

# Monad P3 : Existential Types (1D)

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# Based on

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Haskell in 5 steps

[https://wiki.haskell.org/Haskell\\_in\\_5\\_steps](https://wiki.haskell.org/Haskell_in_5_steps)

# Three different usages for **forall**

Basically, there are 3 different common uses for the forall keyword (or at least so it seems), and each has its own Haskell extension:

## ScopedTypeVariables

specify types for code inside **where** clauses

## RankNTypes/Rank2Types,

The type is labeled "**Rank-N**" where N is the number of **forall**s which are nested and cannot be merged with a previous one.

## ExistentialQuantification

<https://stackoverflow.com/questions/3071136/what-does-the-forall-keyword-in-haskell-ghc-do>

# Existential Quantification

<https://stackoverflow.com/questions/3071136/what-does-the-forall-keyword-in-haskell-ghc-do>

# Hiding a type variable (1)

Normally when creating a new type using **type**, **newtype**, **data**, etc., every **type variable** that appears on the right-hand side must also appear on the left-hand side.

```
newtype ST s a = ST (State# s -> (# State# s, a #))
```

**Existential types** are a way of escaping

Existential types can be used for several different purposes. But what they do is to **hide a type variable** on the right-hand side.

[https://wiki.haskell.org/Existential\\_type](https://wiki.haskell.org/Existential_type)

# Hiding a type variable (2)

Normally, any type variable appearing on the right must also appear on the left:

```
data Worker x y = Worker {buffer :: b, input :: x, output :: y}
```

This is an **error**, since the **type b** of the **buffer** is not specified on the right (**b** is a **type variable** rather than a **type**) but also is not specified on the left (there's no **b** in the left part).

In Haskell98, you would have to write

```
data Worker b x y = Worker {buffer :: b, input :: x, output :: y}
```

[https://wiki.haskell.org/Existential\\_type](https://wiki.haskell.org/Existential_type)

# Hiding a type variable (3)

However, suppose that a **Worker** can use any type **b** so long as it belongs to some particular class.

Then every **function** that uses a **Worker** will have a type like

```
foo :: (Buffer b) => Worker b Int Int
```

In particular, failing to write an **explicit type signature** `(Buffer b)` will invoke the dreaded monomorphism restriction.

Using **existential types**, we can avoid this:

[https://wiki.haskell.org/Existential\\_type](https://wiki.haskell.org/Existential_type)



# Hiding a type variable (4)

Using existential type :

```
data Worker x y = forall b. Buffer b => Worker {buffer :: b, input :: x, output :: y}
foo :: Worker Int Int
```

The **type** of the **buffer** (**Buffer**) now does not appear in the **Worker** type at all.    **Worker x y**

Explicit type signature :

```
data Worker b x y = Worker {buffer :: b, input :: x, output :: y}
foo :: (Buffer b) => Worker b Int Int
```

[https://wiki.haskell.org/Existential\\_type](https://wiki.haskell.org/Existential_type)

# Hiding a type variable (5)

- it is now impossible for a function to demand a **Worker** having a specific type of **buffer**.
- the **type** of **foo** can now be derived automatically without needing an explicit type signature.  
(No monomorphism restriction.)
- since code now has no idea what **type** the buffer function returns, you are more limited in what you can do to it.

```
data Worker x y = forall b. Buffer b => Worker {buffer :: b, input :: x, output :: y}
foo :: Worker Int Int
```

[https://wiki.haskell.org/Existential\\_type](https://wiki.haskell.org/Existential_type)

# Hiding a type variable (6)

In general, when you use a **hidden type** in this way, you will usually want that **type** to belong to a **specific class**, or you will want to **pass some functions** along that can work on that type.

Otherwise you'll have some value belonging to a **random unknown type**, and you won't be able to do anything to it!

[https://wiki.haskell.org/Existential\\_type](https://wiki.haskell.org/Existential_type)

# Less specific types (1)

Note: You can use **existential types** to **convert** a **more specific type** into a **less specific one**.

**constrained type variables**

There is no way to perform the reverse conversion!

