Laurent Series and z-Transform - Geometric Series Double Pole Examples B

20180522

2 formulas of z

① $$\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$$\Big\updownarrow z^{-1}$$

② $$\frac{3}{2}\,\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right)$$

$$\boxed{\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} \quad \xleftrightarrow{z^{-1}} \quad \frac{3}{2}\,\frac{-z^2}{(z-2)(z-0.5)}}$$

$$
\begin{aligned}
\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} &= \frac{3}{2}\,\frac{2}{3}\left(\frac{1}{z-0.5}-\frac{1}{z-2}\right) \\
&= \left(\frac{1}{z-0.5}-\frac{1}{z-2}\right)
\end{aligned}
$$

$$\Big\downarrow z^{-1}$$

$$
\begin{aligned}
\frac{3}{2}\,\frac{-1}{(z^{-1}-0.5)(z^{-1}-2)} &= \frac{3}{2}\,\frac{2}{3}\left(\frac{1}{z^{-1}-0.5}-\frac{1}{z^{-1}-2}\right) \\
&= \left(\frac{2}{2z^{-1}-1}-\frac{0.5}{0.5z^{-1}-1}\right) \\
&= \left(\frac{2z}{2-z}-\frac{0.5z}{0.5-z}\right) \\
&= \left(\frac{-2z}{z-2}+\frac{0.5z}{z-0.5}\right) \\
&= z\left(\frac{-2}{z-2}+\frac{0.5}{z-0.5}\right) \\
&= z\left(\frac{-\frac{3}{2}z}{(z-2)(z-0.5)}\right) \\
&= \frac{3}{2}\,\frac{-z^2}{(z-2)(z-0.5)}
\end{aligned}
$$

$$\frac{3}{2}\,\frac{-z^2}{(z-2)(z-0.5)} = \frac{3}{2}\,\frac{2}{3}\left(\frac{0.5z}{(z-0.5)}-\frac{2z}{(z-2)}\right)$$

$\boxed{\text{Laurent}}$ $f(z), g(z)$: causal , $f(z^{-1}), g(z^{-1})$: anti-causal

$\boxed{z\text{- Trans}}$ $X(z), Y(z)$: causal, $X(z^{-1}), Y(z^{-1})$: anti-causal

$$\begin{tikzcd}
\boxed{\dfrac{1}{z-p}} \arrow[r, shift left, "\frac{p^{-1}}{p^{-1}}"] \arrow[d, shift right, "\frac{z^{-1}}{z^{-1}}"'] & \boxed{-\dfrac{p^{-1}}{1-p^{-1}z}} \arrow[l, shift left] \\
\boxed{\dfrac{z^{-1}}{1-pz^{-1}}} \arrow[u, shift right] &
\end{tikzcd}$$

$= f(z)$ ↔ $-(p^{-1})^{n+1}$

$Y(z^{-1})$

$\| $

$g(z^{-1})$

$X(z)$ ↔ $(p)^{n-1}$

$$\begin{tikzcd}
\boxed{\dfrac{1}{z^{-1}-p}} \arrow[r, shift left, "\frac{p^{-1}}{p^{-1}}"] \arrow[d, shift right, "\frac{z^{-1}}{z^{-1}}"'] & \boxed{-\dfrac{p^{-1}}{1-p^{-1}z^{-1}}} \arrow[l, shift left] \\
\boxed{\dfrac{z}{1-pz}} \arrow[u, shift right] &
\end{tikzcd}$$

$= f(z^{-1})$

$Y(z)$ ↔ $-(p^{-1})^{n+1}$

$\|$

$g(z)$ ↔ $(p)^{n-1}$

$X(z^{-1})$

2 formulas of z : $f(z)$, $g(z)$

2 representations : $f(z^{-1})$, $g(z^{-1})$

* Simple Pole Forms

p	$\boxed{\dfrac{1}{z-p}}$	$\xleftrightarrow{z^{-1}}$	$\boxed{\dfrac{1}{z^{-1}-p}}$	p^{-1}
	$\dfrac{z^{-1}}{z^{-1}} \cdot \quad \cdot \dfrac{z}{z}$		$\dfrac{z}{z} \cdot \quad \cdot \dfrac{z^{-1}}{z^{-1}}$	
p	$\boxed{\dfrac{z^{-1}}{1-pz^{-1}}}$	$\xleftrightarrow{z^{-1}}$	$\boxed{\dfrac{z}{1-pz}}$	p^{-1}

* Geometric Series Forms

$n \geq 0$	$\lvert z \rvert < p$		$\lvert z \rvert > p^{-1}$	$n \geq 0$
$-(p^{-1})^{n+1}$ causal f(z)	$\boxed{-\dfrac{p^{-1}}{1-p^{-1}z}}$	$\xleftrightarrow{z^{-1}}$	$\boxed{-\dfrac{p^{-1}}{1-p^{-1}z^{-1}}}$	causal X(z) $-(p^{-1})^{n+1}$
	$\cdot \dfrac{p}{p}\dfrac{z^{-1}}{z^{-1}}$		$\cdot \dfrac{p}{p}\dfrac{z}{z}$	
causal X(z) $(p)^{n-1}$	$\boxed{\dfrac{z^{-1}}{1-pz^{-1}}}$	$\xleftrightarrow{z^{-1}}$	$\boxed{\dfrac{z}{1-pz}}$	$(p)^{n-1}$ causal f(z)
$n \geq 1$	$\lvert z \rvert > p$		$\lvert z \rvert < p^{-1}$	$n \geq 1$

Ⓐ $f(z)$ for $|z|<p$, $g(z)$ for $|z|<p^{-1}$ Laurent S

Geometric Series Forms

$|z|<p$

p $\boxed{f(z)} = \boxed{-\dfrac{p^{-1}}{1-p^{-1}z}}$ $\qquad \boxed{-\dfrac{p^{-1}}{1-p^{-1}z^{-1}}}$

$\boxed{\dfrac{z^{-1}}{1-pz^{-1}}} \qquad \boxed{\dfrac{z}{1-pz}} = \boxed{g(z)}$ p^{-1}

$|z|<p^{-1}$

Ⓑ $f(z^{-1})$ for $|z|>p^{-1}$, $g(z^{-1})$ for $|z|>p$

Geometric Series Forms

$|z|<p$ $\qquad |z|>p^{-1}$

$f(z) = \boxed{-\dfrac{p^{-1}}{1-p^{-1}z}} \xleftrightarrow{z^{-1}} \boxed{-\dfrac{p^{-1}}{1-p^{-1}z^{-1}}} = \boxed{f(z^{-1})}$

$\boxed{g(z^{-1})} = \boxed{\dfrac{z^{-1}}{1-pz^{-1}}} \xleftrightarrow{z^{-1}} \boxed{\dfrac{z}{1-pz}} = g(z)$

$|z|>p$ $\qquad |z|<p^{-1}$

Laurent Series $a_n \leftrightarrow f(z)$ $b_n \leftrightarrow g(z)$

Geometric Series Forms

$|z| < p$

$$f(z) = \boxed{-\frac{p^{-1}}{1-p^{-1}z}} \xleftrightarrow{z^{-1}} \boxed{-\frac{p^{-1}}{1-p^{-1}z^{-1}}} = f(z^{-1})$$

$|z| > p^{-1}$

$\cdot \frac{p}{p}\frac{z^{-1}}{z^{-1}}$ $\qquad$ $\cdot \frac{p}{p}\frac{z}{z}$

$$g(z^{-1}) = \boxed{\frac{z^{-1}}{1-pz^{-1}}} \xleftrightarrow{z^{-1}} \boxed{\frac{z}{1-pz}} = g(z)$$

$|z| > p$ $\qquad$ $|z| < p^{-1}$

$$\boxed{\begin{array}{l} f(z) \ (|z| < p) \\ a_n \ (n \geq 0) \end{array}} \xleftrightarrow{z^{-1}} \boxed{\begin{array}{l} f(z^{-1}) \ (|z^{-1}| < p) \\ a_{-n} \ (n < 1) \end{array}} \boxed{\begin{array}{l} g(z) \ (|z| > p^{-1}) \\ -b_n \ (n < 1) \end{array}}$$

$$\boxed{\begin{array}{l} f(z) \ (|z| > p) \\ -a_n \ (n < 0) \end{array}} \boxed{\begin{array}{l} g(z^{-1}) \ (|z^{-1}| < p^{-1}) \\ b_{-n} \ (n < 0) \end{array}} \xleftrightarrow{z^{-1}} \boxed{\begin{array}{l} g(z) \ (|z| < p^{-1}) \\ b_n \ (n \geq 1) \end{array}}$$

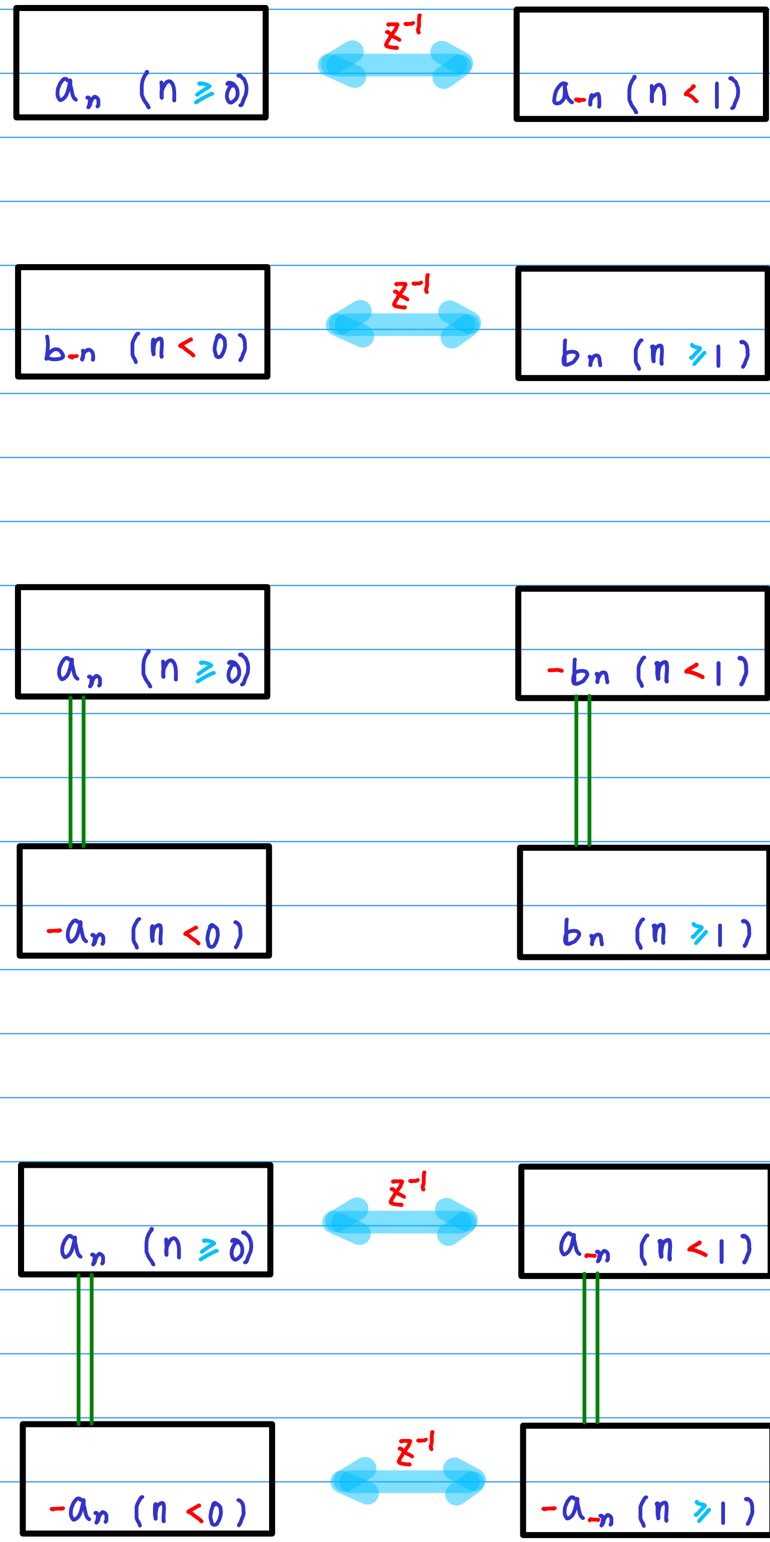
Laurent Series using only $\boxed{a_n} \leftrightarrow \boxed{f(z)}$

