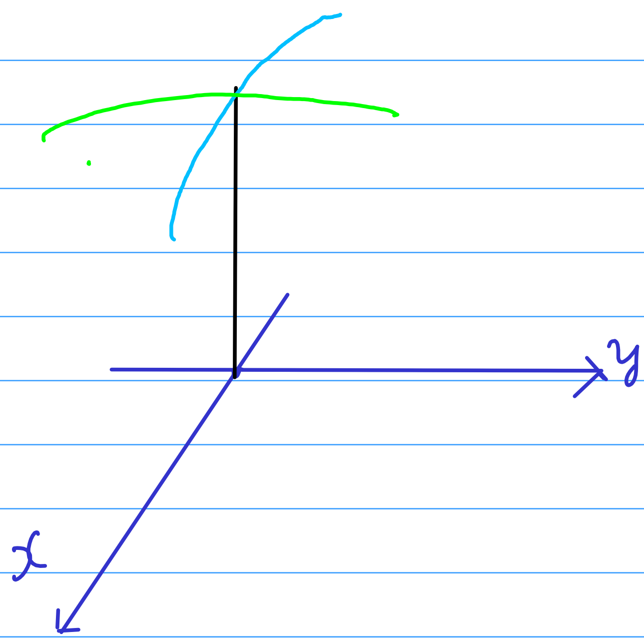
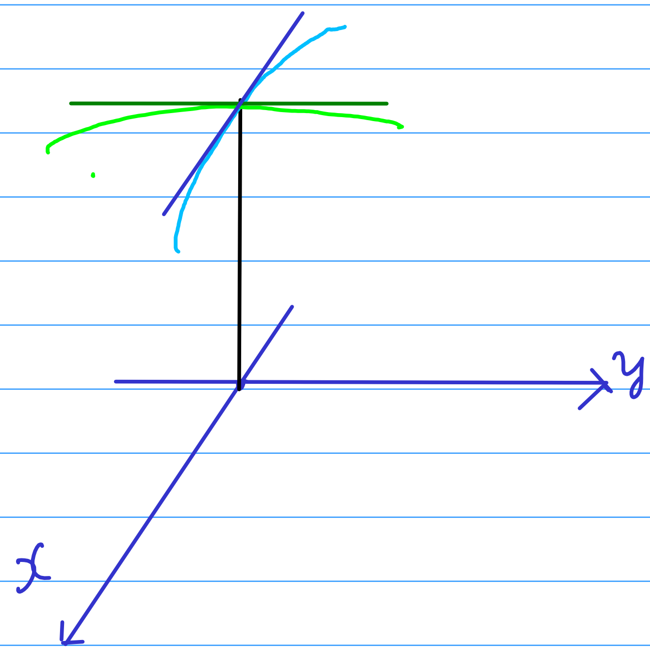
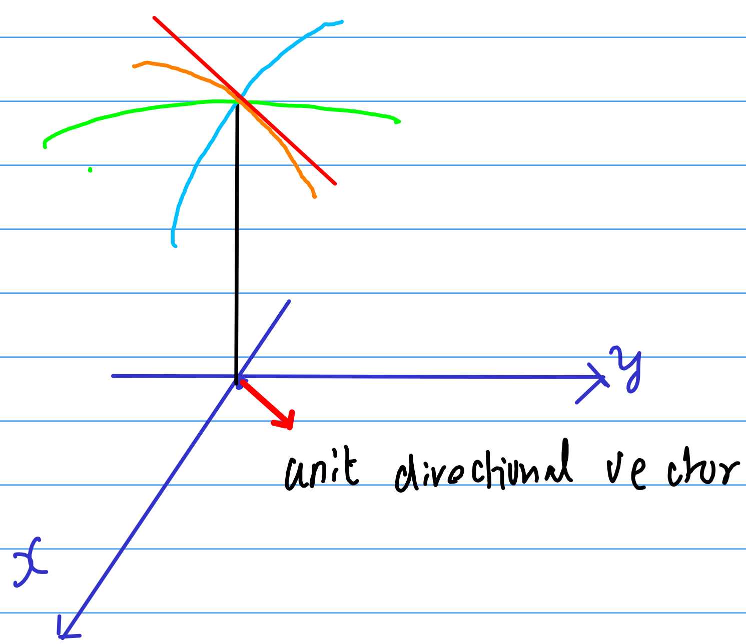
Vector Calculus (H.1)
Partial Derivatives

