Mon, 25 Aug 08: EAS 4200 C
Aerospace Structures
Course website: www.mae.ufl.edu

Finite element method (FEM): widespread use in all areas of engineering, applied math, geologic exploration of oil reservoirs, etc...

Aerospace structures (Structural comp.) governed by Partial Diff. Eqs, which can be conveniently solved using FEM.

Emphasis in this course:
* understanding of mechanics
* ability to formulate problems
* judge correctness of solution (FEM)
* Avoid old ad hoc methods of structural analysis.

Ref: Sun [2006], Preface
Importance of correctness of FEA: Collapse of Minnesota bridge (Javier)
Wed, 27 Aug 08: EAS 4200C

Deadline for team structure: mtg

Wed, 3 Sep 08.

Each team (5 or 6 students)

Submit a paper:

* Team name: 4 char, preferred max 6 char.
* Member names + UF ID numbers
* Wikipedia usernames

real name = WP class username
Fri, 29 Aug 08:

* **Wikipedia**:
  - Confidentiality: usernames (new struct)

  * Goals: MediaWiki is a well-known team collaboration and widely used software for collaboration.
  - Class collaboration
  - Prof.-class collaboration
  - Wikiversity lectures in distant future.

* Students can develop their own Wikipedia articles or other media wiki articles

* Expensive textbooks: benefit UF students and students all over the world.

Collaborative software.
Fri, 29 Aug 08: EAS 4200C

Plan: Vision: Big picture re WP
textbooks, Wikiversity
MIT's OpenCourseWare

- Confidentiality
- Method of work (Jeff)
- E-learning (?)
- Old approach.

Vision: mediawiki
- Wikiversity, MIT's OpenCourseWare
- Collaboration between students
  in a team, collaboration of
  whole class, collaboration of
  prof. and class.

WP user namespaces ≠ WP articles

Confidentiality: course number, semester,
team name, A, B
Old approach:
10% + HW, 30% + 20% + 30% Exam.

Different time zones: UTC, EDT, EST (Draw)
Wed, 3 Sep 08; EAS 4200c
Lect. presentation: Wiki
Stiffness: $\sigma$

Metals: $\sigma$
(Yield stress)$\sigma_y$

Elastic resp. $E$
Plastic resp. $E$

Ultimate stress $\sigma_u$

Softening
Rupture

F18 E + hornet:
Aluminum: Skin
Steel: Landing gears
Titanium: Engine encasing, struct and other structural parts (stabilator, wing, cockpit, gear)
Horie, stabilizer, elevator
Carbon/epoxy: fuselage aileron (control surf. along trailing edge of a wing) stabilator, vert. stabilizer.

Shortcuts:
Edit article: Alt + Shift + E
Save: Alt + Shift + S
Preview: Alt + I
Fri, 5 Sep 08: EAS 4200c

Problem 1.1: Sun [200 c]

Box beam = \frac{Shell beam}{(MIT's OCW)}

\[ t \ll a \]
\[ t \ll b \]

↑ "Very small compared to"

\[ L = 2(a+b) \quad (Assumption 1) \]

Q: Find optimal \( \frac{b}{a} \) to maximize load-bearing capacity of beam assuming \[ M = T \quad (Assumption 2) \]

Contrib. Tallowable = \frac{2}{T} \text{ Tallowable} (Assumption 3)
Motivation:

Study box beam or rectangular cross section as a model for aerospace structures (fuselage, wings)

Fig. Boeing 777 fuselage
Fig. Wing structure
In other words, find opt. cross section carrying max. bending mom. and max. torque.

Shear stress due to torsion:

\[
T = \frac{\tau}{2ab}t
\]

Explanation: Since cross-section walls are very thin, can assume shear stress dist. to be uniform along the wall.

