Laurent Series and z-Transform - Geometric Series Applications Ⓐ

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2 formulas Simple Pole Form

$$\boxed{\frac{1}{z-p}} \qquad \boxed{\frac{1}{z^{-1}-p}}$$

2 representations each Geometric Series Form

$$\begin{tikzcd}
 & \boxed{-\frac{p^{-1}}{1-p^{-1}z}} \triangleq \overset{\text{causal}}{f(z)} = \overset{\text{anti- causal}}{X(z^{-1})} \\
\boxed{\frac{1}{z-p}} \arrow[ur, dash] \arrow[dr, dash] & \qquad\qquad\qquad \| \qquad\qquad \| \\
 & \boxed{\frac{z^{-1}}{1-pz^{-1}}} \triangleq \underset{\text{causal}}{Y(z)} = \underset{\text{anti- causal}}{g(z^{-1})}
\end{tikzcd}$$

$$\begin{tikzcd}
 & \boxed{-\frac{p^{-1}}{1-p^{-1}z^{-1}}} \triangleq \overset{\text{causal}}{X(z)} = \overset{\text{anti- causal}}{f(z^{-1})} \\
\boxed{\frac{1}{z^{-1}-p}} \arrow[ur, dash] \arrow[dr, dash] & \qquad\qquad\qquad \| \qquad\qquad \| \\
 & \boxed{\frac{z}{1-pz}} \triangleq \underset{\text{causal}}{g(z)} = \underset{\text{anti- causal}}{Y(z^{-1})}
\end{tikzcd}$$

Simple Pole Form | Geometric Series Form

Geometric Series

2 formulas

Simple Pole Form

$$\boxed{\frac{1}{z-p}} \qquad \boxed{\frac{1}{z^{-1}-p}}$$

2 representations each

Geometric Series Form

$$\begin{tikzcd}
 & \boxed{-\frac{p^{-1}}{1-p^{-1}z}} & \boxed{-\frac{1}{1-p^{-1}z}} \arrow[l, "/p"'] \\
\boxed{\frac{1}{z-p}} \arrow[ur, dash] \arrow[dr, dash] & p^{-n-1}u(n) & p^{-n}u(n) \\
 & \boxed{\frac{z^{-1}}{1-pz^{-1}}} & \boxed{\frac{1}{1-pz^{-1}}} \arrow[l, "/z"'] \\
 & p^{-n-1}u(-n-1) & p^{-n}u(-n)
\end{tikzcd}$$

$$\begin{tikzcd}
 & \boxed{-\frac{p^{-1}}{1-p^{-1}z^{-1}}} & \boxed{-\frac{1}{1-p^{-1}z^{-1}}} \arrow[l, "/p"'] \\
\boxed{\frac{1}{z^{-1}-p}} \arrow[ur, dash] \arrow[dr, dash] & p^{n-1}u(-n) & p^{n}u(-n) \\
 & \boxed{\frac{z}{1-pz}} & \boxed{\frac{1}{1-pz}} \arrow[l, "*z"'] \\
 & p^{n-1}u(n-1) & p^{n}u(n)
\end{tikzcd}$$

Simple Pole Form | Geometric Series Form

2 formulas Simple Pole Form

$\boxed{\frac{1}{z-p}}$ $\boxed{\frac{1}{z^{-1}-p}}$

2 representations each Geometric Series Form

$$\begin{tikzcd}
 & \boxed{-\frac{p^{-1}}{1-p^{-1}z}} & \boxed{-\frac{p^{-1}z}{1-p^{-1}z}} \arrow[l, "/z"'] \\
\boxed{\frac{1}{z-p}} \arrow[ur, dash] \arrow[dr, dash] & & \\
 & \boxed{\frac{z^{-1}}{1-pz^{-1}}} & \boxed{\frac{pz^{-1}}{1-pz^{-1}}} \arrow[l, "/p"']
\end{tikzcd}$$

$$\begin{tikzcd}
 & \boxed{-\frac{p^{-1}}{1-p^{-1}z^{-1}}} & \boxed{-\frac{p^{-1}z^{-1}}{1-p^{-1}z^{-1}}} \arrow[l, "*z"'] \\
\boxed{\frac{1}{z^{-1}-p}} \arrow[ur, dash] \arrow[dr, dash] & & \\
 & \boxed{\frac{z}{1-pz}} & \boxed{\frac{pz}{1-pz}} \arrow[l, "/p"']
\end{tikzcd}$$

Simple Pole Form | Geometric Series Form

Geometric Series Form Combinations with a unit start term

$$\boxed{-\frac{1}{1-\boxed{az}}}$$

$$\boxed{-\frac{1}{1-\boxed{az^{-1}}}}$$

$$\boxed{+\frac{1}{1-\boxed{a^{-1}z^{-1}}}}$$

$$\boxed{+\frac{1}{1-\boxed{a^{-1}z}}}$$

$$\boxed{-\frac{1}{1-\boxed{a^{-1}z}}}$$

$$\boxed{-\frac{1}{1-\boxed{a^{-1}z^{-1}}}}$$

$$\boxed{+\frac{1}{1-\boxed{az^{-1}}}}$$

$$\boxed{+\frac{1}{1-\boxed{az}}}$$

Geometric Series Form Combinations with a unit start term

$$\boxed{-\frac{1}{1-\boxed{az}}} \qquad \boxed{-\frac{1}{1-\boxed{az^{-1}}}}$$

$$\boxed{+\frac{1}{1-\boxed{a^{-1}z^{-1}}}} \qquad \boxed{+\frac{1}{1-\boxed{a^{-1}z}}}$$

$$\boxed{-\frac{1}{1-\boxed{a^{-1}z}}} \qquad \boxed{-\frac{1}{1-\boxed{a^{-1}z^{-1}}}}$$

$$\boxed{+\frac{1}{1-\boxed{az^{-1}}}} \qquad \boxed{+\frac{1}{1-\boxed{az}}}$$

Geometric Series with a unit start term

Laurent Series

$$\boxed{a\,z} \quad \boxed{|z| < a^{-1}}$$

$$-(a^0z^0 + a^1z^1 + a^2z^2 + \cdots)$$

$$\boxed{a_n = -a^n \quad (n \geqslant 0)}$$

$$\boxed{a\,z^{\boxed{-1}}} \quad \boxed{|z| > a}$$

$$-(a^0z^0 + a^1z^{-1} + a^2z^{-2} + \cdots)$$

$$-((\tfrac{1}{a})^0z^0 + (\tfrac{1}{a})^{-1}z^{-1} + (\tfrac{1}{a})^{-2}z^{-2} + \cdots)$$

$$\boxed{a_n = -(\tfrac{1}{a})^n \quad (n < 1)}$$

$$\boxed{a^{\boxed{-1}}z^{\boxed{-1}}} \quad \boxed{|z| > a^{-1}}$$

$$(a^0z^0 + a^{-1}z^{-1} + a^{-2}z^{-2} + \cdots)$$

$$\boxed{a_n = a^n \quad (n < 1)}$$

$$\boxed{a^{\boxed{-1}}z} \quad \boxed{|z| < a}$$

$$(a^0z^0 + a^{-1}z^1 + a^{-2}z^2 + \cdots)$$

$$((\tfrac{1}{a})^0z^0 + (\tfrac{1}{a})^1z^1 + (\tfrac{1}{a})^2z^2 + \cdots)$$

$$\boxed{a_n = (\tfrac{1}{a})^n \quad (n \geq 0)}$$

$$\boxed{a^{\boxed{-1}}z} \quad \boxed{|z| < a}$$

$$-(a^0z^0 + a^{-1}z^1 + a^{-2}z^2 + \cdots)$$

$$-((\tfrac{1}{a})^0z^0 + (\tfrac{1}{a})^1z^1 + (\tfrac{1}{a})^2z^2 + \cdots)$$

$$\boxed{a_n = -(\tfrac{1}{a})^n \quad (n \geqslant 0)}$$

$$\boxed{a^{\boxed{-1}}z^{\boxed{-1}}} \quad \boxed{|z| > a^{-1}}$$

$$-(a^0z^0 + a^{-1}z^{-1} + a^{-2}z^{-2} + \cdots)$$

$$\boxed{a_n = -a^n \quad (n < 1)}$$

$$\boxed{a\,z^{\boxed{-1}}} \quad \boxed{|z| > a}$$

$$(a^0z^0 + a^1z^{-1} + a^2z^{-2} + \cdots)$$

$$((\tfrac{1}{a})^0z^0 + (\tfrac{1}{a})^{-1}z^{-1} + (\tfrac{1}{a})^{-2}z^{-2} + \cdots)$$

$$\boxed{a_n = (\tfrac{1}{a})^n \quad (n < 1)}$$

$$\boxed{a\,z} \quad \boxed{|z| < a^{-1}}$$

$$(a^0z^0 + a^1z^1 + a^2z^2 + \cdots)$$

$$\boxed{a_n = a^n \quad (n \geq 0)}$$

Geometric Series with a unit start term

z-Transform

$a\,z$ $\quad |z| < a^{-1}$

$$-\left(a^0 z^0 + a^1 z^1 + a^2 z^2 + \cdots\right)$$

$$-\left(\left(\tfrac{1}{a}\right)^0 z^0 + \left(\tfrac{1}{a}\right)^{-1} z^1 + \left(\tfrac{1}{a}\right)^{-2} z^2 + \cdots\right)$$

$a_n = -a^{-n}$	$(-n \geq 0)$
$a_n = -\left(\frac{1}{a}\right)^n$	$(n < 1)$

$a\,z^{\boxed{-1}}$ $\quad |z| > a$

$$-\left(a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \cdots\right)$$

$$-\left(\left(\tfrac{1}{a}\right)^0 z^0 + \left(\tfrac{1}{a}\right)^{-1} z^{-1} + \left(\tfrac{1}{a}\right)^{-2} z^{-2} + \cdots\right)$$

$a_n = -\left(\frac{1}{a}\right)^{-n}$	$(-n < 1)$
$a_n = -a^n$	$(n \geq 0)$

$a^{\boxed{-1}}\,z^{\boxed{-1}}$ $\quad |z| > a^{-1}$

$$\left(a^0 z^0 + a^{-1} z^{-1} + a^{-2} z^{-2} + \cdots\right)$$

$$\left(\left(\tfrac{1}{a}\right)^0 z^0 + \left(\tfrac{1}{a}\right)^1 z^{-1} + \left(\tfrac{1}{a}\right)^2 z^{-2} + \cdots\right)$$

