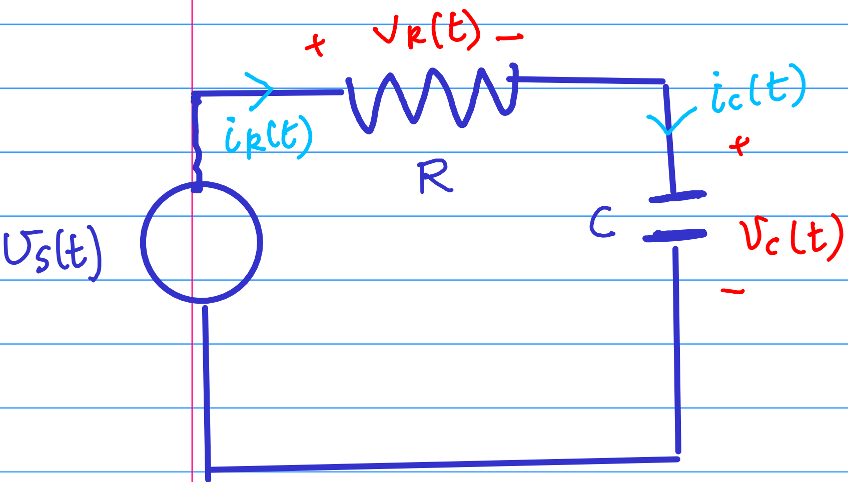
Filter (H.1)

20150610

20150613 10:00 ~ 12:00 115

1st Order Differential Equations

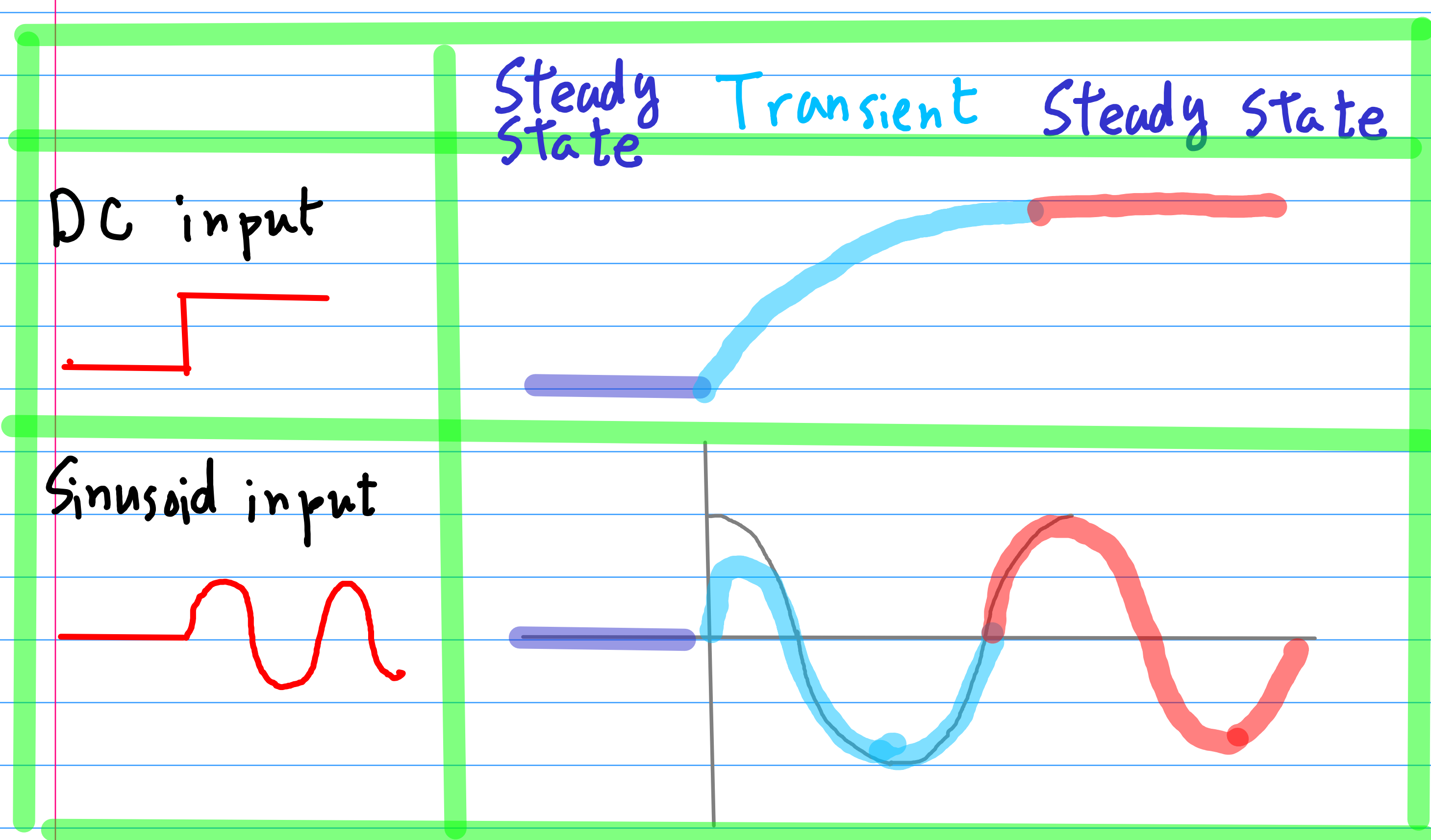


① KCL

② KVL

$$RC\frac{dv_c}{dt} + 1\cdot v_c(t) = v_s(t) \qquad v_c(t)$$

$$RC\frac{di_c}{dt} + 1\cdot i_c(t) = C\frac{dv_s}{dt} \qquad i_c(t)$$



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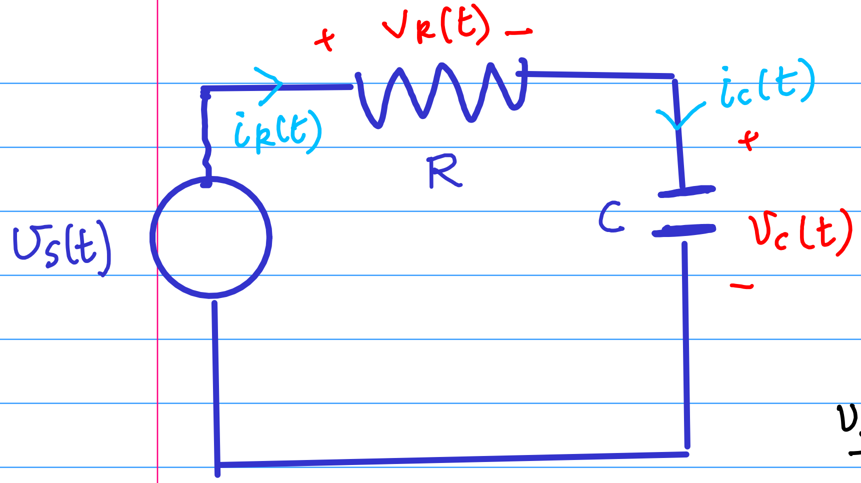