	$f(z)$ $(\lvert z\rvert < p)$ a_n $(n \geq 0)$	$\xleftrightarrow{z^{-1}}$	$f(z^{-1})$ $(\lvert z^{-1}\rvert < p)$ a_{-n} $(n < 1)$	$g(z)$ $(\lvert z\rvert > p^{-1})$ $-b_n$ $(n < 1)$
$f(z)$ $(\lvert z\rvert > p)$ $-a_n$ $(n < 0)$	$g(z^{-1})$ $(\lvert z^{-1}\rvert < p^{-1})$ b_{-n} $(n < 0)$	$\xleftrightarrow{z^{-1}}$	$g(z)$ $(\lvert z\rvert < p^{-1})$ b_n $(n \geq 1)$	

$$a_{-n} = -b_n \qquad -a_{-n} = b_n$$

$f(z)$ $(\lvert z\rvert < p)$ a_n $(n \geq 0)$	$\xleftrightarrow{z^{-1}}$	$f(z^{-1})$ $(\lvert z\rvert > p^{-1})$ a_{-n} $(n < 1)$
$f(z)$ $(\lvert z\rvert > p)$ $-a_n$ $(n < 0)$	$\xleftrightarrow{z^{-1}}$	$f(z^{-1})$ $(\lvert z\rvert < p^{-1})$ $-a_{-n}$ $(n \geq 1)$

Laurent Series $\boxed{a_n} \leftrightarrow f(z)$



Laurent Series $a_n \leftrightarrow \boxed{f(z)}$

$$\begin{tikzcd} \boxed{f(z)\ \ (|z|<p)} \arrow[r, leftrightarrow, "z^{-1}"] & \boxed{f(z^{-1})\ (|z^{-1}|<p)} \end{tikzcd}$$

$$\begin{tikzcd} \boxed{g(z^{-1})\ (|z^{-1}|<p^{-1})} \arrow[r, leftrightarrow, "z^{-1}"] & \boxed{g(z)\ (|z|<p^{-1})} \end{tikzcd}$$

$$\begin{tikzcd} \boxed{f(z)\ \ (|z|<p)} \arrow[d, equal] & \boxed{f(z^{-1})\ (|z|>p^{-1})} \arrow[d, equal] \\ \boxed{g(z^{-1})\ \ (|z|>p)} & \boxed{g(z)\ \ (|z|<p^{-1})} \end{tikzcd}$$

$$\begin{tikzcd} \boxed{f(z)\ \ (|z|<p)} \arrow[r, leftrightarrow, "z^{-1}"] \arrow[d, equal] & \boxed{f(z^{-1})\ (|z|>p^{-1})} \arrow[d, equal] \\ \boxed{f(z)\ \ (|z|>p)} \arrow[r, leftrightarrow, "z^{-1}"] & \boxed{f(z^{-1})\ (|z|<p^{-1})} \end{tikzcd}$$

Z- Transform $\quad X(z) \leftrightarrow x_n \quad Y(z) \leftrightarrow y_n$

Geometric Series Forms

$$
\begin{tikzcd}[column sep=huge, row sep=huge]
Y(z^{-1}) = \underset{|z|<p}{\boxed{-\dfrac{p^{-1}}{1-p^{-1}z}}} \arrow[r, leftrightarrow, "z^{-1}"] \arrow[d, equal, "\cdot\frac{p}{p}\frac{z^{-1}}{z^{-1}}"] & \underset{|z|>p^{-1}}{\boxed{-\dfrac{p^{-1}}{1-p^{-1}z^{-1}}}} = Y(z) \arrow[d, equal, "\cdot\frac{p}{p}\frac{z}{z}"'] \\
X(z) = \underset{|z|>p}{\boxed{\dfrac{z^{-1}}{1-pz^{-1}}}} \arrow[r, leftrightarrow, "z^{-1}"] & \underset{|z|<p^{-1}}{\boxed{\dfrac{z}{1-pz}}} = X(z^{-1})
\end{tikzcd}
$$

causal $\quad X(z),\ Y(z)$

anti - causal $\quad X(z^{-1}),\ Y(z^{-1})$

$g(z^{-1})$

$$
\begin{tikzcd}[column sep=large, row sep=huge]
\underset{n<1}{\boxed{\text{anti- causal } -x_n}} & \underset{|z|<p}{\boxed{Y(z^{-1})}} \arrow[r, leftrightarrow, "z^{-1}"] \arrow[d, equal, "\cdot\frac{p}{p}\frac{z^{-1}}{z^{-1}}"] & \underset{|z|>p^{-1}}{\boxed{Y(z)}} \arrow[d, equal, "\cdot\frac{p}{p}\frac{z}{z}"'] & \underset{n\geq 0}{\boxed{\text{causal } y_n}} \\
\underset{n\geq 1}{\boxed{\text{causal } x_n}} & \underset{|z|>p}{\boxed{X(z)}} \arrow[r, leftrightarrow, "z^{-1}"] & \underset{|z|<p^{-1}}{\boxed{X(z^{-1})}} & \underset{n<0}{\boxed{\text{anti- causal } -y_n}}
\end{tikzcd}
$$

(A) $X(z)$ for $|z|>p$, $Y(z)$ for $|z|>p^{-1}$ z-Transform

Geometric Series Forms

$$-\frac{p^{-1}}{1-p^{-1}z}$$

$|z|>p^{-1}$

$$-\frac{p^{-1}}{1-p^{-1}z^{-1}} = \boxed{Y(z)} \quad p^{-1}$$

$|z|>p$

$$p \quad \boxed{X(z)} = \frac{z^{-1}}{1-pz^{-1}}$$

$$\frac{z}{1-pz}$$

(B) $X(z^{-1})$ for $|z|<p^{-1}$, $Y(z^{-1})$ for $|z|<p$

Geometric Series Forms

$|z|<p$ $|z|>p^{-1}$

$$\boxed{X(z^{-1})} = -\frac{p^{-1}}{1-p^{-1}z} \xleftrightarrow{z^{-1}} -\frac{p^{-1}}{1-p^{-1}z^{-1}} = Y(z)$$

$$X(z) = \frac{z^{-1}}{1-pz^{-1}} \xleftrightarrow{z^{-1}} \frac{z}{1-pz} = \boxed{Y(z^{-1})}$$

$|z|>p$ $|z|<p^{-1}$

Z- Transform $\quad X(z) \leftrightarrow x_n \quad Y(z) \leftrightarrow y_n$

Geometric Series Forms

$|z| < p \qquad\qquad |z| > p^{-1}$

$$Y(z^{-1}) = \boxed{-\frac{p^{-1}}{1 - p^{-1} z}} \quad \xleftrightarrow{z^{-1}} \quad \boxed{-\frac{p^{-1}}{1 - p^{-1} z^{-1}}} = Y(z)$$

$\cdot \frac{p}{p} \frac{z^{-1}}{z^{-1}} \qquad\qquad \cdot \frac{p}{p} \frac{z}{z}$

$$X(z) = \boxed{\frac{z^{-1}}{1 - p z^{-1}}} \quad \xleftrightarrow{z^{-1}} \quad \boxed{\frac{z}{1 - p z}} = X(z^{-1})$$

$|z| > p \qquad\qquad |z| < p^{-1}$

$X(z)$ $(\lvert z\rvert < p)$ $-x_n$ $(n < 1)$	$Y(z^{-1})$ $(\lvert z^{-1}\rvert > p^{-1})$ y_{-n} $(n < 1)$	$\xleftrightarrow{z^{-1}}$	$Y(z)$ $(\lvert z\rvert > p^{-1})$ y_n $(n \geq 0)$

$X(z)$ $(\lvert z\rvert > p)$ x_n $(n \geq 1)$	$\xleftrightarrow{z^{-1}}$	$X(z^{-1})$ $(\lvert z^{-1}\rvert > p)$ x_{-n} $(n < 0)$	$Y(z)$ $(\lvert z\rvert < p^{-1})$ $-y_n$ $(n < 0)$

z-Transform using only $\boxed{x_n} \leftrightarrow \boxed{X(z)}$

$$\boxed{\begin{array}{c} X(z)\ (|z|<p) \\ -x_n\ (n<1) \end{array}}\ \boxed{\begin{array}{c} Y(z^{-1})\ (|z^{-1}|>p^{-1}) \\ y_{-n}\ (n<1) \end{array}} \xleftrightarrow{z^{-1}} \boxed{\begin{array}{c} Y(z)\ (|z|>p^{-1}) \\ y_n\ (n\geq 0) \end{array}}$$

$$\boxed{\begin{array}{c} X(z)\ (|z|>p) \\ x_n\ (n\geq 1) \end{array}} \xleftrightarrow{z^{-1}} \boxed{\begin{array}{c} X(z^{-1})\ (|z^{-1}|>p) \\ x_{-n}\ (n<0) \end{array}}\ \boxed{\begin{array}{c} Y(z)\ (|z|<p^{-1}) \\ -y_n\ (n<0) \end{array}}$$

$$x_{-n} = -y_n \qquad -x_{-n} = y_n$$

$$\boxed{\begin{array}{c} X(z)\ (|z|<p) \\ -x_n\ (n<1) \end{array}} \xleftrightarrow{z^{-1}} \boxed{\begin{array}{c} X(z^{-1})\ (|z|>p^{-1}) \\ -x_{-n}\ (n\geq 0) \end{array}}$$

y_n

$$\boxed{\begin{array}{c} X(z)\ (|z|>p) \\ x_n\ (n\geq 1) \end{array}} \xleftrightarrow{z^{-1}} \boxed{\begin{array}{c} X(z^{-1})\ (|z|<p^{-1}) \\ x_{-n}\ (n<0) \end{array}}$$

z - Transform $\boxed{x_n} \leftrightarrow X(z)$

$$\boxed{y_{-n}\ (n<1)} \xleftrightarrow{z^{-1}} \boxed{y_n\ (n \geq 0)}$$

$$\boxed{x_n\ (n \geq 1)} \xleftrightarrow{z^{-1}} \boxed{x_{-n}\ (n < 0)}$$

$$\begin{tikzcd}
\boxed{-x_n\ (n<1)} \arrow[d, equal] & \boxed{y_n\ (n \geq 0)} \arrow[d, equal] \\
\boxed{x_n\ (n \geq 1)} & \boxed{-y_n\ (n<0)}
\end{tikzcd}$$

$$\begin{tikzcd}
\boxed{-x_n\ (n<1)} \arrow[r, leftrightarrow, "z^{-1}"] \arrow[d, equal] & \boxed{-x_{-n}\ (n \geq 0)} \arrow[d, equal] \\
\boxed{x_n\ (n \geq 1)} \arrow[r, leftrightarrow, "z^{-1}"] & \boxed{x_{-n}\ (n<0)}
\end{tikzcd}$$

z - Transform $\quad x_n \leftrightarrow \boxed{X(z)}$

$$Y(z^{-1})\ (|z^{-1}| > p^{-1}) \xleftrightarrow{z^{-1}} Y(z)\ (|z| > p^{-1})$$

$$X(z)\ (|z| > p) \xleftrightarrow{z^{-1}} X(z^{-1})\ (|z^{-1}| > p)$$

$$\begin{tikzcd}
X(z)\ (|z| < p) \arrow[d, equal] & Y(z)\ (|z| > p^{-1}) \arrow[d, equal] \\
X(z)\ (|z| > p) & Y(z)\ (|z| < p)
\end{tikzcd}$$

$$\begin{tikzcd}
X(z)\ (|z| < p) \arrow[r, leftrightarrow, "z^{-1}"] \arrow[d, equal] & X(z^{-1})\ (|z| > p^{-1}) \arrow[d, equal] \\
X(z)\ (|z| > p) \arrow[r, leftrightarrow, "z^{-1}"] & X(z^{-1})\ (|z| < p)
\end{tikzcd}$$

Laurent Series & Z- Transform ①

Laurent Series $a_n \leftrightarrow f(z)$ $b_n \leftrightarrow g(z)$

$n \geq 0$	$\lvert z\rvert < p$		$\lvert z\rvert > p^{-1}$	$n < 1$
causal a_n	$f(z)$	$\xleftrightarrow{z^{-1}}$	$f(z^{-1})$	anti-causal $-b_n$
	$\cdot\frac{p}{p}\,\frac{z^{-1}}{z^{-1}}$		$\cdot\frac{p}{p}\,\frac{z}{z}$	
$-a_n$ anti-causal	$g(z^{-1})$	$\xleftrightarrow{z^{-1}}$	$g(z)$	b_n causal
$n < 0$	$\lvert z\rvert > p$		$\lvert z\rvert < p^{-1}$	$n \geq 1$