$a_n = a^{-n}$	$(-n < 1)$
$a_n = \left(\frac{1}{a}\right)^n$	$(n \geq 0)$

$a^{\boxed{-1}}\,z$ $\quad |z| < a$

$$\left(a^0 z^0 + a^{-1} z^1 + a^{-2} z^2 + \cdots\right)$$

$$\left(\left(\tfrac{1}{a}\right)^0 z^0 + \left(\tfrac{1}{a}\right)^1 z^1 + \left(\tfrac{1}{a}\right)^2 z^2 + \cdots\right)$$

$a_n = \left(\frac{1}{a}\right)^{-n}$	$(-n \geq 0)$
$a_n = a^n$	$(n < 1)$

$a^{\boxed{-1}}\,z$ $\quad |z| < a$

$$-\left(a^0 z^0 + a^{-1} z^1 + a^{-2} z^2 + \cdots\right)$$

$$-\left(\left(\tfrac{1}{a}\right)^0 z^0 + \left(\tfrac{1}{a}\right)^1 z^1 + \left(\tfrac{1}{a}\right)^2 z^2 + \cdots\right)$$

$a_n = -\left(\frac{1}{a}\right)^{-n}$	$(-n \geq 0)$
$a_n = -a^n$	$(n < 1)$

$a^{\boxed{-1}}\,z^{\boxed{-1}}$ $\quad |z| > a^{-1}$

$$-\left(a^0 z^0 + a^{-1} z^{-1} + a^{-2} z^{-2} + \cdots\right)$$

$$-\left(\left(\tfrac{1}{a}\right)^0 z^0 + \left(\tfrac{1}{a}\right)^1 z^{-1} + \left(\tfrac{1}{a}\right)^2 z^{-2} + \cdots\right)$$

$a_n = -a^{-n}$	$(-n < 1)$
$a_n = -\left(\frac{1}{a}\right)^n$	$(n \geq 0)$

$a\,z^{\boxed{-1}}$ $\quad |z| > a$

$$\left(a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \cdots\right)$$

$$\left(\left(\tfrac{1}{a}\right)^0 z^0 + \left(\tfrac{1}{a}\right)^{-1} z^{-1} + \left(\tfrac{1}{a}\right)^{-2} z^{-2} + \cdots\right)$$

$a_n = \left(\frac{1}{a}\right)^{-n}$	$(-n < 1)$
$a_n = a^n$	$(n \geq 0)$

$a\,z$ $\quad |z| < a^{-1}$

$$\left(a^0 z^0 + a^1 z^1 + a^2 z^2 + \cdots\right)$$

$$\left(\left(\tfrac{1}{a}\right)^0 z^0 + \left(\tfrac{1}{a}\right)^{-1} z^1 + \left(\tfrac{1}{a}\right)^{-2} z^2 + \cdots\right)$$

$a_n = a^{-n}$	$(-n \geq 0)$
$a_n = \left(\frac{1}{a}\right)^n$	$(n < 1)$

Geometric Series with a unit start term

Laurent Series vs. z-Transform

$a\,z$ $\quad |z| < a^{-1}$

$-\left(a^0 z^0 + a^1 z^1 + a^2 z^2 + \cdots\right)$

$-\left(\left(\frac{1}{a}\right)^0 z^0 + \left(\frac{1}{a}\right)^{-1} z^1 + \left(\frac{1}{a}\right)^{-2} z^2 + \cdots\right)$

$a\,z^{\boxed{-1}}$ $\quad |z| > a$

$-\left(a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \cdots\right)$

$-\left(\left(\frac{1}{a}\right)^0 z^0 + \left(\frac{1}{a}\right)^{-1} z^{-1} + \left(\frac{1}{a}\right)^{-2} z^{-2} + \cdots\right)$

Laurent Series	$a_n = -a^n$	$(n \geq 0)$	$a_n = -\left(\frac{1}{a}\right)^n$	$(n < 1)$
z-Transform	$a_n = -\left(\frac{1}{a}\right)^n$	$(n < 1)$	$a_n = -a^n$	$(n \geq 0)$

$a^{\boxed{-1}} z^{\boxed{-1}}$ $\quad |z| > a^{-1}$

$\left(a^0 z^0 + a^{-1} z^{-1} + a^{-2} z^{-2} + \cdots\right)$

$\left(\left(\frac{1}{a}\right)^0 z^0 + \left(\frac{1}{a}\right)^1 z^{-1} + \left(\frac{1}{a}\right)^2 z^{-2} + \cdots\right)$

$a^{\boxed{-1}} z$ $\quad |z| < a$

$\left(a^0 z^0 + a^{-1} z^1 + a^{-2} z^2 + \cdots\right)$

$\left(\left(\frac{1}{a}\right)^0 z^0 + \left(\frac{1}{a}\right)^1 z^1 + \left(\frac{1}{a}\right)^2 z^2 + \cdots\right)$

Laurent Series	$a_n = a^n$	$(n < 1)$	$a_n = \left(\frac{1}{a}\right)^n$	$(n \geq 0)$
z-Transform	$a_n = \left(\frac{1}{a}\right)^n$	$(n \geq 0)$	$a_n = a^n$	$(n < 1)$

$a^{\boxed{-1}} z$ $\quad |z| < a$

$-\left(a^0 z^0 + a^{-1} z^1 + a^{-2} z^2 + \cdots\right)$

$-\left(\left(\frac{1}{a}\right)^0 z^0 + \left(\frac{1}{a}\right)^1 z^1 + \left(\frac{1}{a}\right)^2 z^2 + \cdots\right)$

$a^{\boxed{-1}} z^{\boxed{-1}}$ $\quad |z| > a^{-1}$

$-\left(a^0 z^0 + a^{-1} z^{-1} + a^{-2} z^{-2} + \cdots\right)$

$-\left(\left(\frac{1}{a}\right)^0 z^0 + \left(\frac{1}{a}\right)^1 z^{-1} + \left(\frac{1}{a}\right)^2 z^{-2} + \cdots\right)$

Laurent Series	$a_n = -\left(\frac{1}{a}\right)^n$	$(n \geq 0)$	$a_n = -a^n$	$(n < 1)$
z-Transform	$a_n = -a^n$	$(n < 1)$	$a_n = -\left(\frac{1}{a}\right)^n$	$(n \geq 0)$

$a\,z^{\boxed{-1}}$ $\quad |z| > a$

$\left(a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \cdots\right)$

$\left(\left(\frac{1}{a}\right)^0 z^0 + \left(\frac{1}{a}\right)^{-1} z^{-1} + \left(\frac{1}{a}\right)^{-2} z^{-2} + \cdots\right)$

$a\,z$ $\quad |z| < a^{-1}$

$\left(a^0 z^0 + a^1 z^1 + a^2 z^2 + \cdots\right)$

$\left(\left(\frac{1}{a}\right)^0 z^0 + \left(\frac{1}{a}\right)^{-1} z^1 + \left(\frac{1}{a}\right)^{-2} z^2 + \cdots\right)$

Laurent Series	$a_n = \left(\frac{1}{a}\right)^n$	$(n < 1)$	$a_n = a^n$	$(n \geq 0)$
z-Transform	$a_n = a^n$	$(n \geq 0)$	$a_n = \left(\frac{1}{a}\right)^n$	$(n < 1)$

Geometric Series with a non-unit start term

Laurent Series

$\boxed{a^{\boxed{-1}}\,z^{\boxed{-1}}}$ $\boxed{|z| > a^{-1}}$

$(a^{-1}z^{-1} + a^{-2}z^{-2} + a^{-3}z^{-3} + \cdots)$

$\boxed{a_n = a^{n}}\boxed{(n < 0)}$

$\boxed{a^{\boxed{-1}}\,z}$ $\boxed{|z| < a}$

$(a^{-1}z^{1} + a^{-2}z^{2} + a^{-3}z^{3} + \cdots)$

$\left(\left(\frac{1}{a}\right)^{1}z^{1} + \left(\frac{1}{a}\right)^{2}z^{2} + \left(\frac{1}{a}\right)^{3}z^{3} + \cdots\right)$

$\boxed{a_n = \left(\frac{1}{a}\right)^{n}}\boxed{(n \geq 1)}$

$\boxed{a\,z}$ $\boxed{|z| < a^{-1}}$

$-(a^{1}z^{1} + a^{2}z^{2} + a^{3}z^{3} + \cdots)$

$\boxed{a_n = -a^{n}}\boxed{(n \geq 1)}$

$\boxed{a\,z^{\boxed{-1}}}$ $\boxed{|z| > a}$

$-(a^{1}z^{-1} + a^{2}z^{-2} + a^{3}z^{-3} + \cdots)$

$-\left(\left(\frac{1}{a}\right)^{-1}z^{-1} + \left(\frac{1}{a}\right)^{-2}z^{-2} + \left(\frac{1}{a}\right)^{-3}z^{-3} + \cdots\right)$

$\boxed{a_n = -\left(\frac{1}{a}\right)^{n}}\boxed{(n < 0)}$

$\boxed{a\,z^{\boxed{-1}}}$ $\boxed{|z| > a}$

$(a^{1}z^{-1} + a^{2}z^{-2} + a^{3}z^{-3} + \cdots)$

$\left(\left(\frac{1}{a}\right)^{-1}z^{-1} + \left(\frac{1}{a}\right)^{-2}z^{-2} + \left(\frac{1}{a}\right)^{-3}z^{-3} + \cdots\right)$

$\boxed{a_n = \left(\frac{1}{a}\right)^{n}}\boxed{(n < 0)}$

$\boxed{a\,z}$ $\boxed{|z| < a^{-1}}$

$(a^{1}z^{1} + a^{2}z^{2} + a^{3}z^{3} + \cdots)$

$\boxed{a_n = a^{n}}\boxed{(n \geq 1)}$

$\boxed{a^{\boxed{-1}}\,z}$ $\boxed{|z| < a}$

$-(a^{-1}z^{1} + a^{-2}z^{2} + a^{-3}z^{3} + \cdots)$

$-\left(\left(\frac{1}{a}\right)^{1}z^{1} + \left(\frac{1}{a}\right)^{2}z^{2} + \left(\frac{1}{a}\right)^{3}z^{3} + \cdots\right)$

