

# Digital Signal Octave Codes (0A)

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- Periodic Conditions

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Based on

M.J. Roberts, Fundamentals of Signals and Systems

S.K. Mitra, Digital Signal Processing : a computer-based approach 2<sup>nd</sup> ed

S.D. Stearns, Digital Signal Processing with Examples in MATLAB

# Sampling and Normalized Frequency

$$\omega_0 t = 2\pi f_0 t$$



$$\omega_0 nT_s = 2\pi f_0 nT_s$$

$$= \frac{2\pi}{T_0} nT_s$$

$$= 2\pi n \frac{T_s}{T_0}$$

$$= 2\pi n F_0$$

$$t = nT_s$$

$$f_0 = \frac{1}{T_0}$$

$$\frac{T_s}{T_0} = \frac{f_0}{f_s}$$

$$F_0 = f_0 T_s = \frac{f_0}{f_s}$$

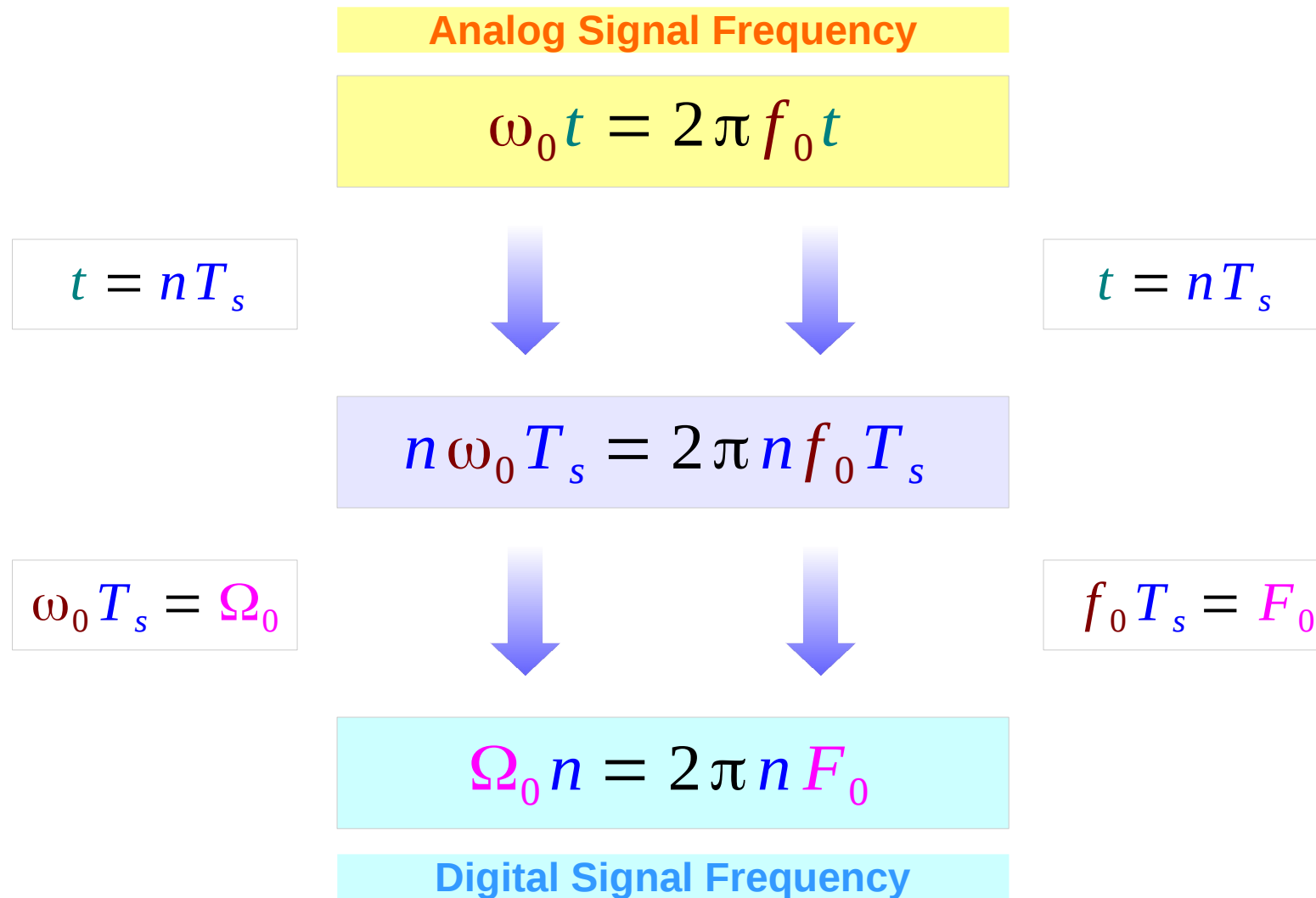
$T_s$  : sampling period

$T_0$  : signal period

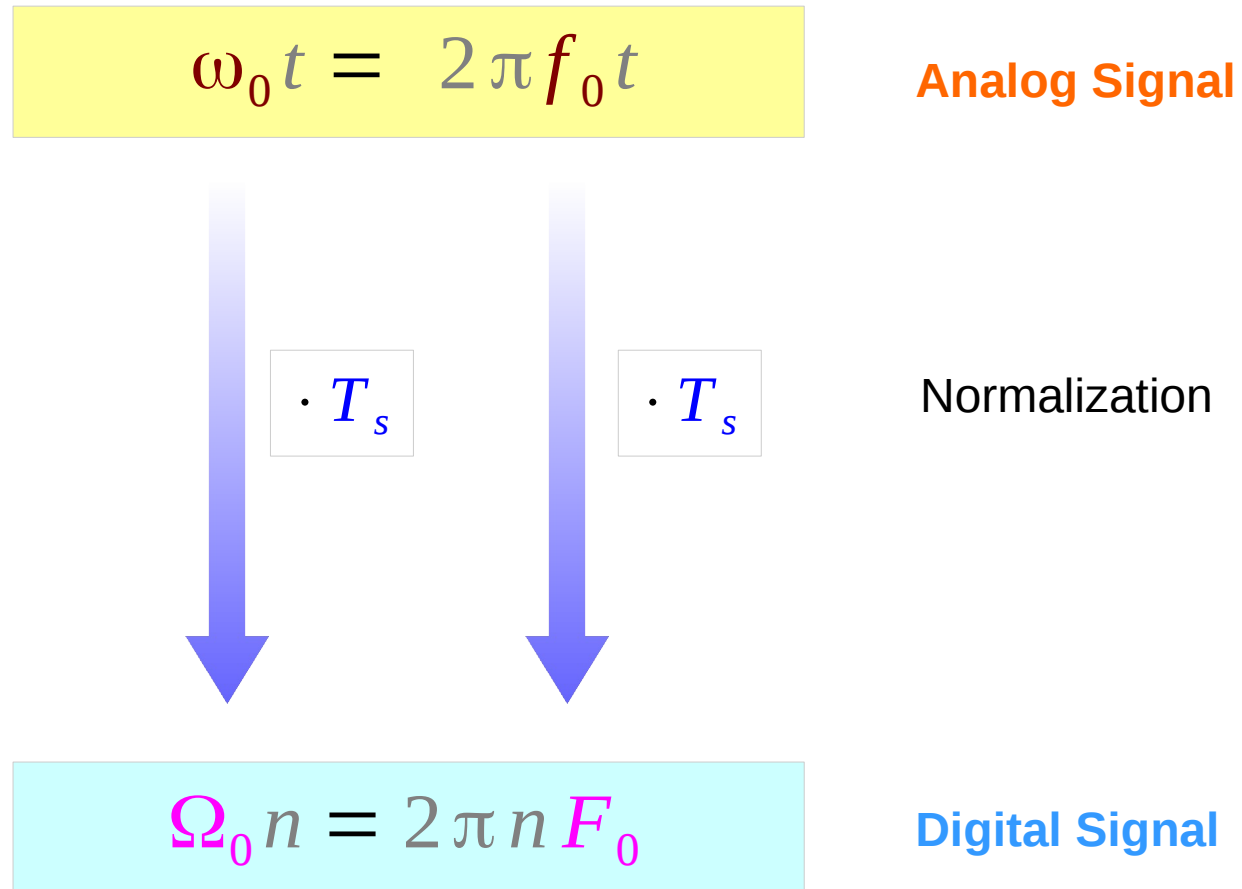
*normalization*

*normalization*

# Analog and Digital Frequencies



# Multiplying by $T_s$ – Normalization



# Normalization

$$F_0 = f_0 \cdot T_s$$

$$= f_0 / f_s$$

$$= T_s / T_0$$

$$f_0 \cdot T_s$$

*Multiplied by  $T_s$*

$$f_0 / f_s$$

*Divided by  $f_s$*

$$\Omega_0 = 2\pi F_0$$

$$f_s > 2 \cdot f_0$$

$$f_0 / f_s < 0.5$$

*Sampling Rate  
Minimum*

# Normalized Cyclic and Radian Frequencies

## Normalized Cyclic Frequency

$$F_0 \text{ cycles/sample} = \frac{f_0 \text{ cycles/second}}{f_s \text{ samples/second}}$$

