DT Sinusoidal Function (1B)

- Discrete Time Sinusoidal Function

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Exponential Functions

McClellan Style

$$x[n] = x(nT_s)$$
$$= A\cos(\omega n T_s + \Phi)$$
$$= A\cos(\hat{\omega} n + \Phi)$$

$$\hat{\omega} = \omega T_s = \omega / f_s$$

$$\hat{\omega} = 2\pi \hat{f}$$
$$\hat{f} = f / f_s$$

$$x[n] = A\cos(2\pi n f / f_s + \Phi)$$
$$= A\cos(\hat{\omega} n + \Phi)$$

Roberts' Style

$$g[n] = A e^{\beta n} = A z^n \qquad e^{\beta} = z$$
$$= A\cos(2\pi F_0 n + \theta)$$
$$= A\cos(\Omega_0 n + \theta)$$

$$\Omega_0 = 2\pi F_0$$

$$g[n] = A\cos(2\pi n q / N_0 + \theta)$$

$$\Omega_0 = 2\pi F_0$$
$$F_0 = q / N_0$$

$$g[n] = A\cos(2\pi n F_0 + \theta)$$
$$= A\cos(\Omega_0 n + \theta)$$

DT Signal Fundamental Period

Fundamental Period N_0

$$g[n] = Ae^{\beta n} = Az^n \qquad e^{\beta} = z$$

$$= A\cos(2\pi F_0 n + \theta)$$

$$= A\cos(\Omega_0 n + \theta)$$

Periodic Condition

for some discrete time *n*

and some integer *m*

$$2\pi F_0 n = 2\pi m \Rightarrow F_0 n = m$$

$F_0 = m/n$ **a rational number**

$F_0 = f_0/f_s$ **periodic**

the smallest positive discrete time

DT Signal Fundamental Period

Fundamental Period N_0

$$1/N_0 = F_0 = \Omega_0/2\pi \quad \textbf{when} \quad q = 1 \qquad g[n] = A\cos(2\pi n/N_0 + \theta)$$

$$q/N_0 = F_0 = \Omega_0/2\pi \quad \textbf{when} \quad q \neq 1 \qquad g[n] = A\cos(2\pi n q/N_0 + \theta)$$

reduced form

reduced form $\frac{q}{N_0} n = m$ ➡ **fundamental period** $n = N_0$ ~~$\frac{N_0}{q}$~~

reduced form $\frac{5}{17} n = m$ ➡ **fundamental period** $n = 17$

DT Signal Frequency

discrete time n

a time index not time itself

units of samples

$$g(t) = \sin(2\pi \cdot f \cdot t) \Rightarrow g[n] = \sin(2\pi \cdot f \cdot T_s \cdot n)$$

Normalized Cyclic Frequency

$$F_0 = \frac{f_0}{f_s}$$

$$\text{cycles/sample} = \frac{\text{cycles/second}}{\text{samples/second}}$$

$$g[n] = \sin(2\pi \cdot F_0 \cdot n)$$

Normalized Radian Frequency

$$\Omega_0 = \frac{\omega_0}{f_s}$$

$$\text{radians/sample} = \frac{\text{radians/second}}{\text{samples/second}}$$

$$g[n] = \sin(\omega_0 \cdot n)$$

reduced form $F_0 = \dfrac{q}{N_0}$ ⇒ $\not{2\pi} \dfrac{q}{N_0} n = \not{2\pi} m$ ⇒ fundamental period $n = N_0$

DT Signal Period : Samples & Cycles

$f = 1\ cycles/sec$

$1\ Hz$

$$g(t) = \sin(2\pi \cdot f \cdot t) \Rightarrow g[n] = \sin(2\pi \cdot f \cdot T_s \cdot n)$$