$\boxed{a_n = -\left(\frac{1}{a}\right)^{n}}\boxed{(n \geq 1)}$

$\boxed{a^{\boxed{-1}}\,z^{\boxed{-1}}}$ $\boxed{|z| > a^{-1}}$

$-(a^{-1}z^{-1} + a^{-2}z^{-2} + a^{-3}z^{-3} + \cdots)$

$\boxed{a_n = -a^{n}}\boxed{(n < 0)}$

Geometric Series with a non-unit start term

z-Transform

$\boxed{a^{\boxed{-1}} z^{\boxed{-1}}}$ $\boxed{|z| > a^{-1}}$

$(a^{-1} z^{-1} + a^{-2} z^{-2} + a^{-3} z^{-3} + \cdots)$

$((\frac{1}{a})^{1} z^{-1} + (\frac{1}{a})^{2} z^{-2} + (\frac{1}{a})^{3} z^{-3} + \cdots)$

$a_n = a^{-n}$	$(-n < 0)$
$a_n = (\frac{1}{a})^{n}$	$(n \geq 1)$

$\boxed{a^{\boxed{-1}} z}$ $\boxed{|z| < a}$

$(a^{-1} z^{1} + a^{-2} z^{2} + a^{-3} z^{3} + \cdots)$

$((\frac{1}{a})^{1} z^{1} + (\frac{1}{a})^{2} z^{2} + (\frac{1}{a})^{3} z^{3} + \cdots)$

$a_n = (\frac{1}{a})^{-n}$	$(-n \geq 1)$
$a_n = a^{n}$	$(n < 0)$

$\boxed{a\ z}$ $\boxed{|z| < a^{-1}}$

$-(a^{1} z^{1} + a^{2} z^{2} + a^{3} z^{3} + \cdots)$

$-((\frac{1}{a})^{-1} z^{1} + (\frac{1}{a})^{-2} z^{2} + (\frac{1}{a})^{-3} z^{3} + \cdots)$

$a_n = -a^{-n}$	$(-n \geq 1)$
$a_n = -(\frac{1}{a})^{n}$	$(n < 0)$

$\boxed{a\ z^{\boxed{-1}}}$ $\boxed{|z| > a}$

$-(a^{1} z^{-1} + a^{2} z^{-2} + a^{3} z^{-3} + \cdots)$

$-((\frac{1}{a})^{-1} z^{-1} + (\frac{1}{a})^{-2} z^{-2} + (\frac{1}{a})^{-3} z^{-3} + \cdots)$

$a_n = -(\frac{1}{a})^{-n}$	$(-n < 0)$
$a_n = -a^{n}$	$(n \geq 1)$

$\boxed{a\ z^{\boxed{-1}}}$ $\boxed{|z| > a}$

$(a^{1} z^{-1} + a^{2} z^{-2} + a^{3} z^{-3} + \cdots)$

$((\frac{1}{a})^{-1} z^{-1} + (\frac{1}{a})^{-2} z^{-2} + (\frac{1}{a})^{-3} z^{-3} + \cdots)$

$a_n = (\frac{1}{a})^{-n}$	$(-n < 0)$
$a_n = a^{n}$	$(n \geq 1)$

$\boxed{a\ z}$ $\boxed{|z| < a^{-1}}$

$(a^{1} z^{1} + a^{2} z^{2} + a^{3} z^{3} + \cdots)$

$((\frac{1}{a})^{-1} z^{1} + (\frac{1}{a})^{-2} z^{2} + (\frac{1}{a})^{-3} z^{3} + \cdots)$

$a_n = a^{-n}$	$(-n \geq 1)$
$a_n = (\frac{1}{a})^{n}$	$(n < 0)$

$\boxed{a^{\boxed{-1}} z}$ $\boxed{|z| < a}$

$-(a^{-1} z^{1} + a^{-2} z^{2} + a^{-3} z^{3} + \cdots)$

$-((\frac{1}{a})^{1} z^{1} + (\frac{1}{a})^{2} z^{2} + (\frac{1}{a})^{3} z^{3} + \cdots)$

$a_n = -(\frac{1}{a})^{-n}$	$(-n \geq 1)$
$a_n = -a^{n}$	$(n < 0)$

$\boxed{a^{\boxed{-1}} z^{\boxed{-1}}}$ $\boxed{|z| > a^{-1}}$

$-(a^{-1} z^{-1} + a^{-2} z^{-2} + a^{-3} z^{-3} + \cdots)$

$-((\frac{1}{a})^{1} z^{-1} + (\frac{1}{a})^{2} z^{-2} + (\frac{1}{a})^{3} z^{-3} + \cdots)$

$a_n = -a^{-n}$	$(-n < 0)$
$a_n = -(\frac{1}{a})^{n}$	$(n \geq 1)$

Geometric Series with a non-unit start term

Laurent Series vs. z-Transform

$a^{\boxed{-1}} z^{\boxed{-1}}$ $\quad |z| > a^{-1}$

$(a^{-1}z^{-1} + a^{-2}z^{-2} + a^{-3}z^{-3} + \cdots)$

$((\frac{1}{a})^{1}z^{-1} + (\frac{1}{a})^{2}z^{-2} + (\frac{1}{a})^{3}z^{-3} + \cdots)$

Laurent Series	$a_n = a^n$	$(n < 0)$
z-Transform	$a_n = (\frac{1}{a})^n$	$(n \geqslant 1)$

$a^{\boxed{-1}} z$ $\quad |z| < a$

$(a^{-1}z^{1} + a^{-2}z^{2} + a^{-3}z^{3} + \cdots)$

$((\frac{1}{a})^{1}z^{1} + (\frac{1}{a})^{2}z^{2} + (\frac{1}{a})^{3}z^{3} + \cdots)$

Laurent Series	$a_n = (\frac{1}{a})^n$	$(n \geqslant 1)$
z-Transform	$a_n = a^n$	$(n < 0)$

$a\ z$ $\quad |z| < a^{-1}$

$-(a^{1}z^{1} + a^{2}z^{2} + a^{3}z^{3} + \cdots)$

$-((\frac{1}{a})^{-1}z^{1} + (\frac{1}{a})^{-2}z^{2} + (\frac{1}{a})^{-3}z^{3} + \cdots)$

Laurent Series	$a_n = -a^n$	$(n \geqslant 1)$
z-Transform	$a_n = -(\frac{1}{a})^n$	$(n < 0)$

$a\ z^{\boxed{-1}}$ $\quad |z| > a$

$-(a^{1}z^{-1} + a^{2}z^{-2} + a^{3}z^{-3} + \cdots)$

$-((\frac{1}{a})^{-1}z^{-1} + (\frac{1}{a})^{-2}z^{-2} + (\frac{1}{a})^{-3}z^{-3} + \cdots)$

Laurent Series	$a_n = -(\frac{1}{a})^n$	$(n < 0)$
z-Transform	$a_n = -a^n$	$(n \geqslant 1)$

$a\ z^{\boxed{-1}}$ $\quad |z| > a$

$(a^{1}z^{-1} + a^{2}z^{-2} + a^{3}z^{-3} + \cdots)$

$((\frac{1}{a})^{-1}z^{-1} + (\frac{1}{a})^{-2}z^{-2} + (\frac{1}{a})^{-3}z^{-3} + \cdots)$

Laurent Series	$a_n = (\frac{1}{a})^n$	$(n < 0)$
z-Transform	$a_n = a^n$	$(n \geqslant 1)$

$a\ z$ $\quad |z| < a^{-1}$

$(a^{1}z^{1} + a^{2}z^{2} + a^{3}z^{3} + \cdots)$

$((\frac{1}{a})^{-1}z^{1} + (\frac{1}{a})^{-2}z^{2} + (\frac{1}{a})^{-3}z^{3} + \cdots)$

Laurent Series	$a_n = a^n$	$(n \geqslant 1)$
z-Transform	$a_n = (\frac{1}{a})^n$	$(n < 0)$

$a^{\boxed{-1}} z$ $\quad |z| < a$

$-(a^{-1}z^{1} + a^{-2}z^{2} + a^{-3}z^{3} + \cdots)$

$-((\frac{1}{a})^{1}z^{1} + (\frac{1}{a})^{2}z^{2} + (\frac{1}{a})^{3}z^{3} + \cdots)$

Laurent Series	$a_n = -(\frac{1}{a})^n$	$(n \geqslant 1)$
z-Transform	$a_n = -a^n$	$(n < 0)$

$a^{\boxed{-1}} z^{\boxed{-1}}$ $\quad |z| > a^{-1}$

$-(a^{-1}z^{-1} + a^{-2}z^{-2} + a^{-3}z^{-3} + \cdots)$

$-((\frac{1}{a})^{1}z^{-1} + (\frac{1}{a})^{2}z^{-2} + (\frac{1}{a})^{3}z^{-3} + \cdots)$

Laurent Series	$a_n = -a^n$	$(n < 0)$
z-Transform	$a_n = -(\frac{1}{a})^n$	$(n \geqslant 1)$

Complemnt ROC Pairs - Original Geometric Series Form Combinations

unit	$-\frac{1}{1-az}$ $\lvert z\rvert<a^{-1}$	$-a^n$ $(n\geq 0)$	$-\frac{1}{1-az^{-1}}$ $\lvert z\rvert>a$	$-\left(\frac{1}{a}\right)^n$ $(n<1)$
non-unit	$\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$	a^n $(n<0)$	$\frac{a^{-1}z}{1-a^{-1}z}$ $\lvert z\rvert<a$	$\left(\frac{1}{a}\right)^n$ $(n\geq 1)$

unit	$\frac{1}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$	a^n $(n<1)$	$\frac{1}{1-a^{-1}z}$ $\lvert z\rvert<a$	$\left(\frac{1}{a}\right)^n$ $(n\geq 0)$
non-unit	$-\frac{az}{1-az}$ $\lvert z\rvert<a^{-1}$	$-a^n$ $(n\geq 1)$	$\frac{az^{-1}}{1-az^{-1}}$ $\lvert z\rvert>a$	$-\left(\frac{1}{a}\right)^n$ $(n<0)$

unit	$-\frac{1}{1-a^{-1}z}$ $\lvert z\rvert<a$	$-\left(\frac{1}{a}\right)^n$ $(n\geq 0)$	$-\frac{1}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$	$-a^n$ $(n<1)$
non-unit	$\frac{az^{-1}}{1-az^{-1}}$ $\lvert z\rvert>a$	$\left(\frac{1}{a}\right)^n$ $(n<0)$	$\frac{az}{1-az}$ $\lvert z\rvert<a^{-1}$	a^n $(n\geq 1)$

unit	$\frac{1}{1-az^{-1}}$ $\lvert z\rvert>a$	$\left(\frac{1}{a}\right)^n$ $(n<1)$	$\frac{1}{1-az}$ $\lvert z\rvert<a^{-1}$	a^n $(n\geq 0)$
non-unit	$-\frac{a^{-1}z}{1-a^{-1}z}$ $\lvert z\rvert<a$	$-\left(\frac{1}{a}\right)^n$ $(n\geq 1)$	$-\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$	$-a^n$ $(n<0)$

start term

Complemnt ROC Pairs -
Shifted Geometric Series Form Combinations

$-\frac{a}{1-az}$ $\|z\|<a^{-1}$	$-a^{n+1}$ $(n\geq 0)$
$\frac{z^{-1}}{1-a^{-1}z^{-1}}$ $\|z\|>a^{-1}$	a^{n+1} $(n<0)$