## Normalized Radian Frequency

$$\Omega_0 \text{ cycles/sample} = \frac{\omega_0 \text{ cycles/second}}{f_s \text{ samples/second}}$$



# Periodic Relation : $N_0$ and $F_0$

$$e^{j(2\pi(n+N_0)F_0)} = e^{j(2\pi nF_0)}$$

$$e^{j2\pi m} = 1$$



Digital Signal Period  $N_0$   
: the smallest integer

$$e^{j2\pi N_0 F_0}$$



$$e^{j2\pi m} = 1$$

Periodic Condition  
: integer  $m$

$$2\pi N_0 F_0 = 2\pi m$$

$$N_0 F_0 = m$$

# Periodic Condition : $N_0$ and $F_0$

$$2\pi N_0 F_0 = 2\pi m$$

$$N_0 F_0 = m$$

$$N_0 = \frac{m}{F_0} = m \cdot \frac{T_0}{T_s}$$

Integer \* Rational :  
must be an Integer

Digital Signal Period  $N_0$   
: the smallest integer

Periodic Condition  
: the smallest integer  $m$

$$m \neq T_s$$

$$m = p$$

reduced form

# Periodic Condition : $N_0$ and $F_0$ in a reduced form

Integer

reduced form

$$N_0 = \frac{m}{F_0} = m \cdot \frac{T_0}{T_s} = m \cdot \frac{q}{p}$$

Integer \* Rational : must be an Integer

Rational numbers

# $N_0$ and $F_0$ in a reduced form : Examples

reduced form

$$F_0 = \frac{p}{q}$$

Rational

$$N_0 = \frac{m}{F_0} = m \cdot \frac{q}{p}$$

integer

$$\begin{aligned} N_0 &\rightarrow q \\ m &\rightarrow p \end{aligned}$$

integers

*the smallest integer  $m$*

$$\frac{1}{F_0} = \frac{2.678}{4.017} = \frac{2 \cdot 1.339}{3 \cdot 1.339} = \frac{2}{3}$$

$$m = 3 \quad m \neq 4.017$$

$$N_0 = 2 \quad N_0 \neq 2.678$$

$$\frac{1}{F_0} = \frac{10}{15} = \frac{2 \cdot 5}{3 \cdot 5} = \frac{2}{3}$$

$$m = 3 \quad m \neq 15$$

$$N_0 = 2 \quad N_0 \neq 10$$

# Periodic Relations – Analog and Digital Cases

$$e^{j(2\pi(n+N_0)F_0)} = e^{j(2\pi nF_0)}$$

Digital Signal Period  $N_0$   
: the smallest integer

$$N_0 = \frac{m}{F_0} = m \cdot \frac{T_0}{T_s}$$

$$k N_0 F_0 = k \cdot m$$

integer multiple of  $m$   
: some integers  $m$

$$e^{j(2\pi f_0)(t+T_0)} = e^{j(2\pi f_0)t}$$

Analog Signal Period  $T_0$   
: the smallest real number

$$T_0 = \frac{1}{f_0}$$

$$k T_0 f_0 = k \cdot 1$$

all integers

# Periodic Conditions – Analog and Digital Cases

$$N F_0 = k \cdot m$$

$$N = \frac{k \cdot m}{F_0} \quad \begin{array}{l} \text{Integer } N_0 \\ \text{Rational } F_0 \end{array}$$

**Minimum Integer  $N_0$**

$$N_0 = q \quad F_0 = \frac{p}{q}$$
$$m = p \quad \text{reduced form}$$

$$N_0 = \frac{m}{p/q}$$

$$T f_0 = k \cdot 1$$

$$T = \frac{k \cdot 1}{f_0} \quad \begin{array}{l} \text{Real } T_0 \\ \text{Real } f_0 \end{array}$$

