* Introduction
* Motivation of FEM
* Course organization

Deadline for submitting team structure:
Wed, 3 Sep 08.

Each team (5 or 6 students)
submit a paper of team info:

* team name: 4 char, max 6 char.
* member names + UF ID numb.
* Wikipedia class usernames

real name = WP class username

Plan: * Vision: Big picture re mediawiki and WP
Wikipedia

textbooks, Wikiversity
MIT's OpenCourseWare

* Confidentiality
* Method of work ← E-learning?
* Old approach

Vision: - Mediawiki
- Wikiversity, MIT's OpenCourseWare
- collaboration
  * members of a team
  * teams in a class
  * prof + students in class.

WP Policies:
WP username sp ≠ WP articles

Confidentiality: course numb. semester. team name.

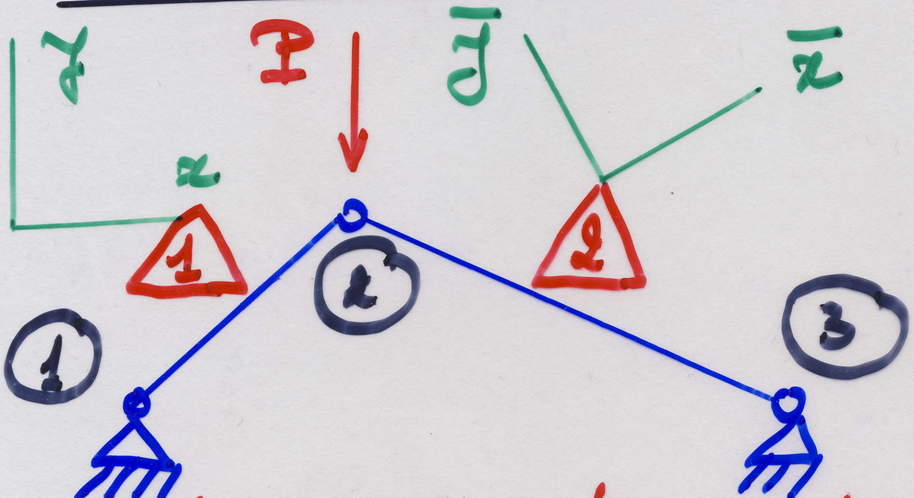
Old approach:

10% HW, 30% + 30% + 30% Exams.

Wed, 3 Sep 08 : EML 4500

Trusses, matrix method. Book, chap 4

Also MIT's OpenCourseWare : Class wiki page.

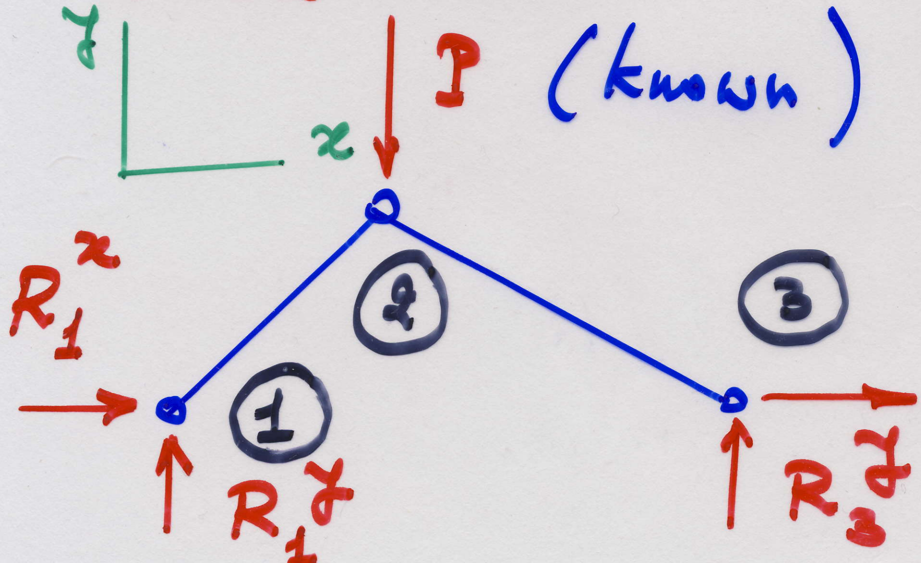


Truss w/ 2 elastic bars $\underbrace{\text{elastic}}_{\text{deformable}}$

disp fixed (constrained) to zero in both x and y (in both $\bar{x}$ and $\bar{y}$)

actually stat. determ.

Global FBD : $\underbrace{\text{Global}}_{\text{whole struct}}$



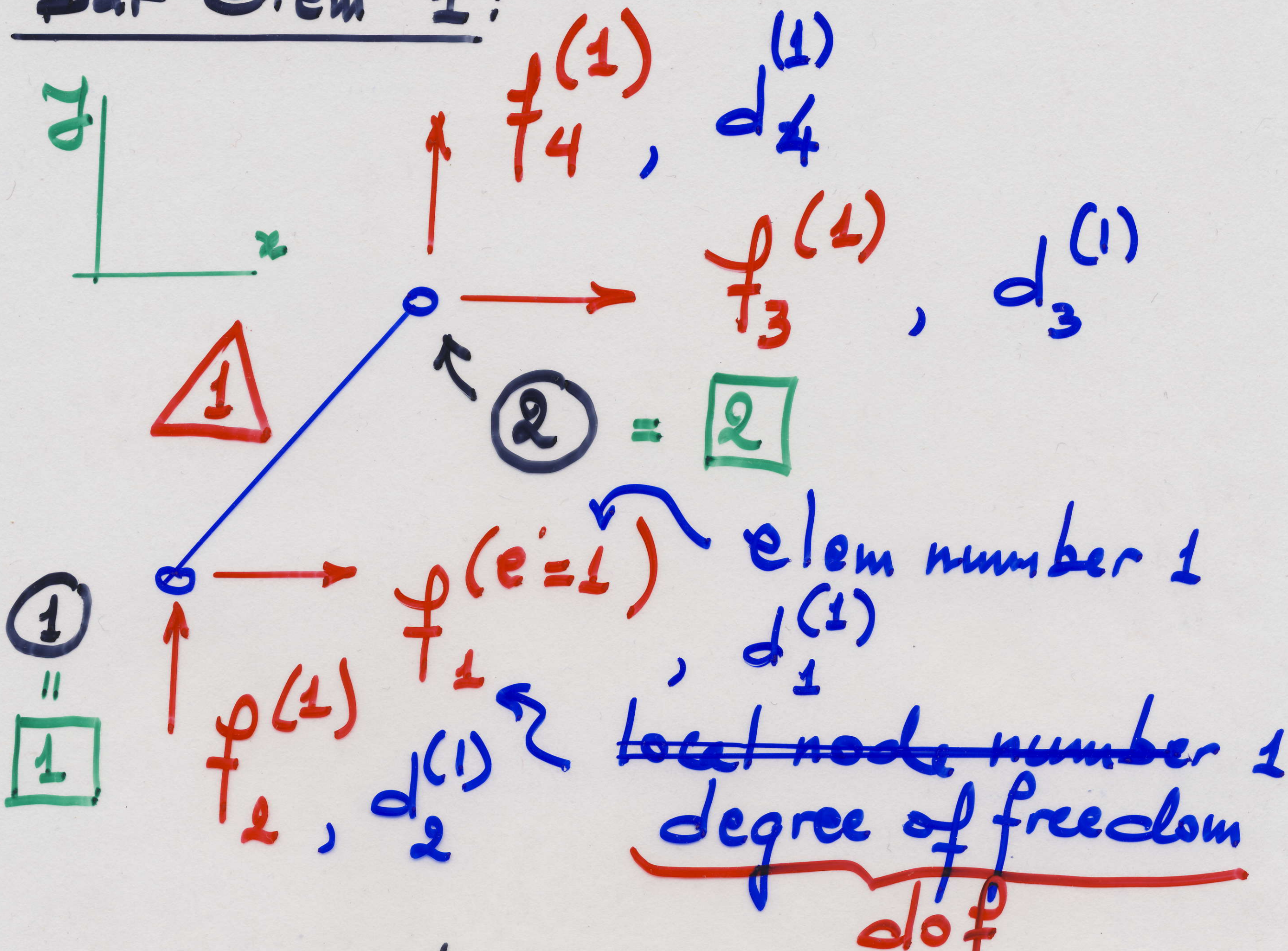
4 unknown reactions :