$-\frac{a}{1-az^{-1}}$ $\|z\|>a$	$-(\frac{1}{a})^{n-1}$ $(n<1)$
$\frac{z}{1-a^{-1}z}$ $\|z\|<a$	$(\frac{1}{a})^{n-1}$ $(n\geq 1)$

$\frac{z^{-1}}{1-a^{-1}z^{-1}}$ $\|z\|>a^{-1}$	a^{n+1} $(n<0)$
$-\frac{a}{1-az}$ $\|z\|<a^{-1}$	$-a^{n+1}$ $(n\geq 0)$

$\frac{z}{1-a^{-1}z}$ $\|z\|<a$	$(\frac{1}{a})^{n-1}$ $(n\geq 1)$
$\frac{a}{1-az^{-1}}$ $\|z\|>a$	$-(\frac{1}{a})^{n-1}$ $(n<1)$

$-\frac{a^{-1}}{1-a^{-1}z}$ $\|z\|<a$	$-(\frac{1}{a})^{n+1}$ $(n\geq 0)$
$\frac{z^{-1}}{1-az^{-1}}$ $\|z\|>a$	$(\frac{1}{a})^{n+1}$ $(n<0)$

$-\frac{a^{-1}}{1-a^{-1}z^{-1}}$ $\|z\|>a^{-1}$	$-a^{n-1}$ $(n<1)$
$\frac{z}{1-az}$ $\|z\|<a^{-1}$	a^{n-1} $(n\geq 1)$

$\frac{z^{-1}}{1-az^{-1}}$ $\|z\|>a$	$(\frac{1}{a})^{n+1}$ $(n<0)$
$-\frac{a^{-1}}{1-a^{-1}z}$ $\|z\|<a$	$-(\frac{1}{a})^{n+1}$ $(n\geq 0)$

$\frac{z}{1-az}$ $\|z\|<a^{-1}$	a^{n-1} $(n\geq 1)$
$-\frac{a^{-1}}{1-a^{-1}z^{-1}}$ $\|z\|>a^{-1}$	$-a^{n-1}$ $(n<1)$

Complemnt ROC Pairs - Reduced Shifted Geometric Series Form Combinations

$-\frac{a}{1-az}$ $\lvert z\rvert < a^{-1}$	$-a^{n+1}$ $(n \geqslant 0)$
$\frac{z^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert > a^{-1}$	a^{n+1} $(n < 0)$

$-\frac{a}{1-az^{-1}}$ $\lvert z\rvert > a$	$-\left(\frac{1}{a}\right)^{n-1}$ $(n < 1)$
$\frac{z}{1-a^{-1}z}$ $\lvert z\rvert < a$	$\left(\frac{1}{a}\right)^{n-1}$ $(n \geq 1)$

$-\frac{a^{-1}}{1-a^{-1}z}$ $\lvert z\rvert < a$	$-\left(\frac{1}{a}\right)^{n+1}$ $(n \geqslant 0)$
$\frac{z^{-1}}{1-az^{-1}}$ $\lvert z\rvert > a$	$\left(\frac{1}{a}\right)^{n+1}$ $(n < 0)$

$-\frac{a^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert > a^{-1}$	$-a^{n-1}$ $(n < 1)$
$\frac{z}{1-az}$ $\lvert z\rvert < a^{-1}$	a^{n-1} $(n \geq 1)$

$\boxed{2z}$ $\boxed{2z^{-1}}$ $\boxed{2^{-1}z^{-1}}$ $\boxed{2^{-1}z}$

$\boxed{|z|<0.5}$ $\boxed{|z|>2}$ $\boxed{|z|>0.5}$ $\boxed{|z|<2}$

$$\begin{tikzcd}
-\frac{2}{1-2z} \arrow[r, leftrightarrow, "z^{-1}"] \arrow[d, shift right, "\cdot\frac{(2z)^{-1}}{(2z)^{-1}}"'] & -\frac{2}{1-2z^{-1}} \arrow[d, shift right, "\cdot\frac{(2z^{-1})^{-1}}{(2z^{-1})^{-1}}"'] \\
+\frac{z^{-1}}{1-0.5z^{-1}} \arrow[u, shift right, "\cdot\frac{(2z)}{(2z)}"'] \arrow[r, leftrightarrow, "z^{-1}"] & +\frac{z}{1-0.5z} \arrow[u, shift right, "\cdot\frac{(2z^{-1})}{(2z^{-1})}"']
\end{tikzcd}$$

		scale(a)		scale(a)
	$-\frac{1}{1-az}$ $\lvert z\rvert<a^{-1}$	$-\frac{a}{1-az}$ $\lvert z\rvert<a^{-1}$	$-\frac{1}{1-az^{-1}}$ $\lvert z\rvert>a$	$-\frac{a}{1-az^{-1}}$ $\lvert z\rvert>a$
Comp.ROC	$\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$	$\frac{z^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$	$\frac{a^{-1}z}{1-a^{-1}z}$ $\lvert z\rvert<a$	$\frac{z}{1-a^{-1}z}$ $\lvert z\rvert<a$

		scale(1/z)		scale(z)
	$\frac{1}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$	$\frac{z^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$	$\frac{1}{1-a^{-1}z}$ $\lvert z\rvert<a$	$\frac{z}{1-a^{-1}z}$ $\lvert z\rvert<a$
Comp.ROC	$-\frac{az}{1-az}$ $\lvert z\rvert<a^{-1}$	$-\frac{a}{1-az}$ $\lvert z\rvert<a^{-1}$	$\frac{az^{-1}}{1-az^{-1}}$ $\lvert z\rvert>a$	$\frac{a}{1-az^{-1}}$ $\lvert z\rvert>a$

		scale(1/a)		scale(1/a)
	$-\frac{1}{1-a^{-1}z}$ $\lvert z\rvert<a$	$-\frac{a^{-1}}{1-a^{-1}z}$ $\lvert z\rvert<a$	$-\frac{1}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$	$-\frac{a^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$
Comp.ROC	$\frac{az^{-1}}{1-az^{-1}}$ $\lvert z\rvert>a$	$\frac{z^{-1}}{1-az^{-1}}$ $\lvert z\rvert>a$	$\frac{az}{1-az}$ $\lvert z\rvert<a^{-1}$	$\frac{z}{1-az}$ $\lvert z\rvert<a^{-1}$

		scale(1/z)		scale(z)
	$\frac{1}{1-az^{-1}}$ $\lvert z\rvert>a$	$\frac{z^{-1}}{1-az^{-1}}$ $\lvert z\rvert>a$	$\frac{1}{1-az}$ $\lvert z\rvert<a^{-1}$	$\frac{z}{1-az}$ $\lvert z\rvert<a^{-1}$
Comp.ROC	$-\frac{a^{-1}z}{1-a^{-1}z}$ $\lvert z\rvert<a$	$-\frac{a^{-1}}{1-a^{-1}z}$ $\lvert z\rvert<a$	$-\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$	$-\frac{a^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$

	scale(a)		scale(a)	
Comp.ROC	$-\frac{1}{1-az}$ $\lvert z\rvert<a^{-1}$	$-\frac{a}{1-az}$ $\lvert z\rvert<a^{-1}$	$-\frac{1}{1-az^{-1}}$ $\lvert z\rvert>a$	$-\frac{a}{1-az^{-1}}$ $\lvert z\rvert>a$
	$\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$	$\frac{z^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$	$\frac{a^{-1}z}{1-a^{-1}z}$ $\lvert z\rvert<a$	$\frac{z}{1-a^{-1}z}$ $\lvert z\rvert<a$

	scale(1/z)		scale(z)	
Comp.ROC	$\frac{1}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$	$\frac{z^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$	$\frac{1}{1-a^{-1}z}$ $\lvert z\rvert<a$	$\frac{z}{1-a^{-1}z}$ $\lvert z\rvert<a$
	$-\frac{az}{1-az}$ $\lvert z\rvert<a^{-1}$	$-\frac{a}{1-az}$ $\lvert z\rvert<a^{-1}$	$\frac{az^{-1}}{1-az^{-1}}$ $\lvert z\rvert>a$	$\frac{a}{1-az^{-1}}$ $\lvert z\rvert>a$

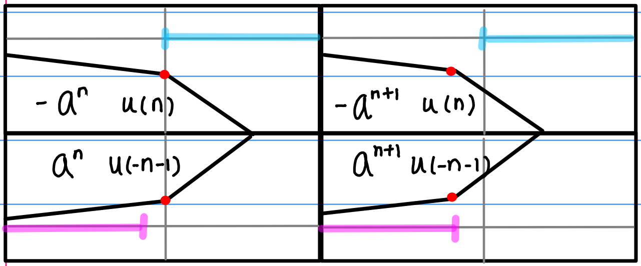
	scale(1/a)		scale(1/a)	
Comp.ROC	$-\frac{1}{1-a^{-1}z}$ $\lvert z\rvert<a$	$-\frac{a^{-1}}{1-a^{-1}z}$ $\lvert z\rvert<a$	$-\frac{1}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$	$-\frac{a^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$
	$\frac{az^{-1}}{1-az^{-1}}$ $\lvert z\rvert>a$	$\frac{z^{-1}}{1-az^{-1}}$ $\lvert z\rvert>a$	$\frac{az}{1-az}$ $\lvert z\rvert<a^{-1}$	$\frac{z}{1-az}$ $\lvert z\rvert<a^{-1}$