**Minimum Real  $T_0$**

$$T_0 = \frac{1}{f_0}$$
$$m = 1$$

# Periodic Conditions Examples

$$N F_0 = k \cdot m$$

$$N_0 = \frac{m}{F_0} \quad \text{Integer } N$$

given

$$F_0 = \frac{36}{19}$$

$km$ : multiples of 36

$$N_0 = 36 \cdot \frac{19}{36} \quad 1 \cdot m = 36$$

$$2 N_0 = 72 \cdot \frac{19}{36} \quad 2 \cdot m = 72$$

$$3 N_0 = 108 \cdot \frac{19}{36} \quad 3 \cdot m = 108$$

$$T f_0 = k \cdot 1$$

$$T_0 = \frac{1}{f_0} \quad \text{Real } T$$

given

$$f_0 = \frac{36}{19}$$

$k$ : all integers

$$T_0 = 1 \cdot \frac{19}{36} \quad 1 \cdot 1 = 1$$

$$2 T_0 = 2 \cdot \frac{19}{36} \quad 2 \cdot 1 = 2$$

$$3 T_0 = 3 \cdot \frac{19}{36} \quad 3 \cdot 1 = 3$$

# Periodic Condition of a Sampled Signal

$$g(nT_s) = A \cos(2\pi f_0 T_s n + \theta)$$

$$F_0 = f_0 T_s = f_0 / f_s$$

$$g[n] = A \cos(2\pi F_0 n + \theta)$$

$$2\pi F_0 n = 2\pi m$$

$$F_0 n = m$$

$$F_0 = \frac{m}{n}$$

integers  $n, m$

Rational Number

$$F_0 = \frac{m}{n}$$

integers

$n, m$

The Smallest Integer  $n$

$$N_0 = \min(n) \quad F_0 = \frac{m}{N_0}$$



# $F_0$ and $N_0$ of a Sampled Signal

## Rational Number $F_0$

$$F_0 = \frac{m}{n} = \frac{p}{q} \quad \text{integer } n, m, p, q$$

$$F_0 = \frac{f_0}{f_s} = \frac{T_s}{T_0} \quad \text{real } f_0, f_s, T_s, T_0$$

$$2\pi F_0 n$$

## Integer $N_0$

$$N_0 F_0 = m$$

$$N_0 = \frac{m}{F_0} = m \cdot \frac{T_0}{T_s} = m \cdot \frac{f_s}{f_0} = m \cdot \frac{q}{p}$$

$$2\pi f_0 T_s n$$

# A cosine waveform example

$$\begin{aligned} n &= [0:19]; \\ x &= \cos(2\pi \cdot 1 \cdot (n/10)); \end{aligned} \quad \equiv \quad 2\pi F_0 n = 2\pi f_0 T_s n \quad \equiv$$

$$\begin{aligned} n &= [0:19]; \\ x &= \cos(2\pi \cdot (1/10) \cdot n); \end{aligned}$$

$$nT_s = n \cdot \frac{1}{10}$$

$$F_0 = f_0 T_s = \frac{f_0}{f_s} = \frac{T_s}{T_0}$$

$$nT_s = n \cdot 1$$

$$\begin{aligned} 2\pi f_0 n T_s \\ = 2\pi \cdot 1 \cdot n \cdot \frac{1}{10} \end{aligned}$$

$$\begin{aligned} 2\pi f_0 n T_s \\ = 2\pi \cdot \frac{1}{10} \cdot n \cdot 1 \end{aligned}$$