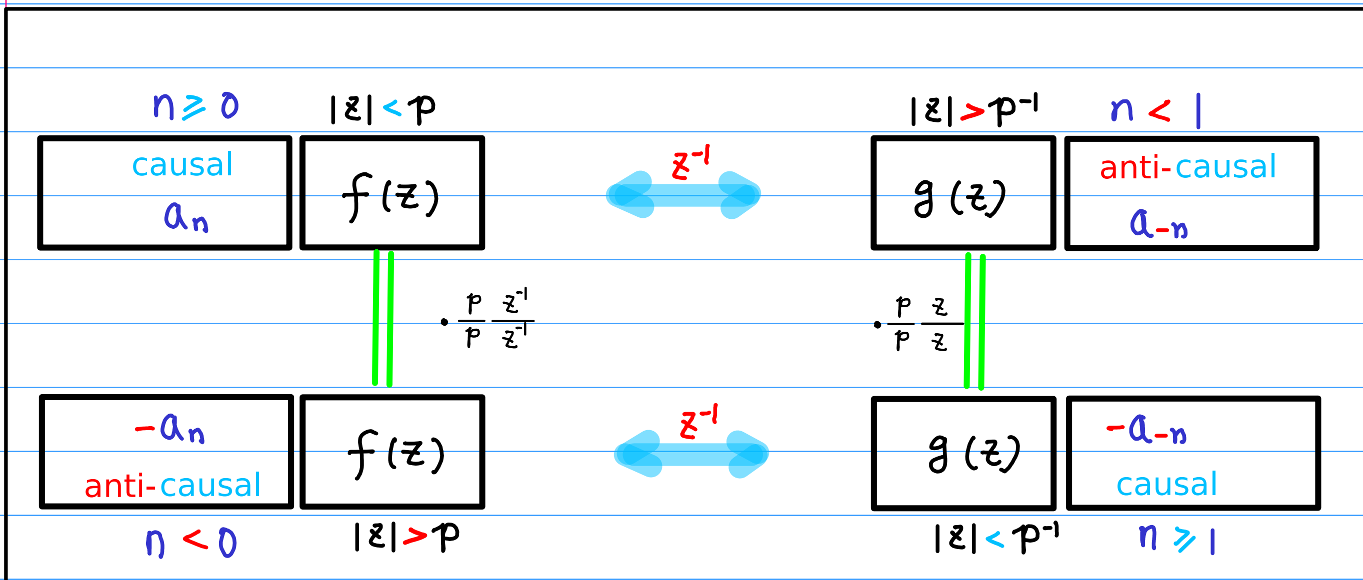
Z- Transform $X(z) \leftrightarrow x_n$ $Y(z) \leftrightarrow y_n$

$g(z^{-1})$

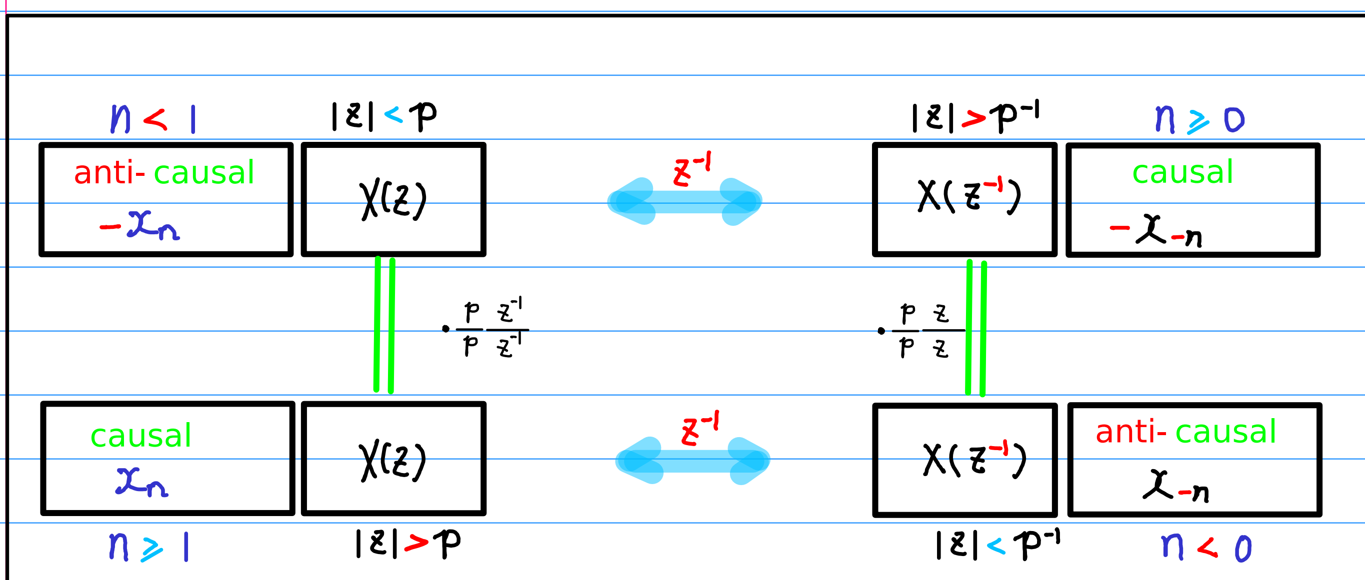
$n < 1$	$\lvert z\rvert < p$		$\lvert z\rvert > p^{-1}$	$n \geq 0$
anti- causal $-x_n$	$Y(z^{-1})$	$\xleftrightarrow{z^{-1}}$	$Y(z)$	causal y_n
	$\cdot\frac{p}{p}\,\frac{z^{-1}}{z^{-1}}$		$\cdot\frac{p}{p}\,\frac{z}{z}$	
causal x_n	$X(z)$	$\xleftrightarrow{z^{-1}}$	$X(z^{-1})$	anti- causal $-y_n$
$n \geq 1$	$\lvert z\rvert > p$		$\lvert z\rvert < p^{-1}$	$n < 0$

Laurent Series & Z-Transform (2)

Laurent Series $a_n \leftrightarrow f(z)$



Z- Transform $X(z) \leftrightarrow x_n$



causal $f(z)$ $(|z|<p)$ anti-causal $f(z)$ $(|z|>p)$

$f(z) \longleftrightarrow a_n$ $(n \geq 0)$ $g(z^{-1}) \longleftrightarrow -a_n$ $(n<0)$

$n \geq 0$ $|z|<p$

$$\boxed{\begin{matrix}\text{causal } f(z)\\ -(p^{-1})^{n+1}\end{matrix}} \quad \boxed{-\frac{p^{-1}}{1-p^{-1}z}}$$

$\cdot(-1)$ $\cdot \frac{p}{p}\frac{z^{-1}}{z^{-1}}$

$$\boxed{\begin{matrix}(p)^{-n-1}\\ \text{anti-causal } f(z)\end{matrix}} \quad \boxed{\frac{z^{-1}}{1-pz^{-1}}}$$

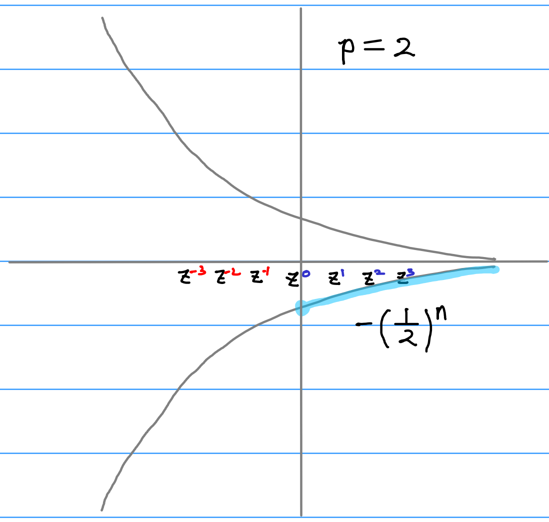
$n<0$ $|z|>p$

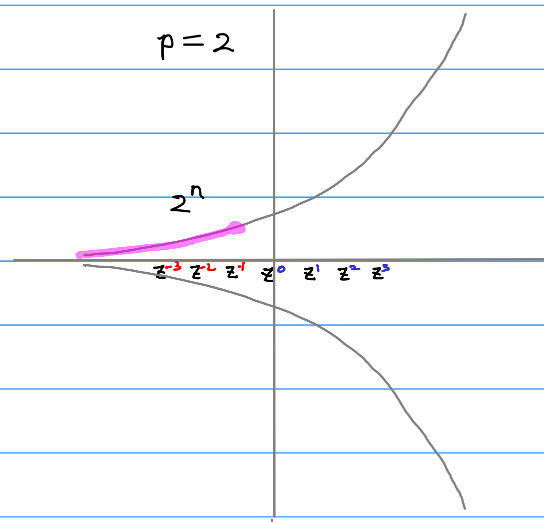
$$-(p^{-1}+p^{-2}z^{1}+p^{-3}z^{2}+\cdots)=\sum_{n=0}^{\infty}-(p^{-1})^{n+1}z^n \qquad n \geq 0$$

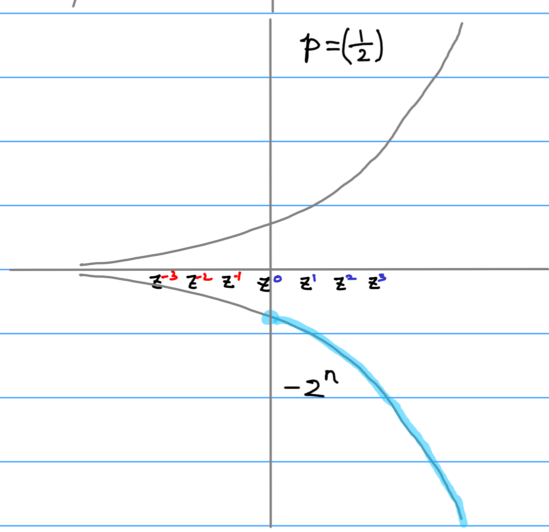
$$p^0z^{-1}+p^1z^{-2}+p^2z^{-3}+\cdots = \sum_{n=-1}^{-\infty}(p)^{-n-1}z^n \qquad n<0$$

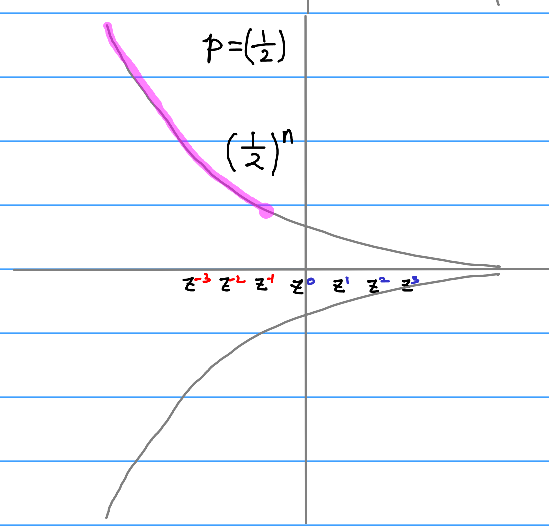
causal $n=0,+1,+2,+3,\ldots.$ $-(p^{-1}, p^{-2}, p^{-3}, \cdots)$

anti-causal $n=-1,-2,-3,\ldots.$ $(p^0, p^1, p^2, \cdots)$









anti-causal $g(z)$ $(|z|>p^{-1})$ causal $g(z)$ $(|z|<p^{-1})$

$f(z^{-1}) \leftrightarrow -b_n$ $(n<1)$ $g(z) \leftrightarrow b_n$ $(n \geq 1)$

$|z|>p^{-1}$ $n<1$

$n<1$ $-(p^{-1}+p^{-2}z^{-1}+p^{-3}z^{-2}+\cdots) = \sum_{n=0}^{-\infty} -(p)^{n-1} z^{n}$ $\boxed{-\dfrac{p^{-1}}{1-p^{-1}z^{-1}}}$ $\boxed{\text{anti-causal } f(z) \quad -(p^{-1})^{-n+1}}$

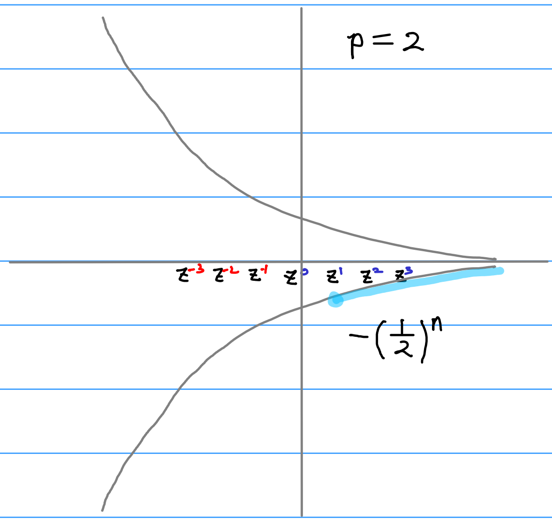
$\cdot \dfrac{p}{p}\dfrac{z}{z}$ $\cdot(-1)$