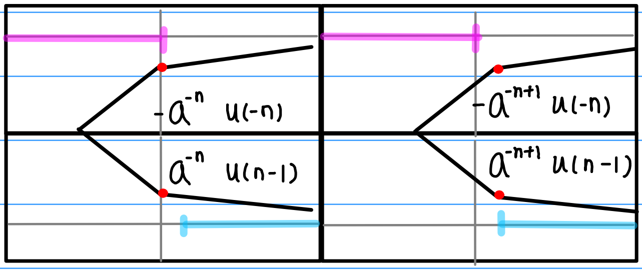
	scale(1/z)		scale(z)	
Comp.ROC	$\frac{1}{1-az^{-1}}$ $\lvert z\rvert>a$	$\frac{z^{-1}}{1-az^{-1}}$ $\lvert z\rvert>a$	$\frac{1}{1-az}$ $\lvert z\rvert<a^{-1}$	$\frac{z}{1-az}$ $\lvert z\rvert<a^{-1}$
	$-\frac{a^{-1}z}{1-a^{-1}z}$ $\lvert z\rvert<a$	$-\frac{a^{-1}}{1-a^{-1}z}$ $\lvert z\rvert<a$	$-\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$	$-\frac{a^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$

SHL.Seq

SHR.Seq

-Comp.Rng

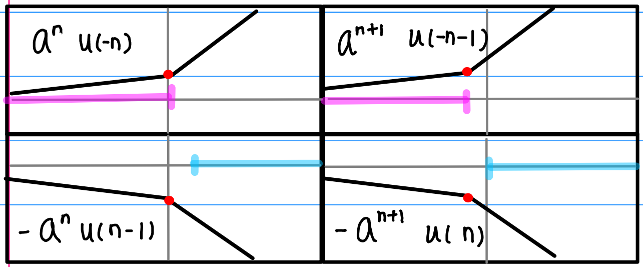

$-a^{n}$ u(n)
a^{n} u(-n-1)
$-a^{n+1}$ u(n)
a^{n+1} u(-n-1)

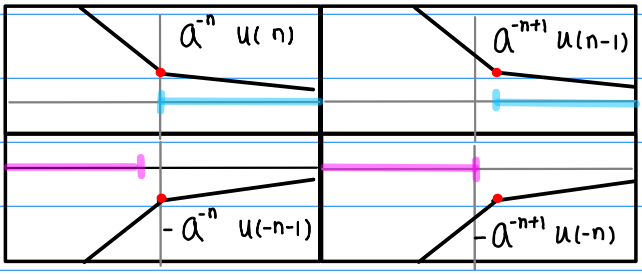

$-a^{-n}$ u(-n)
a^{-n} u(n-1)
$-a^{-n+1}$ u(-n)
a^{-n+1} u(n-1)

SHL.Seq, SHL.Rng

SHR.Seq, SHR.Rng

-Comp.Rng

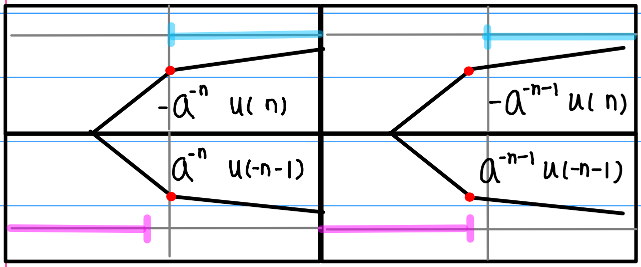

a^{n} u(-n)
$-a^{n}$ u(n-1)
a^{n+1} u(-n-1)
$-a^{n+1}$ u(n)

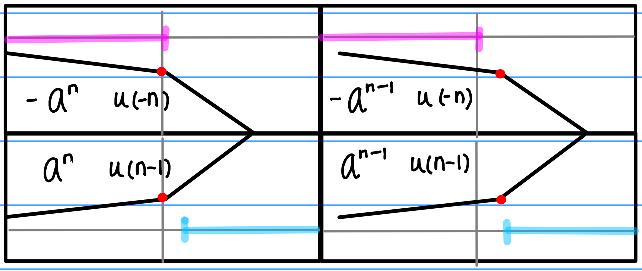

a^{-n} u(n)
$-a^{-n}$ u(-n-1)
a^{-n+1} u(n-1)
$-a^{-n+1}$ u(-n)

SHL.Seq

SHR.Seq

-Comp.Rng

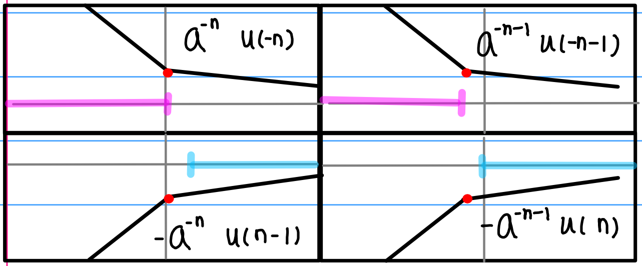

$-a^{-n}$ u(n)
a^{-n} u(-n-1)
$-a^{-n-1}$ u(n)
a^{-n-1} u(-n-1)

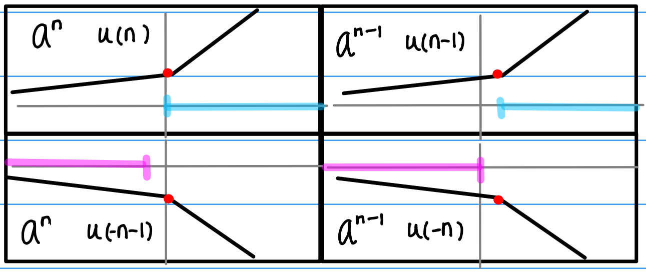

$-a^{n}$ u(-n)
a^{n} u(n-1)
$-a^{n-1}$ u(-n)
a^{n-1} u(n-1)

SHL.Seq, SHL.Rng

SHR.Seq, SHR.Rng

-Comp.Rng


a^{-n} u(-n)
$-a^{-n}$ u(n-1)
a^{-n-1} u(-n-1)
$-a^{-n-1}$ u(n)


a^{n} u(n)
a^{n} u(-n-1)
a^{n-1} u(n-1)
a^{n-1} u(-n)

Left Shifted Sequence

Right Shifted Sequence

		scale(a)		scale(a)
	$-\frac{1}{1-az}$ $\lvert z\rvert<a^{-1}$	$-\frac{a}{1-az}$ $\lvert z\rvert<a^{-1}$	$-\frac{1}{1-az^{-1}}$ $\lvert z\rvert>a$	$-\frac{a}{1-az^{-1}}$ $\lvert z\rvert>a$
Comp.ROC	$\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$	$\frac{z^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$	$\frac{a^{-1}z}{1-a^{-1}z}$ $\lvert z\rvert<a$	$\frac{z}{1-a^{-1}z}$ $\lvert z\rvert<a$

		scale(1/z)		scale(z)
	$\frac{1}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$	$\frac{z^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$	$\frac{1}{1-a^{-1}z}$ $\lvert z\rvert<a$	$\frac{z}{1-a^{-1}z}$ $\lvert z\rvert<a$
Comp.ROC	$-\frac{az}{1-az}$ $\lvert z\rvert<a^{-1}$	$-\frac{a}{1-az}$ $\lvert z\rvert<a^{-1}$	$\frac{az^{-1}}{1-az^{-1}}$ $\lvert z\rvert>a$	$\frac{a}{1-az^{-1}}$ $\lvert z\rvert>a$

		scale(1/a)		scale(1/a)
	$-\frac{1}{1-a^{-1}z}$ $\lvert z\rvert<a$	$-\frac{a^{-1}}{1-a^{-1}z}$ $\lvert z\rvert<a$	$-\frac{1}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$	$-\frac{a^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$
Comp.ROC	$\frac{az^{-1}}{1-az^{-1}}$ $\lvert z\rvert>a$	$\frac{z^{-1}}{1-az^{-1}}$ $\lvert z\rvert>a$	$\frac{az}{1-az}$ $\lvert z\rvert<a^{-1}$	$\frac{z}{1-az}$ $\lvert z\rvert<a^{-1}$

		scale(1/z)		scale(z)
	$\frac{1}{1-az^{-1}}$ $\lvert z\rvert>a$	$\frac{z^{-1}}{1-az^{-1}}$ $\lvert z\rvert>a$	$\frac{1}{1-az}$ $\lvert z\rvert<a^{-1}$	$\frac{z}{1-az}$ $\lvert z\rvert<a^{-1}$
Comp.ROC	$-\frac{a^{-1}z}{1-a^{-1}z}$ $\lvert z\rvert<a$	$-\frac{a^{-1}}{1-a^{-1}z}$ $\lvert z\rvert<a$	$-\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$	$-\frac{a^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$

SHL.Seq

	$-a^n \ (n \geq 0)$	$-a^{n+1} \ (n \geq 0)$
	$-(a^0, a^1, a^2, \cdots)$	$-(a^1, a^2, a^3, \cdots)$
-Comp.Rng	$a^n \ (n < 0)$	$a^{n+1} \ (n < 0)$
	$(\cdots, \frac{1}{a^3}, \frac{1}{a^2}, \frac{1}{a^1})$	$(\cdots, \frac{1}{a^2}, \frac{1}{a^1}, \frac{1}{a^0})$

SHR.Seq

$-(\frac{1}{a})^n \ (n < 1)$	$-(\frac{1}{a})^{n-1} \ (n < 1)$
$-(\cdots, a^2, a^1, a^0)$	$-(\cdots, a^3, a^2, a^1)$
$(\frac{1}{a})^n \ (n \geq 1)$	$(\frac{1}{a})^{n-1} \ (n \geq 1)$
$(\frac{1}{a^1}, \frac{1}{a^2}, \frac{1}{a^3}, \cdots)$	$(\frac{1}{a^0}, \frac{1}{a^1}, \frac{1}{a^2}, \cdots)$

SHL.Seq, SHL.Rng

	$a^n \ (n < 1)$	$a^{n+1} \ (n < 0)$
	$(\cdots, \frac{1}{a^2}, \frac{1}{a^1}, \frac{1}{a^0})$	$(\cdots, \frac{1}{a^2}, \frac{1}{a^1}, \frac{1}{a^0})$
-Comp.Rng	$-a^n \ (n \geq 1)$	$-a^{n+1} \ (n \geq 0)$
	$-(a^1, a^2, a^3, \cdots)$	$-(a^1, a^2, a^3, \cdots)$