$$\begin{aligned} T_s &= 0.1 \\ f_0 &= 1 \quad (T_0 = 1) \end{aligned}$$

$$\begin{aligned} T_s &= 1 \\ f_0 &= 0.1 \quad (T_0 = 10) \end{aligned}$$

$$F_0 = f_0 T_s = 0.1$$

$$F_0 = f_0 T_s = 0.1$$

# Two cases of the same $F_0 = f_0 T_s$

$$\cos(0.2\pi n)$$

$$\cos(2\pi \cdot f_0 \cdot n \cdot T_s)$$

$$\cos(2\pi \cdot 1 \cdot 0.1 \cdot n)$$

$$T_s = 0.1$$

$$f_0 = 1$$

$$F_0 = 0.1$$

$$\cos(0.2\pi n)$$

$$\cos(2\pi \cdot f_0 \cdot n \cdot T_s)$$

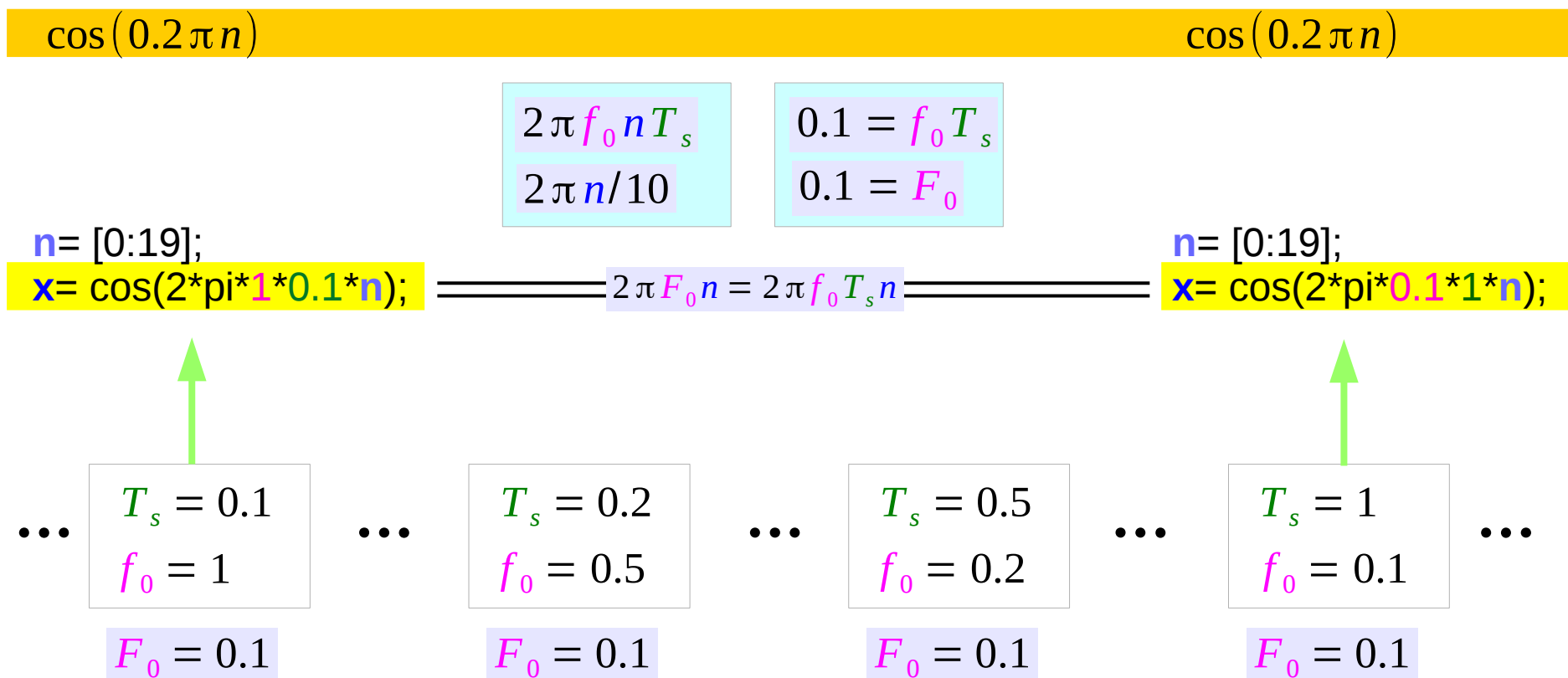
$$\cos(2\pi \cdot 0.1 \cdot 1 \cdot n)$$

$$T_s = 1$$

$$f_0 = 0.1$$

$$F_0 = 0.1$$

# The same sampled waveform examples



# Many waveforms share the same sampled data

The same sampled data

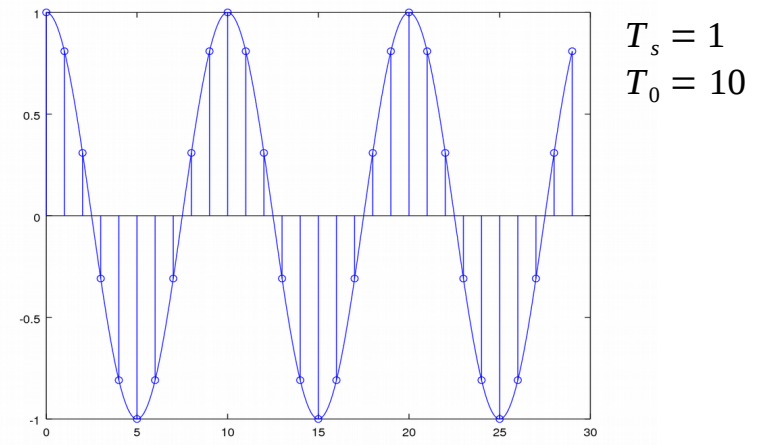
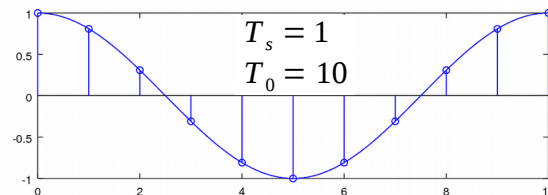
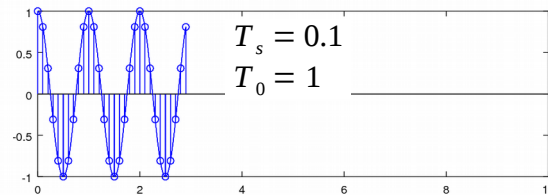
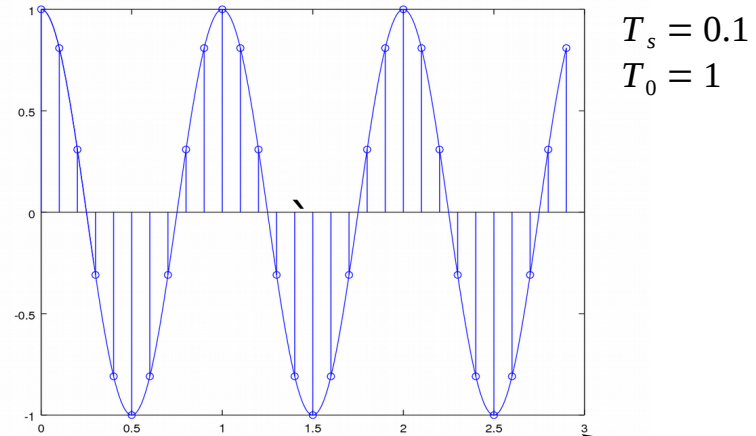
1.00000  
 0.80902  
 0.30902  
 -0.30902  
 -0.80902  
 -1.00000  
 -0.80902  
 -0.30902  
 0.30902  
 0.80902  
 1.00000  
 0.80902  
 0.30902  
 -0.30902  
 -0.80902  
 -1.00000  
 -0.80902  
 -0.30902  
 0.30902  
 0.80902  
 1.00000  
 0.80902  
 0.30902  
 -0.30902  
 -0.80902  
 -1.00000  
 -0.80902  
 -0.30902  
 0.30902  
 0.80902