$T_s = 0.05\ sec$

$f_s = 20\ samples/sec$

$N_0 = 20\ samples$

$1\ cycle$

$T_s = 0.2\ sec$

$f_s = 5\ samples/sec$

$N_0 = 5\ samples$

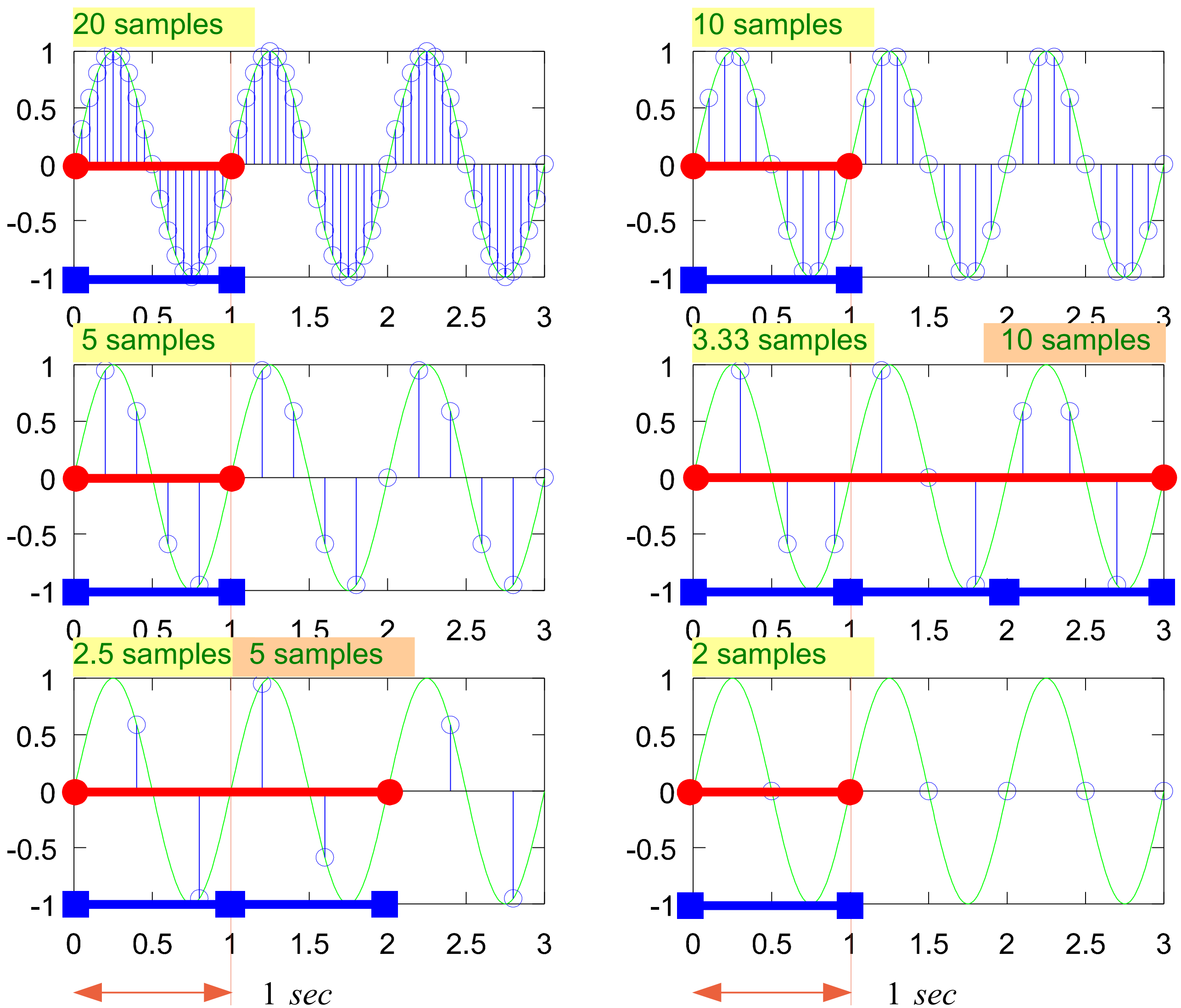
$1\ cycle$

$T_s = 0.4\ sec$

$f_s = 2.5\ samples/sec$

$N_0 = 5\ samples$

$2\ cycles$



$T_s = 0.1\ sec$

$f_s = 10\ samples/sec$

$N_0 = 10\ samples$

$1\ cycle$

$T_s = 0.3\ sec$

$f_s = 3.33\ samples/sec$

$N_0 = 10\ samples$

$3\ cycles$

$T_s = 0.5\ sec$

$f_s = 2\ samples/sec$

$N_0 = 2\ samples$

$1\ cycle$

DT Signal Normalized Cyclic Frequency

$f = 1 \; cycles/sec$

$1 \; Hz$

$$g(t) = \sin(2\pi \cdot f \cdot t) \;\Rightarrow\; g[n] = \sin(2\pi \cdot f \cdot T_s \cdot n)$$