KCL

$$\left.\begin{aligned} i_s(t) &= \frac{v_s(t) - v_c(t)}{R} \\ i_c(t) &= C\frac{dv_c}{dt} \end{aligned}\right)$$

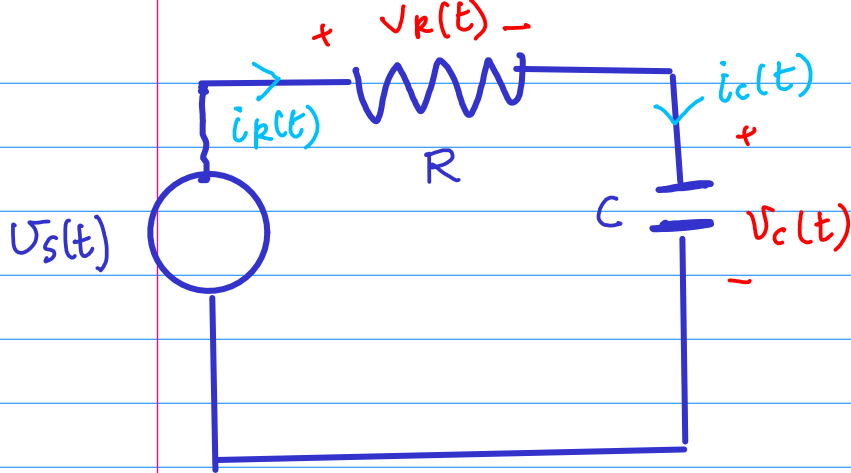
$$\frac{v_s(t) - v_c(t)}{R} = C\frac{dv_c}{dt}$$

$$C\frac{dv_c}{dt} + \frac{1}{R}v_c(t) = \frac{1}{R}v_s(t)$$

$$\left\{\begin{aligned} &1 \cdot \frac{dv_c}{dt} + \frac{1}{RC}v_c(t) = \frac{1}{RC}v_s(t) \\ &RC\frac{dv_c}{dt} + 1 \cdot v_c(t) = v_s(t) \end{aligned}\right.$$

②

$\boxed{KVL}$



$$-v_s(t) + v_R(t) + v_c(t) = 0$$

$$i_c = C \frac{dv_c}{dt}$$

$$\int_{-\infty}^{t} i_c(t)dt = C\, v_c(t)$$

$$-v_s(t) + R i_c(t) + \frac{1}{C}\int_{-\infty}^{t} i_c(t)dt = 0$$

$$\frac{d}{dt}\left\{ -v_s(t) + R i_c(t) + \frac{1}{C}\int_{-\infty}^{t} i_c(t)dt = 0 \right\}$$

$$-\frac{d\,v_s(t)}{dt} + R\frac{d\,i_c(t)}{dt} + \frac{1}{C} i_c(t) = 0$$

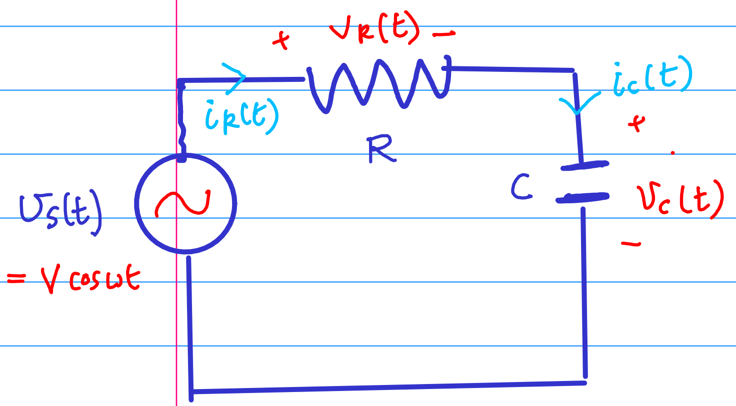
$$\left\{ \begin{array}{l} 1 \cdot \dfrac{di_c}{dt} + \dfrac{1}{RC} i_c(t) = \dfrac{1}{R}\dfrac{dv_s}{dt} \\[2em] RC\dfrac{di_c}{dt} + 1 \cdot i_c(t) = C\dfrac{dv_s}{dt} \end{array} \right.$$

③ $v_s(t) = V\cos\omega t$ Sinusoid input

find forced respone ...... particular solution

...... Steady State

.... $t = \infty$



$$1.\ \frac{dv_c}{dt} + \frac{1}{RC} v_c(t) = \frac{1}{RC} v_s(t)$$

$$1.\ \frac{dv_c}{dt} + \frac{1}{RC} v_c(t) = \frac{1}{RC} \boxed{V\cos\omega t}$$

particular solution $v_{cp}(t)$

assume $\boxed{v_c(t) = A\sin\omega t + B\cos\omega t}$

$= C\cos(\omega t + \phi)$

$$A\omega\cos\omega t - B\omega\sin\omega t \leftarrow \frac{dv_c}{dt}$$

$$\frac{1}{RC}A\sin\omega t + \frac{1}{RC}B\cos\omega t \leftarrow \frac{1}{RC}v_c(t)$$

$$\left(\frac{A}{RC} - B\omega\right)\sin\omega t + \left(\frac{B}{RC} + A\omega\right)\cos\omega t = \frac{V}{RC}\cos\omega t$$

$$\begin{cases} \frac{A}{RC} - B\omega = 0 \\ \frac{B}{RC} + A\omega - \frac{V}{RC} = 0 \end{cases}$$

$A = RC\omega B$

$$\frac{B}{RC} + RC\omega^2 B - \frac{V}{RC} = 0$$

$$\cancel{\frac{1}{RC}}(1 + R^2C^2\omega^2)B = \frac{V}{\cancel{RC}}$$

$$A = \frac{V\omega RC}{1 + \omega^2 (RC)^2}$$

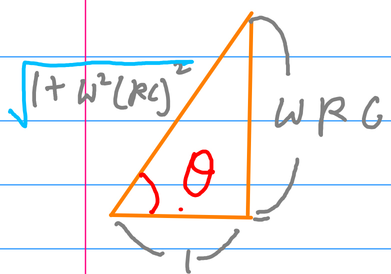
$$B = \frac{V}{1 + \omega^2 (RC)^2}$$

$$\boxed{v_c(t) = \frac{V\omega RC}{1 + \omega^2(RC)^2}\sin\omega t + \frac{V}{1 + \omega^2(RC)^2}\cos\omega t}$$

Sinusoidal Steady State Response

Sinusoid (Input) forced response = particular solution

$$\boxed{v_c(t) = \frac{V\omega RC}{1+w^2(RC)^2}\sin \omega t + \frac{V}{1+w^2(RC)^2}\cos \omega t}$$



$$\sin\theta = \frac{\omega RC}{\sqrt{1+w^2(RC)^2}}$$

$$\cos\theta = \frac{1}{\sqrt{1+w^2(RC)^2}}$$

$$v_c(t) = V\left(\frac{\omega RC}{1+w^2(RC)^2}\sin \omega t + \frac{1}{1+w^2(RC)^2}\cos \omega t\right)$$

$$v_c(t) = \frac{V}{\sqrt{1+w^2(RC)^2}}\left(\boxed{\frac{\omega RC}{\sqrt{1+w^2(RC)^2}}}\sin \omega t + \boxed{\frac{1}{\sqrt{1+w^2(RC)^2}}}\cos \omega t\right)$$

$$= \frac{V}{\sqrt{1+w^2(RC)^2}}\left(\boxed{\sin\theta}\sin \omega t + \boxed{\cos\theta}\cos \omega t\right)$$

