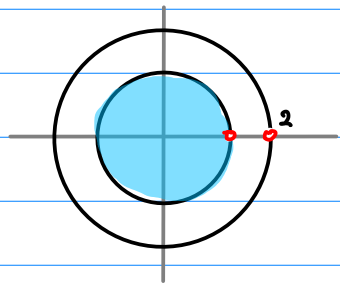
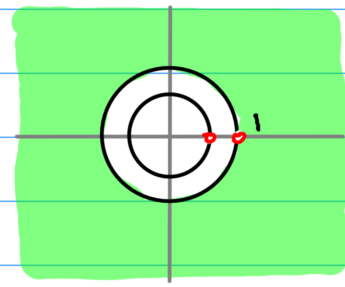
Laurent Series and z-Transform Examples case 0.A

20171030

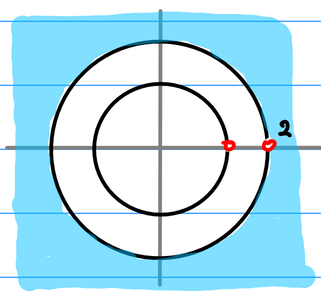
1. A

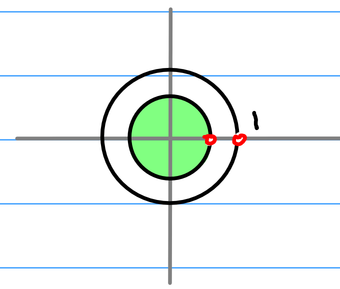
$$f(z) = \frac{-1}{(z-1)(z-2)} \xrightarrow{z^{-1}} X(z) = \frac{-0.5\,z^2}{(z-1)(z-0.5)}$$



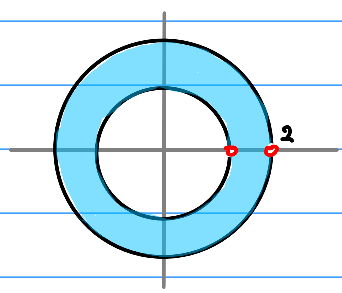


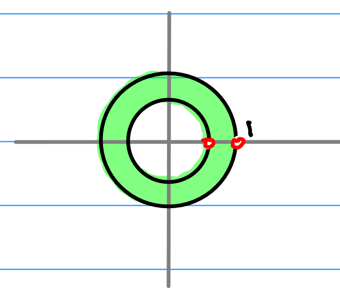
$$\sum_{n=0}^{\infty}\left[\left(\frac{1}{2}\right)^{n+1}-1\right]z^n = \sum_{n=0}^{\infty}\left[\left(\frac{1}{2}\right)^{n+1}-1\right]z^n$$





$$\sum_{n=-1}^{-\infty}\left(1-\left(\frac{1}{2}\right)^{n+1}\right)z^n = \sum_{n=-1}^{-\infty}\left(1-\left(\frac{1}{2}\right)^{n+1}\right)z^n$$

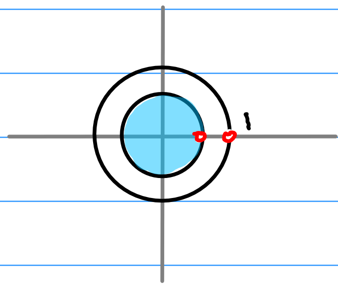


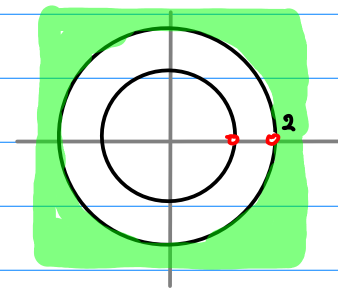


$$\sum_{n=-1}^{-\infty} z^n + \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^{n+1} z^n = \sum_{n=-1}^{-\infty} z^n + \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^{n+1} z^n$$

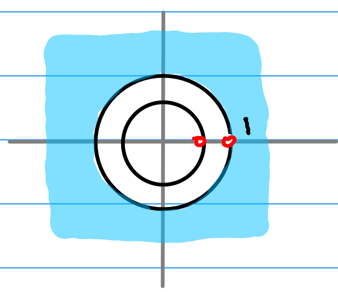
2.A

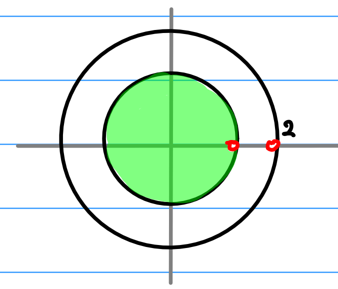
$$f(z) = \frac{-0.5\,z^2}{(z-1)(z-0.5)} \xrightarrow{z^{-1}} X(z) = \frac{-1}{(z-1)(z-2)}$$



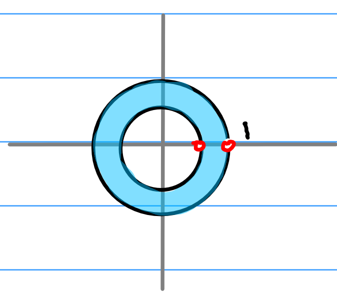


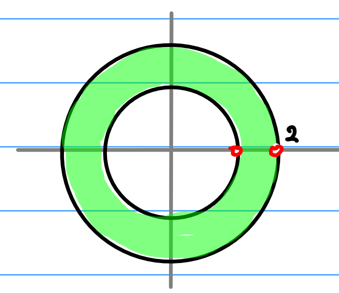
$$\sum_{n=1}^{\infty}\left[1-2^{n-1}\right]z^n = \sum_{n=1}^{\infty}\left[1-2^{n-1}\right]z^n$$





$$\sum_{n=0}^{-\infty}\left[2^{n-1}-1\right]z^n = \sum_{n=0}^{-\infty}\left[2^{n-1}-1\right]z^n$$

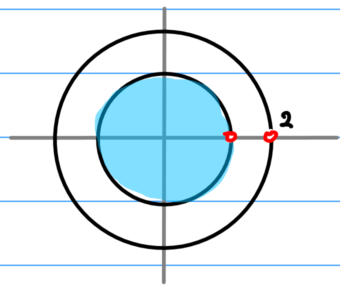


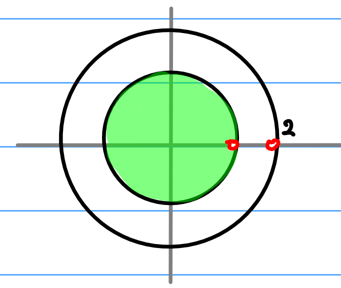


$$\sum_{n=1}^{\infty}1\cdot z^n + \sum_{n=0}^{-\infty}2^{n-1}\cdot z^n = \sum_{n=1}^{\infty}1\cdot z^n + \sum_{n=0}^{-\infty}2^{n-1}\cdot z^n$$