$T_s = 0.05 \; sec$
$f_s = 20 \; samples/sec$

$N_0 = 20 \; samples$

$F_0 = \dfrac{1}{20} \dfrac{cycle}{sample}$

$T_s = 0.1 \; sec$
$f_s = 10 \; samples/sec$

$N_0 = 10 \; samples$

$F_0 = \dfrac{1}{10} \dfrac{cycle}{sample}$

$T_s = 0.2 \; sec$
$f_s = 5 \; samples/sec$

$N_0 = 5 \; samples$

$F_0 = \dfrac{1}{5} \dfrac{cycle}{sample}$

$T_s = 0.3 \; sec$
$f_s = 3.33 \; samples/sec$

$N_0 = 10 \; samples$

$F_0 = \dfrac{3}{10} \dfrac{cycle}{sample}$

$T_s = 0.4 \; sec$
$f_s = 2.5 \; samples/sec$

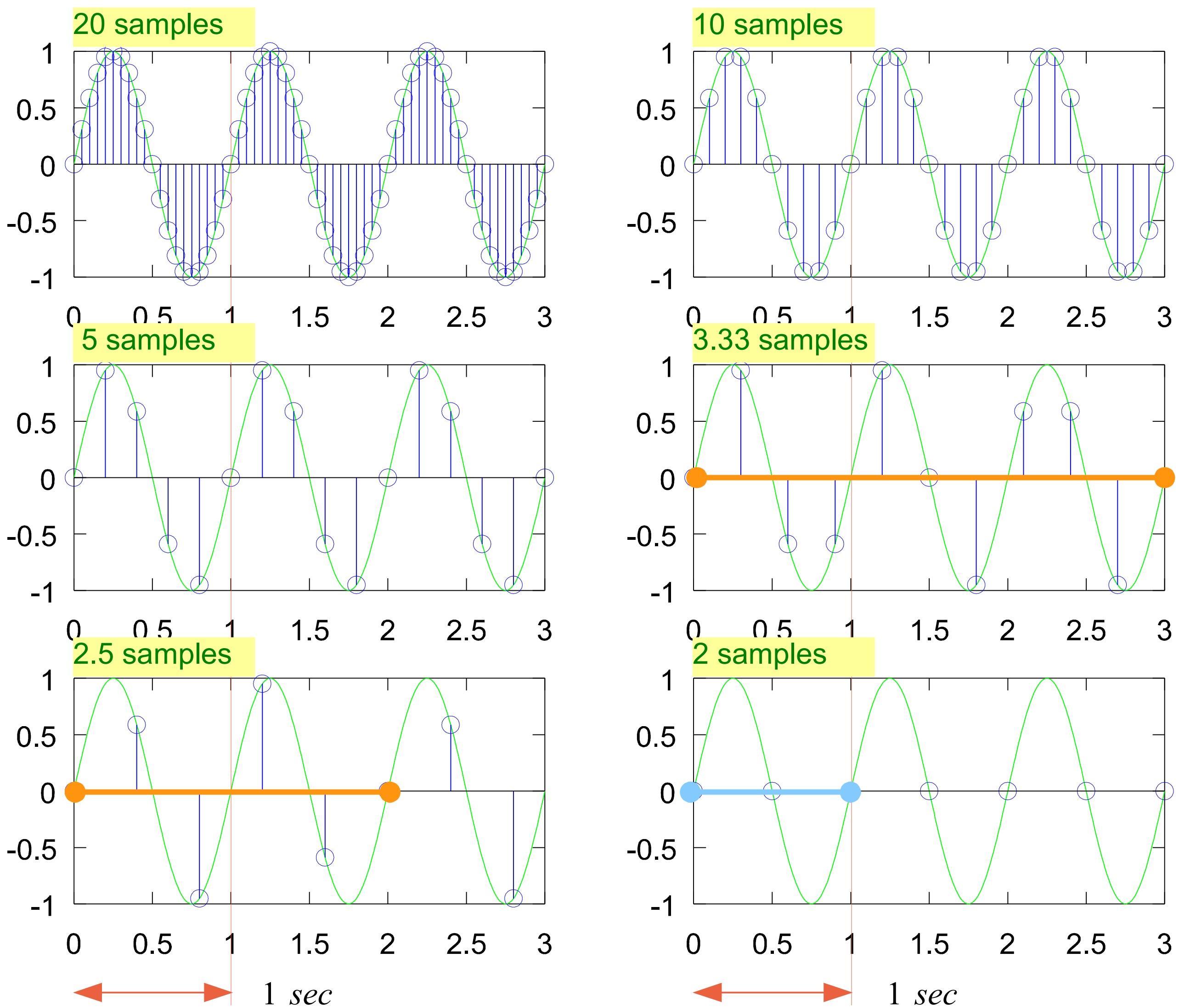
$N_0 = 5 \; samples$

$F_0 = \dfrac{2}{5} \dfrac{cycle}{sample}$

$T_s = 0.5 \; sec$
$f_s = 2 \; samples/sec$

$N_0 = 2 \; samples$

$F_0 = \dfrac{1}{2} \dfrac{cycle}{sample}$



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DT Signal Fundamental Period

$f = 1\ Hz$
$= 1\ cycles/sec$

$$g(t) = \sin(2\pi \cdot f \cdot t) \Rightarrow g[n] = \sin(2\pi \cdot f \cdot T_s \cdot n)$$