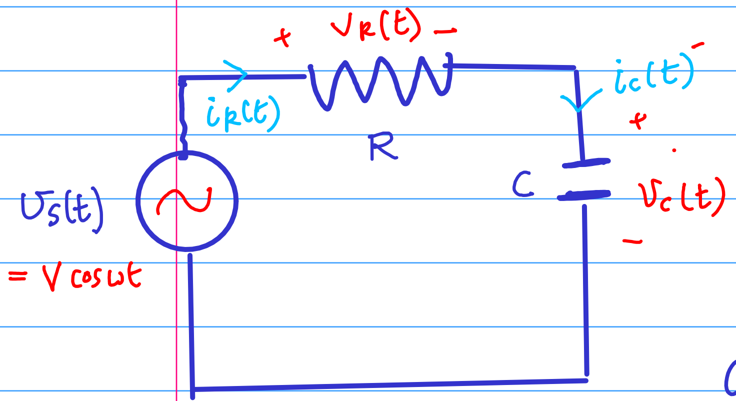
$$= \frac{V}{\sqrt{1+w^2(RC)^2}}\left(\cos \omega t\cos\theta + \sin \omega t\sin\theta\right)$$

$$\boxed{v_c(t) = \frac{V}{\sqrt{1+w^2(RC)^2}}\cos(\omega t - \theta)}$$

$\boxed{\text{Sinusoidal}}$ Steady State Response

Sinusoid (Input)

forced response = particular solution



1. $\dfrac{dv_c}{dt} + \dfrac{1}{RC} v_c(t) = \dfrac{1}{RC} \boxed{V\cos\omega t}$

particular solution $v_{cp}(t)$

assume $\boxed{v_c(t) = A\sin\omega t + B\cos\omega t}$

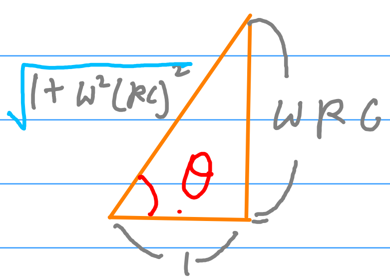
$= C\cos(\omega t + \phi)$

☆ particular sol을 찾았기 때문에

→ $t = \infty$ 일 때

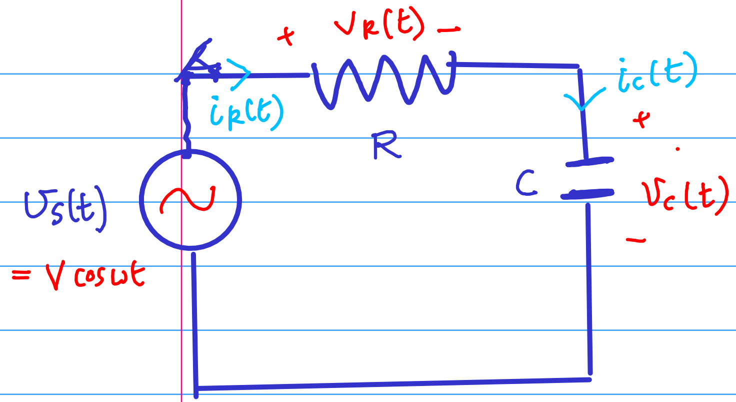
$$\boxed{v_c(t) = \frac{V\omega RC}{1 + \omega^2 (RC)^2}\sin\omega t + \frac{V}{1+\omega^2 (RC)^2}\cos\omega t}$$

$$\boxed{v_c(t) = \frac{V}{\sqrt{1+\omega^2 (RC)^2}}\cos(\omega t - \theta)}$$



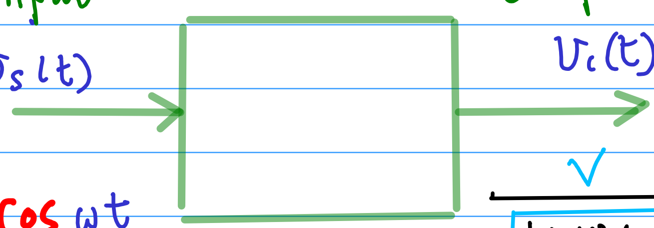
$\sin\theta = \dfrac{\omega RC}{\sqrt{1+\omega^2 (RC)^2}}$

$\cos\theta = \dfrac{1}{\sqrt{1+\omega^2 (RC)^2}}$



input

$v_s(t)$



output

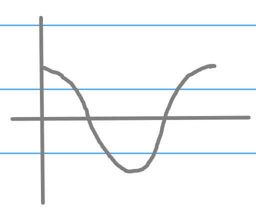
$v_c(t)$

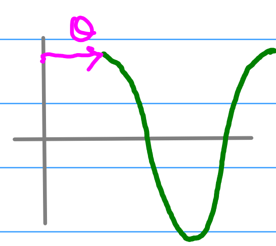
$V\cos\omega t$

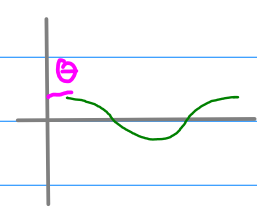
$$\frac{V}{\sqrt{1+\omega^2(RC)^2}}\cos(\omega t-\theta)$$

$$V \longrightarrow \frac{V}{\sqrt{1+\omega^2(RC)^2}}$$

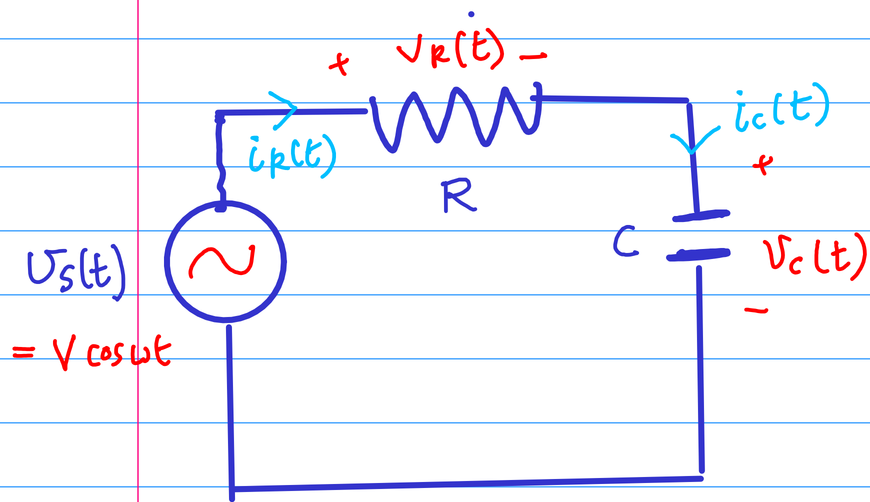
$$\omega t \longrightarrow \omega t-\theta$$







