophysiology



An approximation of the potassium and sodium ion components of a so-called "whole cell" I–V curve of a neuron.

While V–I curves are applicable to any electrical system, they find wide use in the field of biological electricity, particularly in the sub-field of electrophysiology. In this case, the voltage refers to the voltage across a biological membrane, a membrane potential, and the current is the flow of charged ions through channels in this membrane. The current is determined by the conductances of these channels.

In the case of ionic current across biological membranes, currents are measured from inside to outside. That is, positive currents, known as "outward current", corresponding to positively charged ions crossing a cell membrane from the inside to the outside, or a negatively charged ion crossing from the outside to the inside. Similarly, currents with a negative value are referred to as "inward current", corresponding to positively charged ions crossing a cell membrane from the outside to the inside, or a negatively charged ion crossing from inside to outside.

The figure to the right shows an V–I curve that is more relevant to the currents in excitable biological membranes (such as a neuronal axon). The blue line shows the V–I relationship for the potassium ion. Note that it is linear, indicating no voltage-dependent gating of the potassium ion channel. The yellow line shows the V–I relationship for the sodium ion. Note that it is not linear, indicating that the sodium ion channel is voltage-dependent. The green line indicates the I–V relationship derived from summing the sodium and potassium currents. This approximates the actual membrane potential and current relationship of a cell containing both types of channel.

3.3 References

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