20160108

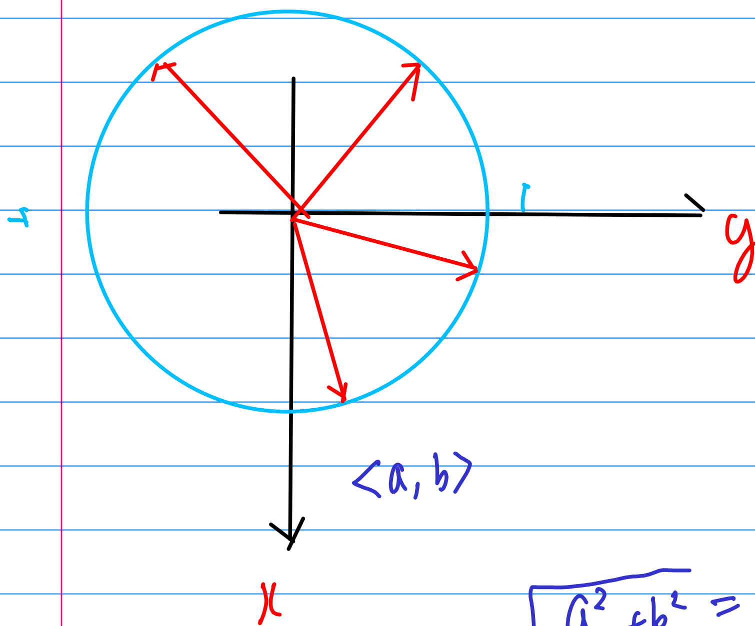
Directional Derivatives







Unit directional vector



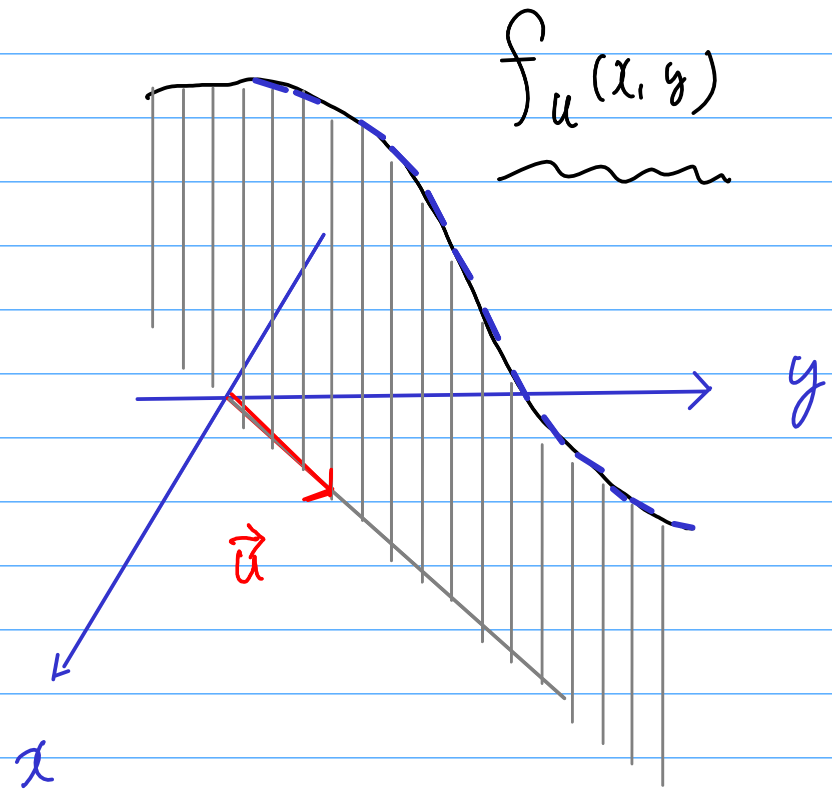
$\vec{u} = \langle a, b \rangle$

unit directional Vector

Magnitude = length = 1

$\sqrt{a^2 + b^2} = 1$ $\quad a^2 + b^2 = 1$

directional derivatives



$\vec{u} = \langle a, b \rangle$

$$D_{\vec{u}} f(x,y) = \lim_{h \to 0} \frac{f(x+ah,\ y+bh) - f(x,y)}{h}$$

$$D_{\vec{u}} f(x,y) = a D_x f(x,y) + b D_y f(x,y)$$

$$= a \boxed{f_x(x,y)} + b \boxed{f_y(x,y)}$$

$$\langle \boxed{f_x(x,y)}, \boxed{f_y(x,y)} \rangle \cdot \underbrace{\langle a, b \rangle}_{\text{vector}}$$

partial derivative $\frac{\partial}{\partial x} f(x,y)$

partial derivative $\frac{\partial}{\partial y} f(x,y)$

$\underbrace{f(x,y)}_{\text{scalar}}$

함수

$\vec{u} = \langle a, b \rangle$

$$D_{\vec{u}}\, f(x, y) = a \boxed{f_x(x, y)} + b \boxed{f_y(x, y)}$$

$$= \langle f_x, f_y \rangle \cdot \langle a, b \rangle$$

$\vec{u} = \langle a, b, c \rangle$

$$D_{\vec{u}}\, f(x, y, z) = a \cdot f_x(x, y, z) + b \cdot f_y(x, y, z) + c \cdot f_z(x, y, z)$$

$$= \langle f_x, f_y, f_z \rangle \cdot \langle a, b, c \rangle$$

$f \to$ 3 variables x, y, z

$$f(x, y, z)$$

consider this as a vector.

$$\left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right\rangle$$

gradient (vector) of f

del

$$\nabla f \triangleq \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right\rangle$$

$$= \frac{\partial f}{\partial x}\vec{i} + \frac{\partial f}{\partial y}\vec{j} + \frac{\partial f}{\partial z}\vec{k}$$

x 방향의 접선 기울기 y 방향의 접선 기울기 z 방향의 접선 기울기

Operator

$$\boxed{\nabla \triangleq \frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j}}$$

Operand.

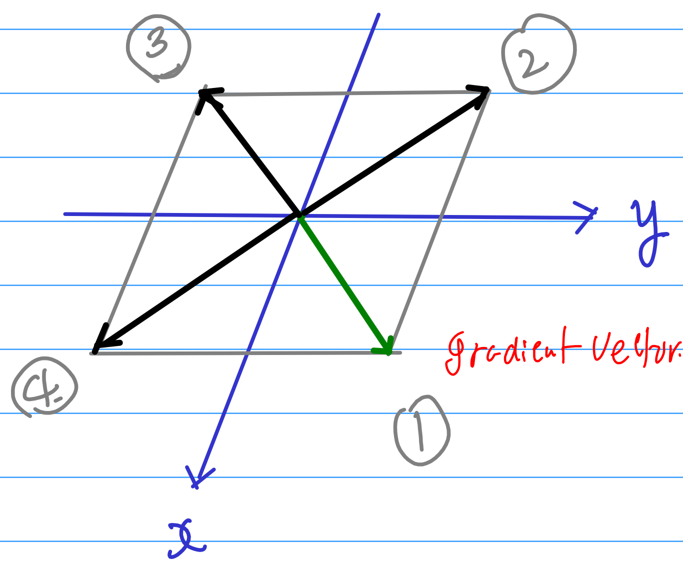
$$\nabla f(x,y) \triangleq \frac{\partial f}{\partial x}\vec{i} + \frac{\partial f}{\partial y}\vec{j} \equiv \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right\rangle$$