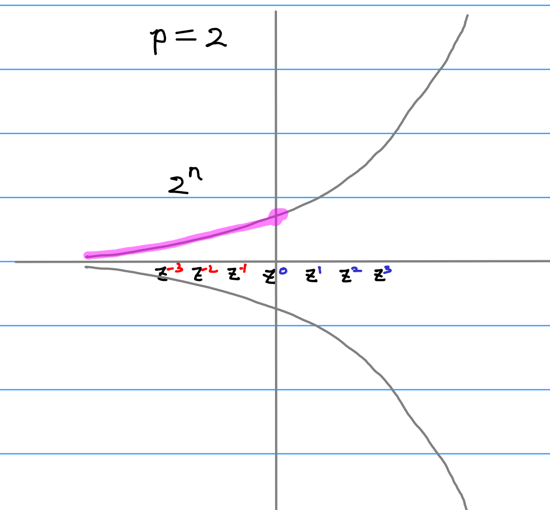
$n \geq 1$ $p^{0}z^{1}+p^{1}z^{2}+p^{2}z^{3}+\cdots = \sum_{n=1}^{\infty} (p)^{n-1} z^{n}$ $\boxed{\dfrac{z}{1-pz}}$ $\boxed{(p)^{n-1} \quad \text{causal } f(z)}$

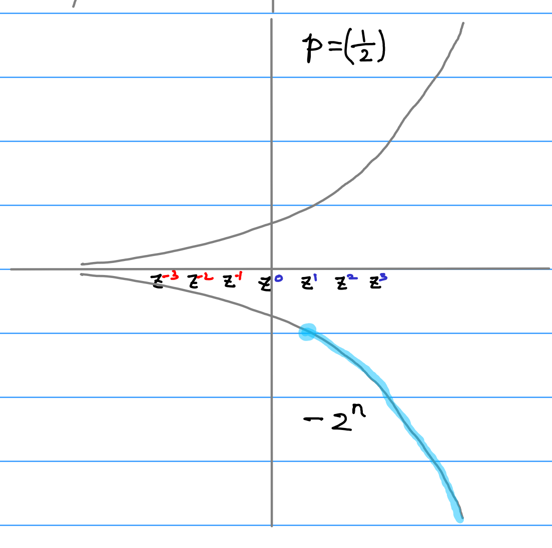
$|z|<p^{-1}$ $n \geq 1$

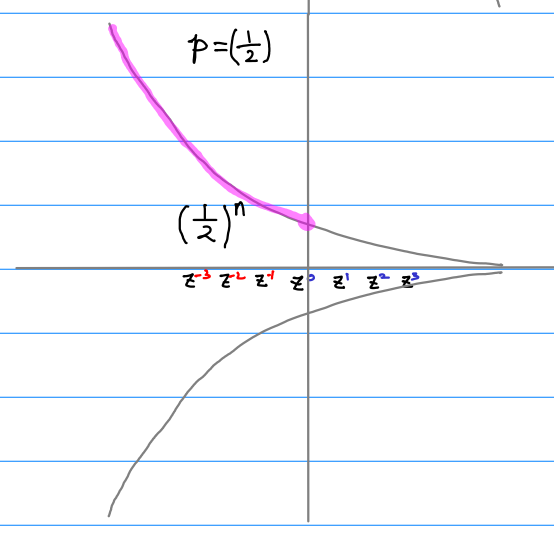
causal n=+1,+2,+3,.... anti-causal n=0,-1,-2,-3,....

$-(p^{0}, p^{1}, p^{2}, \cdots)$ $(p^{-1}, p^{-2}, p^{-3}, \cdots)$



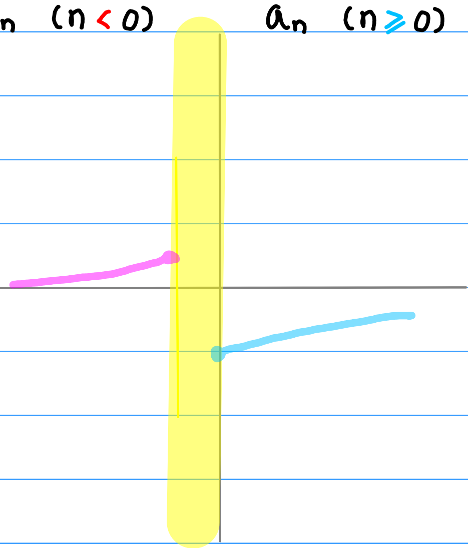






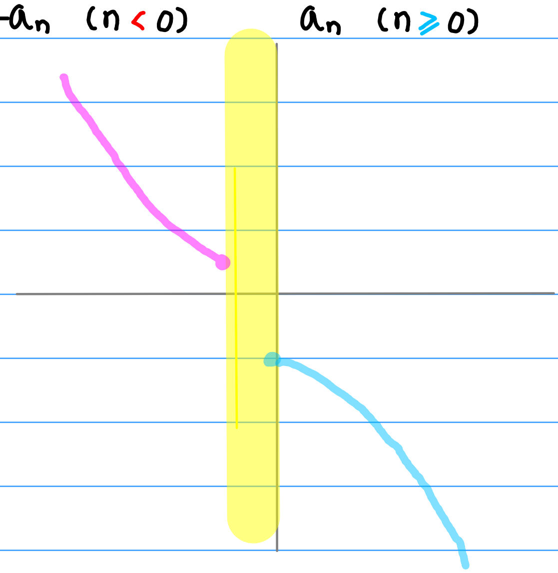
$f(z)$ $(|z|>p)$ $\quad$ $f(z)$ $(|z|<p)$

$-a_n$ $(n<0)$ $\quad$ a_n $(n \geq 0)$



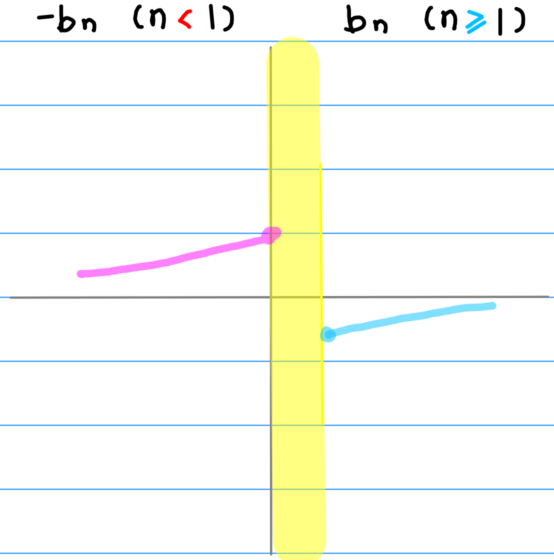
$f(z)$ $(|z|>p)$ $\quad$ $f(z)$ $(|z|<p)$

$-a_n$ $(n<0)$ $\quad$ a_n $(n \geq 0)$



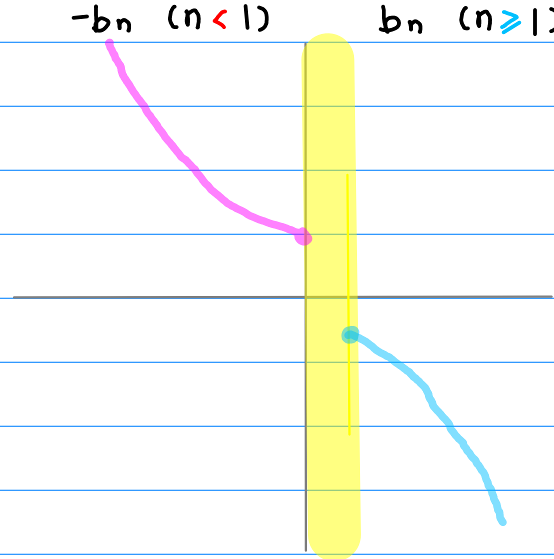
$g(z)$ $(|z|>p^{-1})$ $\quad$ $g(z)$ $(|z|<p^{-1})$

$-b_n$ $(n<1)$ $\quad$ b_n $(n \geq 1)$



$g(z)$ $(|z|>p^{-1})$ $\quad$ $g(z)$ $(|z|<p^{-1})$

$-b_n$ $(n<1)$ $\quad$ b_n $(n \geq 1)$



causal $f(z)$

$f(z) \leftrightarrow a_n \ (n \geq 0)$

anti-causal $f(z)$

$f(z^{-1}) \leftrightarrow a_{-n} \ (n < 1)$

$n \geq 0$ $\quad |z| < p$

causal f(z) $\boxed{-(p^{-1})^{n+1}}$ $\quad \boxed{-\dfrac{p^{-1}}{1-p^{-1}z}}$

$$-(p^{-1}+p^{-2}z^{1}+p^{-3}z^{2}+\cdots) = \sum_{n=0}^{\infty} -(p^{-1})^{n+1} z^{n} \qquad n \geq 0$$

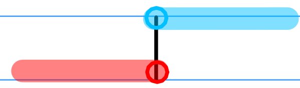
$|z| > p^{-1}$ $\quad n < 1$

$$n<1 \qquad -(p^{-1}+p^{-2}z^{-1}+p^{-3}z^{-2}+\cdots) = \sum_{n=0}^{-\infty} -(p)^{n-1} z^{n}$$

$\boxed{-\dfrac{p^{-1}}{1-p^{-1}z^{-1}}}$ $\quad$ anti-causal f(z) $\boxed{-(p^{-1})^{-n+1}}$

$n \geq 0$

causal f(z) $\boxed{-p^{-n-1}}$



$-n$

$n < 1$

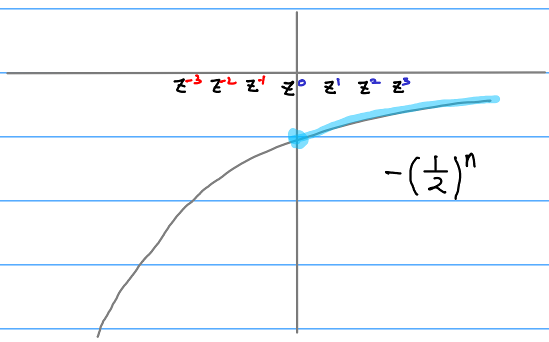
anti-causal f(z) $\boxed{-p^{+n-1}}$

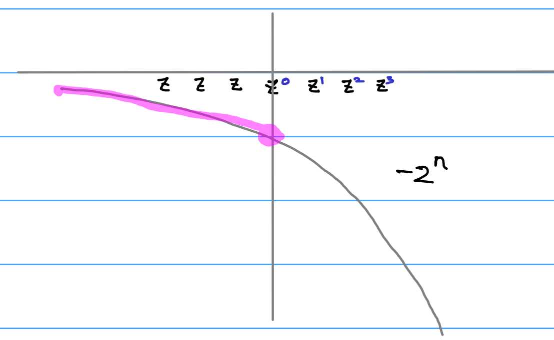
causal $\quad$ n=0,+1,+2,+3,....

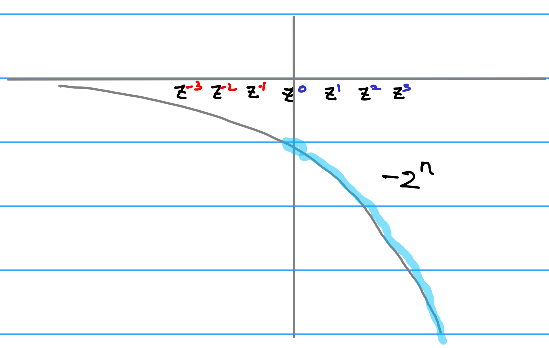
$-(p^{-1}, p^{-2}, p^{-3}, \cdots)$

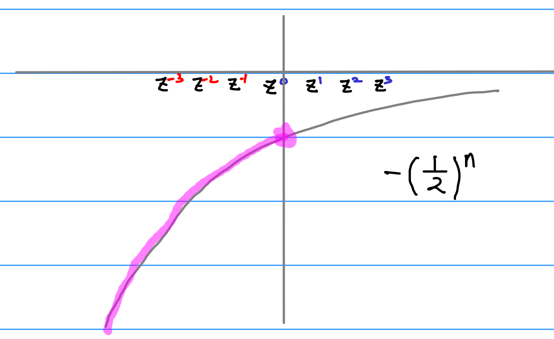
anti-causal $\quad$ n=0,-1,-2,-3,....