$T_s = 0.05\ sec$
$f_s = 20\ samples/sec$

$N_0 = 20\ samples$

$F_0 = \frac{1}{20} \frac{cycle}{sample}$

$$g[n] = \sin\left(2\pi \cdot \frac{1}{20} \cdot n\right)$$

$F_0 = \frac{f}{f_s} = \frac{1}{20} \Rightarrow \frac{q}{N_0} = \frac{1}{20}$

$T_s = 0.1\ sec$
$f_s = 10\ samples/sec$

$$g[n] = \sin\left(2\pi \cdot \frac{1}{10} \cdot n\right)$$

$\frac{q}{N_0} = \frac{1}{10} \Leftarrow F_0 = \frac{f}{f_s} = \frac{1}{10}$

$N_0 = 10\ samples$

$F_0 = \frac{1}{10} \frac{cycle}{sample}$

$T_s = 0.2\ sec$
$f_s = 5\ samples/sec$

$N_0 = 5\ samples$

$F_0 = \frac{1}{5} \frac{cycle}{sample}$

$$g[n] = \sin\left(2\pi \cdot \frac{1}{5} \cdot n\right)$$

$F_0 = \frac{f}{f_s} = \frac{1}{5} \Rightarrow \frac{q}{N_0} = \frac{1}{5}$

$T_s = 0.3\ sec$
$f_s = 3.33\ samples/sec$

$$g[n] = \sin\left(2\pi \cdot \frac{3}{10} \cdot n\right)$$

$\frac{q}{N_0} = \frac{3}{10} \Leftarrow F_0 = \frac{f}{f_s} = \frac{1}{10/3}$

$N_0 = 10\ samples$

$F_0 = \frac{3}{10} \frac{cycle}{sample}$

$T_s = 0.4\ sec$
$f_s = 2.5\ samples/sec$

$N_0 = 5\ samples$

$F_0 = \frac{2}{5} \frac{cycle}{sample}$

$$g[n] = \sin\left(2\pi \cdot \frac{2}{5} \cdot n\right)$$

$F_0 = \frac{f}{f_s} = \frac{1}{2.5} \Rightarrow \frac{q}{N_0} = \frac{2}{5}$

$T_s = 0.5\ sec$
$f_s = 2\ samples/sec$

$$g[n] = \sin\left(2\pi \cdot \frac{1}{2} \cdot n\right)$$

$\frac{q}{N_0} = \frac{1}{2} \Leftarrow F_0 = \frac{f}{f_s} = \frac{1}{2}$

$N_0 = 2\ samples$

$F_0 = \frac{1}{2} \frac{cycle}{sample}$

DT Signal Spectrum Replication

$$2\pi(F+1)n = 2\pi F n + 2\pi n$$

$$(\Omega + 2\pi)n = \Omega n + 2\pi n$$

$$2\pi(F+k)n = 2\pi F n + 2\pi k n$$

$$(\Omega + 2\pi k)n = \Omega n + 2\pi k n$$

$$\cos(2\pi(F+1)n) = \cos(2\pi F n)$$

$$\cos((\Omega + 2\pi)n) = \cos(\Omega n)$$

$$\cos(2\pi(F+k)n) = \cos(2\pi F n)$$

$$\cos((\Omega + 2\pi k)n) = \cos(\Omega n)$$

$$\sin(2\pi(F+1)n) = \sin(2\pi F n)$$

$$\sin((\Omega + 2\pi)n) = \sin(\Omega n)$$

$$\sin(2\pi(F+k)n) = \sin(2\pi F n)$$

$$\sin((\Omega + 2\pi k)n) = \sin(\Omega n)$$

f_0 *cycle / sec*	1 11 …	2 12 …	3 13 …	4 14 …	5 15 …	6 16 …	7 17 …	8 18 …	9 19 …	10 20 …
f_s *sample / sec*	10	10	10	10	10	10	10	10	10	10
$F_0 = \dfrac{f_0}{f_s}\ \dfrac{cycle}{sample}$	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{4}{10}$	$\frac{5}{10}$	$\frac{6}{10}$	$\frac{7}{10}$	$\frac{8}{10}$	$\frac{9}{10}$	$\frac{10}{10}$

DT Signal Fundamental Period

$T_s = 0.1 \; sec$
$f_s = 10 \; samples/sec$

$$g(t) = \sin(2\pi \cdot f \cdot t) \Rightarrow g[n] = \sin(2\pi \cdot f \cdot T_s \cdot n)$$

$f_0 \; cycle/sec$	1	2	3	4	5	6	7	8	9	10
	11	12	13	14	15	16	17	18	19	20
	…	…	…	…	…	…	…	…	…	…
$f_s \; sample/sec$	10	10	10	10	10	10	10	10	10	10
$F_0 = \frac{f_0}{f_s} \frac{cycle}{sample}$	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{4}{10}$	$\frac{5}{10}$	$\frac{6}{10}$	$\frac{7}{10}$	$\frac{8}{10}$	$\frac{9}{10}$	$\frac{10}{10}$
$N_0 = 10 \; samples$	$\frac{1}{10}$		$\frac{3}{10}$				$\frac{7}{10}$		$\frac{9}{10}$	
$N_0 = 5 \; samples$		$\frac{1}{5}$		$\frac{2}{5}$		$\frac{3}{5}$		$\frac{4}{5}$		
$N_0 = 2 \; samples$					$\frac{1}{2}$					
$N_0 = 1 \; samples$										1

DT Signal Fundamental Period

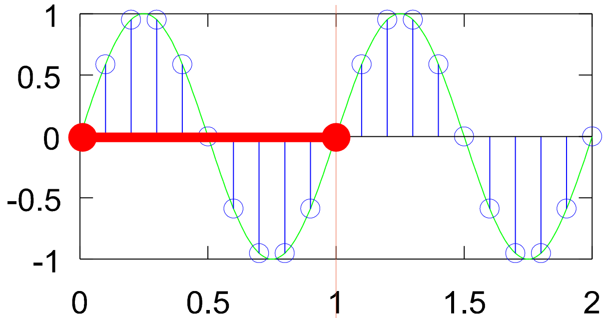
$T_s = 0.1\ sec$
$f_s = 10\ samples/sec$