$$2\pi n/10$$

$$2\pi n f_0 T_s$$

$$0.1 = f_0 T_s$$

$$0.1 = F_0$$



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# Different number of data points

```
x = cos(2*pi*n/10);
```

```
t = [0:19]/10;  
y = cos(2*pi*t);  
stem(t, y)  
hold on  
t2 = [0:199]/100;  
y2 = cos(2*pi*t2);  
plot(t2, y2)
```

`[0:19];`

`[0, ..., 19]` 20 data points

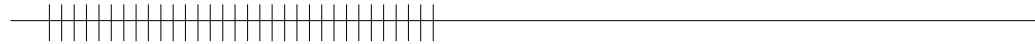
`size([0:19], 2) = 20`



`[0:199];`

`[0, ..., 199]` 200 data points

`size([0:199], 2) = 200`



# Normalized data points

```
x = cos(2*pi*n/10);
```

```
t = [0:19]/10;  
y = cos(2*pi*t);  
stem(t, y)  
hold on  
t2 = [0:199]/100;  
y2 = cos(2*pi*t2);  
plot(t2, y2)
```

$t = [0:19]/10;$      $[0.0, \dots, 1.90]$     20 data points  
coarse resolution

$[0.0, \dots, 1.90] \rightarrow 2 \text{ cycles}$      $[0, \dots, 4\pi]$



$t2 = [0:199]/100;$      $[0.0, \dots, 1.99]$     200 data points  
fine resolution

$[0.0, \dots, 1.99] \rightarrow 2 \text{ cycles}$      $[0, \dots, 4\pi]$



# Different number of data points

`[0:19];`

`[0, ..., 19]` 20 data points

`size([0:19], 2) = 20`



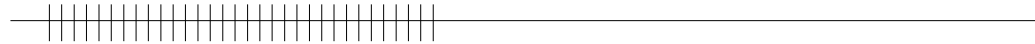
`x = cos(0.2*pi*n);`

```
t = [0:19];  
y = cos(0.2*pi*t);  
stem(t, y)  
hold on  
t2 = [0:199]/10;  
y2 = cos(0.2*pi*t2);  
plot(t2, y2)
```

`[0:199];`

`[0, ..., 199]` 200 data points

`size([0:199], 2) = 200`





# Normalized data points

$t = [0:19];$        $[0.0, \dots, 19.0]$  20 data points  
coarse resolution

$[0.0, \dots, 19.0] \rightarrow 2 \text{ cycles}$        $[0, \dots, 4\pi]$



$x = \cos(0.2 * \pi * n);$

```
t = [0:19];  
y = cos(0.2*pi*t);  
stem(t, y)  
hold on  
t2 = [0:190]/10;  
y2 = cos(0.2*pi*t2);  
plot(t2, y2)
```

$t2 = [0:199]/10;$        $[0.0, \dots, 19.9]$  200 data points  
fine resolution