SHR.Seq, SHR.Rng

$(\frac{1}{a})^n \ (n \geq 0)$	$(\frac{1}{a})^{n-1} \ (n \geq 1)$
$(\frac{1}{a^0}, \frac{1}{a^1}, \frac{1}{a^2}, \cdots)$	$(\frac{1}{a^0}, \frac{1}{a^1}, \frac{1}{a^2}, \cdots)$
$-(\frac{1}{a})^n \ (n < 0)$	$-(\frac{1}{a})^{n-1} \ (n < 1)$
$(\cdots, a^3, a^2, a^1)$	$(\cdots, a^3, a^2, a^1)$

SHL.Seq

	$-(\frac{1}{a})^n \ (n \geq 0)$	$-(\frac{1}{a})^{n+1} \ (n \geq 0)$
	$-(\frac{1}{a^0}, \frac{1}{a^1}, \frac{1}{a^2}, \cdots)$	$-(\frac{1}{a^1}, \frac{1}{a^2}, \frac{1}{a^3}, \cdots)$
-Comp.Rng	$(\frac{1}{a})^n \ (n < 0)$	$(\frac{1}{a})^{n+1} \ (n < 0)$
	$(\cdots, a^3, a^2, a^1)$	$(\cdots, a^2, a^1, a^0)$

SHR.Seq

$-a^n \ (n < 1)$	$-a^{n-1} \ (n < 1)$
$-(\cdots, \frac{1}{a^2}, \frac{1}{a^1}, \frac{1}{a^0})$	$-(\cdots, \frac{1}{a^2}, \frac{1}{a^3}, \frac{1}{a^1})$
$a^n \ (n \geq 1)$	$a^{n-1} \ (n \geq 1)$
$(a^1, a^2, a^3, \cdots)$	$(a^0, a^1, a^2, \cdots)$

SHL.Seq, SHL.Rng

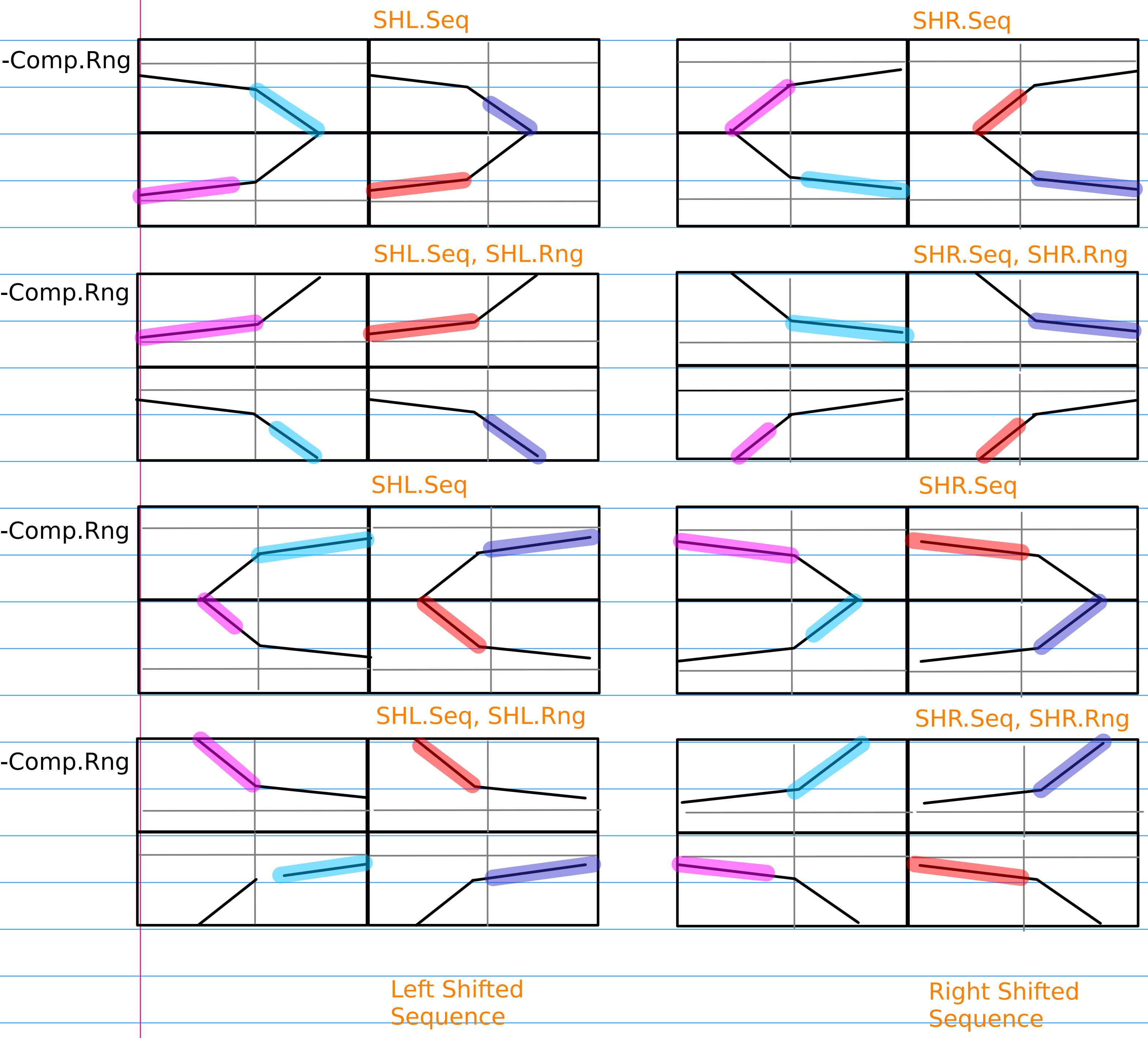
	$(\frac{1}{a})^n \ (n < 1)$	$(\frac{1}{a})^{n+1} \ (n < 0)$
	$(\cdots, a^2, a^1, a^0)$	$(\cdots, a^2, a^1, a^0)$
-Comp.Rng	$-(\frac{1}{a})^n \ (n \geq 1)$	$-(\frac{1}{a})^{n+1} \ (n \geq 0)$
	$-(\frac{1}{a^1}, \frac{1}{a^2}, \frac{1}{a^3}, \cdots)$	$-(\frac{1}{a^1}, \frac{1}{a^2}, \frac{1}{a^3}, \cdots)$

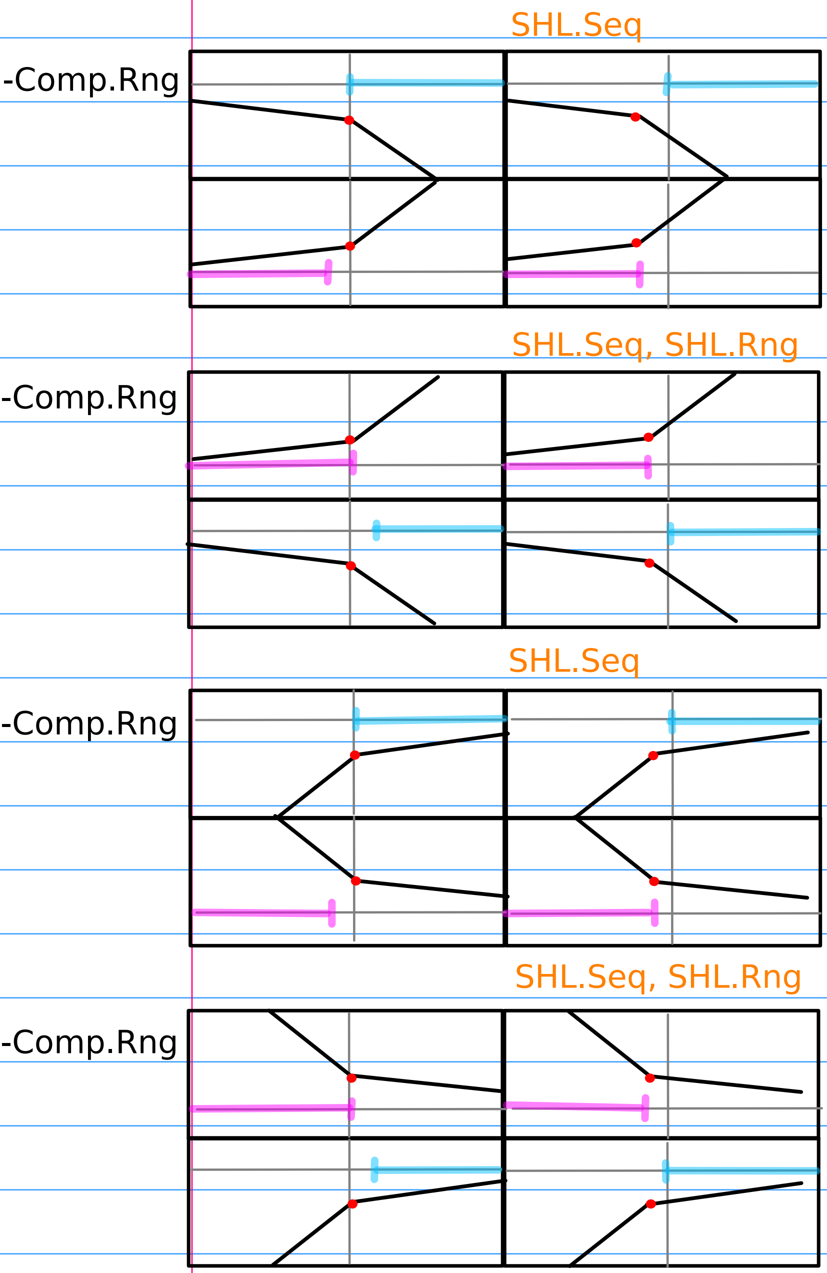
SHR.Seq, SHR.Rng

$a^n \ (n \geq 0)$	$a^{n-1} \ (n \geq 1)$
$(a^0, a^1, a^2, \cdots)$	$(a^0, a^1, a^2, \cdots)$
$-a^n \ (n < 0)$	$-a^{n-1} \ (n < 1)$
$-(\cdots, \frac{1}{a^3}, \frac{1}{a^2}, \frac{1}{a^1})$	$-(\cdots, \frac{1}{a^3}, \frac{1}{a^2}, \frac{1}{a^1})$