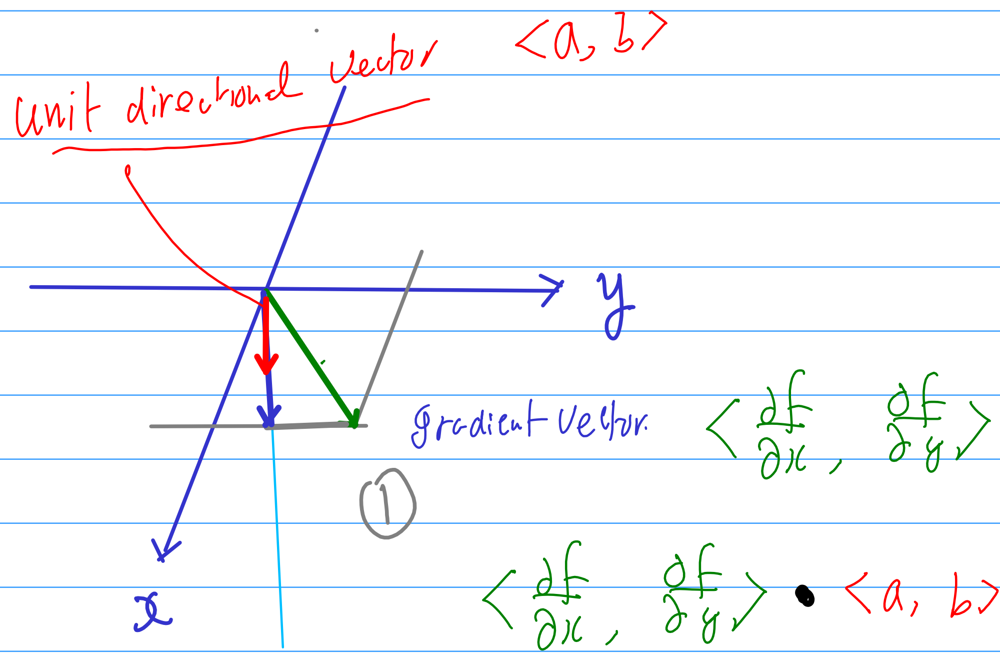
① $\frac{\partial f}{\partial x} = +1$, $\frac{\partial f}{\partial y} = +1$

② $\frac{\partial f}{\partial x} = -1$, $\frac{\partial f}{\partial y} = +1$

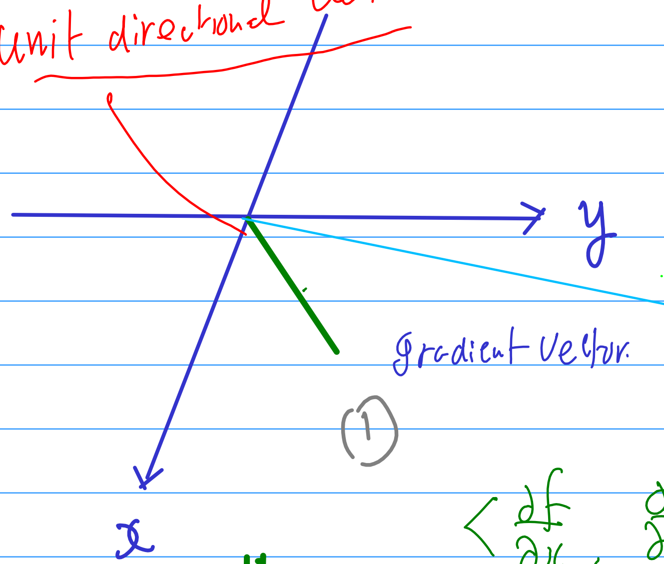
③ $\frac{\partial f}{\partial x} = -1$, $\frac{\partial f}{\partial y} = -1$

④ $\frac{\partial f}{\partial x} = +1$, $\frac{\partial f}{\partial y} = -1$





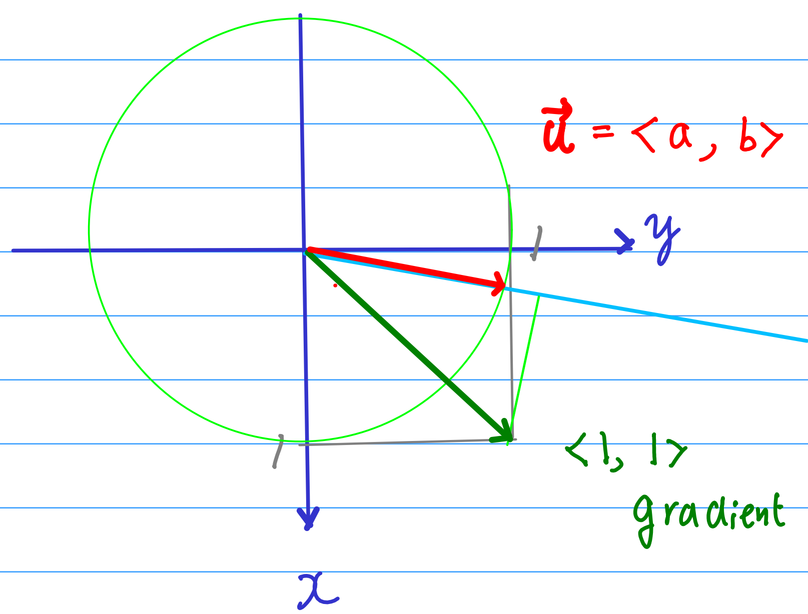
unit directional vector $\langle a, b \rangle$



gradient vector $\left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right\rangle$