3 eqs of equil

$\Rightarrow$ statically indeterm.

2 FBD's of 2 bar elems:

Bar elem 1:



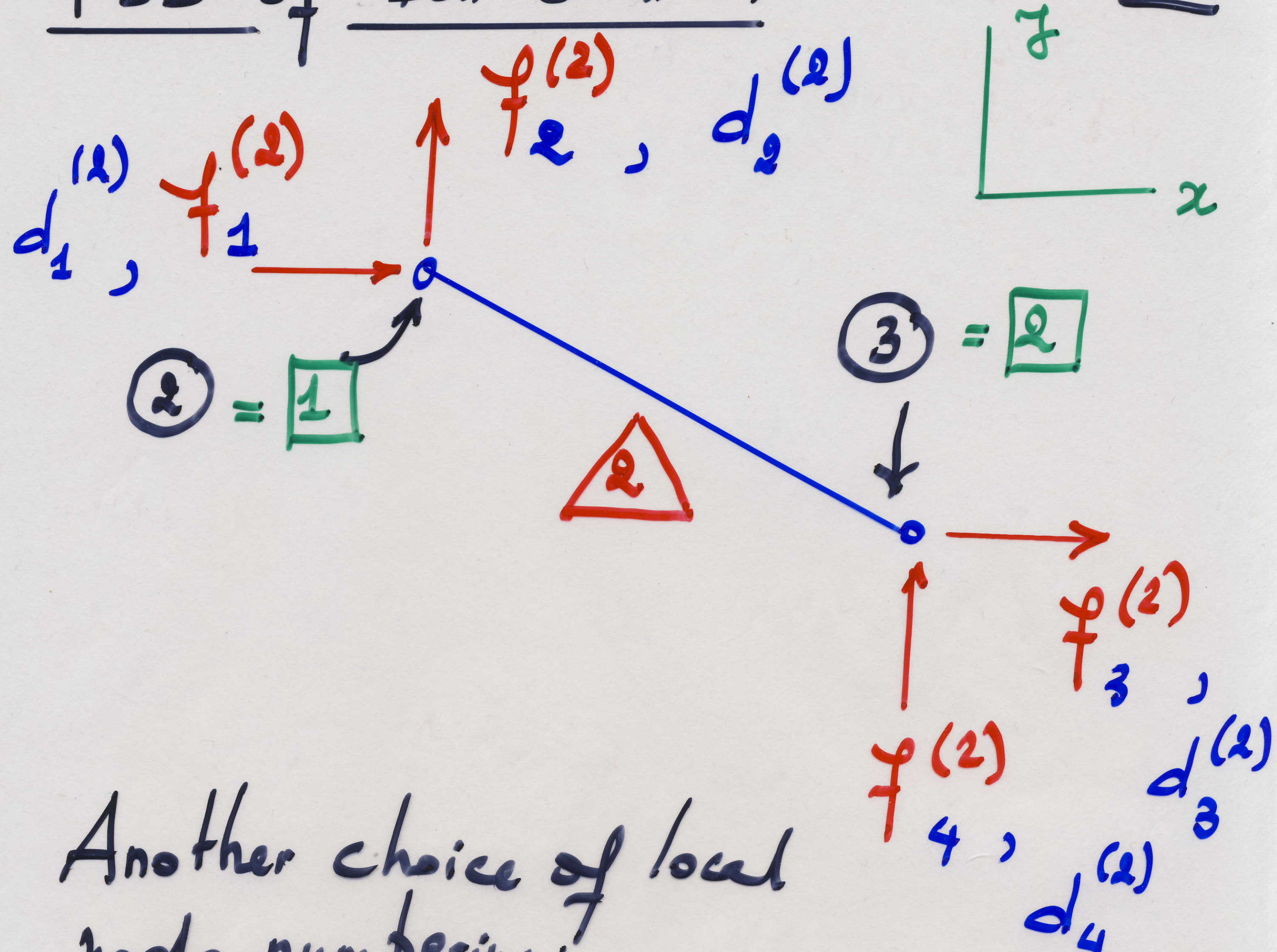
(h) = $\underbrace{\text{global}}_{\text{whole struct}}$ node number n

$\boxed{n}$ = $\underbrace{\text{local}}_{\text{each elem separately}}$ node number n