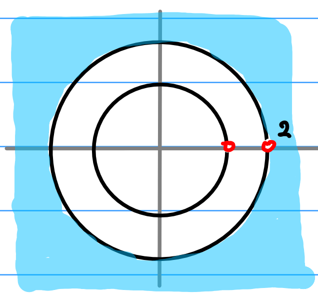
3 . A

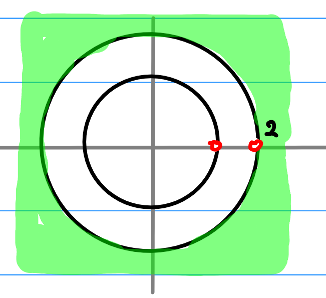
$$f(z) = \frac{-1}{(z-1)(z-2)} = X(z) = \frac{-1}{(z-1)(z-2)}$$



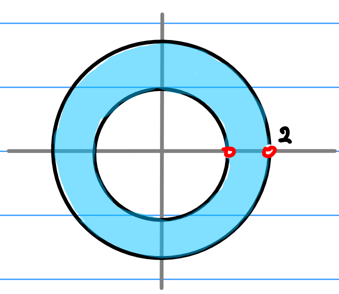


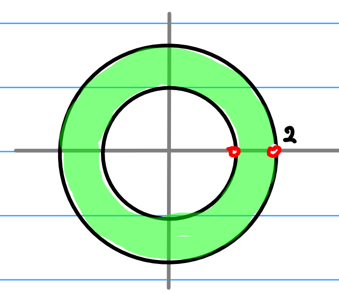
$$\sum_{n=0}^{\infty} \left[\left(\tfrac{1}{2}\right)^{n+1} - 1 \right] z^{n} \xrightarrow{-n} \sum_{n=0}^{-\infty} \left[2^{n-1} - 1 \right] z^{-n}$$





$$\sum_{n=-1}^{-\infty} \left[1 - \left(\tfrac{1}{2}\right)^{n+1} \right] z^{n} \xrightarrow{-n} \sum_{n=1}^{\infty} \left[1 - 2^{n-1} \right] z^{-n}$$

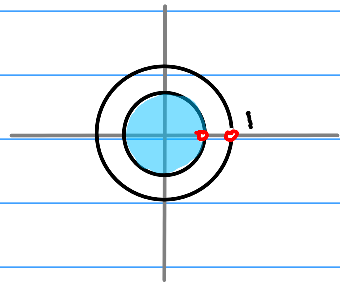


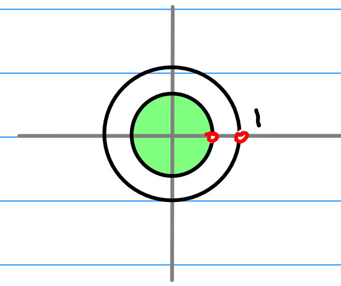


$$\sum_{n=-1}^{-\infty} z^{n} + \sum_{n=0}^{\infty} \left(\tfrac{1}{2}\right)^{n+1} z^{n} \xrightarrow{-n} + \sum_{n=1}^{\infty} z^{-n} + \sum_{n=0}^{\infty} 2^{n-1} z^{-n}$$

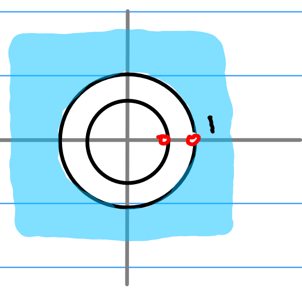
4.A

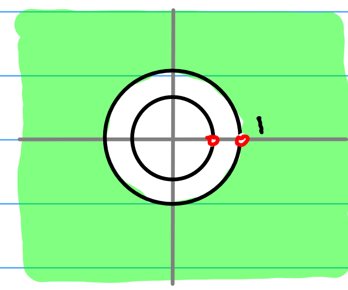
$$f(z) = \frac{-0.5\,z^2}{(z-1)(z-0.5)} = X(z) = \frac{-0.5\,z^2}{(z-1)(z-0.5)}$$



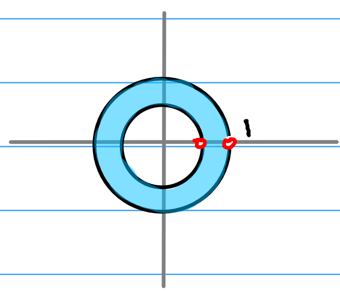


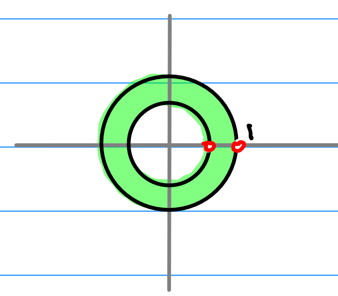
$$\sum_{n=1}^{\infty} [\,1 - 2^{n-1}\,]\, z^{n} \xrightarrow{-n} \sum_{n=-1}^{\infty} \left[\,1 - \left(\tfrac{1}{2}\right)^{n+1}\right] z^{-n}$$





$$\sum_{n=-1}^{-\infty} [\,2^{n-1} - 1\,]\, z^{n} \xrightarrow{-n} \sum_{n=1}^{\infty} \left[\left(\tfrac{1}{2}\right)^{n+1} - 1\right] z^{-n}$$





$$+\sum_{n=1}^{\infty} z^{n} + \sum_{n=0}^{-\infty} 2^{n-1} z^{n} \xrightarrow{-n} +\sum_{n=-1}^{-\infty} z^{-n} + \sum_{n=0}^{\infty} \left(\tfrac{1}{2}\right)^{n+1} z^{-n}$$

1. A

$$f(z) = \frac{-1}{(z-1)(z-2)} \xrightarrow{z^{-1}} X(z) = \frac{-0.5z^2}{(z-1)(z-0.5)}$$

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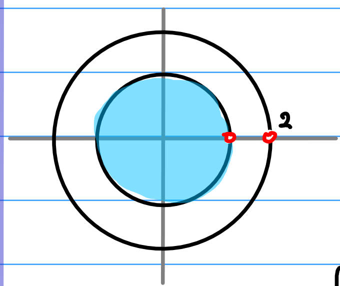

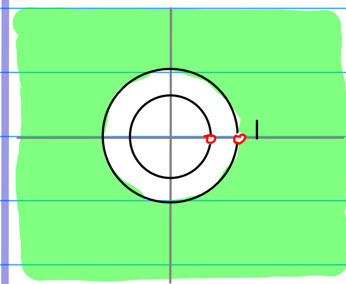

$$a_n = \begin{cases} \left(\frac{1}{2}\right)^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases} \qquad x_n = \begin{cases} 0 & (n > 0) \\ \left(\frac{1}{2}\right)^{n+1} - 1 & (n \leq 0) \end{cases}$$

$$f(z) = \sum_{n=0}^{\infty} \left[\left(\frac{1}{2}\right)^{n+1} - 1\right] z^n \qquad X(z) = \sum_{n=0}^{\infty} \left[\left(\frac{1}{2}\right)^{n+1} - 1\right] z^n$$