Shear force in shear flow

\[
V = \frac{\tau}{t}
\]
\[ T = T_{AB} + T_{BC} + T_{CD} + T_{DA} \]

\[ T_{AB} = \frac{b}{2} \times (v \times a) = \frac{1}{2} \tau abt \]

\[ T_{BC} = \frac{a}{2} \times (v \times b) = \frac{1}{2} \tau abt \]

\[ T_{CD} = ? \]

\[ T_{DA} = ? \]

\[ \Rightarrow \quad T = 2 \tau abt \]

\[ \Rightarrow \quad T = \frac{T}{2abt} \]

\[ \text{Case 1: Assume } \sigma \text{ (bending normal stress) reaches allowable first. } \]

\[ \text{Want to ensure that } T < T_{\text{allowable}} \]
Recall: \[ \sigma = \frac{Mz}{I} \]

- \( M \) = bending moment
- \( z \) = ordinate of a point on axis perpendicular to neutral axis

\[ I = \text{2nd area moment of inertia} \]

\[ I = \int \int z^2 \, dy \, dz \]
Mt6: 8 Sep 08 EAS 42000 C5-1

Pb. 1.1: cont'd, take notes

Exams: see e-mails, web site
(what's allowed, what's not)

Wiki presentation.
Case 1: Assume $\sigma_{\text{max}} = \sigma_{\text{allow}}$.

Recall: $\sigma = \frac{Mz}{I}$, $z = \frac{b}{2}$

\[ M = \frac{2I \sigma_{\text{max}}}{b} \]

\[ M = (2 \sigma_{\text{allow}}) \left( \frac{I}{b} \right) \]

**Case 1**

Recall: $L = 2(a + b) = \text{const}$

\[ a = \frac{L}{2} - b \]

\[ \left( \frac{I}{b} \right) \text{ func. of } b, \]

Maximize $M$: $M_{\text{max}} = (2 \sigma_{\text{allow}}) \left( \frac{I}{b} \right)_{\text{max}}$
\[
I = \sum_{i=1}^{4} \frac{1}{12} \left[ b_i (h_i)^3 + A_i (d_i)^2 \right] \\
= 2 \cdot \frac{t}{12} b^3 + 2 \left[ \frac{a t^3}{12} + (at) \left( \frac{b + t}{2} \right) \right]
\]

\[
\gamma = \left[ \frac{a t^3}{12} + (at) \left( \frac{b + t}{2} \right) \right]^{\gamma}
\]

\[
= \frac{a}{12} \left[ t^3 + t \left( \frac{12}{24} \right) b^2 \right] \\
= \frac{at}{12} \left[ t^2 + 3 b^2 \right] \approx 3ab^2t
\]

t \ll b \Rightarrow t^2 \ll b^2 \Rightarrow t^2 \ll 3b^2

\Rightarrow I \approx 2 \cdot \frac{t b^3}{12} + \frac{ab^2 t}{2}

= \frac{b^2 t}{2} \left( \frac{b^3}{3} + a \right) = \frac{t b^2}{6} (3a + b)
\]
\[ f(b) = \frac{t b}{c} \left( 3a + b \right) \]  

"equal by defn"  

\[ a = \frac{1}{2} - b \]  

\[ \frac{t b \left( 3a - 4b \right)}{12} = \beta_0 + \beta_1 b^2 + \beta_2 b^4 \]  

\[ \beta_0 = 0 \]  
\[ \beta_1 = \frac{3L t}{12} \]  

\[ \beta_2 = -4 \frac{t}{12} = -\frac{t}{3} \]  

\[ \frac{d^2 f(b)}{db^2} = 2\beta_2 = -\frac{2t}{3} < 0 \]  

\[ f(b) = 0 \implies b = 0 \quad \text{or} \quad 3L/4 \]  

**Diagram:**  
- \( b = 3L/8 \)  
- \( b = 3L/4 \)  
- \( 2L/4 \)
\[
\frac{d^2 \theta}{db^2} = \beta_1 + 2 \beta_2 \frac{b}{L} = 0
\]

\[
\Rightarrow \quad \frac{d \theta}{db} = -\frac{\beta_1}{2 \beta_2} = +\frac{3L}{8}
\]