Phasor Approach



$$Z = Z_R + Z_c = R + \frac{1}{j\omega C}$$

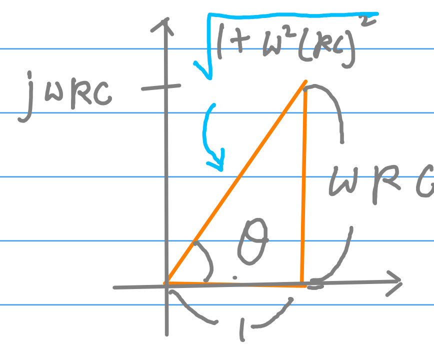
$$V_s = V\angle 0$$

$$V_c = \frac{Z_c}{Z_R + Z_c} V_s$$

phasor phasor

$$V_c = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} V_s = \boxed{\frac{1}{1 + j\omega RC}} V_s$$

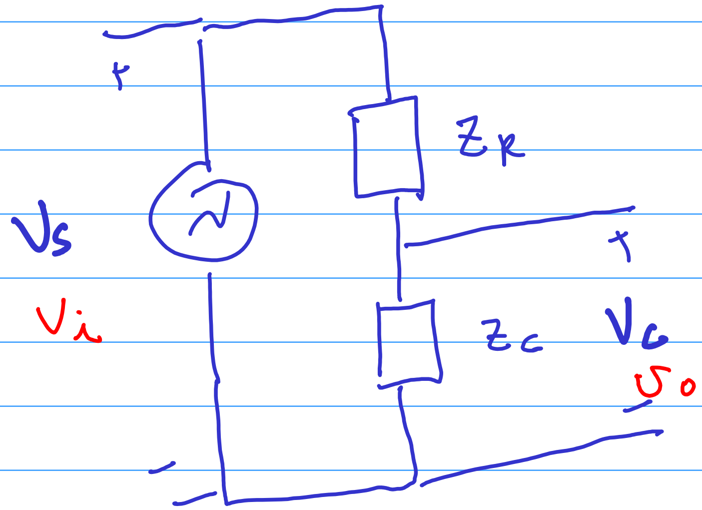
$$\boxed{\frac{1}{1 + j\omega RC} = \frac{1}{\sqrt{1 + (\omega RC)^2}} \angle -\tan^{-1}\omega RC}$$



$$V_c = \left(\boxed{\frac{1}{\sqrt{1 + (\omega RC)^2}} \angle -\tan^{-1}\omega RC}\right)(V\angle 0)$$

$$= \frac{V}{\sqrt{1 + (\omega RC)^2}} \angle -\tan^{-1}\omega RC \Rightarrow \frac{V}{\sqrt{1 + \omega^2 (RC)^2}} \cos(\omega t - \theta)$$

$$r_1 e^{j\theta_1} \; r_2 e^{j\theta_2} = r_1 \cdot r_2 \, e^{j(\theta_1 + \theta_2)}$$



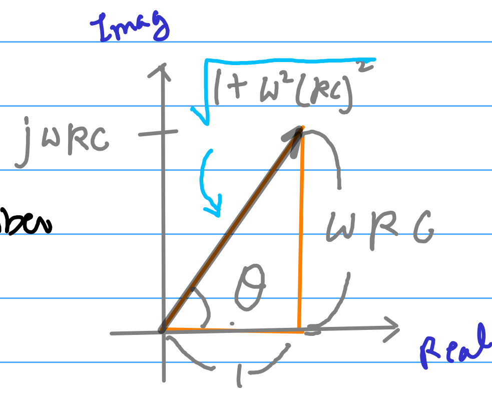
$$V_c = \frac{Z_c}{Z_R + Z_c} V_s$$

$$\frac{1}{\boxed{1 + j\omega RC}}$$

$$\boxed{1 + j\omega RC}$$

a complex number

→ vector



$$\tan\theta = \frac{\omega RC}{1}$$

$r \cdots\rightarrow \; \left| 1 + j\omega RC \right| = \sqrt{1 + (\omega RC)^2}$

$\theta \cdots\rightarrow \; \text{ang}(1 + j\omega RC) = \tan^{-1} \omega RC$

$$\boxed{1 + j\omega RC} = \sqrt{1 + (\omega RC)^2} \; e^{j \tan^{-1} \omega RC} = r e^{j\theta}$$

$$\frac{1}{r e^{i\theta}} = \frac{1}{r} e^{-i\theta}$$

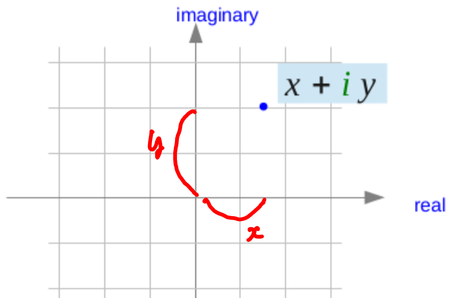
$$\frac{1}{\boxed{1 + \cdot\, \omega RC}} = \frac{1}{\sqrt{1 + (\omega RC)^2}} \angle -\tan^{-1} \omega RC$$

$$x = r\cos\theta$$
$$y = r\sin\theta$$

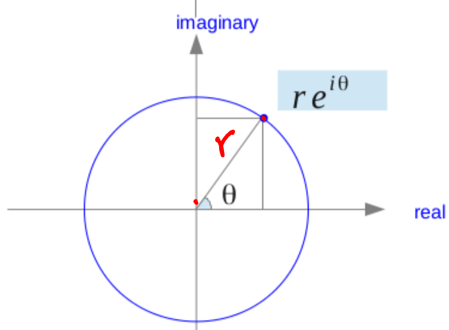
$$r = \sqrt{x^2 + y^2}$$
$$\tan\theta = \frac{y}{x}$$

$$\begin{aligned} x + iy &= r\cos\theta + i r\sin\theta \\ &= r(\cos\theta + i\sin\theta) \\ &= re^{i\theta} \end{aligned}$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$



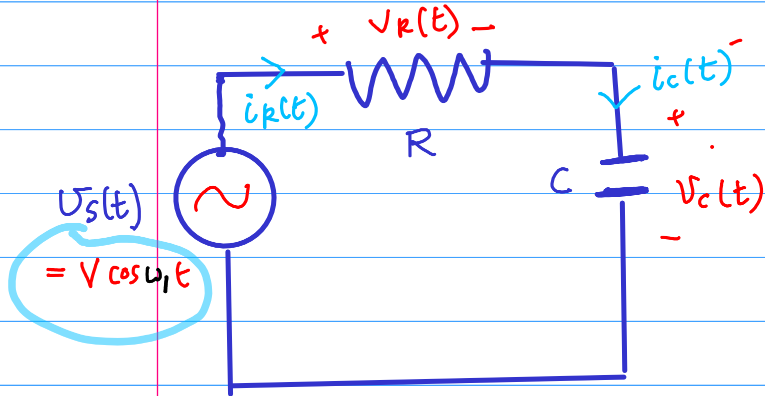
(c) Complex Number (I)