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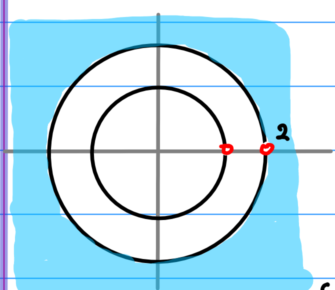

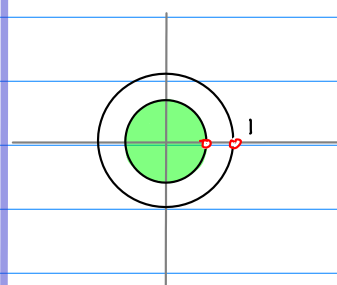

$$a_n = \begin{cases} 0 & (n \geq 0) \\ \left(1 - \left(\frac{1}{2}\right)^{n+1}\right) & (n < 0) \end{cases} \qquad x_n = \begin{cases} \left(1 - \left(\frac{1}{2}\right)^{n+1}\right) & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

$$f(z) = \sum_{n=-1}^{-\infty} \left(1 - \left(\frac{1}{2}\right)^{n+1}\right) z^n \qquad X(z) = \sum_{n=-1}^{-\infty} \left(1 - \left(\frac{1}{2}\right)^{n+1}\right) z^n$$

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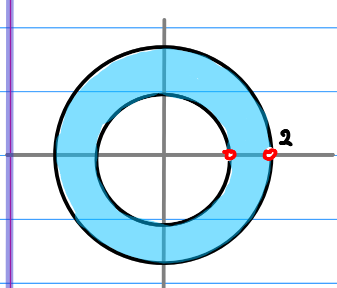

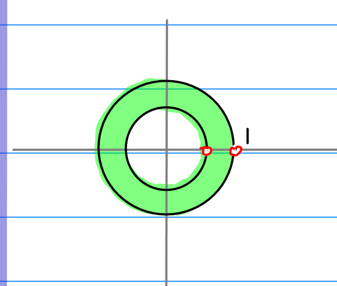

$$a_n = \begin{cases} \left(\frac{1}{2}\right)^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases} \qquad x_n = \begin{cases} 1 & (n > 0) \\ \left(\frac{1}{2}\right)^{n+1} & (n \leq 0) \end{cases}$$

$$f(z) = \sum_{n=-1}^{-\infty} z^n + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^n \qquad X(z) = \sum_{n=-1}^{-\infty} z^n + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^n$$

2.A

$$f(z) = \frac{-0.5z^2}{(z-1)(z-0.5)} \xrightarrow{z^{-1}} X(z) = \frac{-1}{(z-1)(z-2)}$$

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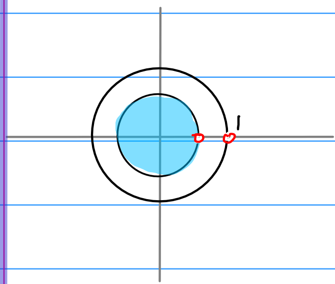

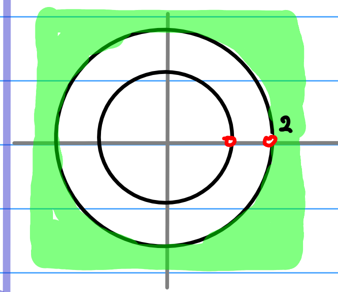

$$a_n = \begin{cases} [1-2^{n-1}] & (n>0) \\ 0 & (n\le 0) \end{cases} \quad \rightarrow \quad x_n = \begin{cases} 0 & (n\ge 0) \\ [1-2^{n-1}] & (n<0) \end{cases}$$

$$f(z) = \sum_{n=1}^{\infty} [1-2^{n-1}]\, z^n \qquad X(z) = \sum_{n=1}^{\infty} [1-2^{n-1}]\, z^n$$

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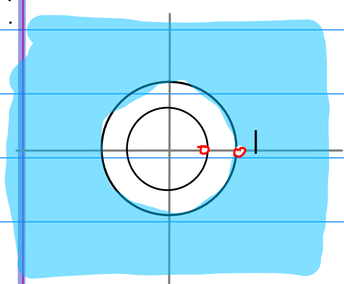

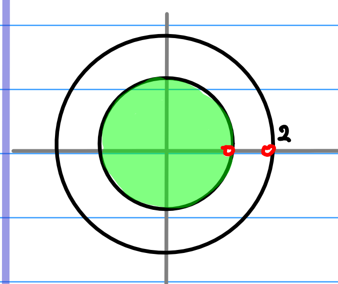

$$a_n = \begin{cases} 0 & (n>0) \\ [2^{n-1}-1] & (n\le 0) \end{cases} \quad \rightarrow \quad x_n = \begin{cases} [2^{n-1}-1] & (n\ge 0) \\ 0 & (n<0) \end{cases}$$

$$f(z) = \sum_{n=0}^{-\infty} [2^{n-1}-1]\, z^n \qquad X(z) = \sum_{n=0}^{-\infty} [2^{n-1}-1]\, z^n$$

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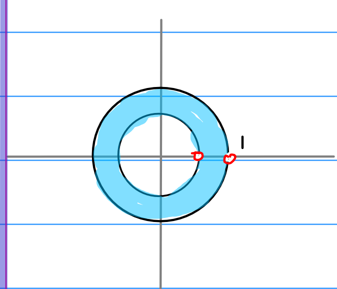

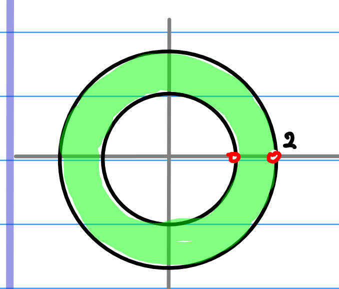

$$a_n = \begin{cases} 1 & (n>0) \\ 2^{n-1} & (n\le 0) \end{cases} \qquad x_n = \begin{cases} 2^{n-1} & (n\ge 0) \\ 1 & (n<0) \end{cases}$$

$$f(z) = \sum_{n=1}^{\infty} 1\cdot z^n + \sum_{n=0}^{-\infty} 2^{n-1}\cdot z^n \qquad X(z) = \sum_{n=1}^{\infty} 1\cdot z^n + \sum_{n=0}^{-\infty} 2^{n-1}\cdot z^n$$

3 . A $\quad f(z) = \dfrac{-1}{(z-1)(z-2)} \quad = \quad X(z) = \dfrac{-1}{(z-1)(z-2)}$

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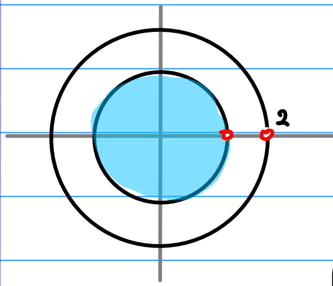