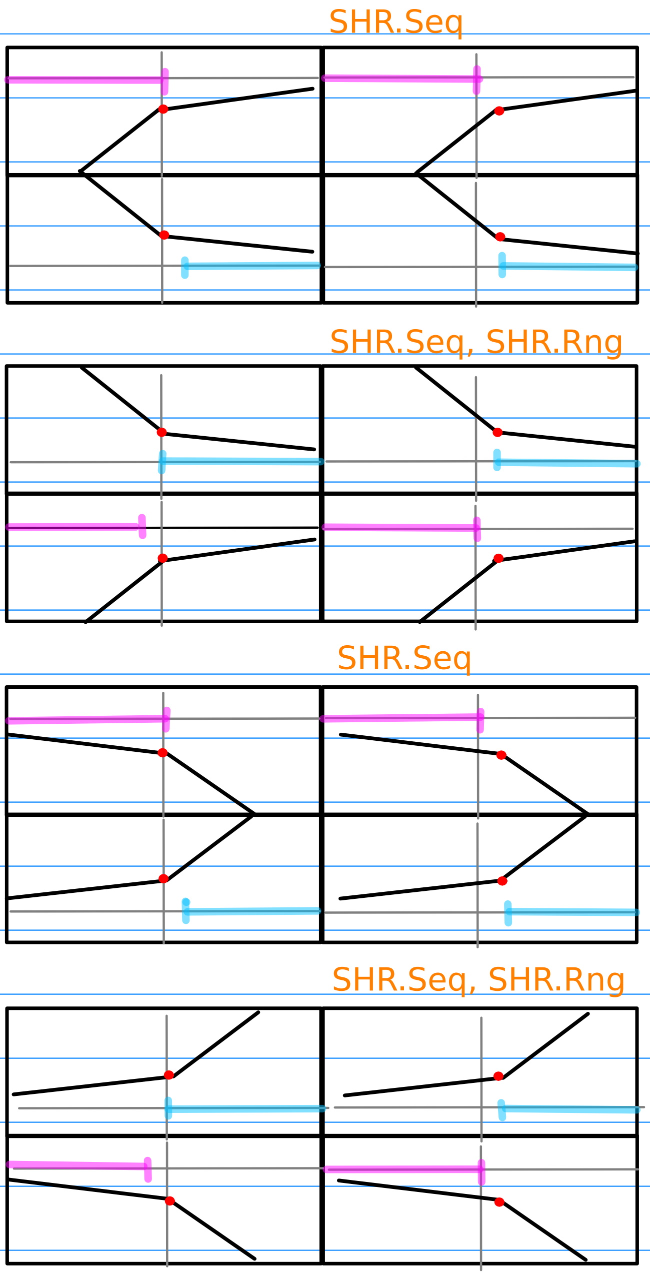
Left Shifted Sequence

Right Shifted Sequence


SHL.Seq
SHR.Seq
-Comp.Rng
SHL.Seq, SHL.Rng
SHR.Seq, SHR.Rng
-Comp.Rng
SHL.Seq
SHR.Seq
-Comp.Rng
SHL.Seq, SHL.Rng
SHR.Seq, SHR.Rng
-Comp.Rng
Left Shifted Sequence
Right Shifted Sequence


SHL.Seq
-Comp.Rng
SHL.Seq, SHL.Rng
-Comp.Rng
SHL.Seq
-Comp.Rng
SHL.Seq, SHL.Rng
-Comp.Rng

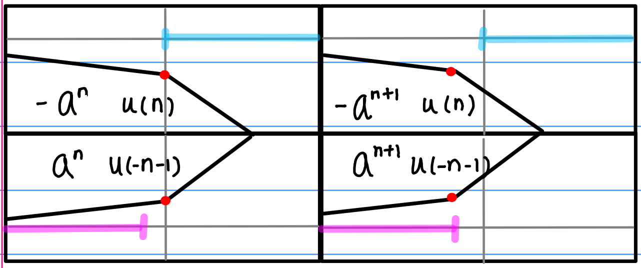
Left Shifted
Sequence


SHR.Seq
SHR.Seq, SHR.Rng
SHR.Seq
SHR.Seq, SHR.Rng

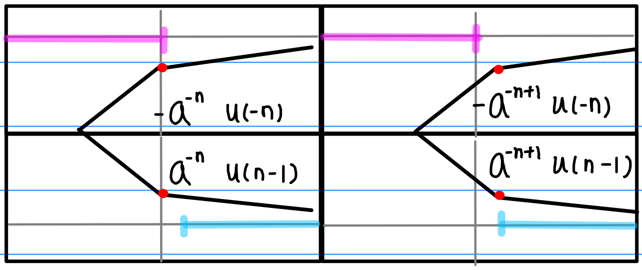
Right Shifted
Sequence

-Comp.Rng

SHL.Seq

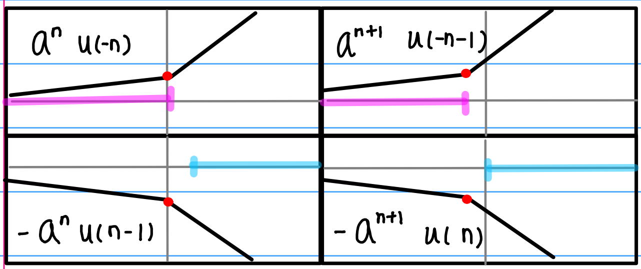

$-a^n \quad u(n)$
$-a^{n+1} \quad u(n)$
$a^n \quad u(-n-1)$
$a^{n+1} \, u(-n-1)$

SHR.Seq

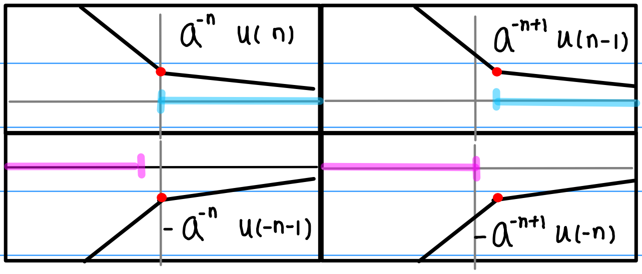

$-a^{-n} \quad u(-n)$
$-a^{-n+1} \, u(-n)$
$a^{-n} \quad u(n-1)$
$a^{-n+1} \, u(n-1)$

-Comp.Rng

SHL.Seq, SHL.Rng

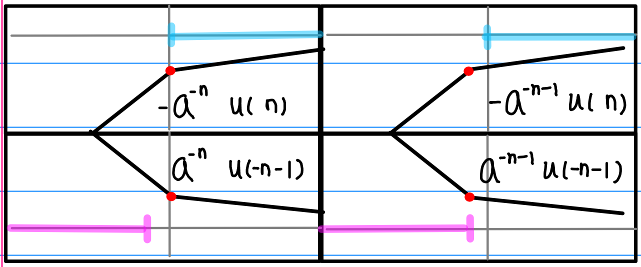

$a^n \, u(-n)$
$a^{n+1} \quad u(-n-1)$
$-a^n \, u(n-1)$
$-a^{n+1} \quad u(n)$

SHR.Seq, SHR.Rng

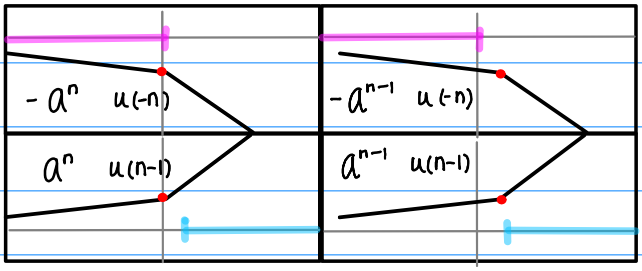

$a^{-n} \quad u(n)$
$a^{-n+1} \, u(n-1)$
$-a^{-n} \quad u(-n-1)$
$-a^{-n+1} \, u(-n)$

-Comp.Rng

SHL.Seq

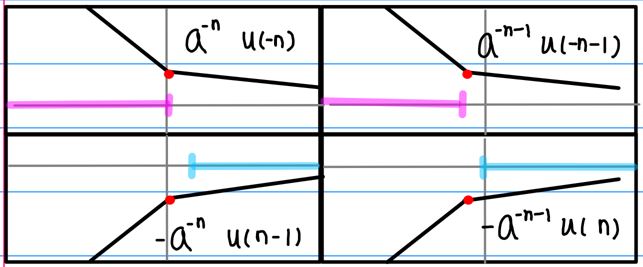

$-a^{-n} \quad u(n)$
$-a^{-n-1} \, u(n)$
$a^{-n} \quad u(-n-1)$
$a^{-n-1} \, u(-n-1)$

SHR.Seq

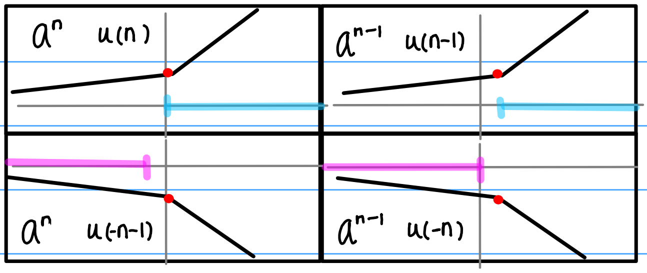

$-a^n \quad u(-n)$
$-a^{n-1} \quad u(-n)$
$a^n \quad u(n-1)$
$a^{n-1} \quad u(n-1)$

-Comp.Rng

SHL.Seq, SHL.Rng


$a^{-n} \quad u(-n)$
$a^{-n-1} \, u(-n-1)$
$-a^{-n} \quad u(n-1)$
$-a^{-n-1} \, u(n)$

SHR.Seq, SHR.Rng

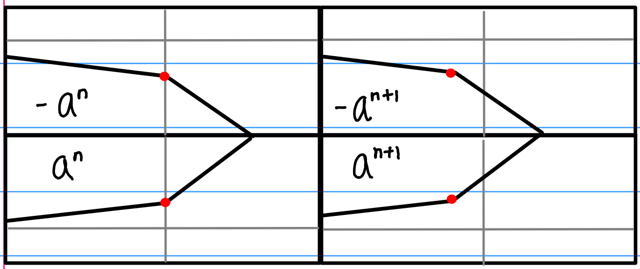

$a^n \quad u(n)$
$a^{n-1} \quad u(n-1)$
$a^n \quad u(-n-1)$
$a^{n-1} \quad u(-n)$