[https://wiki.haskell.org/Existential\\_type](https://wiki.haskell.org/Existential_type)

## Less specific types (2)

This illustrates **creating a heterogeneous list**,  
all of whose members implement "**Show**",  
and progressing through that list to show these items:

```
data Obj = forall a. (Show a) => Obj a
```

```
xs :: [Obj]
```

```
xs = [Obj 1, Obj "foo", Obj 'c']
```

```
doShow :: [Obj] -> String
```

```
doShow [] = ""
```

```
doShow ((Obj x):xs) = show x ++ doShow xs
```

With output: `doShow xs ==> "1\"foo\"'c'"`

[https://wiki.haskell.org/Existential\\_type](https://wiki.haskell.org/Existential_type)

# Existentials in terms of forall (1)

It is also possible to express existentials with RankNTypes as **type expressions** directly (without a **data** declaration)

```
forall r. (forall a. Show a => a -> r) -> r
```

(the leading forall r. is optional unless the expression is part of another expression).

the equivalent type **Obj** :

```
data Obj = forall a. (Show a) => Obj a
```

[https://wiki.haskell.org/Existential\\_type](https://wiki.haskell.org/Existential_type)

# Existentials in terms of forall (2)

The conversions are:

**fromObj :: Obj -> forall r. (forall a. Show a => a -> r) -> r**

**fromObj (Obj x) k = k x**

**toObj :: (forall r. (forall a. Show a => a -> r) -> r) -> Obj**

**toObj f = f Obj**

[https://wiki.haskell.org/Existential\\_type](https://wiki.haskell.org/Existential_type)

# Existentials

**Existential types**, or '**existentials**' for short, provide a way of 'squashing' a group of types into one, single type.

**Existentials** are part of GHC's type system **extensions**.

They aren't part of Haskell98, and as such you'll have

to either compile any code that contains them

with an extra command-line parameter of

**`-XExistentialQuantification`**,

or put at the top of your sources that use existentials.

**`{-# LANGUAGE ExistentialQuantification #-}`**

[https://en.wikibooks.org/wiki/Haskell/Existentially\\_quantified\\_types](https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types)



# forall type variables

Example: A polymorphic function

```
map :: (a -> b) -> [a] -> [b]
```

Example: Explicitly quantifying the type variables

```
map :: forall a b. (a -> b) -> [a] -> [b]
```

**instantiating** the general type of map to a more specific type

**a = Int** and **b = String**

```
(Int -> String) -> [Int] -> [String]
```

Example: Two equivalent type statements

```
id :: a -> a
```

```
id :: forall a . a -> a
```

[https://en.wikibooks.org/wiki/Haskell/Existentially\\_quantified\\_types](https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types)

# Type hider

Suppose we have a group of values.

they may not be all the same **type**,

but they are all **members** of some **class**

thus, they have a certain **property**

It might be useful to throw all these **values** into a **list**.

normally this is impossible because **lists elements**

must be of **the same type**

(**homogeneous** with respect to **types**).

**existential types** allow us to loosen this requirement

by defining a **type hider** or **type box**:

```
data ShowBox = forall s. Show s => SB s
```

```
heteroList :: [ShowBox]
```

```
heteroList = [SB (), SB 5, SB True]
```

[https://en.wikibooks.org/wiki/Haskell/Existentially\\_quantified\\_types](https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types)

# Heterogeneous list example (1)

```
data ShowBox = forall s. Show s => SB s
```

```
-- type hider
```

```
heteroList :: [ShowBox]
```

```
heteroList = [SB (), SB 5, SB True]
```

[SB (), SB 5, SB True] calls the **constructor** on three values of different types, to place them all into a single list virtually **the same type** for each one.

Use the **forall** in the constructor

```
SB :: forall s. Show s => s -> ShowBox.
```

[https://en.wikibooks.org/wiki/Haskell/Existentially\\_quantified\\_types](https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types)

# Heterogeneous list example (2)

```
data ShowBox = forall s. Show s => SB s
heteroList :: [ShowBox]
heteroList = [SB (), SB 5, SB True]
```

When passing **heteroList** type parameters to a function  
we cannot take out the **values** inside the **SB**  
because their type might **Bool**, **Int**, **Char**, ...

**But each of the elements can be**  
converted to a **string** via **show**.

In fact, that's the only thing we know about them.