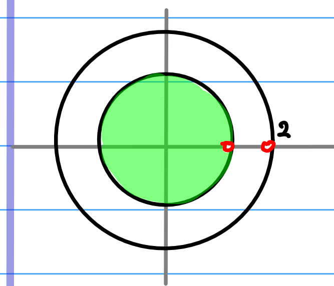

$$a_n = \begin{cases} \left(\frac{1}{2}\right)^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases} \qquad x_n = \begin{cases} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{cases}$$

$$f(z) = \sum_{n=0}^{\infty} \left[\left(\tfrac{1}{2}\right)^{n+1} - 1 \right] z^n \qquad X(z) = \sum_{n=0}^{-\infty} \left[2^{n-1} - 1 \right] z^{-n}$$

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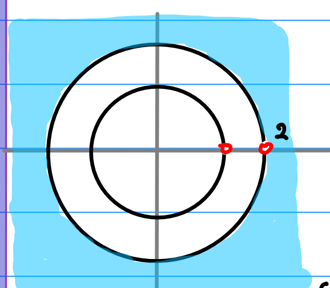

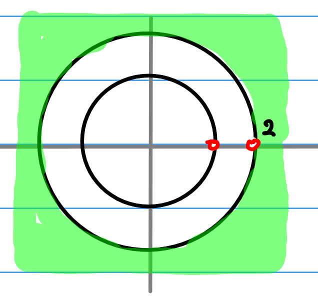

$$a_n = \begin{cases} 0 & (n \geq 0) \\ 1 - \left(\frac{1}{2}\right)^{n+1} & (n < 0) \end{cases} \qquad x_n = \begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

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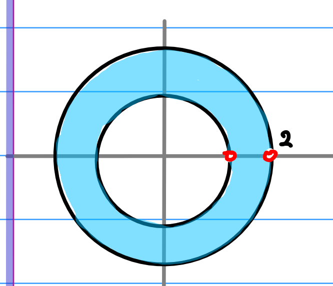

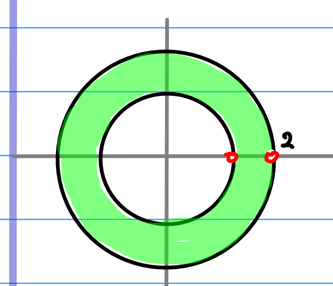

$$a_n = \begin{cases} \left(\frac{1}{2}\right)^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases} \qquad x_n = \begin{cases} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}$$

$$f(z) = \sum_{n=-1}^{-\infty} z^n + \sum_{n=0}^{\infty} \left(\tfrac{1}{2}\right)^{n+1} z^n \qquad X(z) = + \sum_{n=1}^{\infty} z^{-n} + \sum_{n=0}^{\infty} 2^{n-1} z^{-n}$$

4.A

$$f(z) = \frac{-0.5\,z^2}{(z-1)(z-0.5)} = X(z) = \frac{-0.5\,z^2}{(z-1)(z-0.5)}$$

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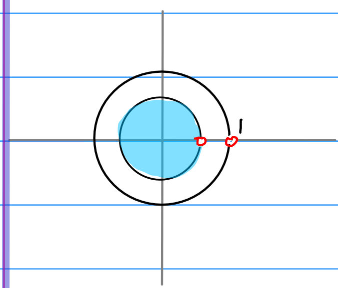

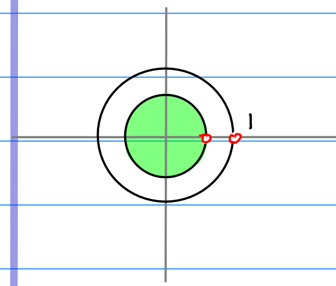

$$a_n = \begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases} \qquad x_n = \begin{cases} 0 & (n \geq 0) \\ 1 - \left(\frac{1}{2}\right)^{n+1} & (n < 0) \end{cases}$$

$$f(z) = \sum_{n=1}^{\infty} \left[1 - 2^{n-1} \right] z^n \qquad X(z) = \sum_{n=-1}^{\infty} \left[1 - \left(\tfrac{1}{2}\right)^{n+1} \right] z^{-n}$$

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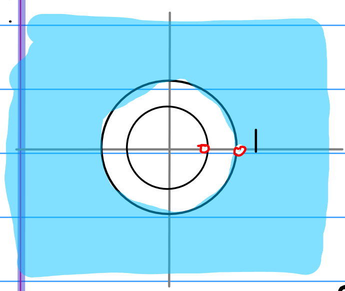

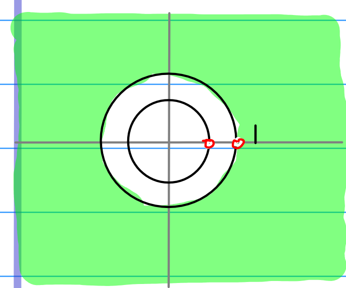

$$a_n = \begin{cases} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{cases} \qquad x_n = \begin{cases} \left(\frac{1}{2}\right)^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

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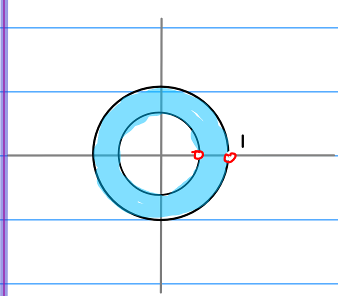
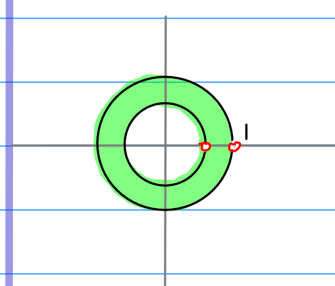




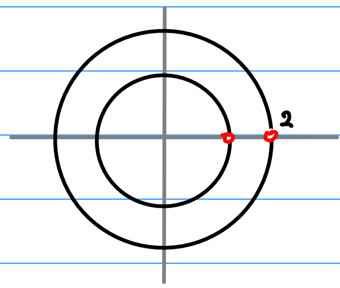
$$a_n = \begin{cases} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases} \qquad x_n = \begin{cases} \left(\frac{1}{2}\right)^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$$

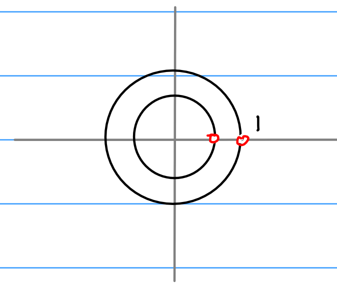
$$f(z) = + \sum_{n=1}^{\infty} z^n + \sum_{n=0}^{-\infty} 2^{n-1} z^n \qquad X(z) = + \sum_{n=-1}^{-\infty} z^{-n} + \sum_{n=0}^{\infty} \left(\tfrac{1}{2}\right)^{n+1} z^{-n}$$

$$f(z) \xrightarrow{Z^{-1}} X(z)$$

1.A

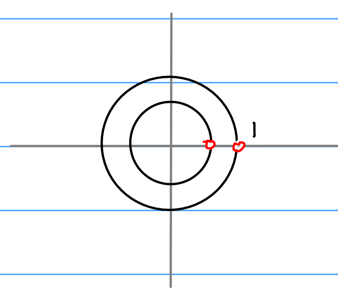
$$\frac{-1}{(z-1)(z-2)} \xrightarrow{Z^{-1}} \frac{-0.5\, z^2}{(z-1)(z-0.5)}$$