$-(p^{-1}, p^{-2}, p^{-3}, \cdots)$









anti-causal $g(z)$ causal $g(z)$

$g(z^{-1}) \leftrightarrow b_{-n}$ $(n<0)$ $g(z) \leftrightarrow b_n$ $(n \geq 1)$

$n<0$ $|z|>p$

$\boxed{(p)^{-n-1}}$ anti-causal f(z) $\boxed{\dfrac{z^{-1}}{1-pz^{-1}}}$

$$p^0 z^{-1} + p^1 z^{-2} + p^2 z^{-3} + \cdots = \sum_{n=-1}^{-\infty} (p)^{-n-1} z^n \quad n<0$$

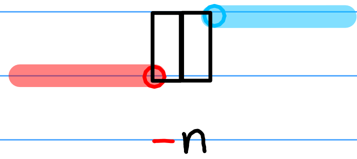
$|z|<p^{-1}$ $n \geq 1$

$$n \geq 1 \quad p^0 z^1 + p^1 z^2 + p^2 z^3 + \cdots = \sum_{n=1}^{\infty} (p)^{n-1} z^n$$

$\boxed{\dfrac{z}{1-pz}}$ $\boxed{(p)^{n-1}}$ causal f(z)

$n<0$

$\boxed{p^{-n-1}}$ anti-causal f(z)

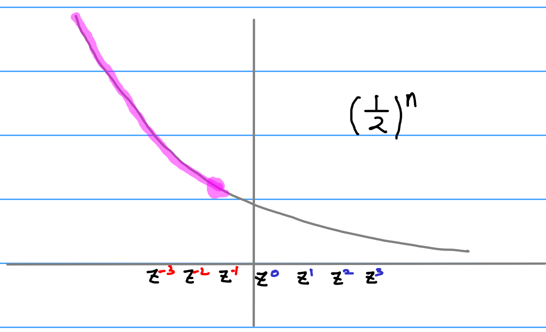


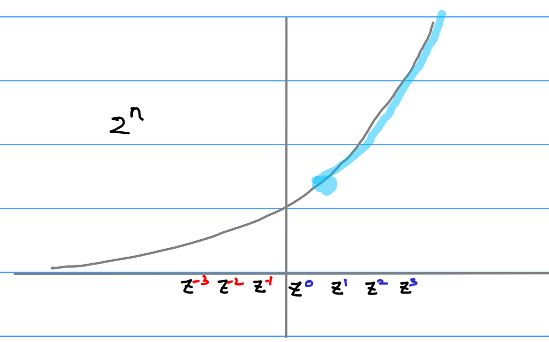
$n \geq 1$

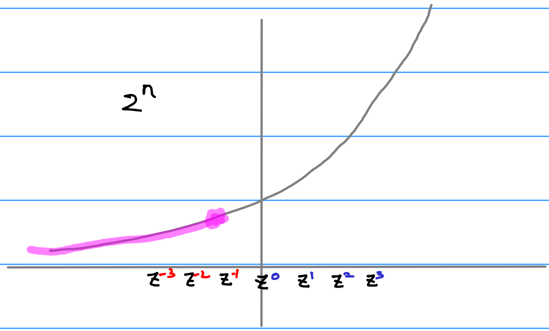
$\boxed{p^{n-1}}$ causal f(z)

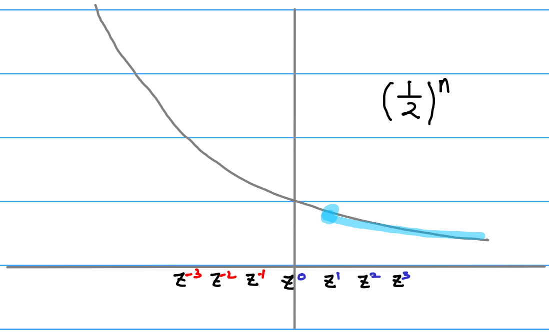
anti-causal n=-1,-2,-3,.... causal n=+1,+2,+3,....

$(p^{-2}, p^{-3}, p^{-4}, \cdots)$ $(p^{-2}, p^{-3}, p^{-4}, \cdots)$









$f(z^{-1})$ $(|z|>\rho^{-1})$ $f(z)$ $(|z|<\rho)$

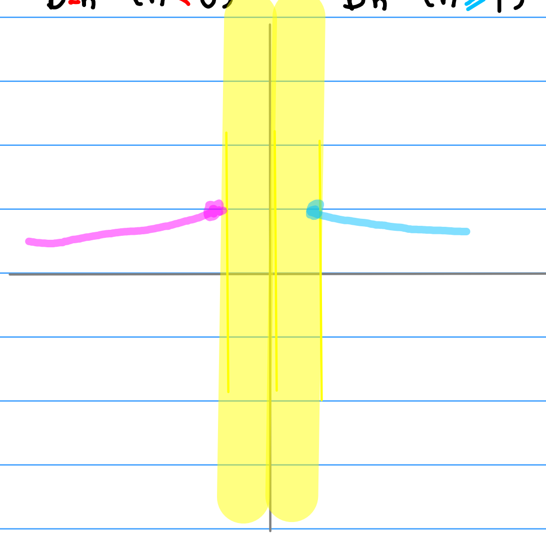
a_{-n} $(n<1)$ a_n $(n\geq 0)$

$f(z^{-1})$ $(|z|>\rho^{-1})$ $f(z)$ $(|z|<\rho)$

a_{-n} $(n<1)$ a_n $(n\geq 0)$

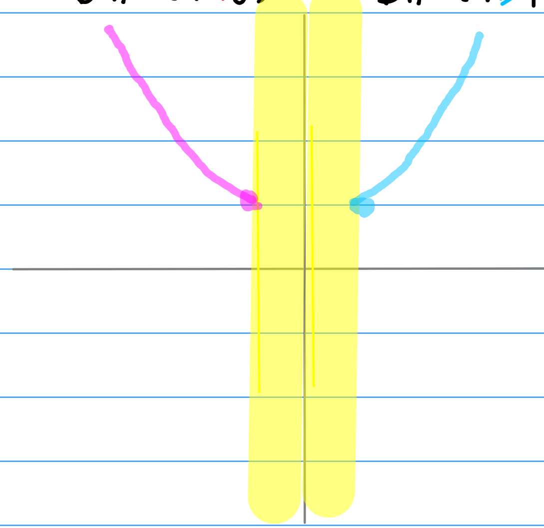
$g(z^{-1})$ $(|z|>\rho)$ $g(z)$ $(|z|<\rho^{-1})$

b_{-n} $(n<0)$ b_n $(n\geq 1)$



$g(z^{-1})$ $(|z|>\rho)$ $g(z)$ $(|z|<\rho^{-1})$

b_{-n} $(n<0)$ b_n $(n\geq 1)$



x_n		a_{-n}	
y_n		b_{-n}	
causal		anti-causal	
$n \geq 0$	$-(p^{-1}, p^{-2}, p^{-3}, \cdots)$	$n < 1$	
$n \geq 1$	$(p^{-2}, p^{-3}, p^{-4}, \cdots)$	$n < 0$	
anti-causal		causal	
$n < 1$	$-(p^{-1}, p^{-2}, p^{-3}, \cdots)$	$n \geq 0$	
$n < 0$	$(p^{-2}, p^{-3}, p^{-4}, \cdots)$	$n \geq 1$	

getting anti-causal sequence

$$\begin{tikzcd}[column sep=huge, row sep=large]
\boxed{\dfrac{1}{z-p}} \arrow[r, leftrightarrow, "\frac{p^{-1}}{p^{-1}}"] \arrow[d, leftrightarrow, "\frac{z^{-1}}{z^{-1}}"'] & \boxed{-\dfrac{p^{-1}}{1-p^{-1}z}} \\
\boxed{\dfrac{z^{-1}}{1-pz^{-1}}} &
\end{tikzcd}$$

$$-\frac{p^{-1}}{1-p^{-1}z} = f(z) \longleftrightarrow -(p^{-1})^{n+1}$$

$$Y(z^{-1}) \longleftrightarrow \boxed{?}$$

$$\frac{z^{-1}}{1-pz^{-1}} = g(z^{-1}) \longleftrightarrow \boxed{?}$$

$$X(z) \longleftrightarrow (p)^{n-1}$$

$$\begin{tikzcd}[column sep=huge, row sep=large]
\boxed{\dfrac{1}{z^{-1}-p}} \arrow[r, leftrightarrow, "\frac{p^{-1}}{p^{-1}}"] \arrow[d, leftrightarrow, "\frac{z^{-1}}{z^{-1}}"'] & \boxed{-\dfrac{p^{-1}}{1-p^{-1}z^{-1}}} \\
\boxed{\dfrac{z}{1-pz}} &
\end{tikzcd}$$

$$-\frac{p^{-1}}{1-p^{-1}z^{-1}} = f(z^{-1}) \longleftrightarrow \boxed{?}$$

$$Y(z) \longleftrightarrow -(p^{-1})^{n+1}$$

$$\frac{z}{1-pz} = g(z) \longleftrightarrow (p)^{n-1}$$

$$X(z^{-1}) \longleftrightarrow \boxed{?}$$

$$\boxed{\frac{1}{z-p}} \xleftrightarrow{\frac{p^{-1}}{p^{-1}}} \boxed{-\frac{p^{-1}}{1-p^{-1}z}} = f(z) \longleftrightarrow -(p^{-1})^{n+1}$$

$$Y(z^{-1}) \longleftrightarrow \boxed{-(p^{-1})^{-n+1}}$$

$$\Big\updownarrow \frac{z^{-1}}{z^{-1}}$$

$$\boxed{\frac{z^{-1}}{1-pz^{-1}}}$$

$$\|$$

$$g(z^{-1}) \longleftrightarrow \boxed{(p)^{-n-1}}$$

$$X(z) \longleftrightarrow (p)^{n-1}$$

z^{-1}

z^{-1}

$$\boxed{\frac{1}{z^{-1}-p}} \xleftrightarrow{\frac{p^{-1}}{p^{-1}}} \boxed{-\frac{p^{-1}}{1-p^{-1}z^{-1}}} = f(z^{-1}) \longleftrightarrow \boxed{-(p^{-1})^{-n+1}}$$

$$Y(z) \longleftrightarrow -(p^{-1})^{n+1}$$

$$\Big\updownarrow \frac{z^{-1}}{z^{-1}}$$

$$\boxed{\frac{z}{1-pz}}$$

$$\|$$

$$g(z) \longleftrightarrow (p)^{n-1}$$

$$X(z^{-1}) \longleftrightarrow \boxed{(p)^{-n-1}}$$

$$\boxed{-\frac{p^{-1}}{1-p^{-1}z^{-1}}} \qquad \boxed{-\frac{p^{-1}}{1-p^{-1}z}}$$

$$\| \qquad\qquad \|$$

$$f(z^{-1}) \longleftrightarrow \boxed{-(p^{-1})^{-n+1}} \qquad f(z) \longleftrightarrow \boxed{-(p^{-1})^{n+1}}$$

$$\boxed{\frac{z^{-1}}{1-pz^{-1}}} \qquad \boxed{\frac{z}{1-pz}}$$

$$\| \qquad\qquad \|$$

$$g(z^{-1}) \longleftrightarrow \boxed{(p)^{-n-1}} \qquad g(z) \longleftrightarrow \boxed{(p)^{n-1}}$$

$$\boxed{\frac{z}{1-pz}} \qquad \boxed{\frac{z^{-1}}{1-pz^{-1}}}$$

$$\| \qquad\qquad \|$$

$$X(z^{-1}) \longleftrightarrow \boxed{(p)^{-n-1}} \qquad X(z) \longleftrightarrow \boxed{(p)^{n-1}}$$

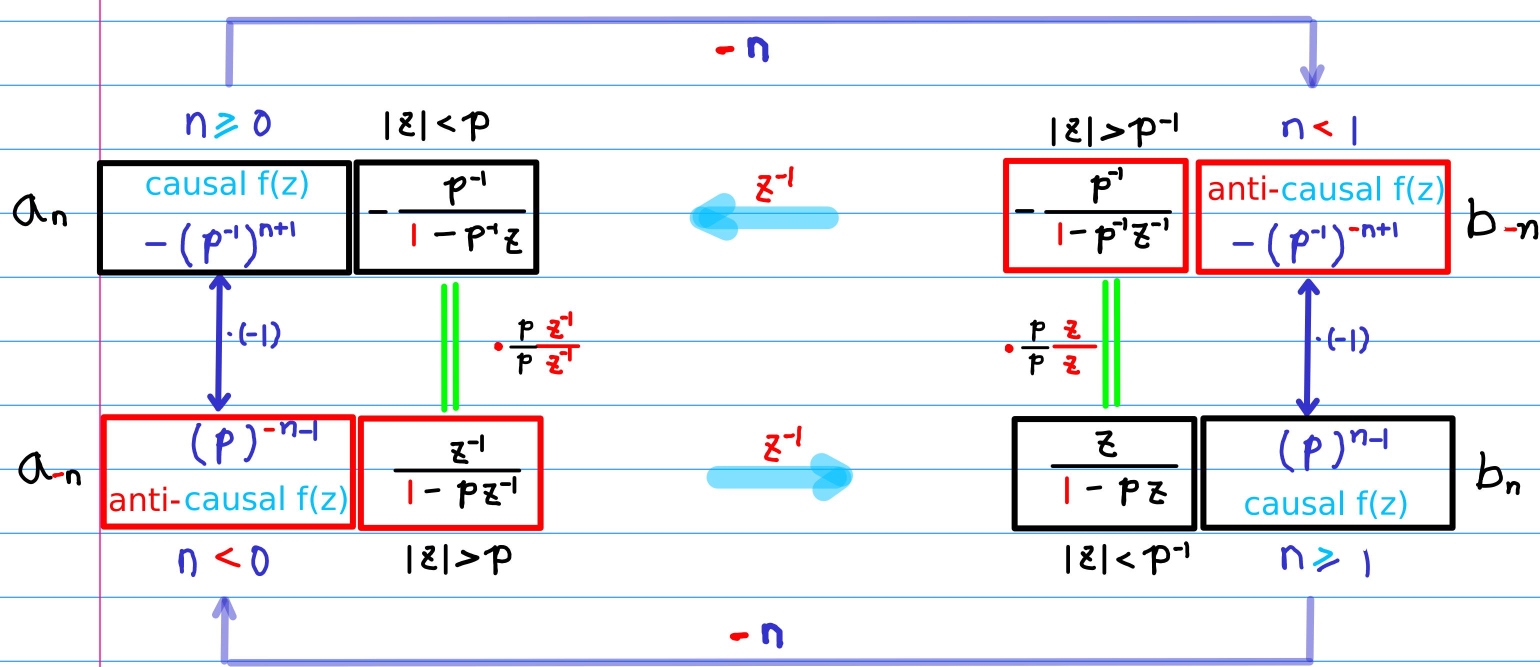
$$\boxed{-\frac{p^{-1}}{1-p^{-1}z}} \qquad \boxed{-\frac{p^{-1}}{1-p^{-1}z^{-1}}}$$