Left Shifted
Sequence

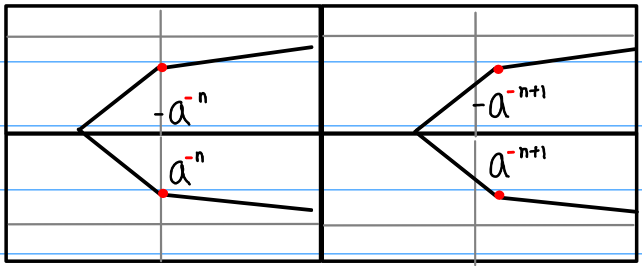
Right Shifted
Sequence

a Sequence Function

-Comp.Rng

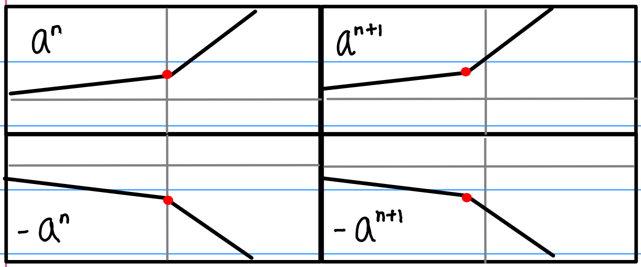

$-a^n$
a^n
$-a^{n+1}$
a^{n+1}

SHL.Seq

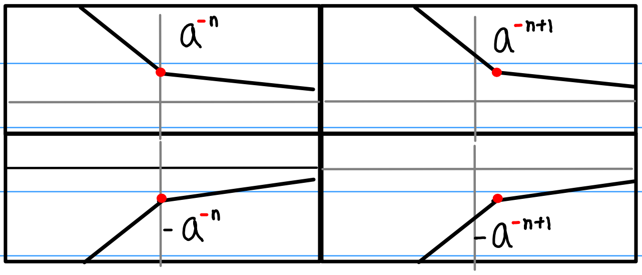

$-a^{-n}$
a^{-n}
$-a^{-n+1}$
a^{-n+1}

SHR.Seq

-Comp.Rng

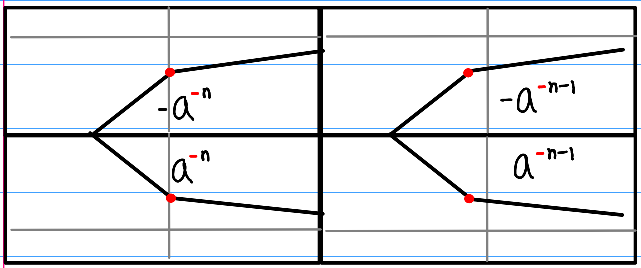

a^n
a^{n+1}
$-a^n$
$-a^{n+1}$

SHL.Seq, SHL.Rng

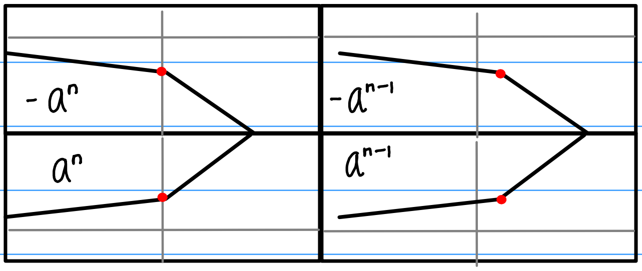

a^{-n}
a^{-n+1}
$-a^{-n}$
$-a^{-n+1}$

SHR.Seq, SHR.Rng

-Comp.Rng

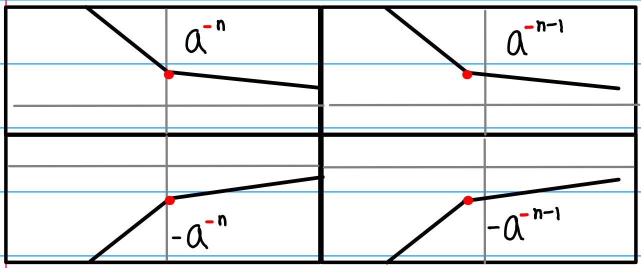

$-a^{-n}$
a^{-n}
$-a^{-n-1}$
a^{-n-1}

SHL.Seq

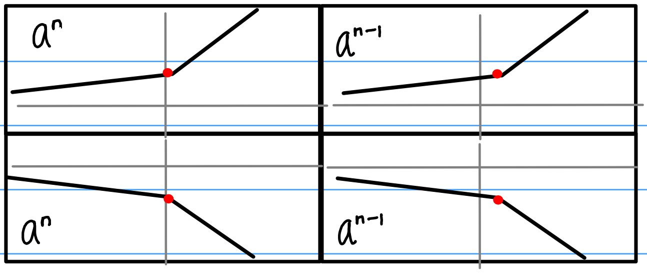

$-a^n$
a^n
$-a^{n-1}$
a^{n-1}

SHR.Seq

-Comp.Rng


a^{-n}
a^{-n-1}
$-a^{-n}$
$-a^{-n-1}$

SHL.Seq, SHL.Rng


a^n
a^{n-1}
a^n
a^{n-1}

SHR.Seq, SHR.Rng

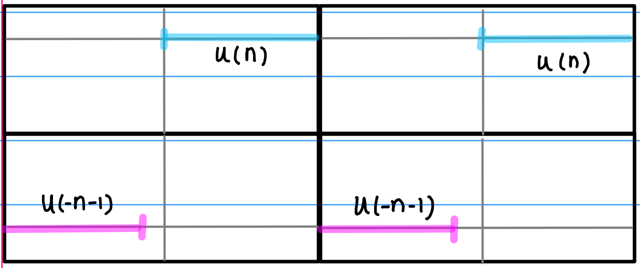
Left Shifted Sequence

Right Shifted Sequence

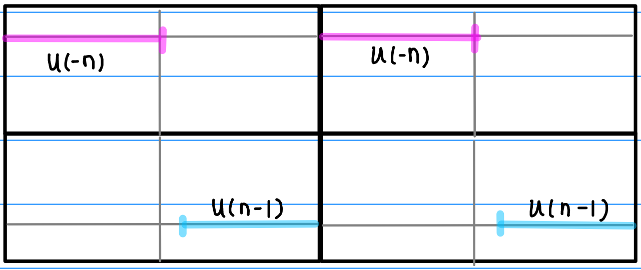
Range of a Sequence

-Comp.Rng

SHL.Seq

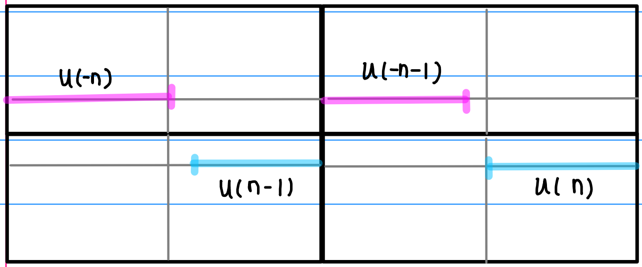

u(n)
u(n)
u(-n-1)
u(-n-1)

SHR.Seq

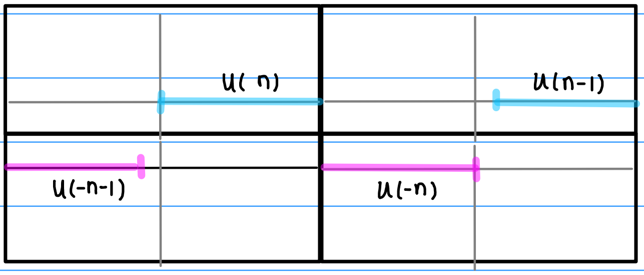

u(-n)
u(-n)
u(n-1)
u(n-1)

-Comp.Rng

SHL.Seq, SHL.Rng

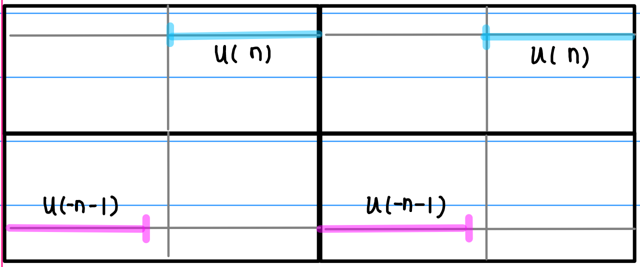

u(-n)
u(-n-1)
u(n-1)
u(n)

SHR.Seq, SHR.Rng

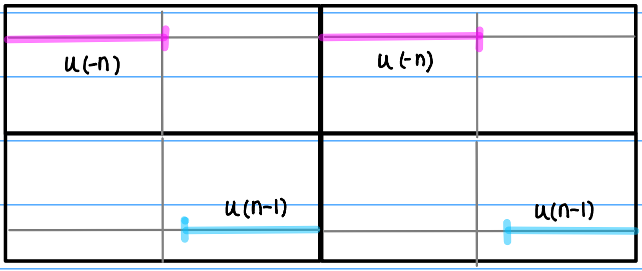

u(n)
u(n-1)
u(-n-1)
u(-n)

-Comp.Rng

SHL.Seq

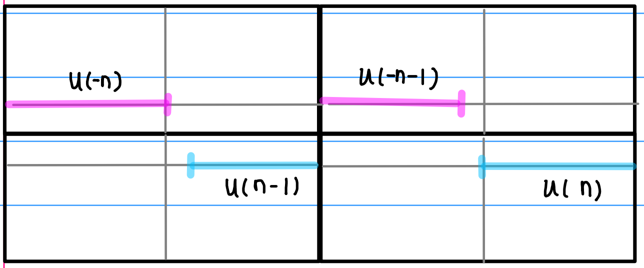

u(n)
u(n)
u(-n-1)
u(-n-1)

SHR.Seq

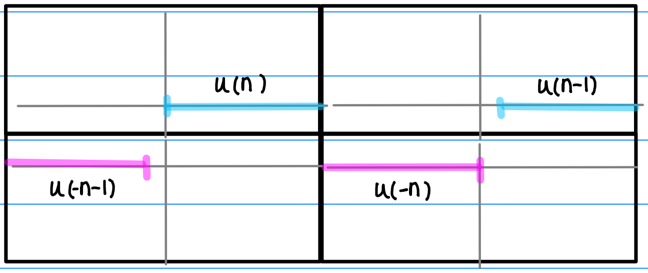

u(-n)
u(-n)
u(n-1)
u(n-1)

-Comp.Rng

SHL.Seq, SHL.Rng


u(-n)
u(-n-1)
u(n-1)
u(n)

SHR.Seq, SHR.Rng


u(n)
u(n-1)
u(-n-1)
u(-n)