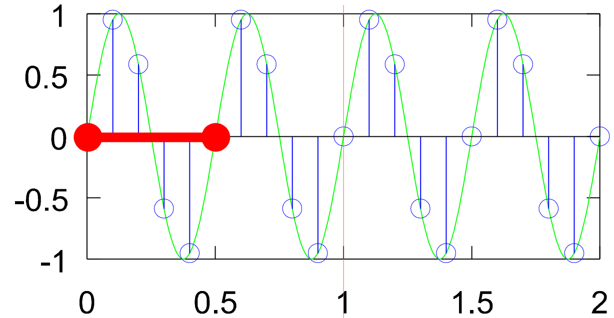
$$g(t) = \sin(2\pi \cdot f \cdot t) \Rightarrow g[n] = \sin(2\pi \cdot f \cdot T_s \cdot n)$$

$f_0 = 1\ cycle/sec$

$N_0 = 10\ samples$

$F_0 = \frac{1}{10}\ \frac{cycle}{sample}$





$f_0 = 2\ cycle/sec$

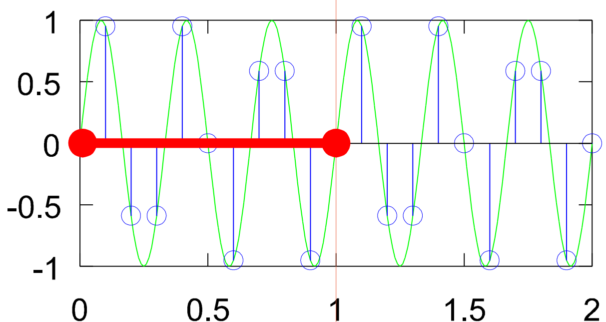
$N_0 = 5\ samples$

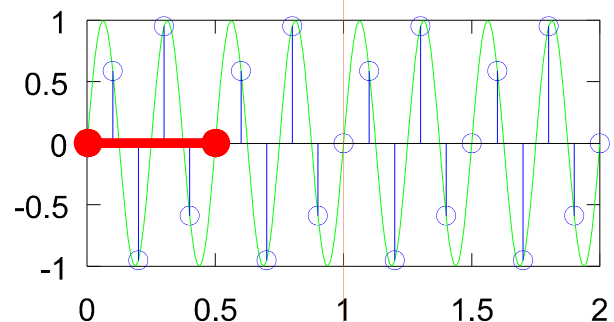
$F_0 = \frac{1}{5}\ \frac{cycle}{sample}$

$f_0 = 3\ cycle/sec$

$N_0 = 10\ samples$

$F_0 = \frac{3}{10}\ \frac{cycle}{sample}$





$f_0 = 4\ cycle/sec$

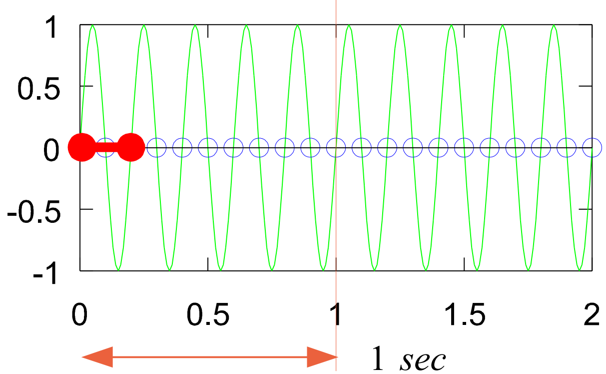
$N_0 = 5\ samples$

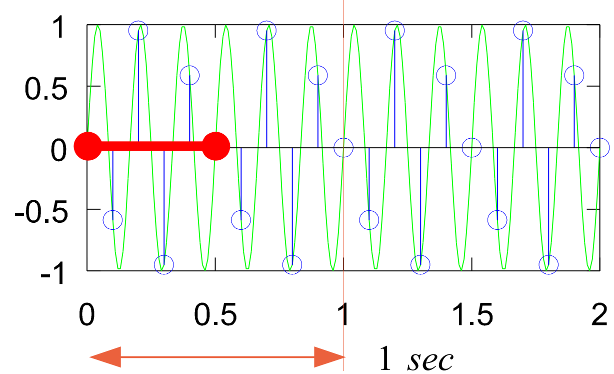
$F_0 = \frac{2}{5}\ \frac{cycle}{sample}$