[https://en.wikibooks.org/wiki/Haskell/Existentially\\_quantified\\_types](https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types)

# Heterogeneous list example (3)

```
instance Show ShowBox where
```

```
  show (SB s) = show s
```

```
f :: [ShowBox] -> IO ()
```

```
f xs = mapM_ print xs
```

```
main = f heteroList
```

[https://en.wikibooks.org/wiki/Haskell/Existentially\\_quantified\\_types](https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types)

# Heterogeneous list example (4)

Example: Using our heterogeneous list

```
instance Show ShowBox where
  show (SB s) = show s      -- (*) see the comment in the text below
f :: [ShowBox] -> IO ()
f xs = mapM_ print xs
main = f heteroList
```

Example: Types of the functions involved

```
print :: Show s => s -> IO ()      -- print x = putStrLn (show x)
mapM_  :: (a -> m b) -> [a] -> m ()
mapM_ print :: Show s => [s] -> IO ()
```

[https://en.wikibooks.org/wiki/Haskell/Existentially\\_quantified\\_types](https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types)

# mapM, mapM\_, and map (1)

The core idea is that **mapM** maps an "action" (ie function of type **a -> m b**) over a list and gives you **all** the results as **m [b]**

**mapM\_** does the same thing, but never collects the results, returning a **m ()**.

If you care about the **results** of your **a -> m b** function, use **mapM**.  
If you only care about the **effect**, but not the resulting value, use **mapM\_**, because it can be more **efficient**

<https://stackoverflow.com/questions/27609062/what-is-the-difference-between-mapm-and-mapm-in-haskell/27609146>

# mapM, mapM\_, and map (2)

Always use **mapM\_** with functions of the type **a -> m ()**,  
like **print** or **putStrLn**.  
these functions return **()** to signify that only the **effect** matters.

If you used **mapM**, you'd get **a list of ()** (ie **[(), (), ()]**),  
which would be completely useless  
but waste some memory.

If you use **mapM\_**, you would just get a **()**,  
but it would still print everything.

<https://stackoverflow.com/questions/27609062/what-is-the-difference-between-mapm-and-mapm-in-haskell/27609146>



# mapM, mapM\_, and map (3)

Normal **map** is something different:

it takes a normal function (**a -> b**)

instead of one using a monad (**a -> m b**).

This means that it cannot have any sort of **effect**

besides returning the **changed list**.

You would use it if you want to **transform a list**

using a normal function.

**map\_** doesn't exist because, since you don't have any effects,  
you always care about the **results** of using **map**.

<https://stackoverflow.com/questions/27609062/what-is-the-difference-between-mapm-and-mapm-in-haskell/27609146>

# A set of possible values

One way to think about **forall** is to consider **types** as a set of possible values.

**Bool** is the set **{True, False, ⊥}**  
(remember that **bottom**, **⊥**, is a member of every type!),

**Integer** is the set of integers (and bottom),

**String** is the set of all possible strings (and bottom), and so on.

[https://en.wikibooks.org/wiki/Haskell/Existentially\\_quantified\\_types](https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types)

# Intersection of the specified types

**forall** serves as a way to assert a **commonality** or **intersection** of the specified types (i.e. sets of values).

**forall a. a** is the **intersection** of all types.

this **subset** turns out to be the set  $\{\perp\}$ ,

since it is an **implicit value in every type**.

that is, [the **type** whose **only available value is bottom**]

However, since **every Haskell type includes bottom,  $\{\perp\}$** ,  
this quantification in fact stipulates all Haskell types.

But the only permissible operations on it are

those available to [a **type** whose **only available value is bottom**]

[https://en.wikibooks.org/wiki/Haskell/Existentially\\_quantified\\_types](https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types)

# A list of bottoms type (1)

1. The list `[forall a. a]`
2. The list `[forall a. Show a => a]`
3. The list `[forall a. Num a => a]`
4. The list `forall a. [a]`

a list of bottoms. `[⊥]`, `[⊥, ⊥]`, ...

[https://en.wikibooks.org/wiki/Haskell/Existentially\\_quantified\\_types](https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types)

# A list of bottoms type (2)

The list, `[forall a. a]`, is the **type of a list**  
whose **elements** all have the type `forall a. a`, i.e.  
**a list of bottoms**. `[⊥]`, `[⊥,⊥]`, ...

The list, `[forall a. Show a => a]`, is the **type of a list**  
whose **elements** all have the type `forall a. Show a => a`.

the **Show** class constraint requires the possible types  
also to be **a member of the class, Show**.