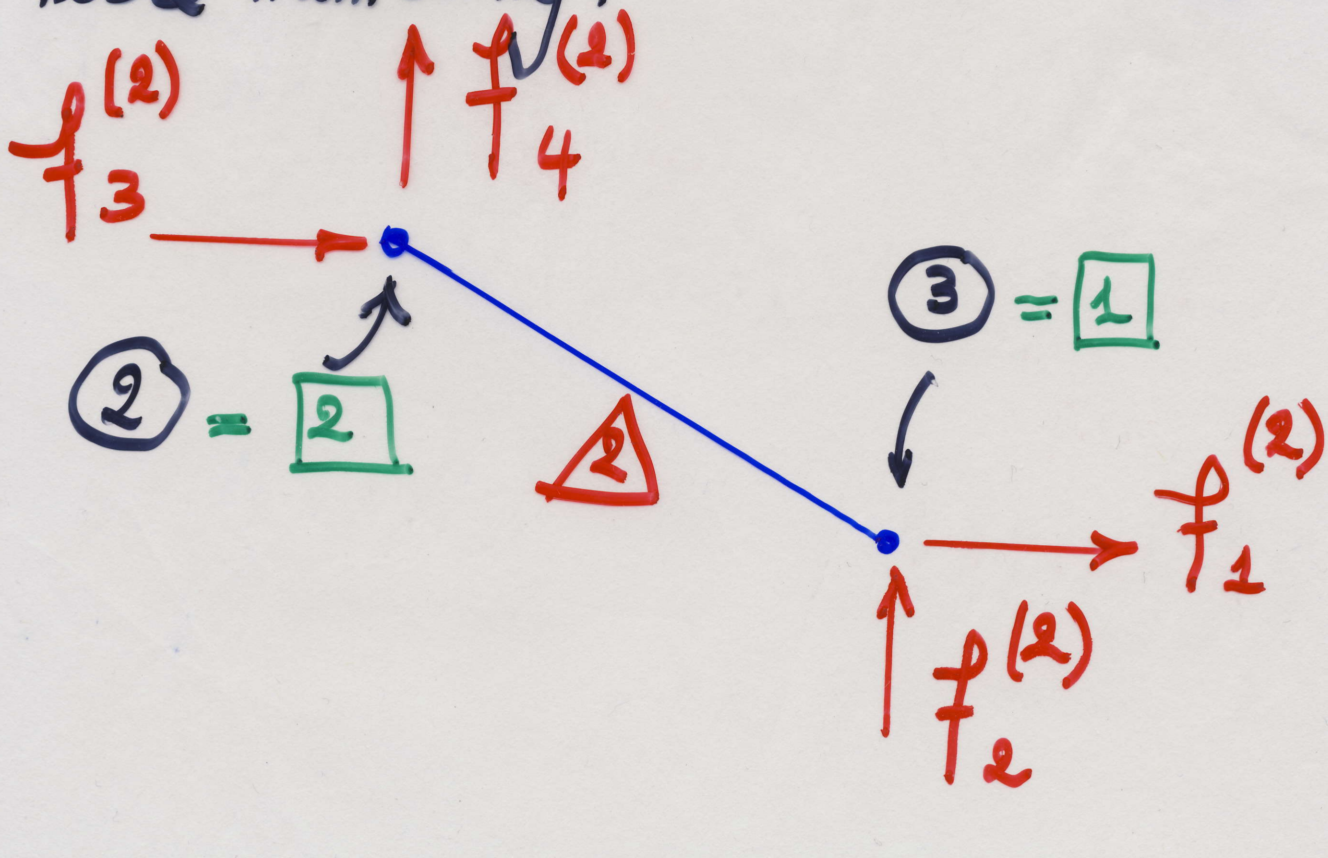
$$f_i^{(e)} = \text{ith internal force of elem } e$$

$i = 1, 2, 3, 4$ } for this example
$e = 1, 2$

FBD of bar elem 2:

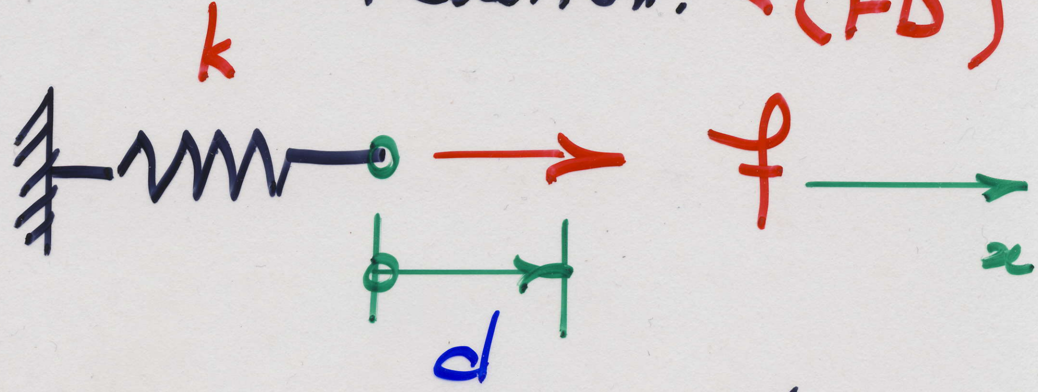


Another choice of local node numbering:



Next big step: $\underbrace{\text{Force-disp.}}$ relation. (FD)

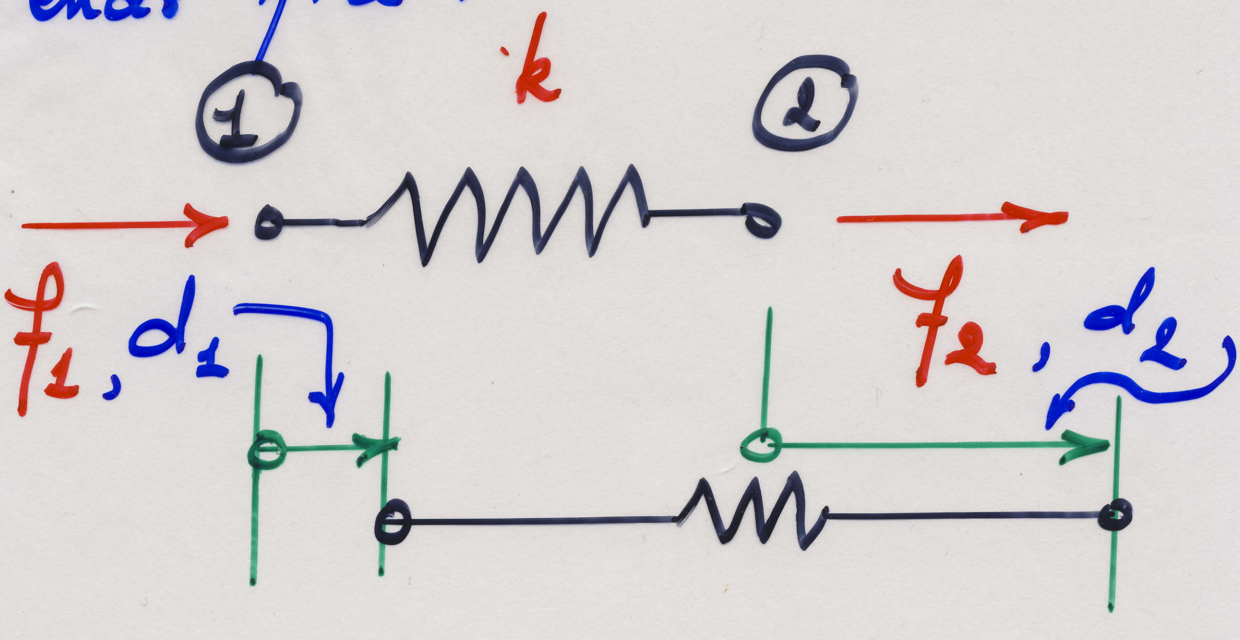
Recall:



FD rel. of 1-D spring elem: (w/ 1 end fixed)

$$\boxed{f = k\,d}$$

FD rel. of 1-D spring with 2 ends free:



$$\boxed{\underbrace{\begin{Bmatrix} f_1 \\ f_2 \end{Bmatrix}}_{2\times 1} = \underbrace{\begin{bmatrix} k & -k \\ -k & k \end{bmatrix}}_{2\times 2} \underbrace{\begin{Bmatrix} d_1 \\ d_2 \end{Bmatrix}}_{2\times 1}}$$

row col

Case 1: Observer sits on node 1

$$f_2 = k\,(d_2 - d_1)$$

Case 2: Obs. sits on node 2

or equil. $f_1 + f_2 = 0$

$$\Rightarrow \quad f_1 = -f_2 = -k\,(d_2 - d_1)$$

$$= k\,(d_1 - d_2)$$

Mtg 5: Fri, 5 Sep 08, EML 4500

Read: Chap 4 (Trusses, beams, frames)

Chap 1: Big picture

Secs 1.1. Discretization

1.1.1 Plane truss elem

1.2. Assembly of elem eqs.

Ex 1.4: Five-bar truss

1.4. Elem soln & model validity

1.4.1. Plane truss elem.

Steps to solve simple truss syst described on p. 4-1 (Recipe)

Mtg 4 ← slide #

1 Global picture (description):

At structure level

* global dofs (disp. dofs) unknowns in general

* global forces

Actually, the disp dofs are partitioned into :

* a known part, e.g., fixed dofs, constraints
* an unknown part : Solved using FEM.

(See truss example on p. 4-1)

Similarly for the global forces :

* a known part : Applied forces
* an unknown part : Reactions

2. Element picture :
   * Elem dofs
   * Elem forces

   } Either in global coord. syst or in local coord. syst.

3. Global FD rel :
   * Elem. stiffness matrices in global coord.
   * Elem. force matrices in global coord.

* Assembly of elem stiff. mat. and elem force mat. into global FD rel. :

"Free-free" syst (unconstrained)

$$\boxed{\underset{n \times n}{\underline{K}}\ \underset{n \times 1}{\underline{d}} = \underset{n \times 1}{\underline{F}}}$$

$\underline{K}$ singular

4. Elimination of known dofs. to reduce the global FD rel.

(stiffness matrix non-singular → invertible)

$$\boxed{\underset{m \times m}{\overline{\underline{K}}}\ \underset{m \times 1}{\overline{\underline{d}}} = \underset{m \times 1}{\overline{\underline{F}}}} \qquad m < n$$

m = n° of unknown disp. dofs

n = n° of both known and unknown disp. dofs.

$\overline{\underline{K}}$ non singular ⇒ $\overline{\underline{K}}^{-1}$ exists

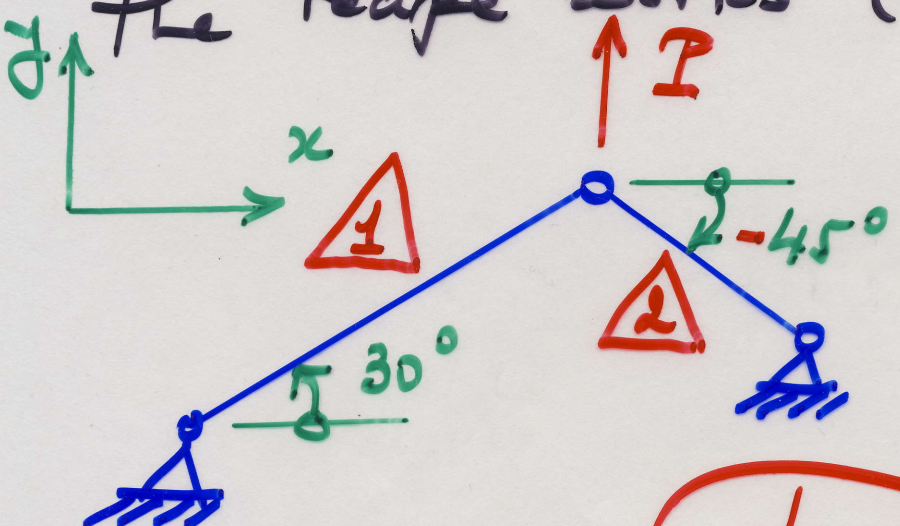
($\overline{\underline{K}}$ invertible)

$$\Rightarrow \boxed{\underset{m \times 1}{\bar{\underline{d}}} = \underset{m \times m}{\bar{\underline{K}}^{-1}} \; \underset{m \times 1}{\bar{\underline{F}}}}$$

5. Compute elem forces from now known $\bar{\underline{d}}$ $\Rightarrow$ elem stresses

6. Compute reactions. (unknown forces)

Take a specific example to see how the recipe works (no justification yet)



Data:

Elem length:

$L^{(1)} = 4$

$L^{(2)} = 2$

elem #

Young's modulus:

$E^{(1)} = 3$

$E^{(2)} = 5$

Cross section area:

$A^{(1)} = 1$

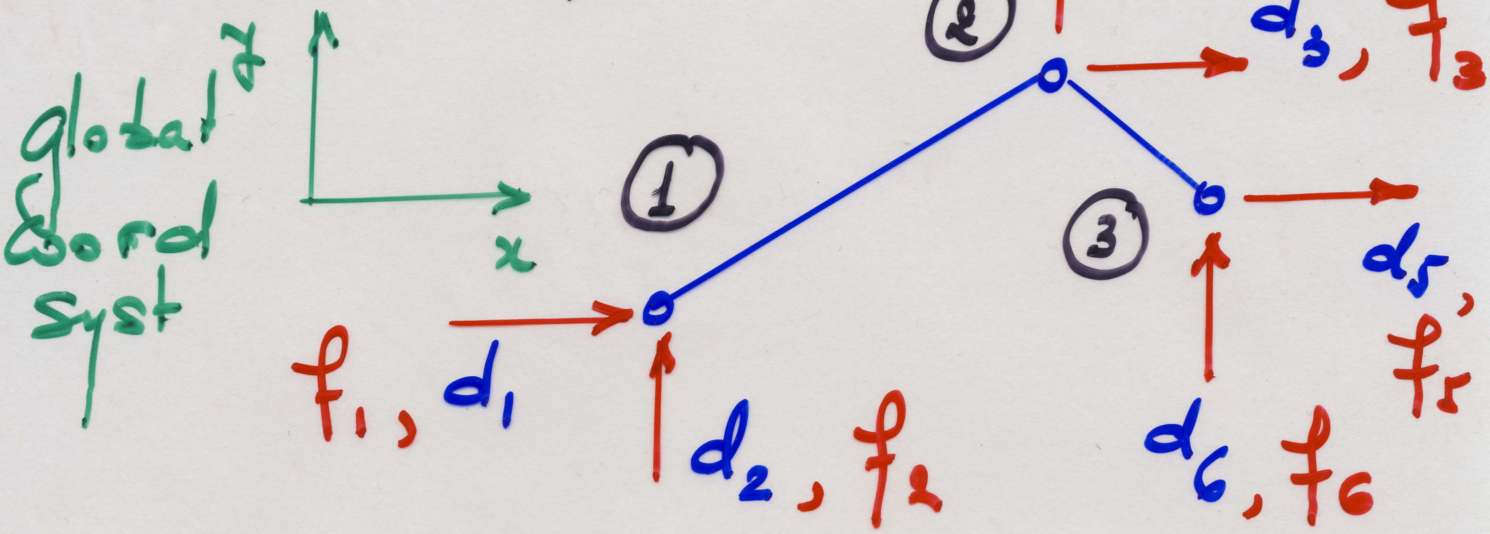
$A^{(2)} = 2$

Inclination angle: $\theta^{(1)} = 30°$

$\theta^{(2)} = -45°$

1). Global picture:

Global dofs



(n) = global node numb. n

Numbering the disp dofs:

* Follow order of global node numb.
* For each node, follow the order of global coord. axes, number the disp. dofs for that node.

Global forces : same thing

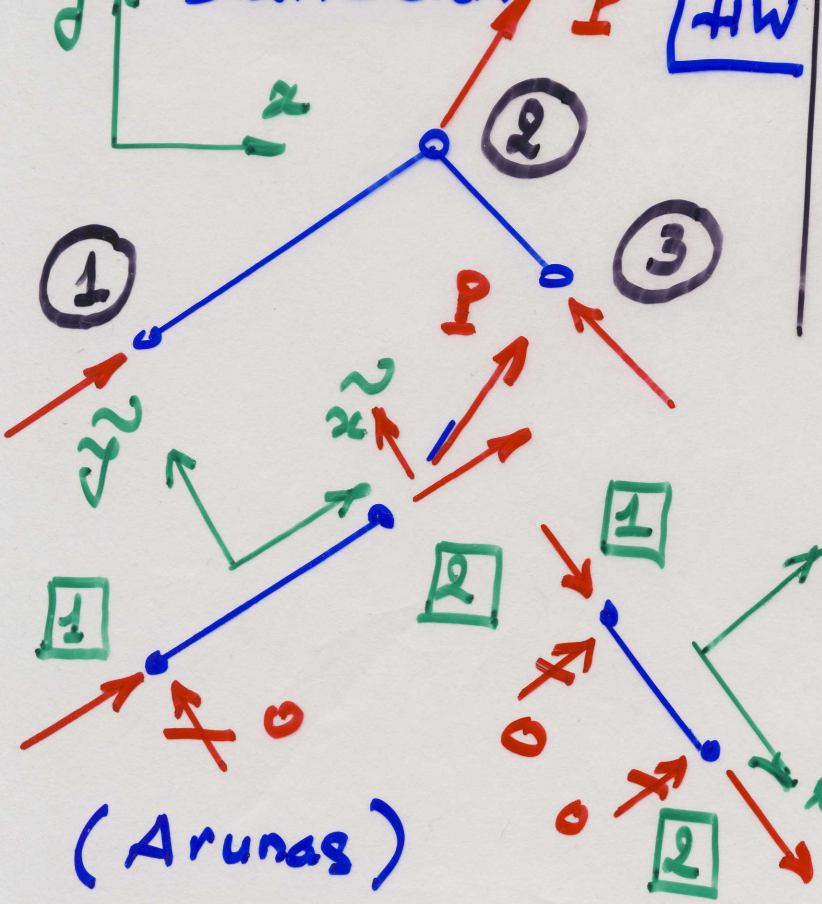
$$\begin{array}{c} F_1 \leftarrow \\ \\ \\ F_6 \leftarrow \end{array} \underbrace{\begin{Bmatrix} f_1 \\ \vdots \\ \vdots \\ f_6 \end{Bmatrix}}_{6\times 1} = \underbrace{\begin{bmatrix} & & \\ & \underline{K} & \\ & & \end{bmatrix}}_{6\times 6} \underbrace{\begin{Bmatrix} d_1 \\ \vdots \\ \vdots \\ d_6 \end{Bmatrix}}_{6\times 1}$$

$\underline{F}$ global force col. matrix

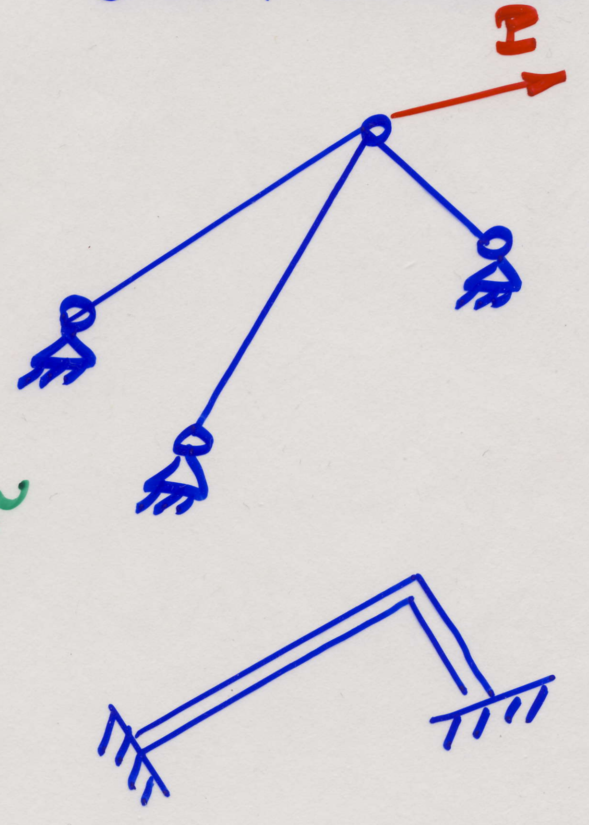
global stiffness matrix

$\underline{d}$ global disp. col. mat.