$f_0 = 5\ cycle/sec$

$N_0 = 2\ samples$

$F_0 = \frac{1}{2}\ \frac{cycle}{sample}$





$f_0 = 6\ cycle/sec$

$N_0 = 5\ samples$

$F_0 = \frac{3}{5}\ \frac{cycle}{sample}$

DT Signal Fundamental Period

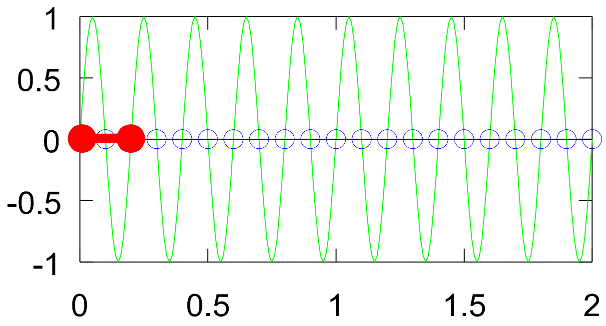
$T_s = 0.1\ sec$
$f_s = 10\ samples/sec$

$$g(t) = \sin(2\pi \cdot f \cdot t) \Rightarrow g[n] = \sin(2\pi \cdot f \cdot T_s \cdot n)$$

$f_0 = 5\ cycle/sec$

$N_0 = 2\ samples$

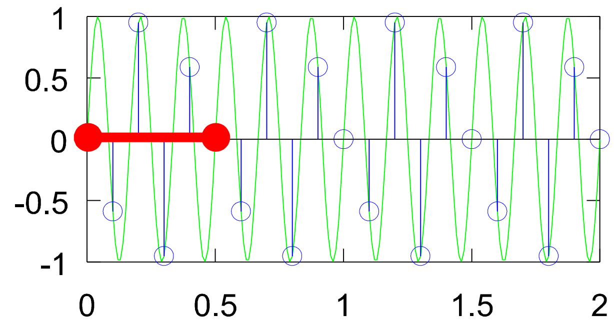
$F_0 = \frac{1}{2}\ \frac{cycle}{sample}$



$f_0 = 6\ cycle/sec$

$N_0 = 5\ samples$

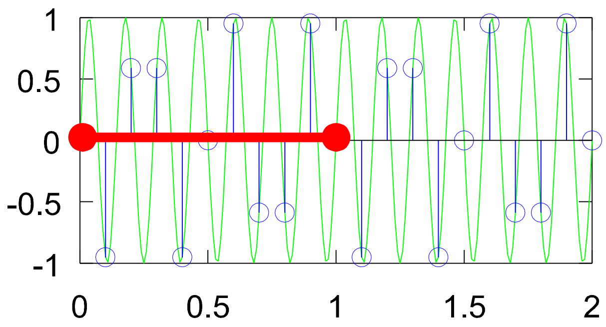
$F_0 = \frac{3}{5}\ \frac{cycle}{sample}$



$f_0 = 7\ cycle/sec$

$N_0 = 10\ samples$

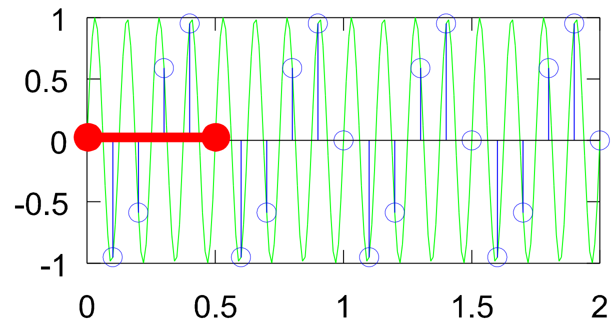
$F_0 = \frac{7}{10}\ \frac{cycle}{sample}$



$f_0 = 8\ cycle/sec$

$N_0 = 5\ samples$

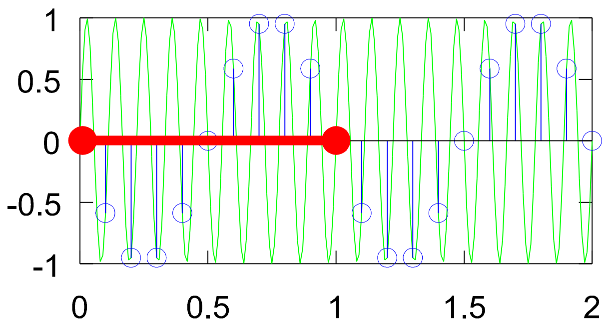
$F_0 = \frac{4}{5}\ \frac{cycle}{sample}$



$f_0 = 9\ cycle/sec$

$N_0 = 10\ samples$

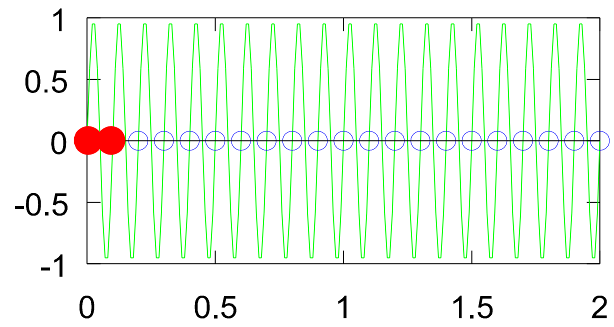
$F_0 = \frac{9}{10}\ \frac{cycle}{sample}$