However, `⊥` is still the only value common to all these types, `{⊥}`,  
so this too is **a list of bottoms**. `[forall a. a]`

[https://en.wikibooks.org/wiki/Haskell/Existentially\\_quantified\\_types](https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types)

# A list of bottoms type (3)

The list, `[forall a. Num a => a]`, requires each element to be a member of the class, Num.

Consequently, the possible values include **numeric literals**, which have the specific type, `forall a. Num a => a`, as well as **bottom**.

`forall a. [a]` is the type of **the list** whose elements all have the same type **a**.

since we cannot presume any particular type at all, this too is **a list of bottoms**.

[https://en.wikibooks.org/wiki/Haskell/Existentially\\_quantified\\_types](https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types)

# Intersections over types

most **intersections over types** just lead to **bottoms**  $\perp\perp\perp\perp$   
**types** generally don't have **any values in common**  
presumptions cannot be made about a **union of their values**.

a **heterogeneous list** using a **type hider**  
**type hider'** functions as a **wrapper type**  
which guarantees certain facilities  
by implying a **predicate** or **constraint** on the permissible **types**.

the purpose of **forall** is to **impose type constraint**  
on the permissible types within a **type declaration**  
guaranteeing certain facilities with such types.

```
data ShowBox = forall s. Show s => SB s
heteroList :: [ShowBox]
heteroList = [SB (), SB 5, SB True]
```

[https://en.wikibooks.org/wiki/Haskell/Existentially\\_quantified\\_types](https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types)

# Summary of heterogeneous list examples (1)

An **existential datatype**

```
data T = forall a. MkT a
```

This defines a **polymorphic constructor**,  
or a family of constructors for **T**

```
MkT :: forall a. (a -> T)
```

**Pattern matching** on our existential constructor

```
foo (MkT x) = ... -- what is the type of x?
```

Constructing the **heterogeneous list**

```
heteroList = [MkT 5, MkT (), MkT True, MkT map]
```

```
data ShowBox = forall s. Show s => SB s
```

```
heteroList :: [ShowBox]
```

```
heteroList = [SB (), SB 5, SB True]
```

[https://en.wikibooks.org/wiki/Haskell/Existentially\\_quantified\\_types](https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types)



# Summary of heterogeneous list examples (2)

A new existential data type, with a **class constraint**

```
data T' = forall a. Show a => MkT' a
```

```
data T = forall a. MkT a
```

Using our new heterogeneous setup

```
heteroList' = [MkT' 5, MkT' (), MkT' True, MkT' "Sartre"]
```

```
main = mapM_ (\(MkT' x) -> print x) heteroList'
```

```
{- prints:
```

```
5
```

```
()
```

```
True
```

```
"Sartre"
```

```
-}
```

[https://en.wikibooks.org/wiki/Haskell/Existentially\\_quantified\\_types](https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types)

# State and ST monads

the **ST** monad is essentially

a more powerful version of the **State** monad:

It was originally written to provide Haskell with **IO**.

**IO** is basically just a **State** monad

with an environment of all the information about the real world.

In fact, inside GHC at least, **ST** is used,

and the environment is a **type** called **RealWorld**.

To get out of the **State** / **ST** monad,

use **runState** / **runST**

[https://en.wikibooks.org/wiki/Haskell/Existentially\\_quantified\\_types](https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types)

# runST – rank-2 polymorphism

```
runST :: forall a. (forall s. ST s a) -> a
```

This is actually an example of **rank-2 polymorphism**

a **forall** appearing within the **left-hand side** of (->)  
cannot be moved up, and therefore forms **another level or rank**  
therefore, there are **2 levels** of universal quantification.

[https://en.wikibooks.org/wiki/Haskell/Existentially\\_quantified\\_types](https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types)

# runST – initial state

```
runST :: forall a. (forall s. ST s a) -> a
```

there is **no parameter** for the **initial state** ... **s**

Indeed, **ST** uses a different notion of state to **State**;

**State** allows you to **get** and **put** the *current state*,

**ST** provides an **interface** to **references**

[https://en.wikibooks.org/wiki/Haskell/Existentially\\_quantified\\_types](https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types)

# runST – reference interfaces

To create references of the type **STRef**

**newSTRef** :: a -> ST s (STRef s a)

To provide an **initial value**

**readSTRef** :: STRef s a -> ST s a

To manipulate them.