$$\| \qquad\qquad \|$$

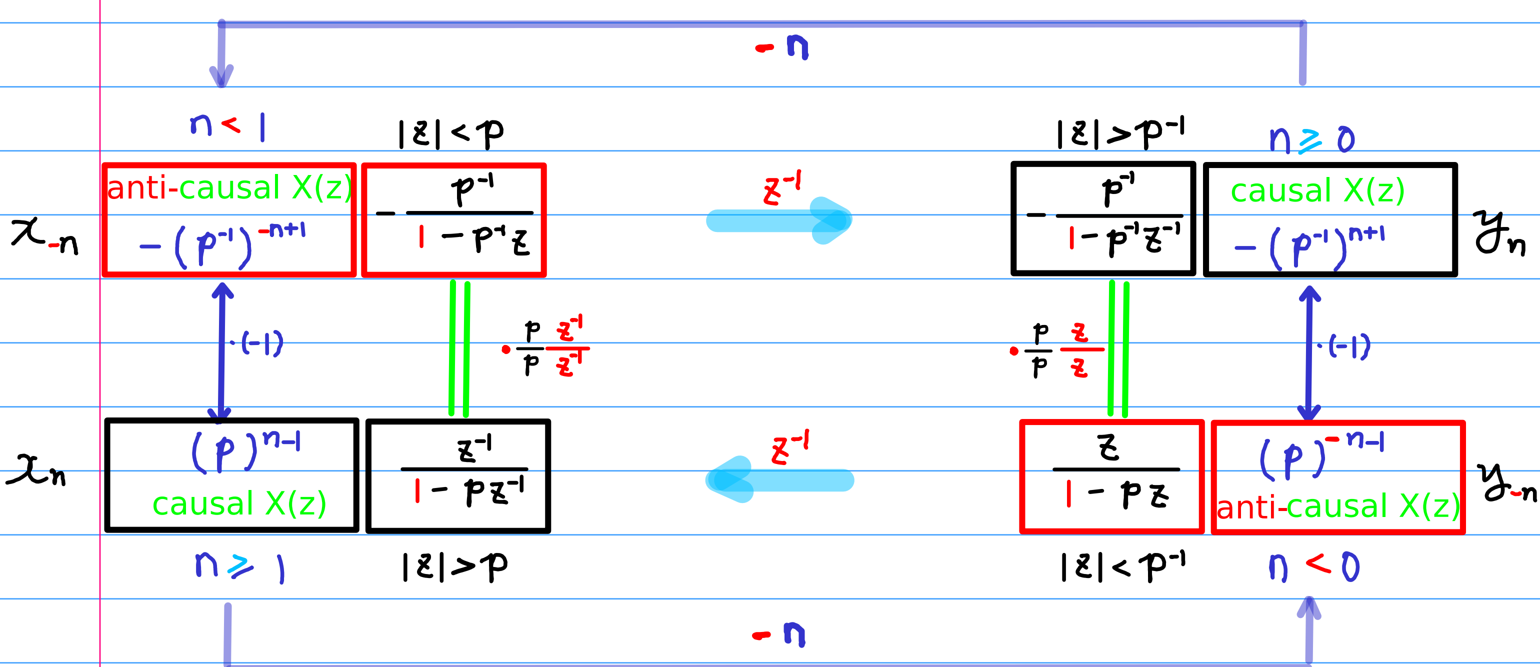
$$Y(z^{-1}) \longleftrightarrow \boxed{-(p^{-1})^{-n+1}} \qquad Y(z) \longleftrightarrow \boxed{-(p^{-1})^{n+1}}$$

Getting anti-causal sequence

Laurent Series



z - Transform



① $z \to z^{-1}$ to get causal $f(z)$, $|z| < a$

② $f(z) \leftrightarrow a_n$ $|z| < a$, $n \geq 0, 1$

③ $n \to -n$ to get anti-causal $n < 0, 1$

① $\dfrac{-1}{(z-1)(z-2)}$

② $\dfrac{-0.5z^2}{(z-1)(z-0.5)}$

$-\dfrac{1}{1-z}+\dfrac{0.5}{1-0.5z}$

$f(z)$ $|z|<1$ causal

$X(z)$ $|z|<1$ anti-causal

$+\dfrac{z}{1-z}-\dfrac{z}{1-2z}$

$f(z)$ $|z|<0.5$ causal

$X(z)$ $|z|<0.5$ anti-causal

$+\dfrac{z^{-1}}{1-z^{-1}}-\dfrac{z^{-1}}{1-2z^{-1}}$

$f(z)$ $|z|>1$ anti-causal

$X(z)$ $|z|>1$ causal

$-\dfrac{1}{1-z^{-1}}+\dfrac{1}{1-0.5z^{-1}}$

$f(z)$ $|z|>2$ anti-causal

$X(z)$ $|z|>2$ causal

$$-\frac{1}{1-z}+\frac{0.5}{1-0.5z}$$

$$f(z) \qquad |z|<1$$

$$+\frac{z}{1-z}-\frac{z}{1-2z}$$

$$f(z) \qquad |z|<0.5$$

$$\boxed{\cdot z \quad n-1}$$

$$+\frac{1}{1-z}-\frac{1}{1-2z}$$

$$g(z) \qquad |z|<0.5$$

$$+\frac{z^{-1}}{1-z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}$$

$$X(z) \qquad |z|>1$$

$$\boxed{\cdot z^{-1} \quad n-1}$$

$$+\frac{1}{1-z^{-1}}-\frac{1}{1-2z^{-1}}$$

$$V(z) \qquad |z|>1$$

$$-\frac{1}{1-z^{-1}}+\frac{1}{1-0.5z^{-1}}$$

$$X(z) \qquad |z|>2$$

$$-\frac{1}{1-z}+\frac{0.5}{1-0.5z}$$

$X(z) \qquad |z|<1$

$\boxed{z^{-1} \quad -n}$

$$-\frac{1}{1-z^{-1}}+\frac{1}{1-0.5z^{-1}}$$

$V(z) \qquad |z|>2$

$$+\frac{z}{1-z}-\frac{z}{1-2z}$$

$X(z) \qquad |z|<0.5$

$\boxed{z^{-1} \quad -n}$

$$+\frac{z^{-1}}{1-z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}$$

$V(z) \qquad |z|>1$

$\boxed{\cdot z^{-1} \quad n-1}$

$$+\frac{1}{1-z^{-1}}-\frac{1}{1-2z^{-1}}$$

$W(z) \qquad |z|>1$

$$+\frac{z^{-1}}{1-z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}$$

$f(z) \qquad |z|>1$

$\boxed{z^{-1} \quad -n}$

$$+\frac{z}{1-z}-\frac{z}{1-2z}$$

$g(z) \qquad |z|<0.5$

$\boxed{\cdot z \quad n-1}$

$$+\frac{1}{1-z}-\frac{1}{1-2z}$$

$h(z) \qquad |z|<0.5$

$$-\frac{1}{1-z^{-1}}+\frac{1}{1-0.5z^{-1}}$$

$f(z) \qquad |z|>2$

$\boxed{z^{-1} \quad -n}$

$$-\frac{1}{1-z}+\frac{0.5}{1-0.5z}$$

$g(z) \qquad |z|<1$

Ⓐ $f(z)$ Ⓑ $X(z)$

① $\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$

①-Ⓐ $|z|<0.5$ $f(z) = -\frac{2}{1-2z} + \frac{0.5}{1-0.5z}$ $\boxed{-2^{n+1} + \left(\frac{1}{2}\right)^{n+1}}$ $(n \geq 0)$

$f(z)$ $|z|>2$ $f(z) = \frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}$ $\boxed{+2^{n+1} - \left(\frac{1}{2}\right)^{n+1}}$ $(n<0)$

①-Ⓑ $|z|<0.5$ $X(z) = -\frac{2}{1-2z} + \frac{0.5}{1-0.5z}$ $\boxed{-\left(\frac{1}{2}\right)^{n-1} + 2^{n-1}}$ $(n<1)$

$X(z)$ $|z|>2$ $X(z) = \frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}$ $\boxed{+\left(\frac{1}{2}\right)^{n-1} - 2^{n-1}}$ $(n \geq 1)$

② $\frac{3}{2}\,\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right)$

②-Ⓐ $|z|<0.5$ $f(z) = -\frac{z}{1-2z} + \frac{z}{1-0.5z}$ $\boxed{-2^{n-1} + \left(\frac{1}{2}\right)^{n-1}}$ $(n \geq 1)$

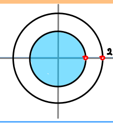
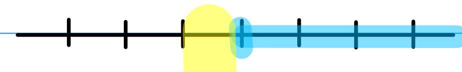
$f(z)$ $|z|>2$ $f(z) = \frac{0.5}{1-0.5z^{-1}} - \frac{2}{1-2z^{-1}}$ $\boxed{+2^{n-1} - \left(\frac{1}{2}\right)^{n-1}}$ $(n<1)$

②-Ⓑ $|z|<0.5$ $X(z) = -\frac{z}{1-2z} + \frac{z}{1-0.5z}$ $\boxed{-\left(\frac{1}{2}\right)^{n+1} + 2^{n+1}}$ $(n<0)$

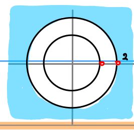
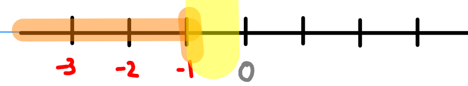
$X(z)$ $|z|>2$ $X(z) = \frac{0.5}{1-0.5z^{-1}} - \frac{2}{1-2z^{-1}}$ $\boxed{+\left(\frac{1}{2}\right)^{n+1} - 2^{n+1}}$ $(n \geq 0)$

Ⓐ $f(z)$ Ⓑ $X(z)$ time domain view

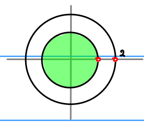
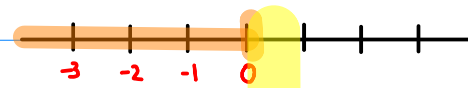
① $\dfrac{-1}{(z-1)(z-2)} = \left(\dfrac{1}{z-1} - \dfrac{1}{z-2}\right)$

①-Ⓐ $|z| < 1$  $-\dfrac{1}{1-z} + \dfrac{0.5}{1-0.5z}$ 

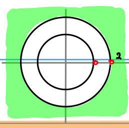
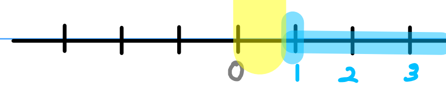
$(n \geq 0)$

$f(z)$ $|z| > 2$  

$(n < 0)$

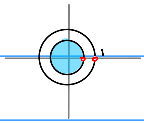
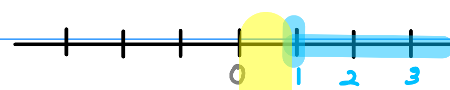
①-Ⓑ $|z| < 1$  

$(n < 1)$

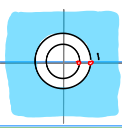
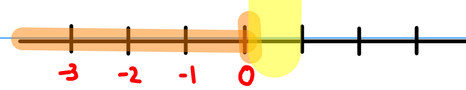
$X(z)$ $|z| > 2$  $\dfrac{z^{-1}}{1-z^{-1}} - \dfrac{z^{-1}}{1-2z^{-1}}$ 

$(n \geq 1)$

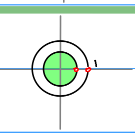
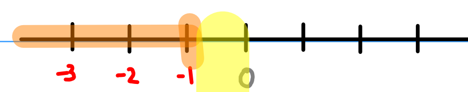
② $\dfrac{-0.5z^2}{(z-1)(z-0.5)} = \left(-\dfrac{z}{(z-1)} + \dfrac{0.5z}{(z-0.5)}\right)$