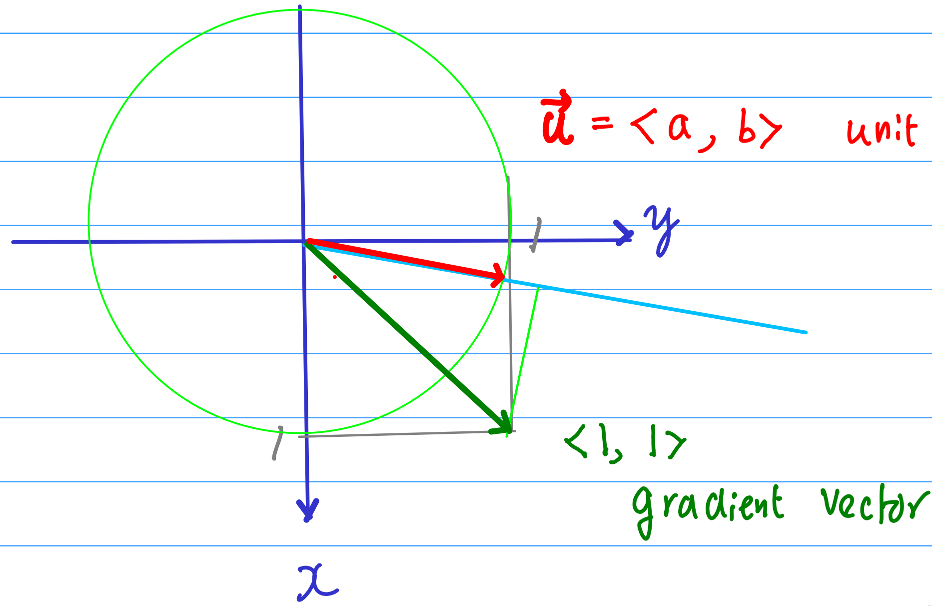
$\left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right\rangle \cdot \langle a, b \rangle$

$\Downarrow$

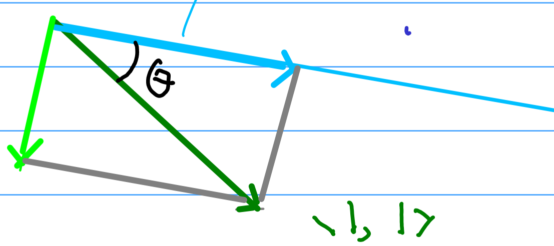


$\vec{u} = \langle a, b \rangle$ unit vector

$\langle 1, 1 \rangle$
gradient vector



gradient vector의 정사영



projection vector

⇓

length ⇒

$\vec{u}$ 방향의 접선의 기울기

방향도함수

$$\boxed{f_{\vec{u}}(x, y)}$$

$$\boxed{\nabla f} = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right\rangle$$