**writeSTRef** :: STRef s a -> a -> ST s ()

**runST** :: forall a. (forall s. ST s a) -> a

[https://en.wikibooks.org/wiki/Haskell/Existentially\\_quantified\\_types](https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types)

# runST – a mapping

the **internal environment** of a **ST** computation

is not one specific value,

but a **mapping** from **references** to **values**.

... (STRef s a)

**newSTRef** :: a -> ST s (STRef s a)

No need to provide an **initial state** to **runST**,

as the **initial state** is just the **empty mapping**

... ()

containing **no references**.

**runST** :: forall a. (forall s. ST s a) -> a

[https://en.wikibooks.org/wiki/Haskell/Existentially\\_quantified\\_types](https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types)

# runST – no specific references

It is not allowed

to create a **reference** in one **ST computation**,  
then to use the created **reference** in another **ST computation**.  
for reasons of **thread-safety**

because no ST computation should be allowed  
to assume that the **initial internal environment**  
contains any specific references.

[https://en.wikibooks.org/wiki/Haskell/Existentially\\_quantified\\_types](https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types)

# runST

```
runST :: forall a. (forall s. ST s a) -> a
newSTRef :: a -> ST s (STRef s a)
readSTRef :: STRef s a -> ST s a
```

Example: Bad ST code

```
let v = runST (newSTRef True) ... one ST computation
in runST (readSTRef v) ... another ST computation
```

Example: Briefer bad ST code

```
... runST (newSTRef True) ...
```

```
newSTRef True :: ST s (STRef s a)
runST (newSTRef True) :: STRef s a
v :: STRef s a
```

```
readSTRef v :: ST s a
runST (readSTRef v) :: a
```

[https://en.wikibooks.org/wiki/Haskell/Existentially\\_quantified\\_types](https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types)



# runST

Example: Bad ST code

```
let v = runST (newSTRef True)
in runST (readSTRef v)
```

**runST** :: forall a. (forall s. ST s a) -> a

the **rank-2 polymorphism** in **runST**'s type  
to constrain the scope of the **type variable s**  
to be within the first parameter (the left hand side of ->)

if the **type variable s** appears in the first parameter  
it cannot also appear in the second.  
(the right hand side of ->)

[https://en.wikibooks.org/wiki/Haskell/Existentially\\_quantified\\_types](https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types)

# runST

Example: Briefer bad ST code

```
... runST (newSTRef True) ...
```

Example: The compiler's typechecking stage

```
newSTRef True :: forall s. ST s (STRef s Bool)
```

```
runST :: forall a. (forall s. ST s a) -> a
```

```
runST (newSTRef True) ::
```

```
(forall s. ST s (STRef s Bool)) -> STRef s Bool
```

```
runST :: forall a. (forall s. ST s a) -> a
```

```
newSTRef :: a -> ST s (STRef s a)
```

```
readSTRef :: STRef s a -> ST s a
```

[https://en.wikibooks.org/wiki/Haskell/Existentially\\_quantified\\_types](https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types)

# forall

The importance of the forall in the first bracket is that we can change the name of the s.

`runST (newSTRef True) ::`

`(forall s. ST s (STRef s Bool)) -> STRef s Bool`

Example: A type mismatch!

`(forall s'. ST s' (STRef s' Bool)) -> STRef s Bool`

This is similar to  $\forall x . x > 5$  is precisely the same as  $\forall y . y > 5$  giving the variable a different label.

[https://en.wikibooks.org/wiki/Haskell/Existentially\\_quantified\\_types](https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types)

# forall

Example: A type mismatch!

```
(forall s'. ST s' (STRef s' Bool)) -> STRef s Bool
```

Notice that as the `forall` does not scope over the return type of `runST`, `STRef s Bool` we don't rename the `s` there as well.

But suddenly, we've got a **type mismatch!**

The result type of the ST computation in the **first parameter** must match the **result type** of `runST`, but now it doesn't!

[https://en.wikibooks.org/wiki/Haskell/Existentially\\_quantified\\_types](https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types)

# forall

The key feature of the existential is that it allows the compiler to generalise the type of the state in the first parameter, and so the result type cannot depend on it.