②-Ⓐ $|z| < 0.5$  $+\dfrac{z}{1-z} - \dfrac{z}{1-2z}$ 

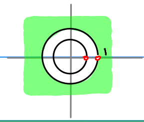
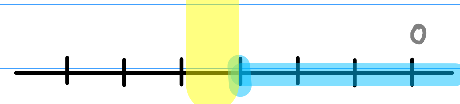
$(n \geq 1)$

$f(z)$ $|z| > 1$  

$(n < 1)$

②-Ⓑ $|z| < 0.5$  

$(n < 0)$

$X(z)$ $|z| > 1$  $-\dfrac{1}{1-z^{-1}} + \dfrac{0.5}{1-0.5z^{-1}}$ 

$(n \geq 0)$

$z^{-1}X(z)$ Shifted Sequence

$$\begin{array}{ccc} & |z|>1 & & |z|>2 & \\ X(z) = & \dfrac{1}{1-z^{-1}} & - & \dfrac{1}{1-2z^{-1}} & (|z|>2) \\ & \downarrow & & \downarrow & \\ a_n = & 1^n & + & 2^n & (n \geq 0) \end{array}$$

$$\bullet z^{-1} \downarrow$$

$$(1^0 z^0 + 1^1 z^{-1} + 1^2 z^{-2} + \cdots) + (2^0 z^0 + 2^1 z^{-1} + 2^2 z^{-2} + \cdots) \quad n \qquad n = 0, 1, 2, \cdots$$

$$1^0 \quad 1^1 \quad 1^2 \quad \cdots \quad 2^0 \quad 2^1 \quad 2^2 \quad \cdots$$

$$(1^0 z^{-1} + 1^1 z^{-2} + 1^2 z^{-3} + \cdots) + (2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots) \quad n-1 \qquad n = 1, 2, 3, \cdots$$

$$\begin{array}{ccc} z^{-1}X(z) = & \dfrac{z^{-1}}{1-z^{-1}} & - & \dfrac{z^{-1}}{1-2z^{-1}} & (|z|>2) \\ & \downarrow & & \downarrow & \\ a_{n-1} = & 1^{n-1} & + & 2^{n-1} & (n \geq 1) \end{array}$$

disjoint domains $\quad D_1 : |z| < p \quad D_2 : |z| > p^{-1} \quad 0 < p < 1$

$\quad D_1 : |z| < p^{-1} \quad D_2 : |z| > p \quad p > 1$

disjoint domains $\quad N_1 : n \geq 0 \quad N_2 : n < 0$

$\quad N_1 : n \geq 1 \quad N_2 : n < 1$

causal $\quad f(z) \quad D_1 : |z| < p \quad 0 < p < 1$

$\quad D_1 : |z| < p^{-1} \quad p > 1$

$$\frac{1}{1 - p^{-1} z} \quad N_1 : n \geq 0$$

$$\frac{z}{1 - p^{-1} z} \quad N_1 : n \geq 1$$

causal $\quad X(z) \quad D_2 : |z| > p^{-1} \quad 0 < p < 1$

$\quad D_2 : |z| > p \quad p > 1$

$$\frac{1}{1 - p z^{-1}} \quad N_1 : n \geq 0$$

$$\frac{z^{-1}}{1 - p z^{-1}} \quad N_1 : n \geq 1$$

$z f(z)$ Shifted Sequence

$$f(z) = (+1)\frac{1}{1-z} - \frac{1}{1-2z} \quad (|z|<0.5)$$

$$a_n = 1^n - 2^n \quad (n \geq 0)$$

$$(1^0 z^0 + 1^1 z^1 + 1^2 z^2 + \cdots) - (2^0 z^0 + 2^1 z^1 + 2^2 z^2 + \cdots) \quad n \quad n = 0, 1, 2, \cdots$$

$\cdot z$ $\quad 1^0 \quad 1^1 \quad 1^2 \quad \cdots \quad 2^0 \quad 2^1 \quad 2^2 \quad \cdots$

$$(1^0 z^1 + 1^1 z^2 + 1^2 z^3 + \cdots) - (2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \cdots) \quad n-1 \quad n = 1, 2, 3, \cdots$$

$$z f(z) = (+1)\frac{z}{1-z} - \frac{z}{1-2z} \quad (|z|<0.5)$$

$$a_{n-1} = 1^{n-1} - 2^{n-1} \quad (n \geq 1)$$

$z^{-1} f(z^{-1})$ Shifted & Reflected Sequence

$$f(z) = \frac{1}{1-z} - \frac{1}{1-2z} \qquad (|z| < 0.5)$$

$$a_n = 1^n - 2^n \qquad (n \geq 0)$$

$$\cdot z$$

$$(1^0 z^0 + 1^1 z^1 + 1^2 z^2 + \cdots) - (2^0 z^0 + 2^1 z^1 + 2^2 z^2 + \cdots) \quad n \qquad n = 0, 1, 2, \cdots$$

$$1^0 \quad 1^1 \quad 1^2 \quad \cdots \qquad 2^0 \quad 2^1 \quad 2^2 \quad \cdots$$

$$(1^0 z^1 + 1^1 z^2 + 1^2 z^3 + \cdots) - (2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \cdots) \quad n-1 \qquad n = 1, 2, 3, \cdots$$

$$z f(z) = \frac{z}{1-z} - \frac{z}{1-2z} \qquad (|z| < 0.5)$$

$$a_{n-1} = 1^{n-1} - 2^{n-1} \qquad (n \geq 1)$$

$$z$$

$$(1^0 z^{-1} + 1^1 z^{-2} + 1^2 z^{-3} + \cdots) + (2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots) \quad n \qquad n = 1, 2, 3, \cdots$$

$$1^0 \quad 1^1 \quad 1^2 \quad \cdots \qquad 2^0 \quad 2^1 \quad 2^2 \quad \cdots$$

$$z^{-1}$$

$$(1^0 z^1 + 1^1 z^2 + 1^2 z^3 + \cdots) + (2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \cdots) \quad -n \qquad n = -1, -2, -3, \cdots$$

$$z^{-1} f(z^{-1}) = \frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \qquad (|z| > 2)$$

$$a_{-n-1} = 1^{n+1} - \left(\frac{1}{2}\right)^{n+1} \qquad (n < 0)$$

$$a_{-(n+1)}$$

$$X(z) = \frac{1}{1-z^{-1}} - \frac{1}{1-2z^{-1}} \quad (|z|>2)$$

$$a_n = 1^n - 2^n \quad (n \geq 0)$$

$\cdot z^{-1}$

$(1^0 z^0 + 1^1 z^{-1} + 1^2 z^{-2} + \cdots) + (2^0 z^0 + 2^1 z^{-1} + 2^2 z^{-2} \quad)$ n $n = 0, 1, 2, \cdots$

$1^0 \quad 1^1 \quad 1^2 \quad \cdots \quad 2^0 \quad 2^1 \quad 2^2 \quad \cdots$

$(1^0 z^{-1} + 1^1 z^{-2} + 1^2 z^{-3} + \cdots) + (2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots)$ $n-1$ $n = 1, 2, 3, \cdots$

$$z^{-1}X(z) = \frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \quad (|z|>2)$$

$$a_{n-1} = 1^{n-1} - 2^{n-1} \quad (n \geq 1)$$

z

$(1^0 z^{-1} + 1^1 z^{-2} + 1^2 z^{-3} + \cdots) + (2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots)$ n $n = 1, 2, 3, \cdots$

$1^0 \quad 1^1 \quad 1^2 \quad \cdots \quad 2^0 \quad 2^1 \quad 2^2 \quad \cdots$

z^{-1}

$(1^0 z^{1} + 1^1 z^{2} + 1^2 z^{3} + \cdots) + (2^0 z^{1} + 2^1 z^{2} + 2^2 z^{3} + \cdots)$ $-n$ $n = -1, -2, -3, \cdots$

$$zX(z^{-1}) = \frac{z}{1-z} - \frac{z}{1-2z} \quad (|z|<0.5)$$

$$a_{-n-1} = 1^{n+1} - \left(\frac{1}{2}\right)^{n+1} \quad (n < 0)$$

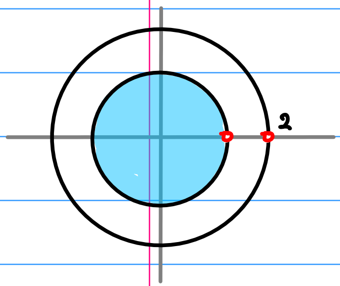
$a_{-(n+1)}$

Causal $\qquad f(z) \qquad X(z)$

$|z| < 1 \qquad |z| > 2$

① − Ⓐ $\quad \dfrac{-1}{(z-1)(z-2)} = \left(\dfrac{1}{z-1} - \dfrac{1}{z-2}\right)$

$|z|<1 \qquad |z|<2$

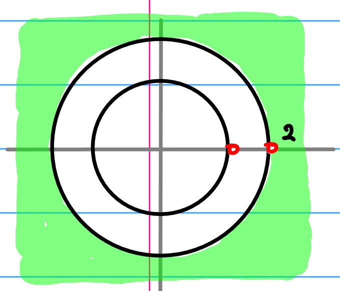


$$f(z) = (-1)\frac{1}{1-z} + \frac{0.5}{1-0.5z} \qquad (|z|<1)$$

$$a_n = -1^{n+1} + \left(\frac{1}{2}\right)^{n+1} \qquad (n \geq 0)$$

① − Ⓑ $\quad \dfrac{-1}{(z-1)(z-2)} = \left(\dfrac{1}{z-1} - \dfrac{1}{z-2}\right)$

$|z|>1 \qquad |z|>2$



$$X(z) = \frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \qquad (|z|>2)$$

$$1^n - 2^n \qquad (n \geq 0)$$

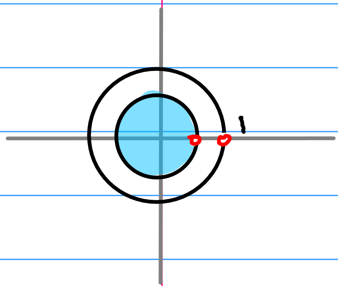
$$a_n = 1^{n-1} - 2^{n-1} \qquad (n \geq 1)$$

Causal $\quad f(z) \quad X(z)$

$|z| < 0.5 \qquad |z| > 1$

②-Ⓐ $\dfrac{-0.5z^2}{(z-1)(z-0.5)} = \left(-\dfrac{z}{(z-1)} + \dfrac{0.5z}{(z-0.5)}\right)$

$|z|<1 \qquad |z|<0.5$



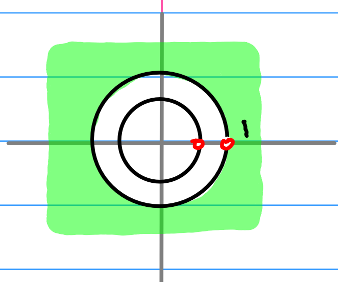
$f(z) = (+1)\dfrac{z}{1-z} - \dfrac{z}{1-2z} \quad (|z|<0.5)$

$1^n - 2^n \quad (n \geq 0)$

$a_n = 1^{n-1} - 2^{n-1} \quad (n \geq 1)$

②-Ⓑ $\dfrac{-0.5z^2}{(z-1)(z-0.5)} = \left(-\dfrac{z}{(z-1)} + \dfrac{0.5z}{(z-0.5)}\right)$

$|z|>1 \qquad |z|>0.5$



$X(z) = -\dfrac{1}{1-z^{-1}} + \dfrac{0.5}{1-0.5z^{-1}} \quad (|z|>1)$

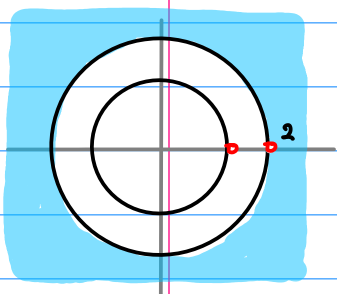
$a_n = -1^{n+1} + \left(\frac{1}{2}\right)^{n+1} \quad (n \geq 0)$

Anti-causal $\quad f(z) \quad X(z)$

$|z| > 2 \qquad |z| < 1$

①-Ⓐ

$$\frac{-1}{(z-1)(z-2)} = \left(\frac{1}{z-1} - \frac{1}{z-2}\right)$$



$|z|>1 \qquad |z|>2$

$$f(z) = \frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \qquad (|z|>2)$$