gradient vect

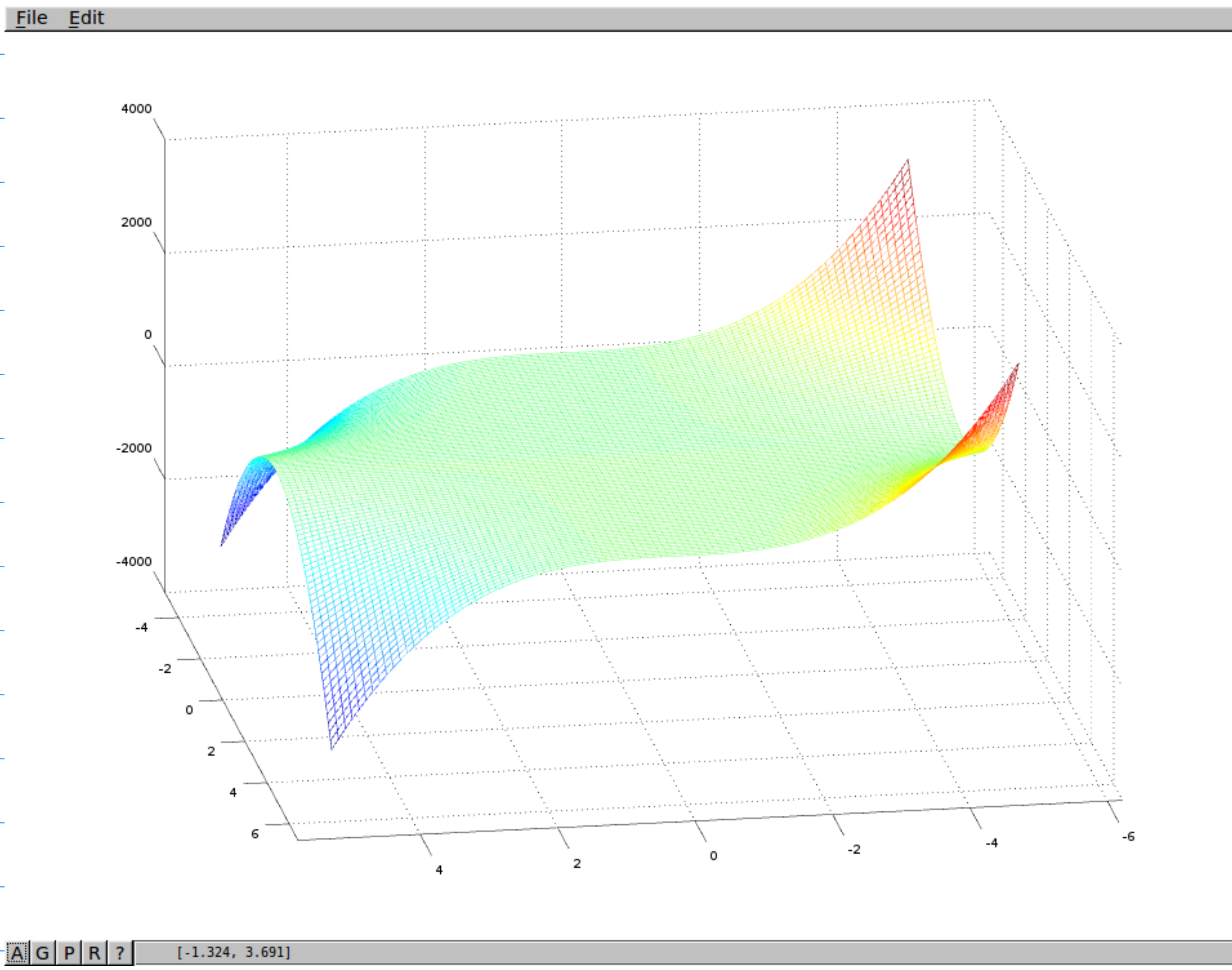
$$\nabla f \cdot u$$

$$= \|\nabla f\| \underbrace{\|\vec{u}\|}\cos\theta$$

$$= |\nabla f| \underbrace{\cos\theta}$$

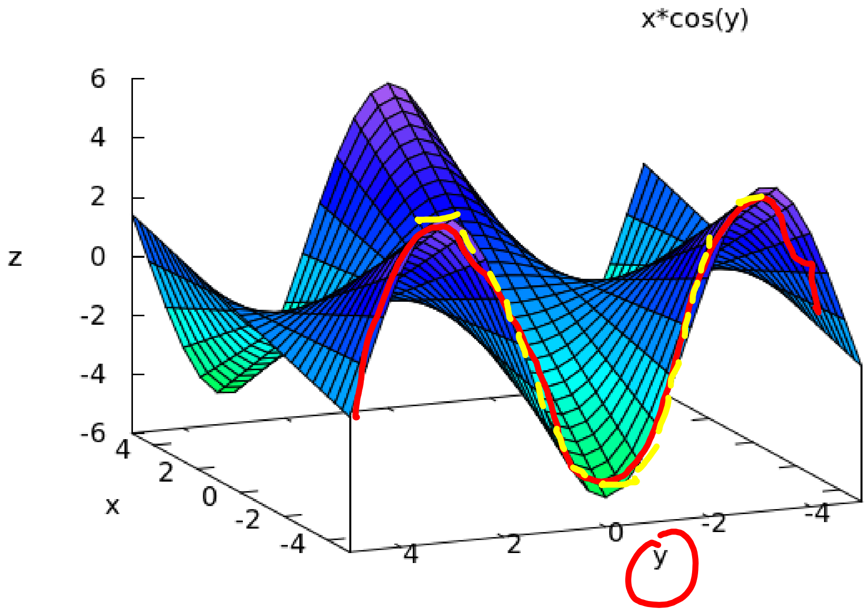
$$f(x,y) = 5y - x^3y^2 \qquad \frac{\partial f}{\partial x} = -3x^2y^2 \qquad \frac{\partial f}{\partial y} = 5 - 2x^3y$$

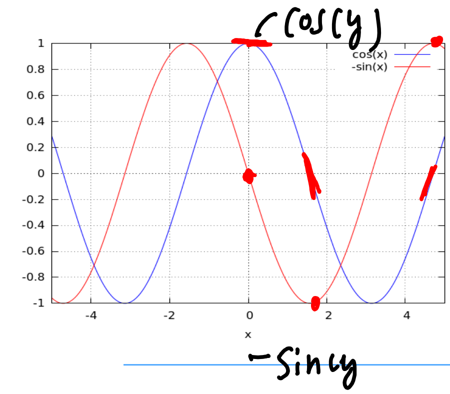
$$\nabla f = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right\rangle = \left\langle -3x^2y^2,\ 5-2x^3y \right\rangle$$