Stat. Deter.



Stat. Indeter.



2) Elem. picture

3) Global FD at elem level:

$$\underset{4\times 4}{\underline{k}^{(e)}}\ \underset{4\times 1}{\underline{d}^{(e)}} = \underset{4\times 1}{\underline{f}^{(e)}}$$

elem stiffness matrix for elem e

e = 1, 2

elem disp mat. of elem e

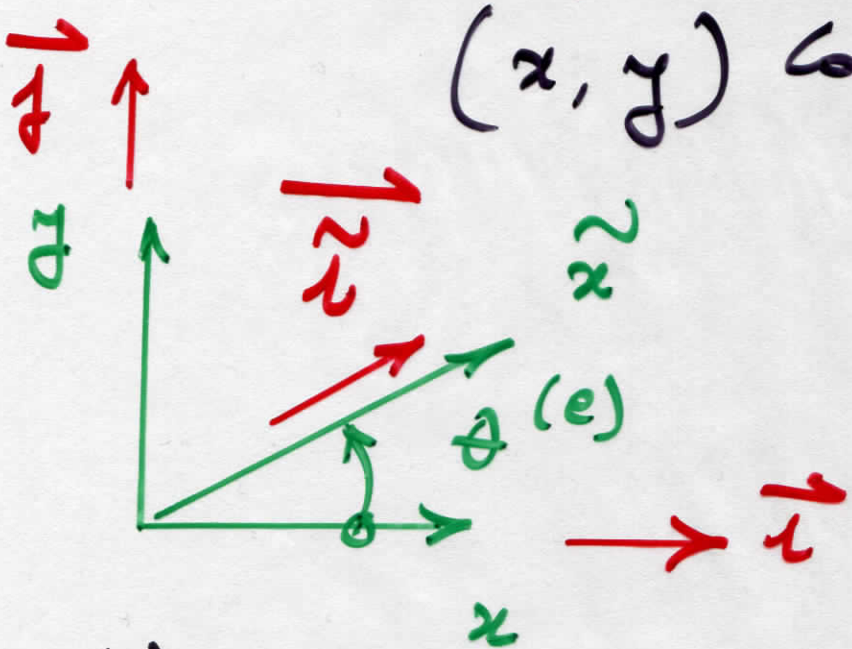
elem force mat.

Book, p. 225, 2nd eq. from bottom:

$$k^{(e)} \begin{array}{c} \begin{array}{cccc} d_1^{(e)} & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} (\ell^{(e)})^2 & \ell^{(e)} m^{(e)} & -(\ell^{(e)})^2 & -\ell^{(e)} m^{(e)} \\ \ell^{(e)} m^{(e)} & (m^{(e)})^2 & -\ell^{(e)} m^{(e)} & -(m^{(e)})^2 \\ -(\ell^{(e)})^2 & -\ell^{(e)} m^{(e)} & (\ell^{(e)})^2 & \ell^{(e)} m^{(e)} \\ -\ell^{(e)} m^{(e)} & -(m^{(e)})^2 & \ell^{(e)} m^{(e)} & (m^{(e)})^2 \end{bmatrix}_{4\times 4} \end{array} = \underline{k}^{(e)}$$

$$k^{(e)} = \frac{E^{(e)} A^{(e)}}{L^{(e)}}$$ axial stiffness of bar elem "e"
$e = 1, 2$

$l^{(e)}, m^{(e)}$ = director cosines of $\tilde{x}$ axis (goes from $\boxed{1}$ to $\boxed{2}$) wrt global (x, y) coord.



$$l^{(e)} = \vec{\tilde{i}} \cdot \vec{i} = \cos\theta^{(e)}$$

$$m^{(e)} = \vec{\tilde{i}} \cdot \vec{j} = \cos\left(\frac{\pi}{2} - \theta^{(e)}\right)$$

$$= \sin\theta^{(e)}$$

$$\vec{\tilde{i}} = \cos\theta^{(e)} \vec{i} + \sin\theta^{(e)} \vec{j}$$

$$\vec{\tilde{i}} \cdot \vec{i} = \cancel{A} \cdot \left(\cos\theta^{(e)} \, \vec{i} + \sin\theta^{(e)} \, \vec{j} \right)^{\text{6-3}} \cdot \vec{i}$$

$$= \cos\theta^{(e)} \, \underbrace{\vec{i} \cdot \vec{i}}_{1} + \sin\theta^{(e)} \, \underbrace{\vec{j} \cdot \vec{i}}_{0}$$

$$\vec{\tilde{i}} \cdot \vec{j} = \text{HW}$$

HW: Write $\underline{k}^{(1)}$, $\underline{k}^{(2)}$: provide numerical values for all coeff.

Note: The director cosines are the components of $\vec{\tilde{i}}$ (unit vector along $\tilde{x}$ axis) wrt basis $(\vec{i}, \vec{j})$

$$\vec{\tilde{i}} = \underbrace{\cos\theta^{(e)}}_{\ell^{(e)}} \, \vec{i} + \underbrace{\sin\theta^{(e)}}_{m^{(e)}} \, \vec{j}$$

Mtg 7: Wed, 10 Sep 08. EML 4500

Model 2-bar truss syst (Cont'd)

Elem 1: $\theta^{(1)} = 30°$

$$\ell^{(1)} = \cos\theta^{(1)} = \cos 30° = \frac{\sqrt{3}}{2}$$

$$m^{(1)} = \sin\theta^{(1)} = \sin 30° = \frac{1}{2}$$

$$k^{(1)} = \frac{E^{(1)}A^{(1)}}{L^{(1)}} = \frac{(3)(1)}{4} = \frac{3}{4}$$

$$\underline{k}^{(1)} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} k_{11}^{(1)} & k_{12}^{(1)} & k_{13}^{(1)} & k_{14}^{(1)} \\ k_{21}^{(1)} & k_{22}^{(1)} & k_{23}^{(1)} & k_{24}^{(1)} \\ k_{31}^{(1)} & k_{32}^{(1)} & k_{33}^{(1)} & k_{34}^{(1)} \\ k_{41}^{(1)} & \cdot & \cdot & k_{44}^{(1)} \end{bmatrix}_{4\times4} \end{array}$$

$$= \left[k_{ij}^{(e)} \right]_{4\times4} \qquad i = 1, 2, \ldots, 4 \qquad j = 1, \ldots, 4$$

row $\rightarrow i$, $j \leftarrow$ col

7.2

$$k^{(1)}_{11} = k^{(1)}\left(l^{(1)}\right)^2 = \left(\frac{3}{4}\right)\left(\frac{\sqrt{3}}{2}\right)^2 = \frac{9}{16}$$

row col

$$k^{(1)}_{12} = k^{(1)}\left(l^{(1)} m^{(1)}\right) = \frac{3\sqrt{3}}{16}$$ (Megan)

$$k^{(1)}_{42} = -k^{(1)}\left(m^{(1)}\right)^2 = -\frac{3}{16}$$ (Megan)

<u>Obs:</u> 1) Only need to compute 3 numbers. Other coeffs have same abs value, just differ by + or −.