This neatly sidesteps our **dependence problems**, and '**compartmentalises**' each call to **runST** into **its own little heap**, with **references** not being able to be shared between different **calls**.

[https://en.wikibooks.org/wiki/Haskell/Existentially\\_quantified\\_types](https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types)

# Quantified Types as Products and Sums

A universally quantified type may be interpreted as an infinite product of types.

Example: Identity function

```
id :: forall a. a -> a
```

```
id a = a
```

a polymorphic function can be understood as a product, or a tuple, of individual functions, one per every possible type  $a$ .

To construct a value of such type, we have to provide all the components of the tuple at once.

[https://en.wikibooks.org/wiki/Haskell/Existentially\\_quantified\\_types](https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types)

# Quantified Types as Products and Sums

In case of function types, this is possible because of polymorphism  
-- one formula generating an infinity of functions.

Example: Polymorphic value

**x :: forall a. Num a => a**

**x = 0**

x may be conceptualized as a tuple consisting  
of an Int value, a Double value, etc.

[https://en.wikibooks.org/wiki/Haskell/Existentially\\_quantified\\_types](https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types)

# Quantified Types as Products and Sums

Similarly, an existentially quantified type may be interpreted as an infinite sum. For instance,

Example: Existential type

```
data ShowBox = forall s. Show s => SB s
```

may be conceptualized as a sum:

Example: Sum type

```
data ShowBox = SBUnit | SBInt Int | SBBool Bool | SBIntList [Int] | ...
```

to construct a value of this type,

we only have to pick one of the constructors.

A polymorphic constructor SB combines all those constructors into one.

[https://en.wikibooks.org/wiki/Haskell/Existentially\\_quantified\\_types](https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types)



# forall

```
{-# LANGUAGE ExistentialQuantification, RankNTypes #-}
```

```
newtype Pair a b = Pair (forall c. (a -> b -> c) -> c)
```

```
makePair :: a -> b -> Pair a b
```

```
makePair a b = Pair $ \f -> f a b
```

[https://en.wikibooks.org/wiki/Haskell/Existentially\\_quantified\\_types](https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types)

# forall

```
λ> :set -XExistentialQuantification
λ> :set -XRankNTypes
λ> newtype Pair a b = Pair {runPair :: forall c. (a -> b -> c) -> c}
λ> makePair a b = Pair $ \f -> f a b
λ> pair = makePair "a" 'b'

λ> :t pair
pair :: Pair [Char] Char

λ> runPair pair (\x y -> x)
"a"

λ> runPair pair (\x y -> y)
'b'
```

[https://en.wikibooks.org/wiki/Haskell/Existentially\\_quantified\\_types](https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types)

# forall – quantifier (1)

quantifier in predicate calculus

type quantifier polymorphic types

to encode a type in **type isomorphism**

**Isomorphism**

from . to = id

<https://stackoverflow.com/questions/3071136/what-does-the-forall-keyword-in-haskell-ghc-do>

# forall – quantifier (2) type isomorphism

the class of **isomorphic types**, i.e. those which can be **cast** to each other without loss of information.

**type isomorphism** is an **equivalence relation** (**reflexive, symmetric, transitive**), but due to the limitations of the type system, only **reflexivity** is implemented for all types

**Isomorphism**

from . to = id

<https://stackoverflow.com/questions/3071136/what-does-the-forall-keyword-in-haskell-ghc-do>

## forall – quantifier (3)

```
foo :: (forall a. a -> a) -> (Char, Bool)
bar :: forall a. ((a -> a) -> (Char, Bool))
```

To call **bar**, any **type a** can be chosen,  
and it is possible to pass a **function** from **type a** to **type a**.

the **function (+1)** or the **function reverse**.

the **forall** is considered to be as saying

"I get to pick the type now". (**instantiating**.)

<https://stackoverflow.com/questions/3071136/what-does-the-forall-keyword-in-haskell-ghc-do>

# forall – quantifier (4)

```
foo :: (forall a. a -> a) -> (Char, Bool)
```

```
bar :: forall a. ((a -> a) -> (Char, Bool))
```

The restrictions on calling **foo** are much more stringent:  
the argument to **foo** must be a **polymorphic function**.

With that type, **the only functions** that can be passed to **foo**  
are **id** or a **function** that always **diverges** or **errors**, like **undefined**.