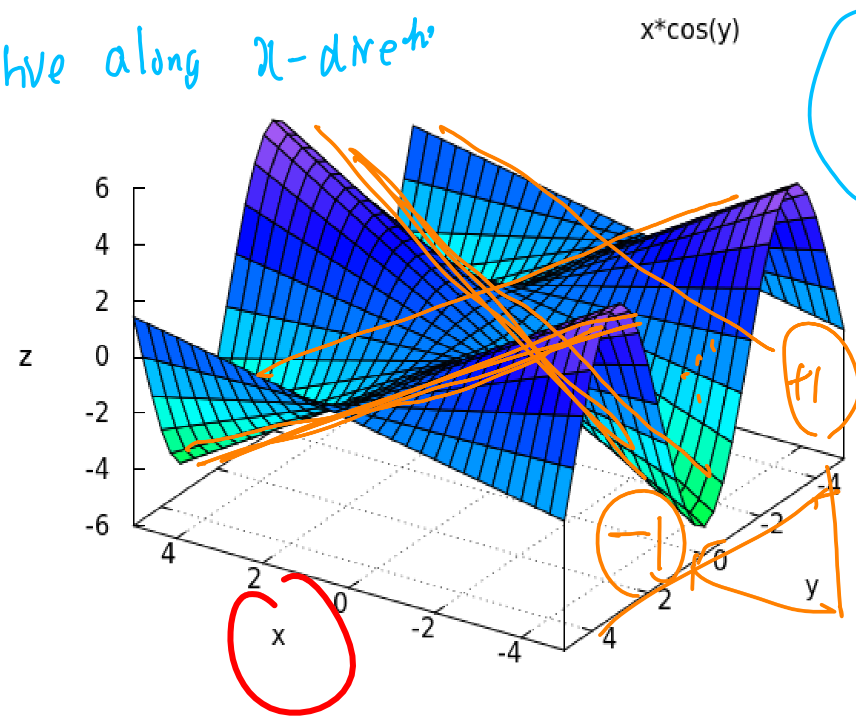
$f(x, y) = x \cdot \cos(y)$

$\frac{\partial}{\partial y} f(x, y) = -x\sin(y)$





derivative along x-directi'



$\frac{\partial}{\partial x} f(x, y) = \cos(y)$

+1

-1

기울기가 상수

y 치는 ''

$$\nabla f(x,y) = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right\rangle$$

$$= \langle \underline{\cos(y)}, \underline{-x\sin(y)} \rangle$$

gradient vector of $f(x,y)$

$$\nabla f(x,y) = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right\rangle$$

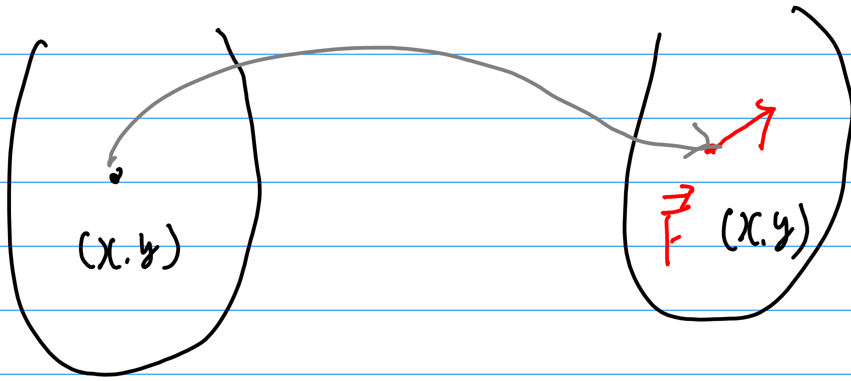
$$= \frac{\partial f}{\partial x}\vec{i} + \frac{\partial f}{\partial y}\vec{j}$$

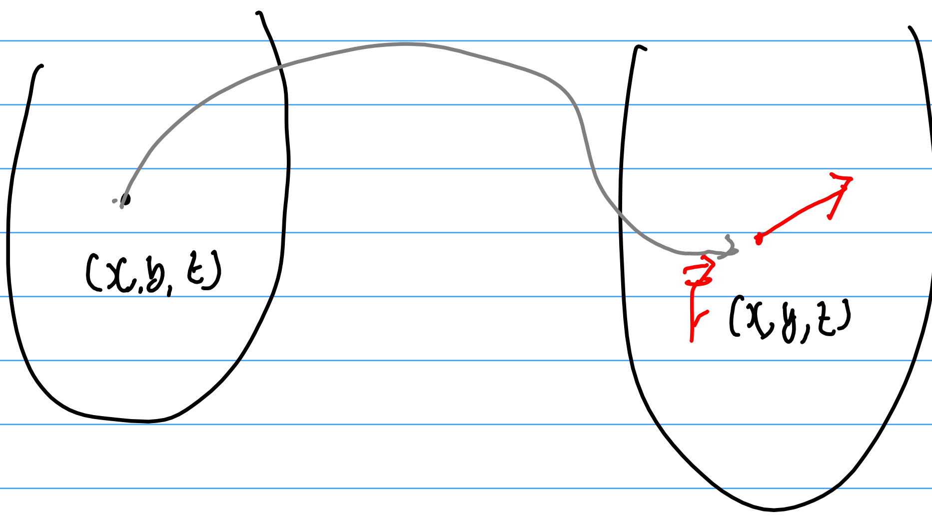
<Vector field> ..... vector valued function

at each point

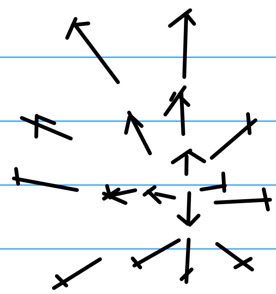
2-d (x, y) a vector (2-d) is assigned

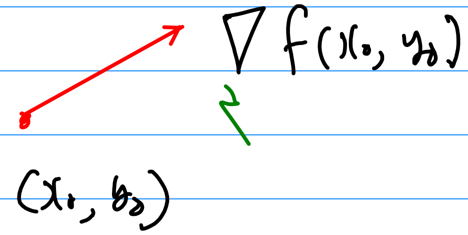
3-d (x, y, z) a vector (3-d) is assigned