2) Mat. $\underline{k}^{(1)}$ is symmetric, i.e.,

$$k^{(1)}_{ij} = k^{(1)}_{ji} \quad \text{e.g.}$$

$$k^{(1)}_{13} = k^{(1)}_{31}$$ (just interchange the row and col. indices)

In general, $k_{ij}^{(e)} = k_{ji}^{(e)}$

or $\underline{k}^{(e)T} = \underline{k}^{(e)}$

(transpose)

the transpose of $\underline{k}^{(e)}$ is equal to $\underline{k}^{(e)}$

$$\underline{k}^{(e)} = \begin{bmatrix} k_{11}^{(e)} & k_{12}^{(e)} & k_{13}^{(e)} & k_{14}^{(e)} \\ & k_{22}^{(e)} & k_{23}^{(e)} & k_{24}^{(e)} \\ & \text{Sym.} & k_{33}^{(e)} & k_{34}^{(e)} \\ & & & k_{44}^{(e)} \end{bmatrix}$$

upper trian-gular part

L7-4

Elem 2: $k^{(2)} = \dfrac{E^{(2)} A^{(2)}}{L^{(2)}} = \dfrac{(5)(2)}{2} = 5$

$$\theta^{(2)} = -\frac{\pi}{4} \Rightarrow l^{(2)} = \cos\left(-\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$

$$m^{(2)} = \sin\left(-\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$$

$$\underline{k}^{(2)} = \left[k_{ij}^{(2)} \right]_{4 \times 4}$$

$$k_{11}^{(2)} = k^{(2)} \left(l^{(2)}\right)^2 = (5)\left(\frac{\sqrt{2}}{2}\right)^2 = 2.5$$

Obs: 1) Abs. values of all coeffs $k_{ij}^{(e)}$, $e = 2$, $(i,j) = 1, \ldots, 4$, are the same $\Rightarrow$ comp. 1 coeff. For other coeff, add $+$ or $-$.

(Luke)

2) $\underline{k}^{(2)T} = \underline{k}^{(2)}$, i.e., $\underline{k}^{(2)}$ sym.

Elem FD rel. : $\boxed{\underset{4\times4}{\underline{k}^{(e)}}\;\underset{4\times1}{\underline{d}^{(e)}} = \underset{4\times1}{\underline{f}^{(e)}}}$ 7-5

$e = 1, 2$

$$\underline{d}^{(e)} = \underset{4\times1}{\begin{Bmatrix} d_1^{(e)} \\ \vdots \\ d_4^{(e)} \end{Bmatrix}} \qquad \underline{f}^{(e)} = \underset{4\times1}{\begin{Bmatrix} f_1^{(e)} \\ \vdots \\ f_4^{(e)} \end{Bmatrix}}$$

Global FD rel : ("free-free" struct)

$$\boxed{\underset{n\times n}{\underline{K}}\;\underset{n\times1}{\underline{d}} = \underset{n\times1}{\underline{F}}}$$

Here, $n = 6$ (p. 5-5)

$$\begin{array}{c} \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \\ \begin{bmatrix} K_{11} & K_{12} & \cdots & \cdots & & K_{16} \\ \vdots & & & & & \vdots \\ \vdots & & & & & \vdots \\ K_{61} & \cdots & \cdots & & & K_{66} \end{bmatrix} \end{array} \begin{Bmatrix} d_1 \\ \vdots \\ \vdots \\ d_6 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ \vdots \\ F_6 \end{Bmatrix}$$

Cont'd from p. 7-5:

In compact notation:

$$\underbrace{[K_{ij}]}_{6\times 6}\{d_j\}_{6\times 1} = \{F_i\}_{6\times 1}$$

(more generally $n \times n$)

$$\sum_{j=1}^{6} K_{ij} d_j = F_i \, , \quad i = 1, \ldots, 6$$

$$\underset{n\times n}{\underline{K}} = [K_{ij}]_{n\times n} = \text{global stiffness mat.}$$

$$\underset{n\times 1}{\underline{d}} = \{d_j\}_{n\times 1} = \text{global disp. mat.}$$

$$\underset{n\times 1}{\underline{F}} = \{F_i\}_{n\times 1} = \text{global force mat.}$$

Recall elem FD rel. (p. 7-5)

$$\boxed{\underline{k}^{(e)}_{4\times4}\,\underline{d}^{(e)}_{4\times1} = \underline{f}^{(e)}_{4\times1}}$$

$$\underline{k}^{(e)}_{4\times4} = \left[k^{(e)}_{ij} \right] = \text{elem stiff. mat.}$$

$$\underline{d}^{(e)}_{4\times1} = \left\{ d^{(e)}_{j} \right\}_{4\times1} = \text{elem disp. mat.}$$

$$\underline{f}^{(e)}_{4\times1} = \left\{ f^{(e)}_{i} \right\}_{4\times1} = \text{elem force mat.}$$

How to go from elem matrices (stiff, disp, force) to global mat?

Through an assembly process:

* Identify the correspondence betw. elem disp dofs and global disp. dofs.