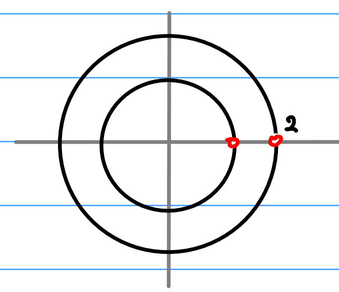




1.B

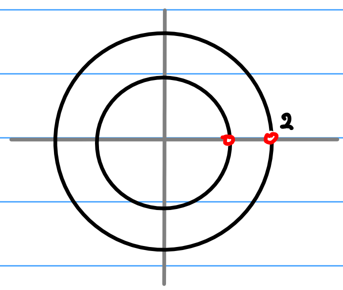
$$\frac{-0.5\, z^2}{(z-1)(z-0.5)} \xrightarrow{Z^{-1}} \frac{-1}{(z-1)(z-2)}$$

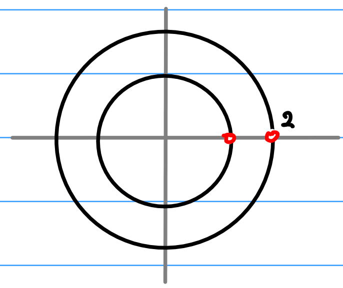




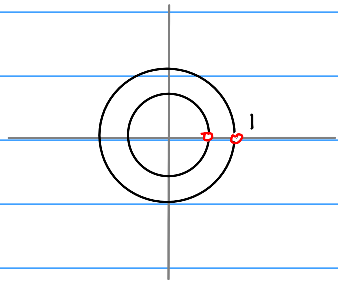
$$f(z) = X(z)$$

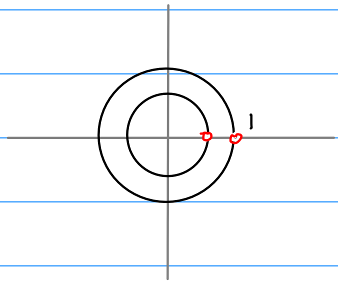
3.A $\quad \dfrac{-1}{(z-1)(z-2)} = \dfrac{-1}{(z-1)(z-2)}$





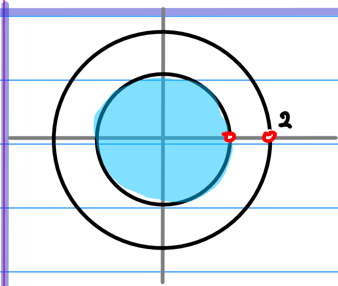
4.B $\quad \dfrac{-0.5z^2}{(z-1)(z-0.5)} = \dfrac{-0.5z^2}{(z-1)(z-0.5)}$



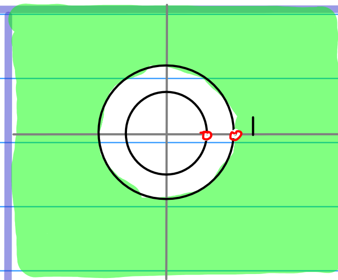


$$f(z) \xrightarrow{Z^{-1}} X(z)$$

Ⅰ

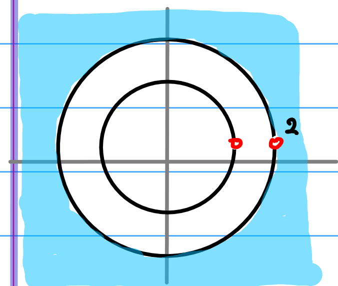


$$\boxed{\begin{matrix} \left(\frac{1}{2}\right)^{n+1}-1 & (n \geq 0) \\ 0 & (n < 0) \end{matrix}}$$

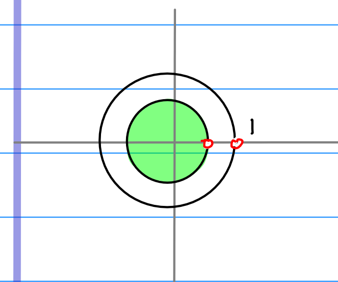


$$\boxed{\begin{matrix} 0 & (n > 0) \\ \left(\frac{1}{2}\right)^{n+1}-1 & (n \leq 0) \end{matrix}}$$

Ⅱ

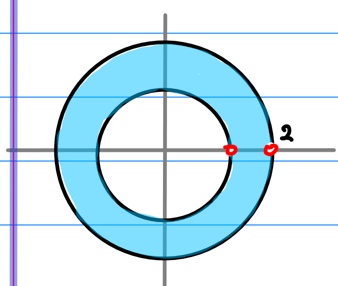


$$\boxed{\begin{matrix} 0 & (n \geq 0) \\ 1-\left(\frac{1}{2}\right)^{n+1} & (n < 0) \end{matrix}}$$

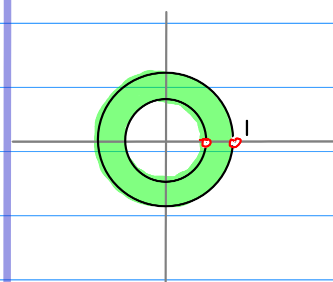


$$\boxed{\begin{matrix} 1-\left(\frac{1}{2}\right)^{n+1} & (n > 0) \\ 0 & (n \leq 0) \end{matrix}}$$

Ⅲ

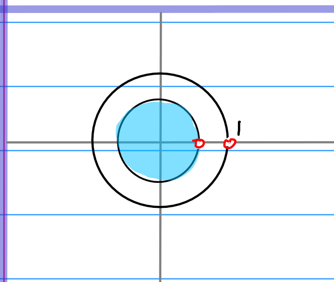


$$\boxed{\begin{matrix} \left(\frac{1}{2}\right)^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{matrix}}$$

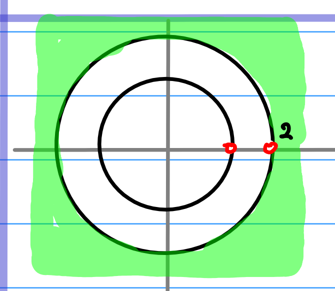


$$\boxed{\begin{matrix} 1 & (n > 0) \\ \left(\frac{1}{2}\right)^{n+1} & (n \leq 0) \end{matrix}}$$

Ⅰ

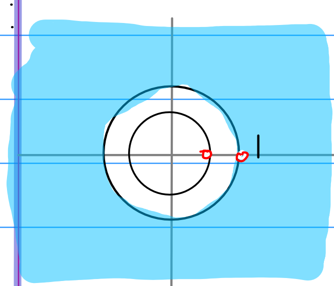


$$\boxed{\begin{matrix} 1-2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{matrix}}$$