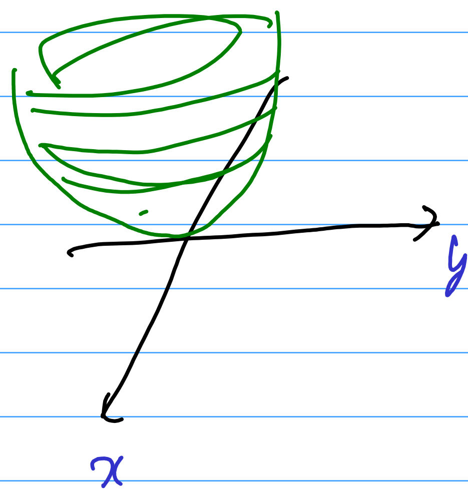


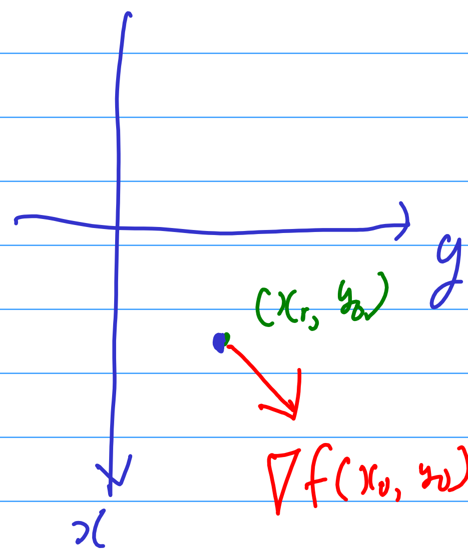
gradient vector field



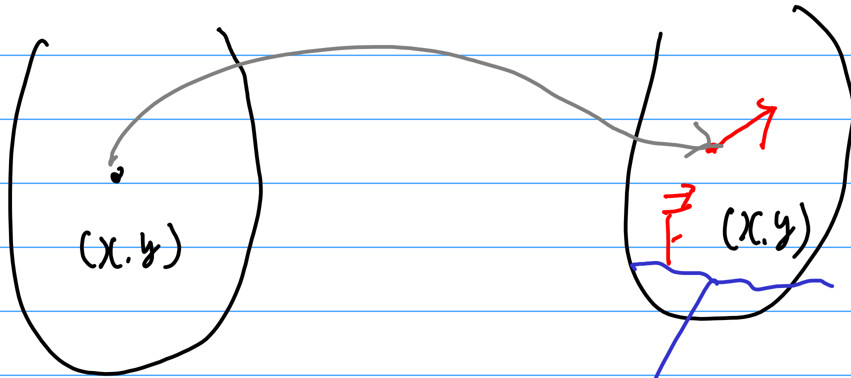


$z = f(x, y)$: 3차원 그림





2-d vector field



at a 2-d point (x, y) the value of a function $\vec{F}$

: a 2-d vector.

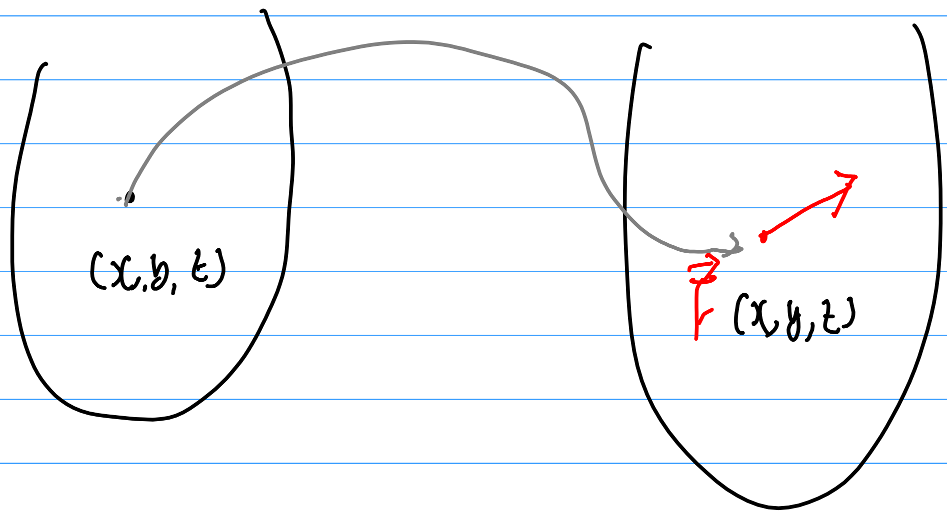
2 component

$$\langle P(x,y), Q(x,y) \rangle$$

$$= \underbrace{P(x,y)}\,\vec{i} + \underbrace{Q(x,y)}\,\vec{j}$$

scalar function

Vector valued function



at a 3-d point (x, y) the value of a function $\vec{F}$

: a 3-d vector.

3 component

$$\langle P(x,y), Q(x,y), R(x,y) \rangle$$

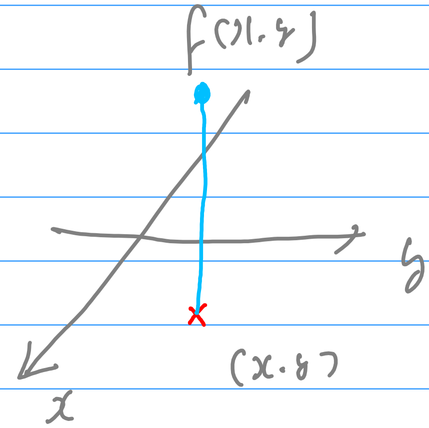
$$= P(x,y)\,\vec{i} + Q(x,y)\,\vec{j} + R(x,y)\,\vec{k}$$