Global level : p. 5-5

$\{d_1, d_2, \dots, d_6\}$

Elem level : p. 4-3 , p. 4-2

* Elem 1 : $\{d_1^{(1)}, d_2^{(1)}, d_3^{(1)}, d_4^{(1)}\}$

* Elem 2 : $\{d_1^{(2)}, \dots, d_4^{(2)}\}$

Identification $\text{global-}\underbrace{\text{local}}_{\text{elem}}$ dofs :

$$\left.\begin{array}{l} d_1 = d_1^{(1)} \\ d_2 = d_2^{(1)} \end{array}\right\} \text{node } 1$$

elem 1

1st dof

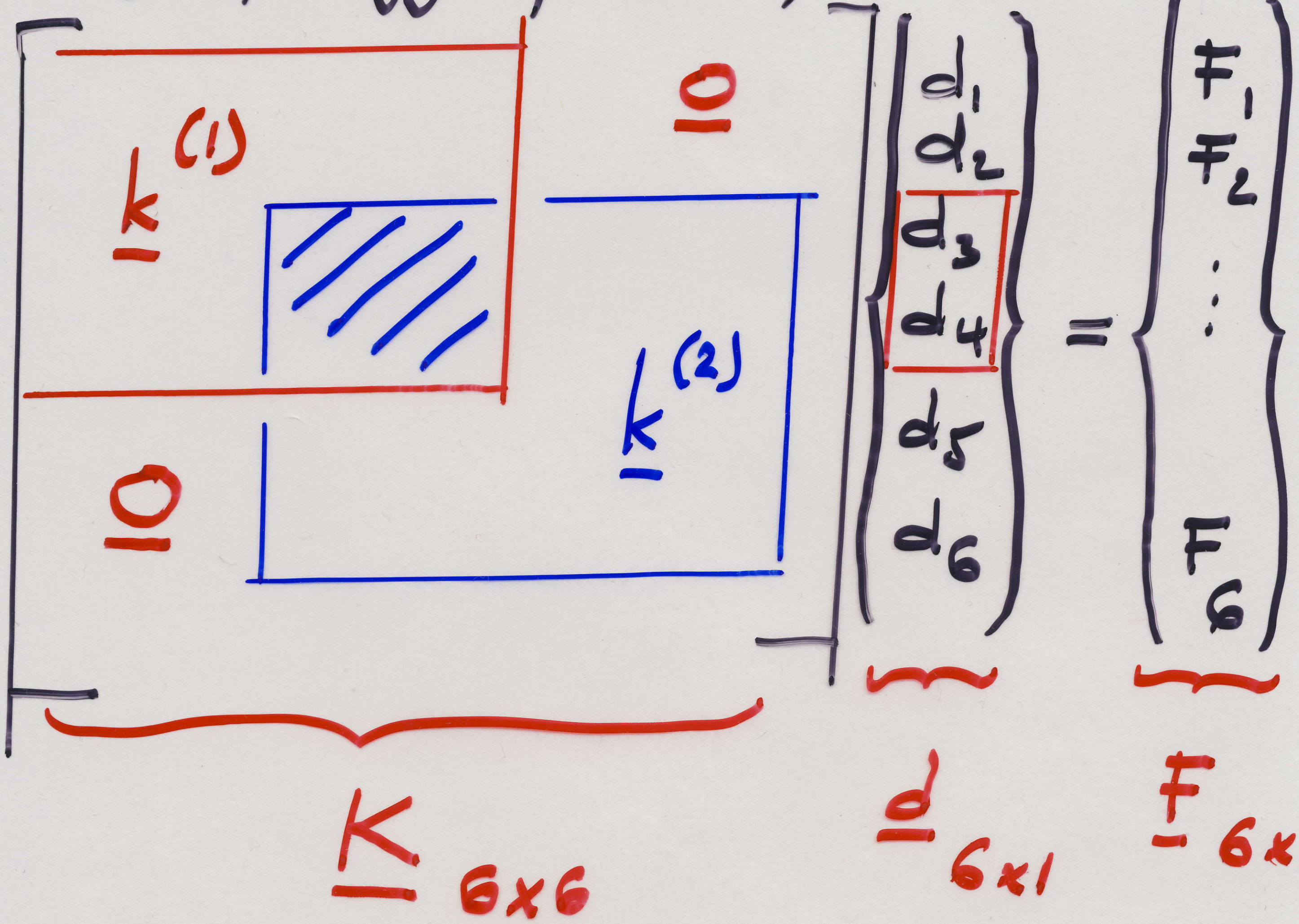
1st global dof

global

$$\left.\begin{array}{l} d_3 = d_3^{(1)} = d_1^{(2)} \\ d_4 = d_4^{(1)} = d_2^{(2)} \end{array}\right\} \text{node } 2$$

$$\left.\begin{aligned} d_5 &= d_3^{(2)} \\ d_6 &= d_4^{(2)} \end{aligned}\right\} \text{node } 3$$

Conceptual step of assembly:
(topology of $\underline{K}$)



$$
\underbrace{
\begin{array}{cccccc}
1 & 2 & 3 & 4 & 5 & 6 \\
\left[\begin{array}{cccccc}
k^{(1)}_{11} & k^{(1)}_{12} & k^{(1)}_{13} & k^{(1)}_{14} & & \\
k^{(1)}_{21} & k^{(1)}_{22} & k^{(1)}_{23} & k^{(1)}_{24} & & \\
k^{(1)}_{31} & k^{(1)}_{32} & (k^{(1)}_{33}+k^{(2)}_{11}) & (k^{(1)}_{34}+k^{(2)}_{12}) & k^{(2)}_{13} & k^{(2)}_{14} \\
k^{(1)}_{41} & k^{(1)}_{42} & (k^{(1)}_{43}+k^{(2)}_{21}) & (k^{(1)}_{44}+k^{(2)}_{22}) & k^{(2)}_{23} & k^{(2)}_{24} \\
 & & k^{(2)}_{31} & k^{(2)}_{32} & k^{(2)}_{33} & k^{(2)}_{34} \\
 & & k^{(2)}_{41} & k^{(2)}_{42} & k^{(2)}_{43} & k^{(2)}_{44}
\end{array}\right]
\end{array}
}_{\underline{K}_{6\times 6}}
$$

Mtg 9: Mon, 15 Sep 08, EML 4500

$$K_{11} = k_{11}^{(1)} = \frac{9}{16}$$

$$K_{12} = k_{12}^{(1)} = \frac{3\sqrt{3}}{16}$$

...

$$K_{33} = k_{33}^{(1)} + k_{11}^{(2)} = \frac{9}{16} + \frac{5}{2} = 3.0625$$

$$K_{34} = k_{34}^{(1)} + k_{12}^{(2)} = \frac{3\sqrt{3}}{16} + \left(-\frac{5}{2}\right)$$

$$\begin{array}{ccccc} K_{43} = K_{34} & = & k_{43}^{(1)} & + & k_{21}^{(2)} \\ \parallel & & \parallel & & \parallel \\ -2.1752 & & k_{34}^{(1)} & & k_{12}^{(2)} \end{array}$$

$$K_{44} = k_{44}^{(1)} + k_{22}^{(2)} = \frac{3}{16} + \frac{5}{2}$$

$$= 2.6875$$

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4) Elimination of known dofs $\Rightarrow$ reduce global FD rel. (p. 5-3)

$$\boxed{d_1 = d_2 = d_5 = d_6 = 0}$$