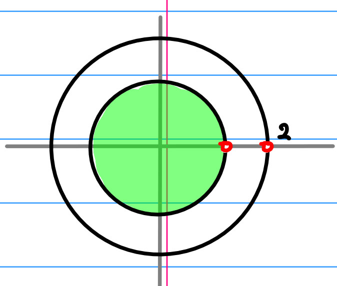
$$1^n - 2^n \qquad (n \geq 0)$$

$$1^{n-1} - 2^{n-1} \qquad (n \geq 1)$$

$$a_n = 1^{n+1} - \left(\tfrac{1}{2}\right)^{n+1} \qquad (n < 0)$$

①-Ⓑ

$$\frac{-1}{(z-1)(z-2)} = \left(\frac{1}{z-1} - \frac{1}{z-2}\right)$$



$|z|<1 \qquad |z|<2$

$$X(z) = -\frac{1}{1-z} + \frac{0.5}{1-0.5z} \qquad (|z|<1)$$

$$-1^{n+1} + \left(\tfrac{1}{2}\right)^{n+1} \qquad (n \geq 0)$$

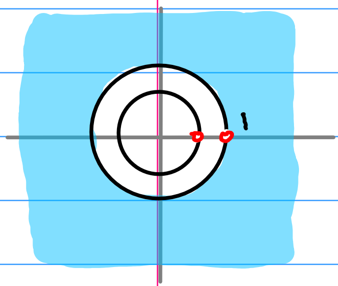
$$a_n = -1^{n-1} + 2^{n-1} \qquad (n < 1)$$

Anti-causal $\quad f(z) \quad X(z)$

$|z| > 1 \qquad |z| < 0.5$

②-Ⓐ $\dfrac{-0.5z^2}{(z-1)(z-0.5)} = \left(-\dfrac{z}{(z-1)} + \dfrac{0.5z}{(z-0.5)}\right)$

$|z|>1 \qquad |z|>0.5$



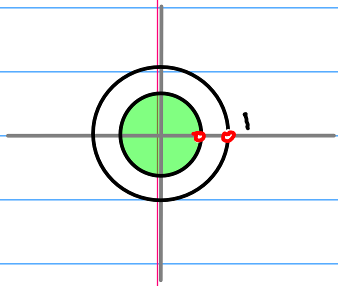
$f(z) = (-1)\dfrac{1}{1-z^{-1}} + \dfrac{0.5}{1-0.5z^{-1}} \quad (|z|>1)$

$-1^{n+1} + \left(\frac{1}{2}\right)^{n+1} \quad (n \geq 0)$

$a_n = -1^{n-1} + 2^{n-1} \quad (n<1)$

②-Ⓑ $\dfrac{-0.5z^2}{(z-1)(z-0.5)} = \left(-\dfrac{z}{(z-1)} + \dfrac{0.5z}{(z-0.5)}\right)$

$|z|<1 \qquad |z|<0.5$



$X(z) = \dfrac{z}{1-z} - \dfrac{z}{1-2z} \quad (|z|<1)$

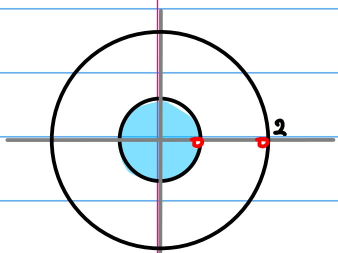
$+1^n - 2^n \quad (n \geq 0)$

$+1^{n-1} - 2^{n-1} \quad (n \geq 1)$

$a_n = 1^{n+1} - \left(\frac{1}{2}\right)^{n+1} \quad (n<0)$

① - Ⓐ $\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \boxed{f(z)}$ $|z| < 0.5$ causal $|z| > 2$ anticausal

$$\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right) = \frac{-2}{1-2z} + \frac{0.5}{1-0.5z}$$

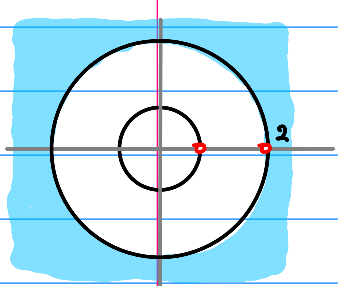


$|z| < 0.5$

$$\boxed{\begin{aligned} f(z) &= \frac{(-2)}{1-(2z)} + \frac{(\frac{1}{2})}{1-(\frac{z}{2})} \\ &= -\sum_{n=0}^{\infty}(2)^{n+1}(z)^n + \sum_{n=0}^{\infty}\left(\tfrac{1}{2}\right)^{n+1}(z)^n \\ &= -\sum_{n=0}^{\infty}(2)^{n+1} z^n + \sum_{n=0}^{\infty}\left(\tfrac{1}{2}\right)^{n+1} z^n \end{aligned}}$$

$(n \geq 0)$ $\quad a_n = -2^{n+1} + \left(\frac{1}{2}\right)^{n+1}$

$$\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right) = \frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}$$



$|z| > 2$

$$\boxed{\begin{aligned} f(z) &= \frac{(\frac{1}{z})}{1-(\frac{1}{2z})} - \frac{(\frac{1}{z})}{1-(\frac{2}{z})} \\ &= \sum_{n=0}^{\infty}\left(\tfrac{1}{2}\right)^n\left(\tfrac{1}{z}\right)^{n+1} - \sum_{n=0}^{\infty}(2)^n\left(\tfrac{1}{z}\right)^{n+1} \\ &= \sum_{n=1}^{\infty}\left(\tfrac{1}{2}\right)^{n-1} z^{-n} - \sum_{n=1}^{\infty}(2)^{n-1} z^{-n} \end{aligned}} \neq$$

$$= \sum_{n=-1}^{-\infty}(2)^{n+1} z^n - \sum_{n=-1}^{-\infty}\left(\tfrac{1}{2}\right)^{n+1} z^n$$

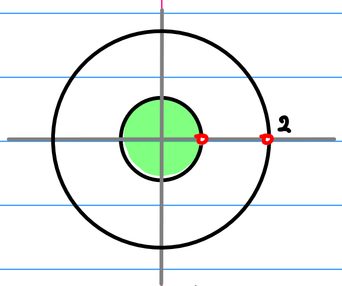
$(n < 0)$ $\quad a_n = 2^{n+1} - \left(\frac{1}{2}\right)^{n+1}$

①-Ⓑ $\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \boxed{X(z)}$

$|z| < 0.5$ anticausal

$|z| > 2$ causal

$$\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right) = \frac{-2}{1-2z} + \frac{0.5}{1-0.5z}$$



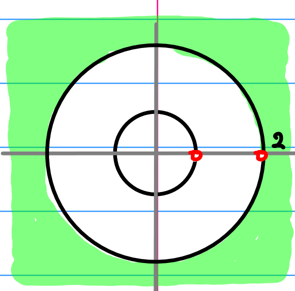
$|z| < 0.5$

$$\boxed{\begin{aligned} X(z) &= \frac{(-2)}{1-(2z)} + \frac{(\frac{1}{2})}{1-(\frac{z}{2})} \\ &= -\sum_{n=0}^{\infty}(2)^{n+1}(z)^n + \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^{n+1}(z)^n \\ &= -\sum_{n=0}^{\infty}(2)^{n+1} z^n + \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^{n+1} z^n \end{aligned}}$$

$$= -\sum_{n=0}^{-\infty}\left(\frac{1}{2}\right)^{n-1} z^{-n} + \sum_{n=0}^{-\infty}(2)^{n-1} z^{-n}$$

$(n \leq 0)$ $\quad a_n = -\left(\frac{1}{2}\right)^{n-1} + 2^{n-1}$

$$\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right) = \frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}$$



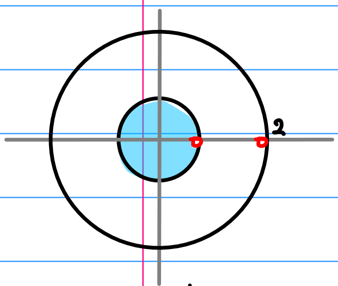
$|z| > 2$

$$\boxed{\begin{aligned} X(z) &= \frac{(\frac{1}{z})}{1-(\frac{1}{2z})} - \frac{(\frac{1}{z})}{1-(\frac{2}{z})} \\ &= \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n\left(\frac{1}{z}\right)^{n+1} - \sum_{n=0}^{\infty}(2)^n\left(\frac{1}{z}\right)^{n+1} \\ &= \sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^{n-1} z^{-n} - \sum_{n=1}^{\infty}(2)^{n-1} z^{-n} \end{aligned}}$$ $\neq$

$(n > 0)$ $\quad a_n = \left(\frac{1}{2}\right)^{n-1} - (2)^{n-1}$

② - Ⓐ $\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)} = \boxed{f(z)}$ $|z| < 0.5$ causal $|z| > 2$ anticausal

$$\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right) = -\frac{z}{1-2z} + \frac{z}{1-0.5z}$$

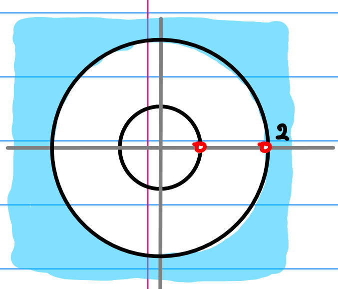


$|z|<0.5$

$$f(z) = \boxed{\begin{aligned} & -\frac{(z)}{1-(2z)} + \frac{(z)}{1-(\frac{z}{2})} \\ &= -\sum_{n=0}^{\infty}(2)^n(z)^{n+1} + \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n(z)^{n+1} \\ &= -\sum_{n=1}^{\infty}(2)^{n-1}z^n + \sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^{n-1}z^n \end{aligned}} \neq$$

$(n > 0)$ $a_n = -2^{n-1} + \left(\frac{1}{2}\right)^{n-1}$

$$\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right) = \frac{0.5}{1-0.5z^{-1}} - \frac{2}{1-2z^{-1}}$$



$|z|>2$

$$f(z) = \boxed{\begin{aligned} & \frac{(\frac{1}{2})}{1-(\frac{1}{2z})} - \frac{(2)}{1-(\frac{2}{z})} \\ &= \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^{n+1}\left(\frac{1}{z}\right)^n - \sum_{n=0}^{\infty}(2)^{n+1}\left(\frac{1}{z}\right)^n \\ &= \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^{n+1}z^{-n} - \sum_{n=0}^{\infty}(2)^{n+1}z^{-n} \end{aligned}}$$

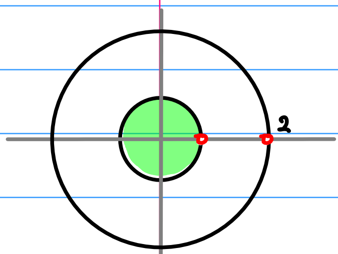
$$= \sum_{n=0}^{-\infty}(2)^{n-1}z^n - \sum_{n=0}^{-\infty}\left(\frac{1}{2}\right)^{n-1}z^n$$

$(n \leq 0)$ $a_n = 2^{n-1} - \left(\frac{1}{2}\right)^{n-1}$

② - ⓑ $\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)} = \boxed{X(z)}$ $|z| < 0.5$ $|z| > 2$

anticausal causal

$$\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right) = -\frac{z}{1-2z} + \frac{z}{1-0.5z}$$



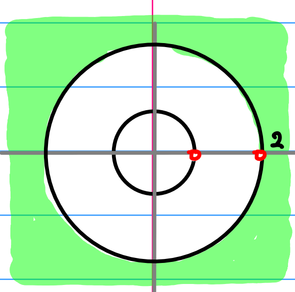
$|z| < 0.5$

$$\begin{aligned} X(z) &= -\frac{(z)}{1-(2z)} + \frac{(z)}{1-(\frac{z}{2})} \\ &= -\sum_{n=0}^{\infty}(2)^n(z)^{n+1} + \sum_{n=0}^{\infty}\left(\tfrac{1}{2}\right)^n(z)^{n+1} \\ &= -\sum_{n=1}^{\infty}(2)^{n-1}z^n + \sum_{n=1}^{\infty}\left(\tfrac{1}{2}\right)^{n-1}z^n \\ &= -\sum_{n=-1}^{-\infty}\left(\tfrac{1}{2}\right)^{n+1}z^{-n} + \sum_{n=-1}^{-\infty}(2)^{n+1}z^{-n} \end{aligned}$$

$\neq$

$(n < 0)$ $a_n = -\left(\frac{1}{2}\right)^{n+1} + 2^{n+1}$

$$\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right) = \frac{0.5}{1-0.5z^{-1}} - \frac{2}{1-2z^{-1}}$$



$|z| > 2$

$$\begin{aligned} X(z) &= \frac{(\frac{1}{2})}{1-(\frac{1}{2z})} - \frac{(2)}{1-(\frac{2}{z})} \\ &= \sum_{n=0}^{\infty}\left(\tfrac{1}{2}\right)^{n+1}\left(\tfrac{1}{z}\right)^n - \sum_{n=0}^{\infty}(2)^{n+1}\left(\tfrac{1}{z}\right)^n \\ &= \sum_{n=0}^{\infty}\left(\tfrac{1}{2}\right)^{n+1}z^{-n} - \sum_{n=0}^{\infty}(2)^{n+1}z^{-n} \end{aligned}$$

$(n \geq 0)$ $a_n = \left(\frac{1}{2}\right)^{n+1} - 2^{n+1}$