scalar function

Vector valued function

2 variable functions

$f(x,y) = x^2+y^2$

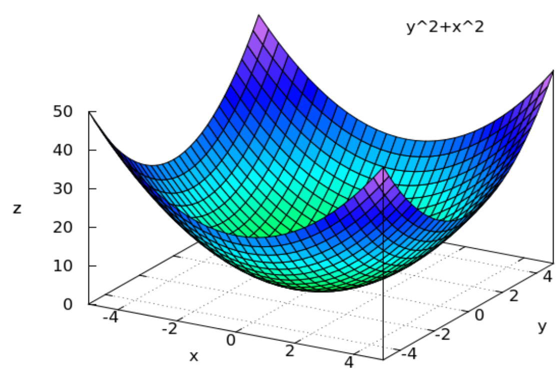


at a 2-d point (x, y),

$\boxed{\text{a value}}$ is assigned x^2+y^2

Scalar function $f(x,y)$

~~Vector~~ function



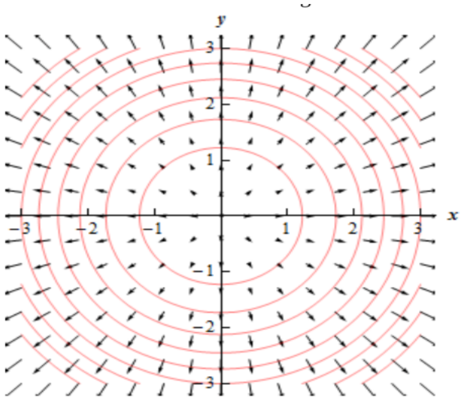
Gradient Vector

$$\vec{F}(x,y) = \left\langle \frac{\partial f}{\partial x}(x,y), \frac{\partial f}{\partial y}(x,y) \right\rangle$$

at a 2-d point (x,y),

a Vector is assigned

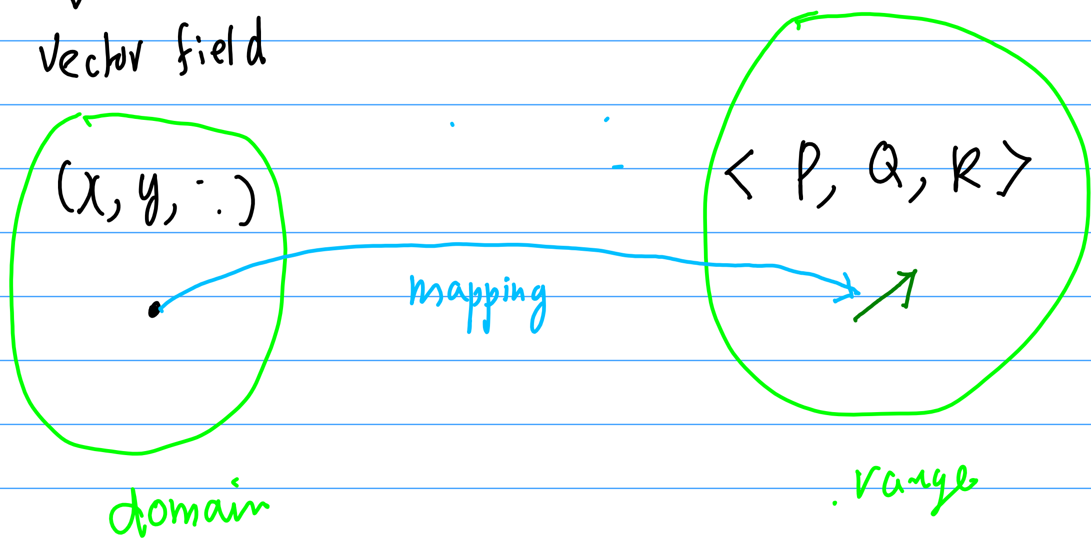
$$\left\langle \frac{\partial f}{\partial x}(x,y), \frac{\partial f}{\partial y}(x,y) \right\rangle$$



$$\vec{F} = P\vec{i} + Q\vec{j} + R\vec{k} \qquad R^3$$

$$= \langle P, \ Q, \ R \rangle$$

Vector field



$$P = P(x, y, z)$$
$$Q = Q(x, y, z)$$
$$R = R(x, y, z)$$

$$\text{curl } \vec{F} = \nabla \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix}$$

$$\text{curl } \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix} \begin{matrix} \vec{i} & \vec{j} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ P & Q \end{matrix}$$

$$\frac{\partial R}{\partial y}\vec{i} + \frac{\partial P}{\partial z}\vec{j} + \frac{\partial Q}{\partial x}\vec{k}$$

$$- \frac{\partial Q}{\partial z}\vec{i} - \frac{\partial R}{\partial x}\vec{j} - \frac{\partial P}{\partial y}\vec{k}$$

$$\text{curl } \vec{F} = \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right)\vec{i} + \left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}\right)\vec{j} + \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\vec{k}$$

$$\nabla = \frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}$$

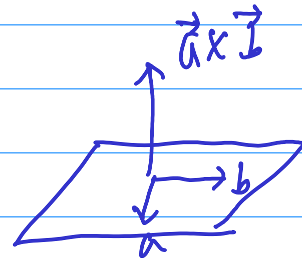
$$\vec{F} = P\vec{i} + Q\vec{j} + R\vec{k}$$

$$\nabla \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix}$$

$$\vec{a} = \langle x_1, y_1, z_1 \rangle = x_1\vec{i} + y_1\vec{j} + z_1\vec{k}$$

$$\vec{b} = \langle x_2, y_2, z_2 \rangle = x_2\vec{i} + y_2\vec{j} + z_2\vec{k}$$

$$\underset{\text{Vec}}{\vec{a}} \times \underset{\text{Vec}}{\vec{b}} \triangleq \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix}$$



Vec

$\vec{a}$에도 수직
$\vec{b}$에도 수직 } 오른손 법칙

$$\int_C f\, ds$$

$f(x, y)$