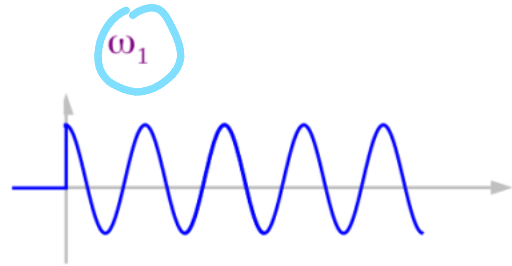


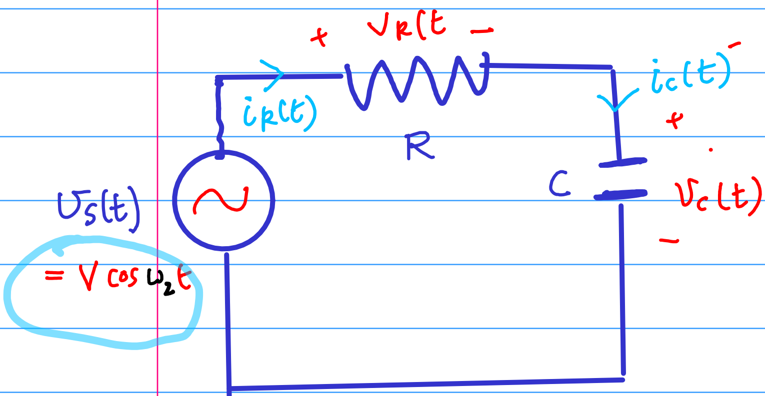
(d) Complex Number (II)