$f_0 = 10\ cycle/sec$

$N_0 = 1\ samples$

$F_0 = 1\ \frac{cycle}{sample}$



DT Signal Aliasing

$$2\pi(1 - F)n = 2\pi n - 2\pi F n$$

$$(2\pi - \Omega)n = 2\pi n - \Omega n$$

$$2\pi(k - F)n = 2\pi k n - 2\pi F n$$

$$(2\pi k - \Omega)n = 2\pi k n - \Omega n$$

$$\cos(2\pi(1 - F)n) = \cos(2\pi F n)$$

$$\cos((2\pi - \Omega)n) = \cos(\Omega n)$$

$$\cos(2\pi(k - F)n) = \cos(2\pi F n)$$

$$\cos((2\pi k - \Omega)n) = \cos(\Omega n)$$

$$\sin(2\pi(1 - F)n) = -\sin(2\pi F n)$$

$$\sin((2\pi - \Omega)n) = -\sin(\Omega n)$$

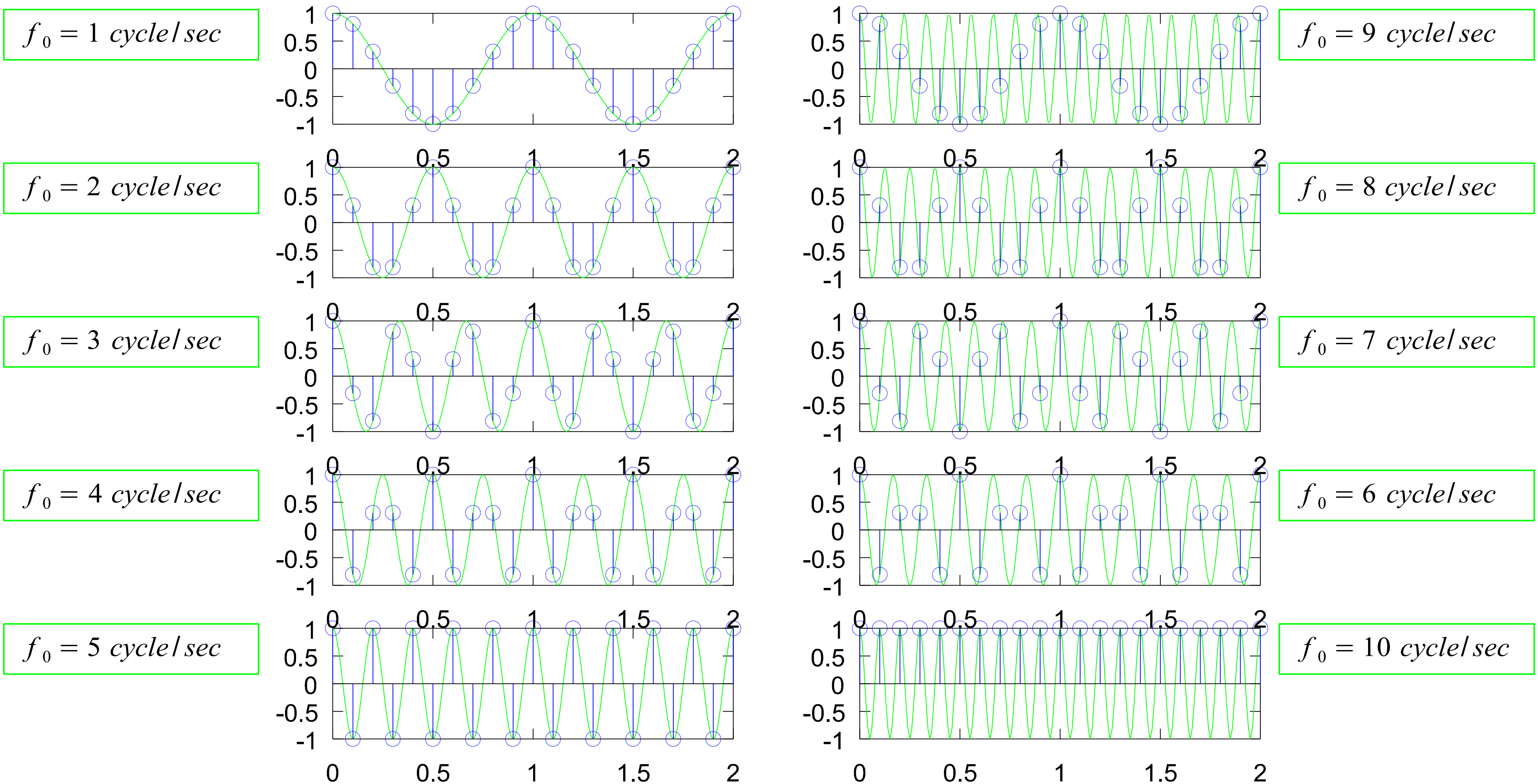
$$\sin(2\pi(k - F)n) = -\sin(2\pi F n)$$

$$\sin((2\pi k - \Omega)n) = -\sin(\Omega n)$$

DT Signal Aliasing - COS

$T_s = 0.1 \ sec$
$f_s = 10 \ samples/sec$

$$g(t) = \cos(2\pi \cdot f \cdot t) \Rightarrow g[n] = \cos(2\pi \cdot f \cdot T_s \cdot n)$$