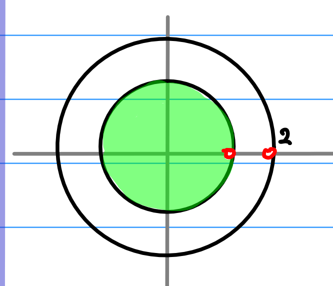


$$\boxed{\begin{matrix} 0 & (n \geq 0) \\ 1-2^{n-1} & (n < 0) \end{matrix}}$$

Ⅱ

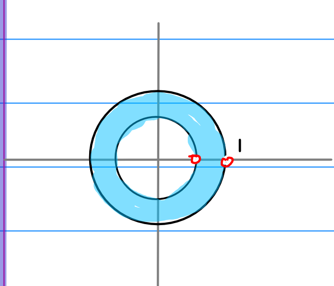


$$\boxed{\begin{matrix} 0 & (n > 0) \\ 2^{n-1}-1 & (n \leq 0) \end{matrix}}$$

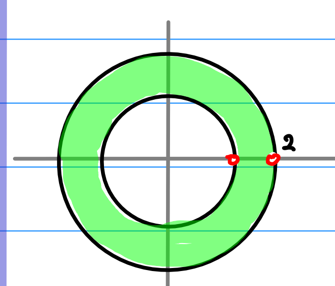


$$\boxed{\begin{matrix} 2^{n-1}-1 & (n \geq 0) \\ 0 & (n < 0) \end{matrix}}$$

Ⅲ



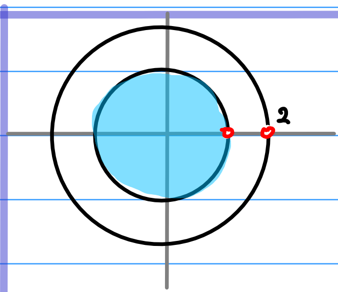
$$\boxed{\begin{matrix} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{matrix}}$$



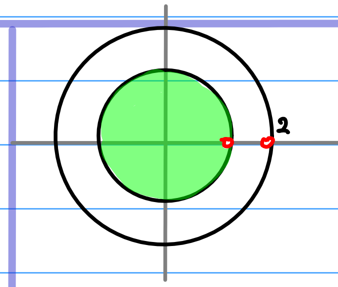
$$\boxed{\begin{matrix} 2^{n-1} & (n \geq 0) \\ 1 & (n < 0) \end{matrix}}$$

$f(z) = X(z)$

I

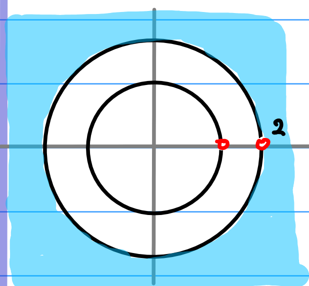


$$\begin{cases} \left(\frac{1}{2}\right)^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

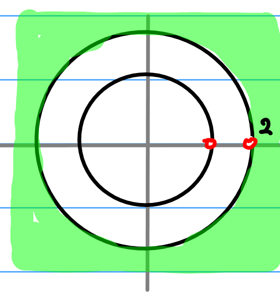


$$\begin{cases} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{cases}$$

II

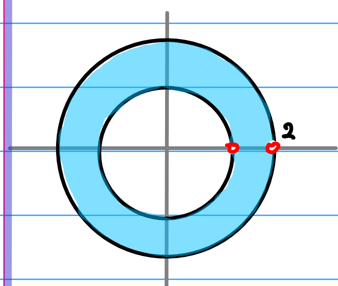


$$\begin{cases} 0 & (n \geq 0) \\ 1 - \left(\frac{1}{2}\right)^{n+1} & (n < 0) \end{cases}$$

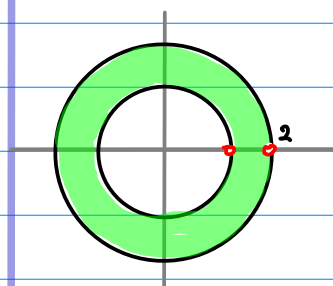


$$\begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

III

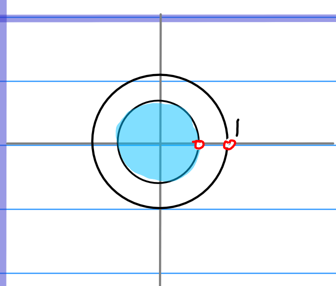


$$\begin{cases} \left(\frac{1}{2}\right)^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$$

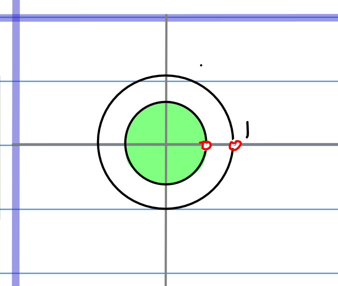


$$\begin{cases} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}$$

I

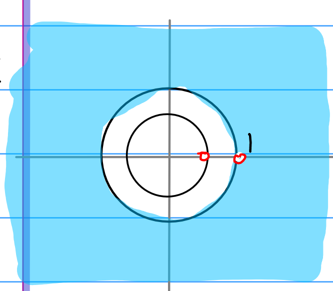


$$\begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

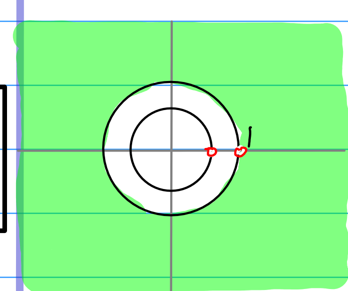


$$\begin{cases} 0 & (n \geq 0) \\ 1 - \left(\frac{1}{2}\right)^{n+1} & (n < 0) \end{cases}$$

II

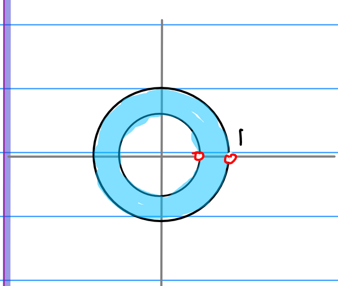


$$\begin{cases} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{cases}$$

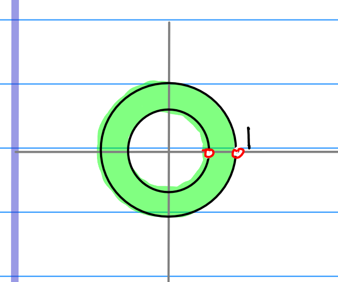


$$\begin{cases} \left(\frac{1}{2}\right)^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

III



$$\begin{cases} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}$$



$$\begin{cases} \left(\frac{1}{2}\right)^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$$

$f(z)$ $\quad p_1 < p_2$

$|z| < p_1$ $\quad \boxed{\begin{array}{l} \left(\frac{1}{p_2}\right)^{n+1} - \left(\frac{1}{p_1}\right)^{n+1} \quad (n \geq 0) \\ 0 \quad (n < 0) \end{array}}$