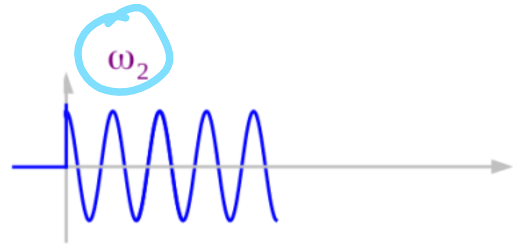
Filter ~ steady state $t = \infty$

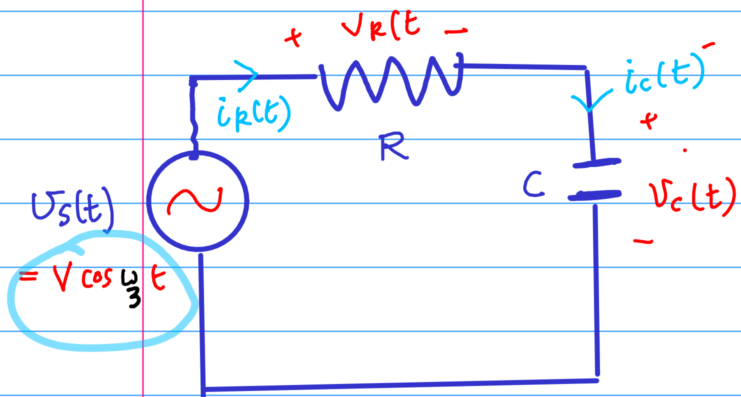
$\omega_1 < \omega_2 < \omega_3$

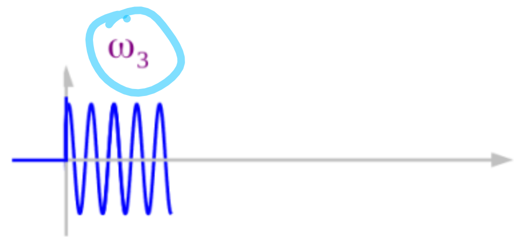


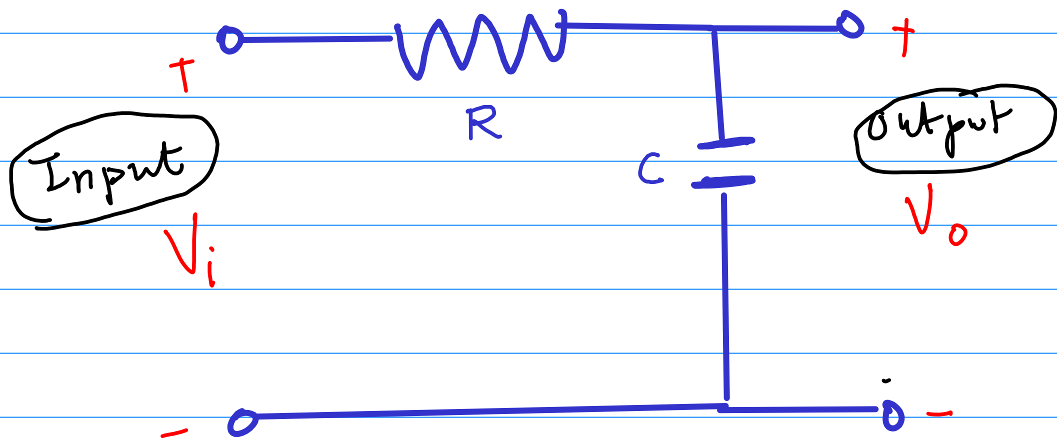




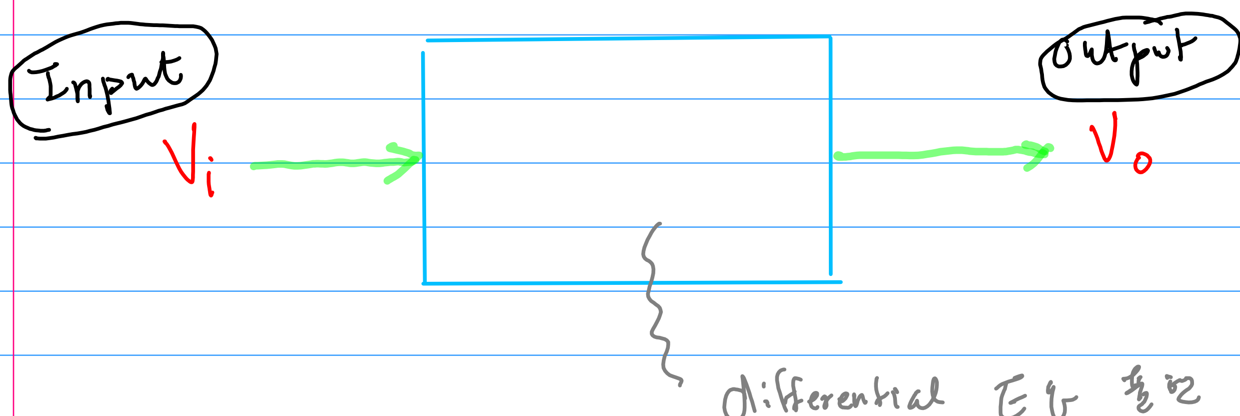






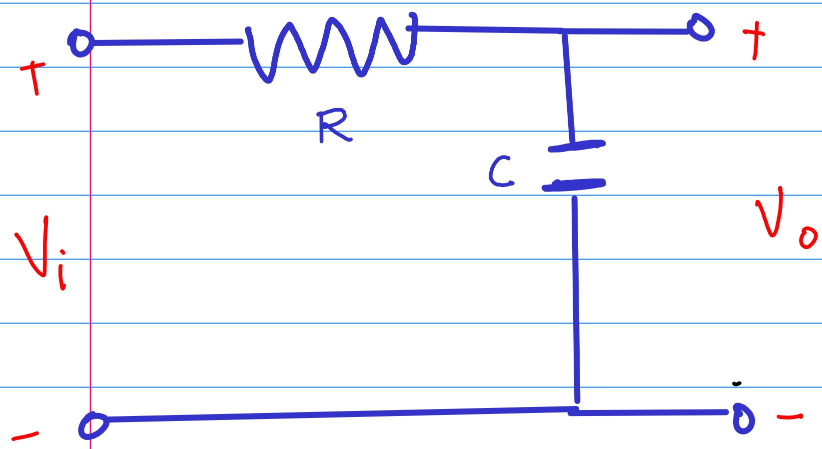


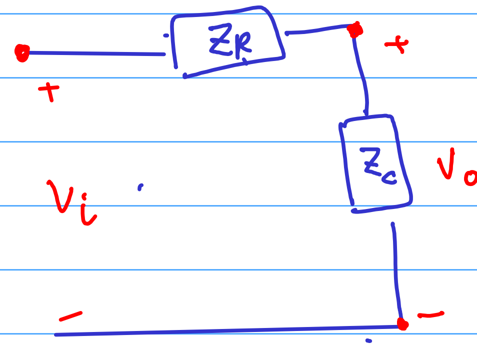
+
Input
V_i
R
C
+
Output
V_o
−
−



Input
V_i
Output
V_o
differential Eq 表记

Low Pass Filter





$$V_o = \frac{Z_C}{Z_R + Z_C} V_i = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} V_i$$

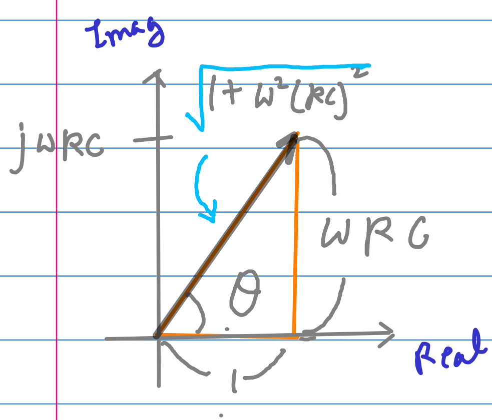
$$V_o = \frac{1}{j\omega RC + 1} V_i$$

$$\frac{V_o}{V_i}$$

$$z = x + iy = \boxed{r \cdot e^{i\theta}}$$

$$\left|\frac{1}{j\omega RC+1}\right|^2 = \frac{1}{(j\omega RC+1)(-j\omega RC+1)} = \frac{1}{1+(\omega RC)^2}$$

$$\left|\frac{1}{j\omega RC+1}\right| = \frac{1}{\sqrt{1+(\omega RC)^2}}$$



$$\tan\theta = \frac{\omega RC}{1}$$

$$\theta = \tan^{-1}(\omega RC)$$

$$\arg(\omega RC + 1) = \tan^{-1}(\omega RC)$$

$$\frac{1}{j\omega RC+1} = \left|\frac{1}{j\omega RC+1}\right| e^{j\arg\left(\frac{1}{j\omega RC+1}\right)}$$

$$= \left|\frac{1}{j\omega RC+1}\right| e^{-j\arg(\omega RC+1)}$$

$$= \left|\frac{1}{j\omega RC+1}\right| e^{-j\tan^{-1}(\omega RC)}$$

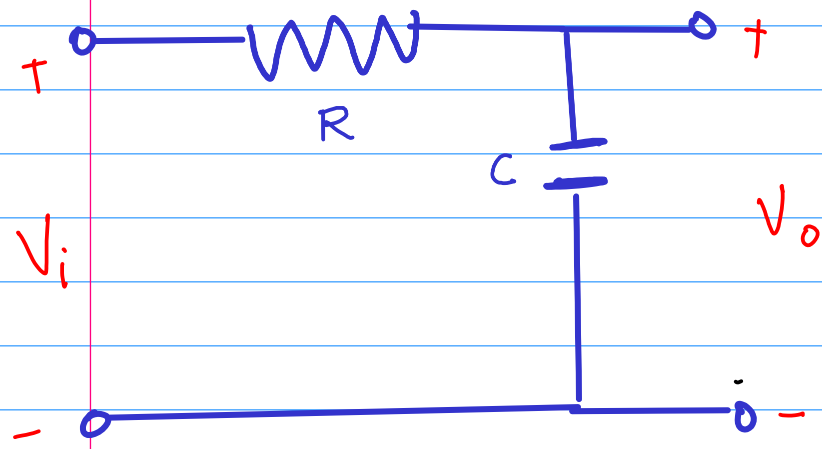
$$|z|^2 = z \cdot \bar{z}$$

$$z = (x + iy)$$
$$\bar{z} = (x - iy)$$

$$z \cdot \bar{z} = (x+iy)(x-iy)$$
$$= x^2 - (iy)^2$$
$$= x^2 - i^2 y^2$$
$$= x^2 + y^2$$

$$|z| = \sqrt{z \cdot \bar{z}} = \sqrt{x^2 + y^2}$$

$$= \left| \frac{1}{j\omega RC + 1} \right| \frac{1}{e^{j \arg(\omega RC + 1)}}$$



rotating phasor

rotating phasor

$$V_o = \frac{1}{j\omega RC + 1} \, V_i$$

$$\boxed{\frac{V_o(j\omega)}{V_i(j\omega)}} = \frac{1}{j\omega RC + 1} = H(j\omega)$$

unitless scaling factor
just a complex number for a given ω

$$H(j\omega) = |H(j\omega)| \cdot e^{j\, ang(H(j\omega))}$$

$$|H(j\omega)| = \left| \frac{1}{j\omega RC + 1} \right|$$

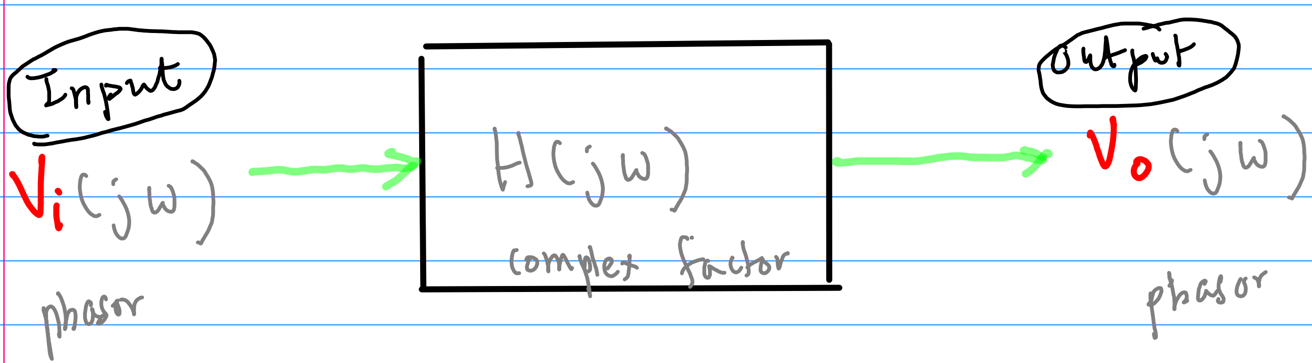
$$ang(H(j\omega)) = e^{-j \tan^{-1}(\omega RC)}$$