\textbf{Sln for } b \text{ for case 1.}

\[
a(t) = \frac{L}{2} - b(t) = \frac{L}{8}
\]

\[
\frac{b(c)}{a(c)} = 3
\]

\[
\left(\frac{I}{b}\right)_{\text{max}} = \left(\frac{I}{b(c)}\right)
\]

\[
M_{\text{max}} = (2 \sigma_{\text{allow}}) \left(\frac{I}{b(c)}\right) = \frac{3tL^2}{32} \sigma_{\text{allow}} = T_{\text{max}}(t) \quad (\text{Assump. 2})
\]
Shear stress due to $T_{\text{max}}$

\[
\tau_{(1)}^{(1)} = \frac{T_{\text{max}}^{(1)}}{2a^{(1)}b^{(1)}t} \quad \text{(Assump. 2)}
\]

\[
M_{\text{max}}^{(1)} = \frac{T_{\text{max}}^{(1)}}{2 \left( \frac{L}{8} \right) \left( \frac{3L}{8} \right) t} = T_{\text{allow}}
\]

\[
T_{\text{max}}^{(1)} = \sigma_{\text{allow}} = 2T_{\text{allow}} > T_{\text{allow}}
\]

\[
\sigma_{\text{allow}} = \frac{T_{\text{allow}}}{2}
\]

\[
T = 0 \quad \sigma = 0
\]

\[
\tau = \sigma_{\text{allow}}
\]

\[
\tau_{\text{allow}} = \frac{T_{\text{allow}}}{2}
\]

\[
F = \sigma A
\]

(Assump. 3) \Rightarrow \tau_{(1)}^{(1)} > \tau_{\text{allow}} \quad \text{not accept.} (Felipe, Eric)
Case 2: Assume \( T_{\text{max}} = T_{\text{allow}} \) (See deriv. p. 5-2)

\[
T = \frac{T}{2abt} = T_{\text{max}} = T_{\text{allow}}
\]

\[
\Rightarrow T = (2t \cdot T_{\text{allow}})(ab)\text{ const} \quad \text{variab.}
\]

\[
\Rightarrow T_{\text{max}} = (2t \cdot T_{\text{allow}})(ab)_{\text{max}}
\]

HW: Show \( a(x) = b(y) = \frac{l}{4} \)
Mtg 9, Mon, 15 Sep 08, EAS 4200C L9-1

Ph 1.1: Cont'd, Case 2.

\[ T_{\text{max}} = (2 + t \cdot \text{allow}) \left( \frac{L}{L} \right)^2 \]
\[ = \frac{1}{8} t \cdot L^2 \cdot \text{allow} = M_{\text{max}} \]

(by Assump. 2)

\[ M_{\text{max}} = \frac{t \cdot L^2}{16} \cdot \text{allow} \]

(by Assump. 3)

\[ \Rightarrow \text{allow} = \frac{16 \cdot M_{\text{max}}^{(2)}}{t \cdot L^2} \]

Recall \( f(b) := \frac{I(b)}{b} \) \( (p. 7-3) \)

\[ \Rightarrow f(b^{(2)}) = \frac{I^{(2)}}{b^{(2)}} = \frac{t \cdot b^{(2)} (3L - 4b^{(2)})}{12} \]
\[ = \frac{t \cdot L^2}{24} \]

(recall \( b^{(2)} = \frac{L}{4} \) \( p. 8-2 \))
\[ \sigma_{\text{max}} = \frac{M_{\text{max}} b^{(2)}}{I^{(2)}} \cdot \frac{12}{2} = M_{\text{max}} \frac{12}{tL^2} \frac{tL^2}{16} \text{Fallow} \]

\[ \sigma = \frac{M_{\text{max}} b^{(2)}}{I} \]

\[ \frac{W}{z} = \frac{b^{(2)}}{2} \]

\[ \Rightarrow \sigma^{(2)}_{\text{max}} = \frac{12}{16} \text{Fallow} < \text{Fallow} \]

\[ \Rightarrow \text{Case 2 acceptable.} \]

**HW:** Stringers (axial members)

**Q:** (Felipe) use bars w/ rectangular cross section for stringers?

**Ans:** No. Use bars w/ open, thin-walled cross sections.

Why? Want stringers to resist bending imposed from external pressure.