<https://stackoverflow.com/questions/3071136/what-does-the-forall-keyword-in-haskell-ghc-do>

# forall – quantifier (5)

```
foo :: (forall a. a -> a) -> (Char, Bool)
```

```
bar :: forall a. ((a -> a) -> (Char, Bool))
```

The reason is that with **foo**, the **forall** is to the **left of the arrow**, so as the **caller** of **foo** I don't get to pick what **a** is —rather it's the **implementation** of **foo** that gets to pick what **a** is.

Because **forall** is to the **left of the arrow**, rather than **above the arrow** as in **bar**, the **instantiation** takes place in the **body** of the **function** rather than at the **call** site.

<https://stackoverflow.com/questions/3071136/what-does-the-forall-keyword-in-haskell-ghc-do>

# forall – quantifier (6) above, below, left

Jargon "**above**", "**below**", "**to the left of**".

nothing to do with the *textual ways* types are written  
everything to do with **abstract-syntax trees**.

In the **abstract syntax**,

- a **forall** takes the **name** of a **type variable**,  
and then there is a **full type** "**below**" the **forall**.
- an **arrow** takes **two types** (**argument** and **result type**)  
and forms a **new type** (the **function type**).
- the **argument type** is "**to the left of**" the **arrow**;
- it is the **arrow's left child** in the **abstract-syntax tree**.

<https://stackoverflow.com/questions/3071136/what-does-the-forall-keyword-in-haskell-ghc-do>



# forall – quantifier (7)

**forall** a . [a] -> [a],

the **forall** is **above the arrow**;

what's to the **left of the arrow** is [a].

**forall** n f e x . (**forall** e x . n e x -> f -> Fact x f)

-> Block n e x -> f -> Fact x f

(**forall** e x . n e x -> f -> Fact x f)

the type in parentheses would be called

"a **forall** to the **left of an arrow**".

<https://stackoverflow.com/questions/3071136/what-does-the-forall-keyword-in-haskell-ghc-do>

# foo :: a -> a (1)

**foo :: a -> a**

given this type signature, there is only one function  
that can satisfy this type and  
the identity function **id**.

**foo 5 = 6**

**foo True = False**

they both satisfy the above type signature,  
then why do Haskell folks claim  
that it is **id** alone which satisfies the type signature?

<https://stackoverflow.com/questions/3071136/what-does-the-forall-keyword-in-haskell-ghc-do>

## foo :: a -> a (2)

That is because there is an implicit forall hidden in the type signature.

```
id :: forall a. a -> a
```

*Constraints liberate, liberties constrain*

A **constraint** at the **type level**,  
becomes a **liberty** at the **term level**

A **liberty** at the **type level**,  
becomes a **constraint** at the **term level**

<https://stackoverflow.com/questions/3071136/what-does-the-forall-keyword-in-haskell-ghc-do>

## foo :: a -> a (3)

A **constraint** at the **type level**..

So putting a constraint on our type signature

```
foo :: (Num a) => a -> a
```

becomes a **liberty** at the term level gives us the liberty or flexibility to write all of these

```
foo 5 = 6
```

```
foo 4 = 2
```

```
foo 7 = 9
```

...

Same can be observed by constraining a with **any other typeclass** etc

A **constraint** at the **type level**,  
becomes a **liberty** at the **term level**

<https://stackoverflow.com/questions/3071136/what-does-the-forall-keyword-in-haskell-ghc-do>

## foo :: a -> a (4)

foo :: (Num a) => a -> a translates to  
 $\exists a, \text{st } a \rightarrow a, \forall a \in \text{Num}$

### existential quantification

which translates to there exists some instances of **a**  
for which a function of **a -> a**  
and those instances all belong to the set of **Numbers**.

adding a **constraint** (**a** should belong to the set of **Nnumbers**),  
**liberates** the **term** level to have multiple possible implementations.

A **constraint** at the **type level**,  
becomes a **liberty** at the **term level**

<https://stackoverflow.com/questions/3071136/what-does-the-forall-keyword-in-haskell-ghc-do>

# foo :: a -> a (5)

the explanation of **forall**:

So now let us **liberate** the the **function** at the **type** level:

**foo :: forall a. a -> a** translates to:

**$\forall a, a -> a$**

the **implementation** of this type signature

should be such that it is **a -> a** for all circumstances.