$$H(j\omega) = |H(j\omega)| \cdot e^{j\,\mathrm{ang}(H(j\omega))}$$

$$\omega_0 = \frac{1}{RC} \quad \therefore \quad \frac{1}{\omega_0} = RC$$

$$|H(j\omega)| = \left|\frac{1}{j\omega RC + 1}\right| = \left|\frac{1}{j(\omega/\omega_0) + 1}\right|$$

$$\mathrm{ang}(H(j\omega)) = e^{-j\tan^{-1}(\omega RC)} = e^{-j\tan^{-1}(\omega/\omega_0)}$$



$$V_o(j\omega) = H(j\omega)\,V_i(j\omega)$$

$$= |H(j\omega)|\,e^{+j\,\mathrm{ang}(H(j\omega))} \cdot |V_i(j\omega)|\,e^{+j\,\mathrm{ang}(V_i(j\omega))}$$

$$= |H(j\omega)| \cdot |V_i(j\omega)|\,e^{+j\left[\mathrm{ang}(H(j\omega)) + \mathrm{ang}(V_i(j\omega))\right]}$$

$$|V_o(j\omega)| = V_o = |H| \cdot V_i$$

$$\arg\{V_o(j\omega)\} = \phi_o = \arg(H) + \arg(V_i)$$

$$v_o = \frac{1}{j\omega RC + 1} v_i$$

$$\frac{1}{j\omega RC+1} = \left|\frac{1}{j\omega RC+1}\right| e^{-j\tan^{-1}(\omega RC)}$$

$$H(j\omega) = \underbrace{\frac{\overset{\text{phasor}}{v_o}}{\underset{\text{phasor}}{v_i}}}_{} = \left|\frac{1}{j\omega RC+1}\right| e^{-j\tan^{-1}(\omega RC)}$$

complex scaling factor

$$H(j\omega) = |H(j\omega)| e^{-j\arg(H(j\omega))}$$

$$|H(j\omega)| = \left|\frac{1}{j\omega RC+1}\right| = \frac{1}{\sqrt{1+\omega^2 C^2 R^2}}$$

$$\arg(H(j\omega)) = -\tan^{-1}(\omega RC)$$

$$\omega_0 = \frac{1}{RC}$$

$$|H(j\omega)| = \frac{1}{\sqrt{1+\omega^2 C^2 R^2}} = \frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_0}\right)^2}}$$

$$\arg(H(j\omega)) = -\tan^{-1}(\omega RC) = -\tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$

$$|H(j\omega)| = \frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_0}\right)^2}}$$

$$\arg(H(j\omega)) = -\tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$

$\omega \gg \omega_0$ $\left(\frac{\omega}{\omega_0} \gg 1\right)$

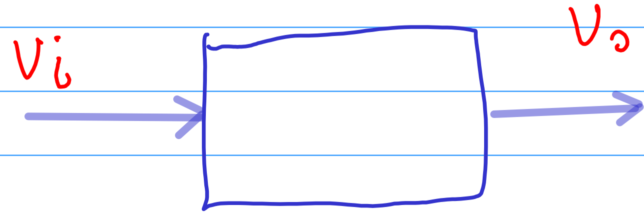
$$1+\frac{\omega}{\omega_0} \approx \frac{\omega}{\omega_0}$$

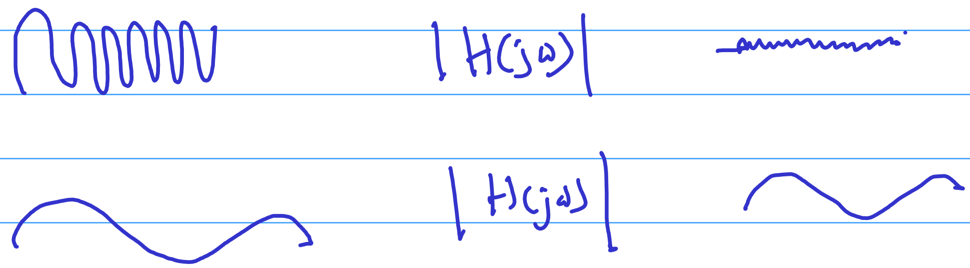
$$|H(j\omega)| = \frac{1}{\left(\frac{\omega}{\omega_0}\right)}$$