DT Signal Aliasing - SIN

$T_s = 0.1 \ sec$
$f_s = 10 \ samples/sec$

$$g(t) = \sin(2\pi \cdot f \cdot t) \Rightarrow g[n] = \sin(2\pi \cdot f \cdot T_s \cdot n)$$

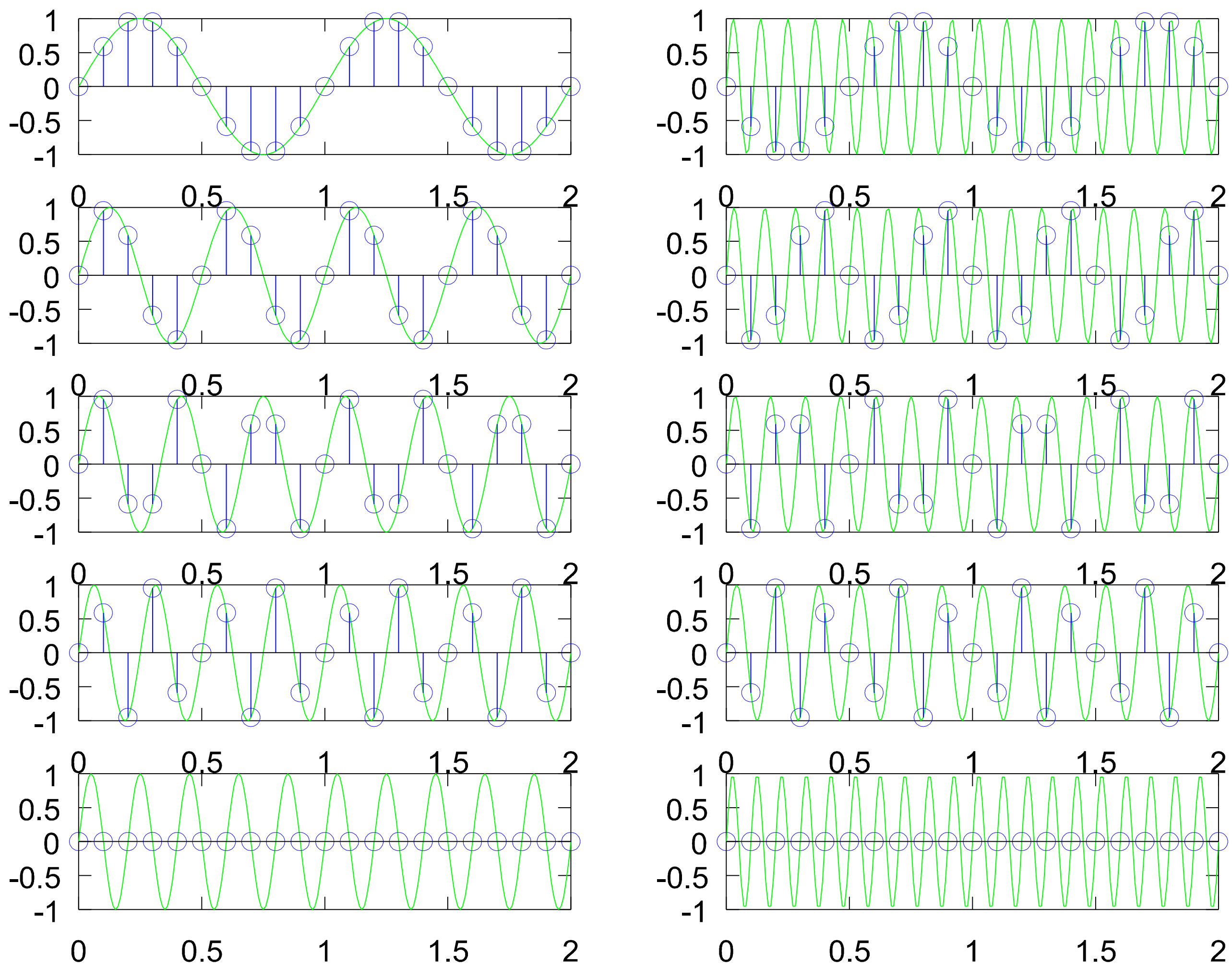
$f_0 = 1 \ cycle/sec$

$f_0 = 2 \ cycle/sec$

$f_0 = 3 \ cycle/sec$

$f_0 = 4 \ cycle/sec$

$f_0 = 5 \ cycle/sec$



$f_0 = 9 \ cycle/sec$

$f_0 = 8 \ cycle/sec$

$f_0 = 7 \ cycle/sec$

$f_0 = 6 \ cycle/sec$

$f_0 = 10 \ cycle/sec$

(1)

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References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
[3] G. Beale, http://teal.gmu.edu/~gbeale/ece_220/fourier_series_02.html
[4] C. Langton, http://www.complextoreal.com/chapters/fft1.pdf