$p_1 = 1 \quad p_2 = 2$

$|z| > p_2$ $\quad \boxed{\begin{array}{l} 0 \quad (n \geq 0) \\ \left(\frac{1}{p_1}\right)^{n+1} - \left(\frac{1}{p_2}\right)^{n+1} \quad (n < 0) \end{array}}$

$p_1 = 1 \quad p_2 = 2$

$p_1 < |z| < p_2$ $\quad \boxed{\begin{array}{l} \left(\frac{1}{p_2}\right)^{n+1} \quad (n \geq 0) \\ \left(\frac{1}{p_1}\right)^{n+1} \quad (n < 0) \end{array}}$

$p_1 = 1 \quad p_2 = 2$

$X(z) \qquad P_1 < P_2$

$|z| < P_1$

$$\boxed{\begin{array}{l} 0 \\ (p_1)^{n+1} - (p_2)^{n+1} \end{array}} \quad \begin{array}{l} (n > 0) \\ (n \leq 0) \end{array}$$

$p_1 = 1/2 \qquad p_2 = 1$

$|z| > P_2$

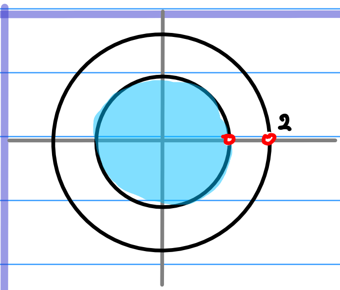
$$\boxed{\begin{array}{l} (p_2)^{n+1} - (p_1)^{n+1} \\ 0 \end{array}} \quad \begin{array}{l} n > 0) \\ n \leq 0) \end{array}$$

$p_1 = 1/2 \qquad p_2 = 1$

$P_1 < |z| < P_2$

$$\boxed{\begin{array}{l} (p_2)^{n+1} \\ (p_1)^{n+1} \end{array}} \quad \begin{array}{l} (n > 0) \\ (n \leq 0) \end{array}$$

Ⅰ

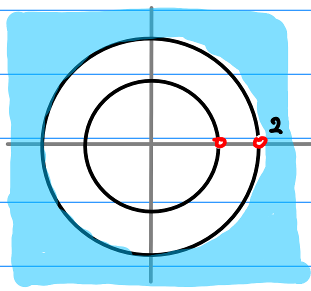


$$\begin{cases} \left(\frac{1}{2}\right)^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

$$\begin{cases} \left(\frac{1}{p_2}\right)^{n+1} - \left(\frac{1}{p_1}\right)^{n+1} \\ 0 \end{cases}$$

$p_1 = 1 \quad p_2 = 2$

Ⅱ

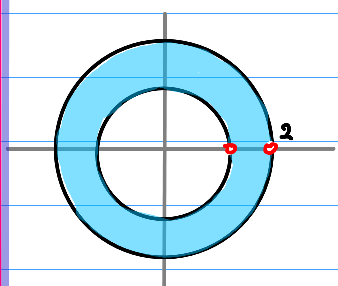


$$\begin{cases} 0 & (n \geq 0) \\ 1 - \left(\frac{1}{2}\right)^{n+1} & (n < 0) \end{cases}$$

$$\begin{cases} 0 \\ \left(\frac{1}{p_1}\right)^{n+1} - \left(\frac{1}{p_2}\right)^{n+1} \end{cases}$$

$p_1 = 1 \quad p_2 = 2$

Ⅲ

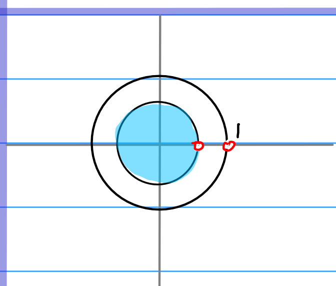


$$\begin{cases} \left(\frac{1}{2}\right)^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$$

$$\begin{cases} \left(\frac{1}{p_2}\right)^{n+1} \\ \left(\frac{1}{p_1}\right)^{n+1} \end{cases}$$

$p_1 = 1 \quad p_2 = 2$

Ⅰ

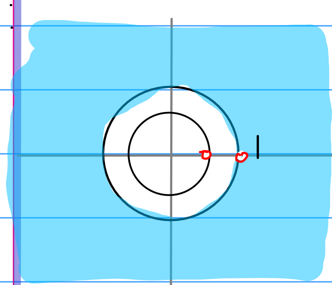


$$\begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

$$\begin{cases} \left(\frac{1}{p_2}\right)^{n+1} - \left(\frac{1}{p_1}\right)^{n+1} \\ 0 \end{cases}$$

$p_1 = \frac{1}{2} \quad p_2 = 1$

Ⅱ

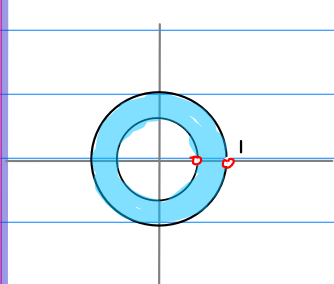


$$\begin{cases} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{cases}$$

$$\begin{cases} 0 \\ \left(\frac{1}{p_1}\right)^{n+1} - \left(\frac{1}{p_2}\right)^{n+1} \end{cases}$$

$p_1 = \frac{1}{2} \quad p_2 = 1$

Ⅲ



$$\begin{cases} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}$$

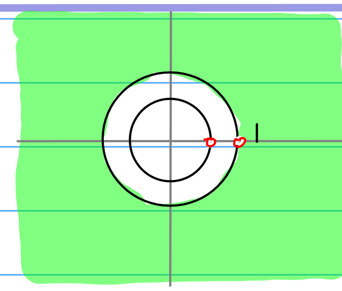
$$\begin{cases} \left(\frac{1}{p_2}\right)^{n+1} \\ \left(\frac{1}{p_1}\right)^{n+1} \end{cases}$$

$p_1 = \frac{1}{2} \quad p_2 = 1$

Ⅰ

$$\boxed{\begin{matrix} 0 \\ (p_1)^{n+1} - (p_2)^{n+1} \end{matrix}}$$

$p_1 = 1/2 \qquad p_2 = 1$

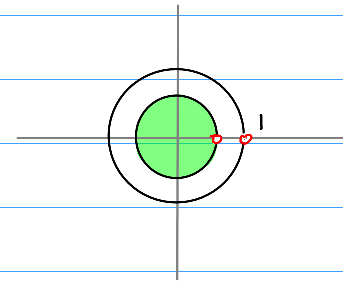


$$\boxed{\begin{matrix} 0 & (n>0) \\ \left(\frac{1}{2}\right)^{n+1} - 1 & (n \leq 0) \end{matrix}}$$

Ⅱ

$$\boxed{\begin{matrix} (p_2)^{n+1} - (p_1)^{n+1} \\ 0 \end{matrix}}$$

$p_1 = 1/2 \qquad p_2 = 1$

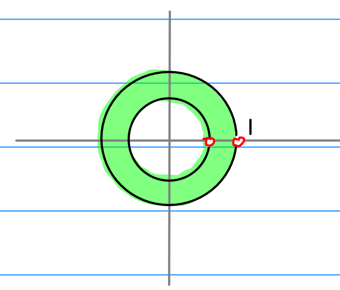


$$\boxed{\begin{matrix} 1 - \left(\frac{1}{2}\right)^{n+1} & (n > 0) \\ 0 & (n \leq 0) \end{matrix}}$$