A **liberty** at the **type level**, becomes  
a **constraint** at the **term level**

<https://stackoverflow.com/questions/3071136/what-does-the-forall-keyword-in-haskell-ghc-do>

## foo :: a -> a (6)

So now this starts **constraining** us at the **term** level.

We can no longer write

**foo 5 = 7**

because this **implementation** would not satisfy  
when a **Bool** type value is passed to **foo**

this is because

**under all circumstances**  $\forall a, a \rightarrow a$

it should return something of the similar type.

**a** can be a **Char** or a **[Char]** or a custom datatype.

A **liberty** at the **type level**, becomes  
a **constraint** at the **term level**

<https://stackoverflow.com/questions/3071136/what-does-the-forall-keyword-in-haskell-ghc-do>

# foo :: a -> a (7)

$\forall a, a \rightarrow a$       the **liberty** at the **type** level  
foo 5 = 7            a constraint at the **term** level  
                         (impossible implementation)

this **liberty** at the **type** level is what is known  
as **Universal Quantification**

the **only** **function** which can satisfy **foo :: forall a. a -> a**

foo a = a            the **identity** function

A **liberty** at the **type** level, becomes  
a **constraint** at the **term** level

<https://stackoverflow.com/questions/3071136/what-does-the-forall-keyword-in-haskell-ghc-do>



# foo :: a -> a (8)

*Runar Bjarnason titled "Constraints Liberate, Liberties Constrain".*

CONSTRAINTS LIBERATE,  
LIBERTIES CONSTRAIN

Its very important to digest and believe this statement

<https://stackoverflow.com/questions/3071136/what-does-the-forall-keyword-in-haskell-ghc-do>

# RunST (1)

```
runST :: forall a. (forall s. ST s a) -> a
```

**runST** has to be able to produce a **value** of **type a**,  
no matter what **type** we give as **a**.

**runST** uses an **argument** of **type (forall s. ST s a)**  
which certainly must somehow produce the **a**.

**runST** must be able to produce a **value** of **type a**  
no matter what **type** the **implementation** of **runST**  
decides to give as **s**.

<https://stackoverflow.com/questions/3071136/what-does-the-forall-keyword-in-haskell-ghc-do>

# RunST (2)

```
runST :: forall a. (forall s. ST s a) -> a
```

the benefit is that this puts a **constraint** on the **caller** of **runST** in that the **type a** cannot involve the **type s** at all.

you can't pass it a value of type **ST s [s]**, for example.

the implementation of **runST** is **free** to perform **mutation** with the value of **type s**.

The **type guarantees** that this **mutation** is **local** to the implementation of **runST**.

<https://stackoverflow.com/questions/3071136/what-does-the-forall-keyword-in-haskell-ghc-do>

# RunST : rank-2 polymorphic type

```
runST :: forall a. (forall s. ST s a) -> a
```

The **type** of **runST** is an example of a **rank-2 polymorphic type** because the **type** of its **argument** contains a **forall** quantifier.

<https://stackoverflow.com/questions/3071136/what-does-the-forall-keyword-in-haskell-ghc-do>

# Existential Quantification

```
-- test.hs
{-# LANGUAGE ExistentialQuantification #-}
data EQList = forall a. EQList [a]
eqListLen :: EQList -> Int
eqListLen (EQList x) = length x

ghci> :l test.hs
ghci> eqListLen $ EQList ["Hello", "World"]
2
```

<https://stackoverflow.com/questions/3071136/what-does-the-forall-keyword-in-haskell-ghc-do>

# Existential Quantification

```
ghci> :set -XRankNTypes
ghci> length (["Hello", "World"] :: forall a. [a])
  Couldn't match expected type 'a' against inferred type '[Char]'
  ...
```

With Rank-N-Types, forall a meant that your expression must fit all possible as. For example:

```
ghci> length ([] :: forall a. [a])
0
```

<https://stackoverflow.com/questions/3071136/what-does-the-forall-keyword-in-haskell-ghc-do>

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## References

- [1] <ftp://ftp.geoinfo.tuwien.ac.at/navratil/HaskellTutorial.pdf>
- [2] <https://www.umiacs.umd.edu/~hal/docs/daume02yaht.pdf>