$[0.0, \dots, 19.9] \rightarrow 2 \text{ cycles}$        $[0, \dots, 4\pi]$



# Plotting sampled cosine waves

```
x = cos(2*pi*n/10);
```

```
t = [0:19]/10;  
y = cos(2*pi*t);  
stem(t, y)  
hold on  
t2 = [0:199]/100;  
y2 = cos(2*pi*t2);  
plot(t2, y2)
```

```
t = [0:19]/10;      [0.0, ..., 1.9] 20 data points
```

```
y = cos(2*pi*t);  stem(t, y)      coarse resolution
```

```
t2 = [0:199]/100; [0.0, ..., 1.99] 200 data points
```

```
y = cos(2*pi*t2); plot(t2, y)    fine resolution
```

```
x = cos(0.2*pi*n);
```

```
t = [0:19];  
y = cos(0.2*pi*t);  
stem(t, y)  
hold on  
t2 = [0:199]/100;  
y2 = cos(0.2*pi*t2);  
plot(t2, y2)
```

```
t = [0:19];      [0.0, ..., 1.9] 20 data points
```

```
y = cos(0.2*pi*t); stem(t, y)      coarse resolution
```

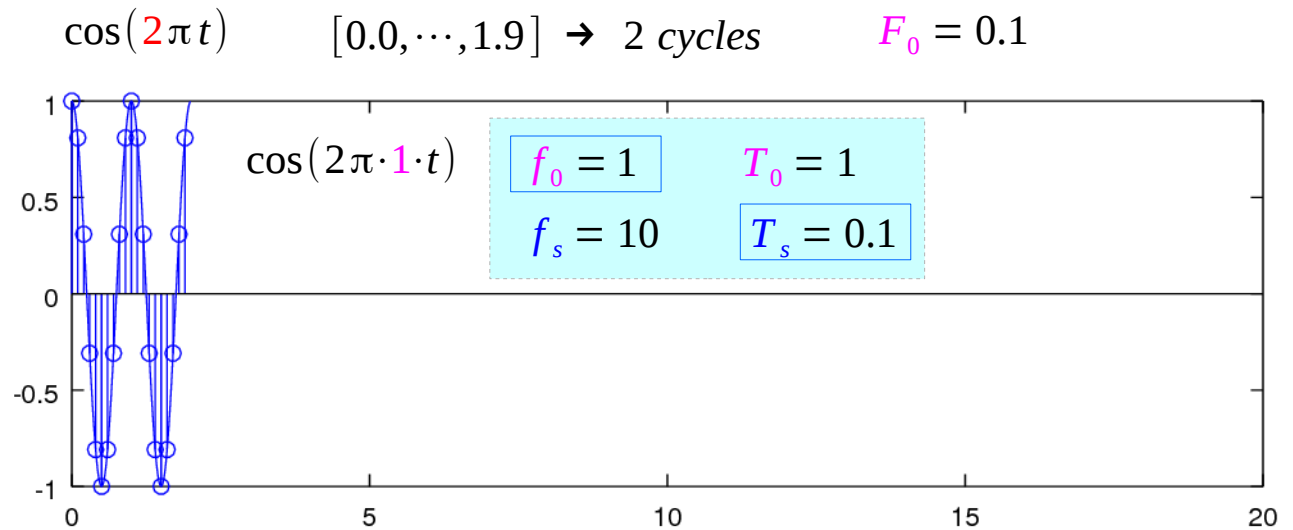
```
t2 = [0:199]/100; [0.0, ..., 1.99] 200 data points
```

```
y = cos(0.2*pi*t2); plot(t2, y)    fine resolution
```

# Two waveforms with the same normalized frequency

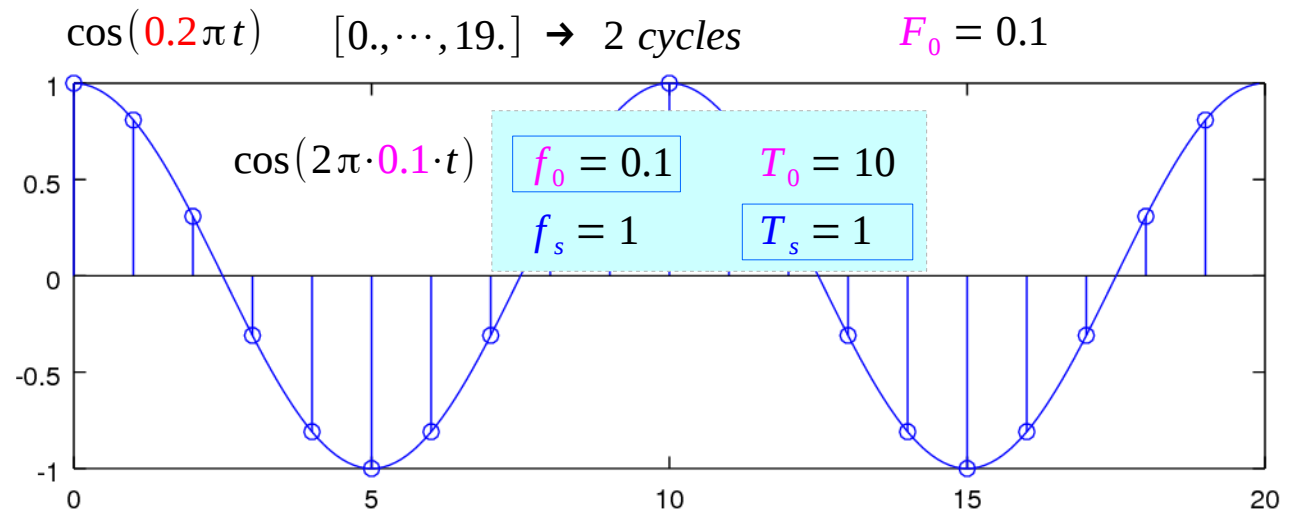
$$x = \cos(2\pi n/10);$$

```
t = [0:19]/10;
y = cos(2*pi*t);
stem(t, y)
hold on
t2 = [0:199]/100;
y2 = cos(2*pi*t2);
plot(t2, y2)
```