$\omega \ll \omega_0$ $\left(\frac{\omega}{\omega_0} \ll 1\right)$

$$1+\frac{\omega}{\omega_0} \approx 1$$

$$|H(j\omega)| = \frac{1}{\sqrt{2}}$$

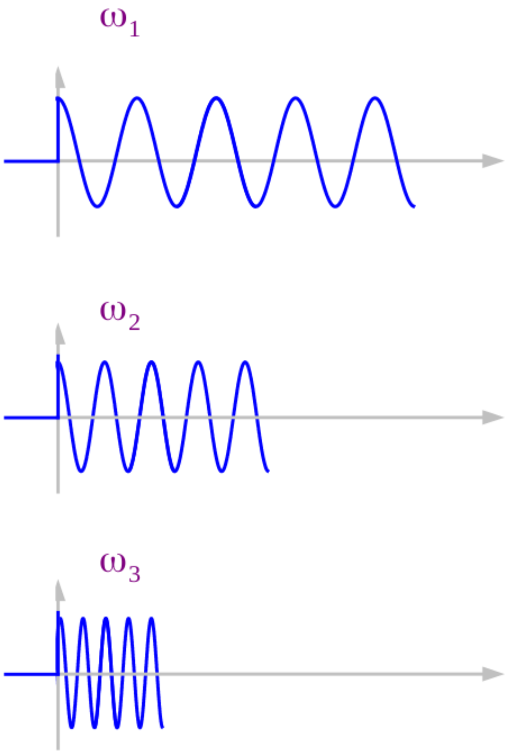

V_i
V_o

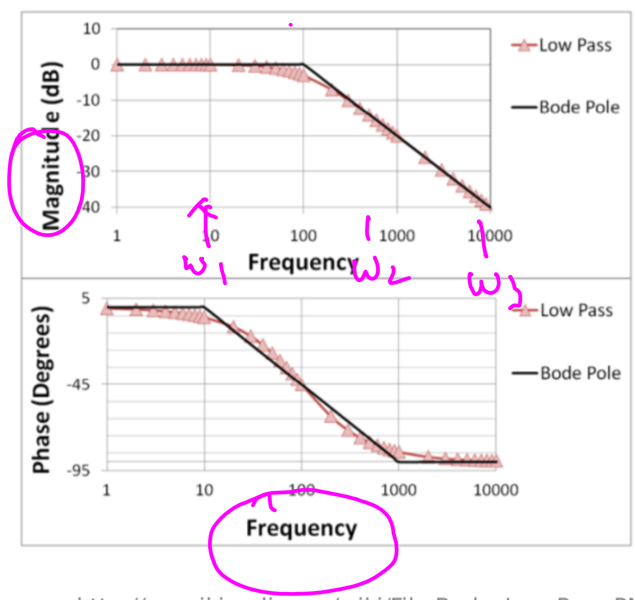

$|H(j\omega)|$
$|H(j\omega)|$

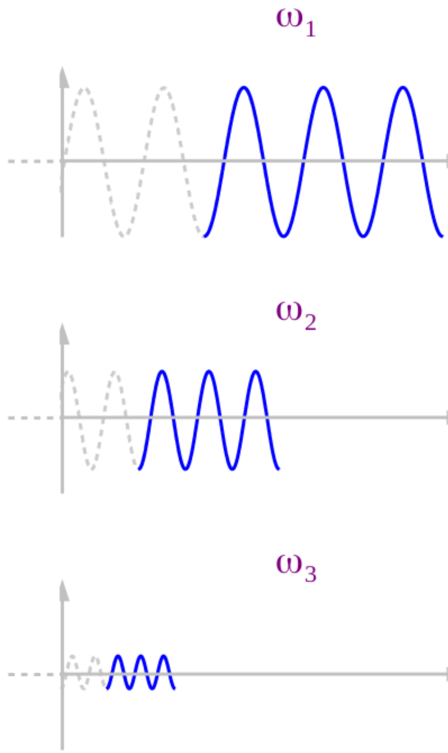
$$y_{ss}(t) = H(j\omega)\cdot A\cos(\omega t)$$
$$= A\cdot H(j\omega)\cos(\omega t)$$

$$x(t) = A\cos(\omega t)$$
$$\xi = j\omega$$

ω_1 ω_2 ω_3

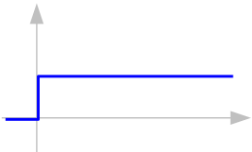






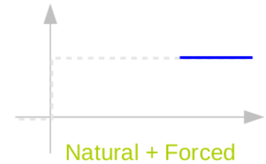
http://en.wikipedia.org/wiki/File:Bode_Low-Pass.PN(

Frequency



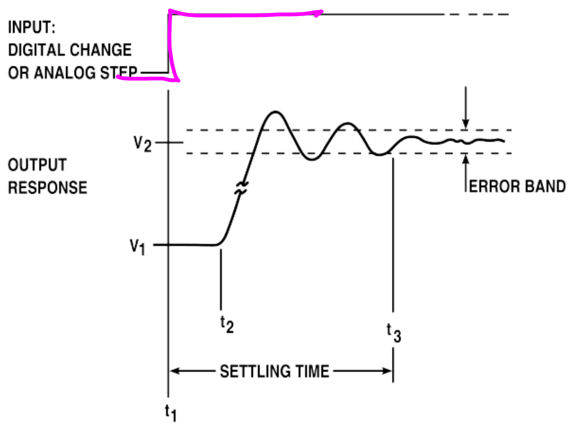
$$y_{ss}(t) = H(0)\cdot A e^{0t} = A\cdot H(0)$$

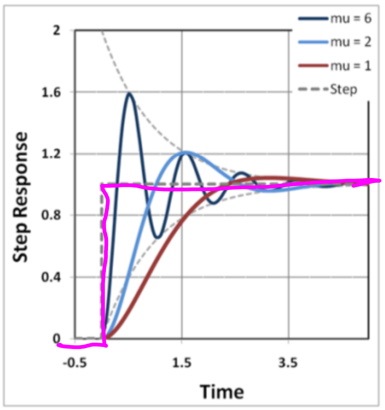
$$x(t) = A \qquad \xi = 0$$



Natural + Forced response

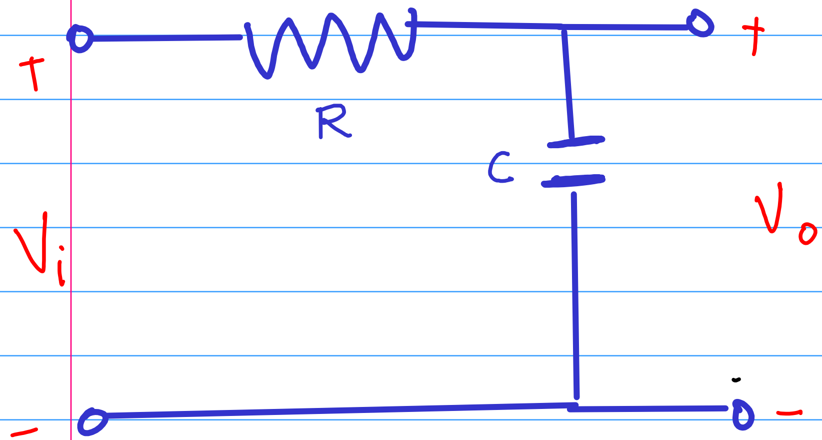
transient response



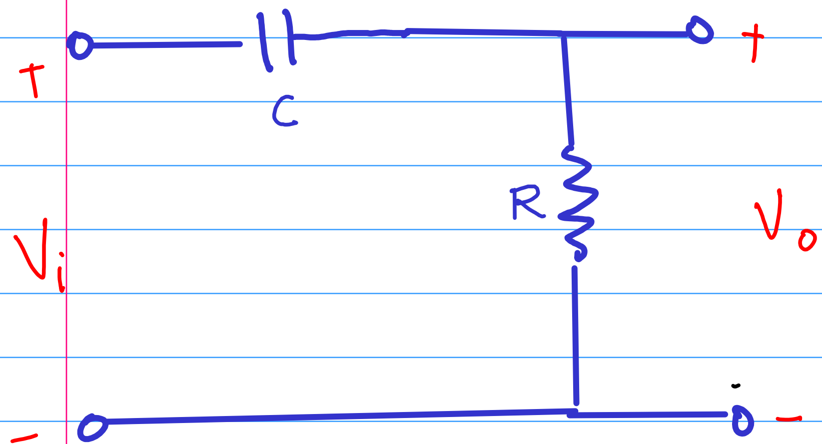


http://en.wikipedia.org/wiki/File:High_accuracy_settling_time_measurements_figure_1.png
http://en.wikipedia.org/wiki/File:Step_response_for_two-pole_feedback_amplifier.PNG

time response



Low Pass



<u>High</u> Pass

$$10 \angle 0 - 4.421 \angle -0.9852$$

$$10 - (4.421 \cos(-0.9852) + j4.421 \sin(-0.9852))$$

$$10 - 4.421 \cos(0.9852) + j4.421 \sin(0.9852))$$

$$7.556 + j\,3.684$$