Ⅲ

$$\boxed{\begin{matrix} (p_2)^{n+1} \\ (p_1)^{n+1} \end{matrix}}$$

$p_1 = 1/2 \qquad p_2 = 1$

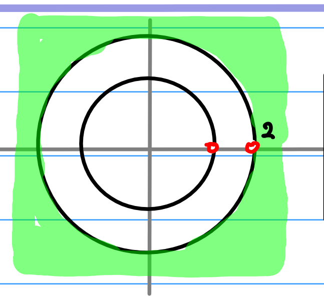


$$\boxed{\begin{matrix} 1 & (n>0) \\ \left(\frac{1}{2}\right)^{n+1} & (n \leq 0) \end{matrix}}$$

Ⅰ

$$\boxed{\begin{matrix} 0 \\ (p_1)^{n+1} - (p_2)^{n+1} \end{matrix}}$$

$p_1 = 1 \qquad p_2 = 2$

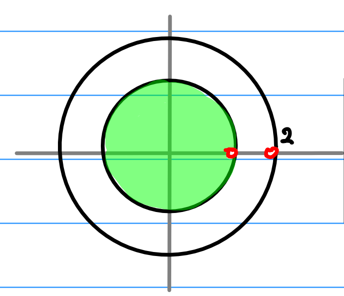


$$\boxed{\begin{matrix} 0 & (n \geq 0) \\ 1 - 2^{n-1} & (n<0) \end{matrix}}$$

Ⅱ

$$\boxed{\begin{matrix} (p_2)^{n+1} - (p_1)^{n+1} \\ 0 \end{matrix}}$$

$p_1 = 1 \qquad p_2 = 2$

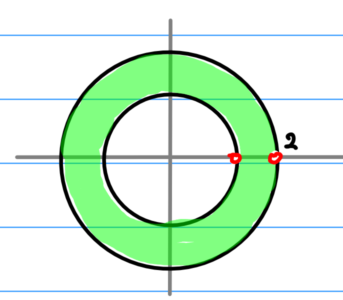


$$\boxed{\begin{matrix} 2^{n-1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{matrix}}$$

Ⅲ

$$\boxed{\begin{matrix} (p_2)^{n+1} \\ (p_1)^{n+1} \end{matrix}}$$

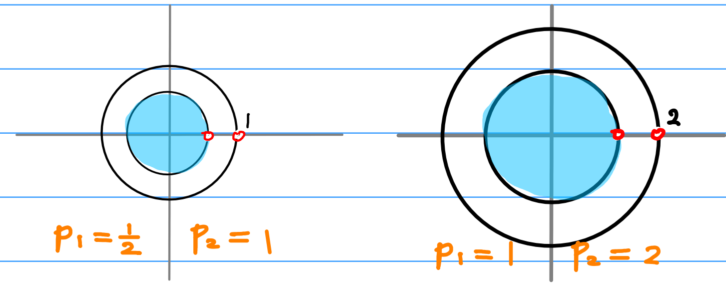
$p_1 = 1 \qquad p_2 = 2$



$$\boxed{\begin{matrix} 2^{n-1} & (n \geq 0) \\ 1 & (n<0) \end{matrix}}$$

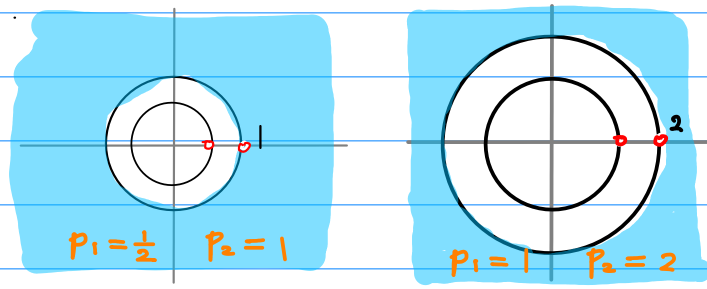
$$\left(\frac{1}{p_2}\right)^{n+1} - \left(\frac{1}{p_1}\right)^{n+1}$$

$$0$$



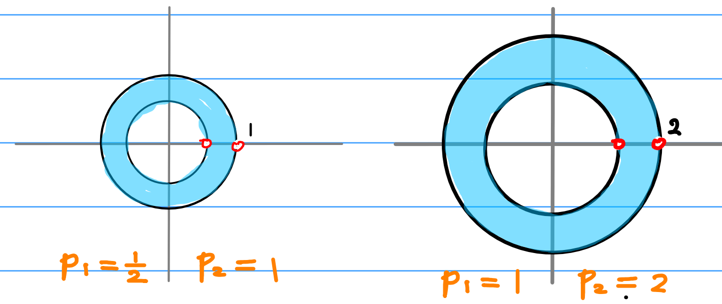
$$0$$

$$\left(\frac{1}{p_1}\right)^{n+1} - \left(\frac{1}{p_2}\right)^{n+1}$$



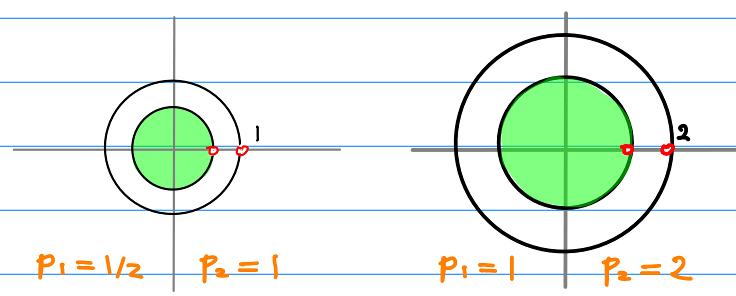
$$\left(\frac{1}{p_2}\right)^{n+1}$$

$$\left(\frac{1}{p_1}\right)^{n+1}$$



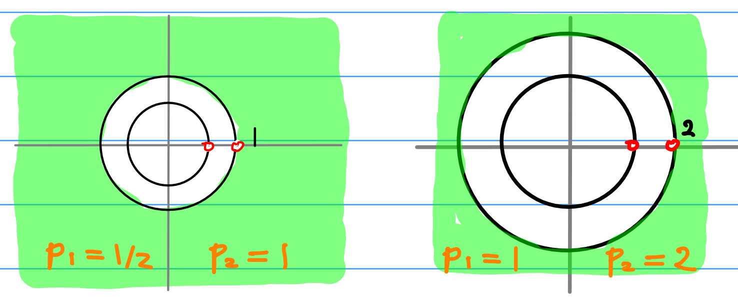
$$(p_2)^{n+1} - (p_1)^{n+1}$$

$$0$$



$$0$$

$$(p_1)^{n+1} - (p_2)^{n+1}$$



$$(p_2)^{n+1}$$

$$(p_1)^{n+1}$$