$$x = \cos(0.2\pi n);$$

```
t = [0:19];
y = cos(0.2*pi*t);
stem(t, y)
hold on
t2 = [0:190]/10;
y2 = cos(0.2*pi*t2);
plot(t2, y2)
```



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# Cosine Wave 1

$$x = \cos(2\pi n/10);$$

```
t = [0:29]/10;  
y = cos(2*pi*t);  
stem(t, y)  
hold on  
t2 = [0:299]/100;  
y2 = cos(2*pi*t2);  
plot(t2, y2)
```

$$f_0 = 1$$

$$T_s = 0.1$$

$$F_0 = f_0 T_s = 0.1$$

$$\cos(2\pi t)$$

$$T_0 = 1$$

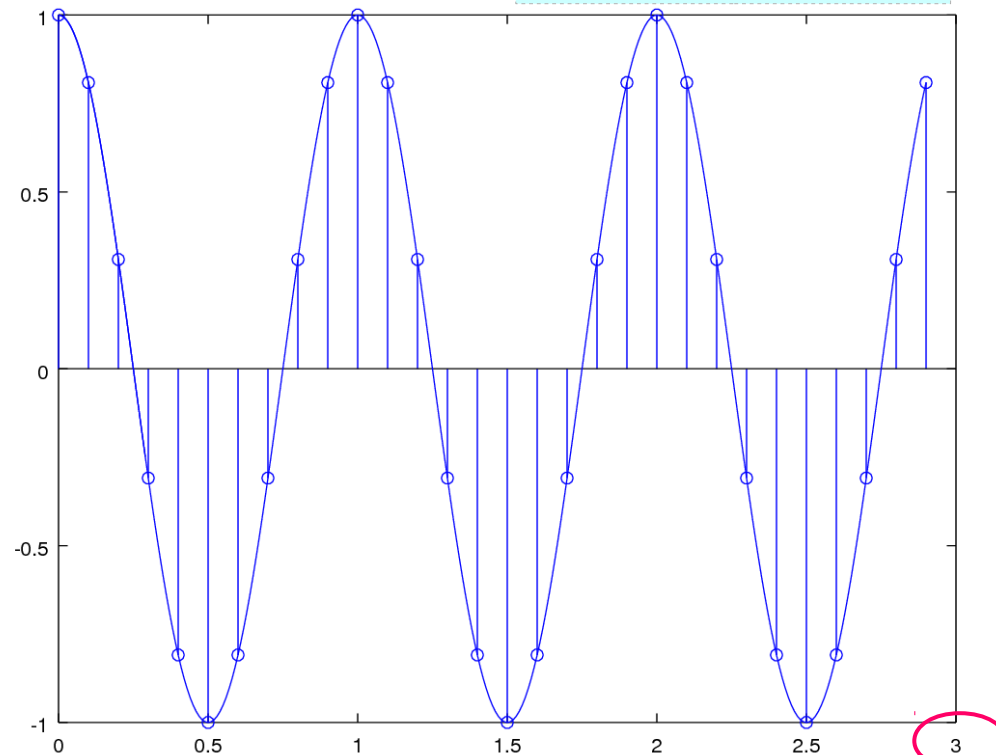
$$\cos(2\pi \cdot 1 \cdot t)$$

$$f_0 = 1$$

$$T_0 = 1$$

$$f_s = 10$$

$$T_s = 0.1$$



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# Cosine Wave 2

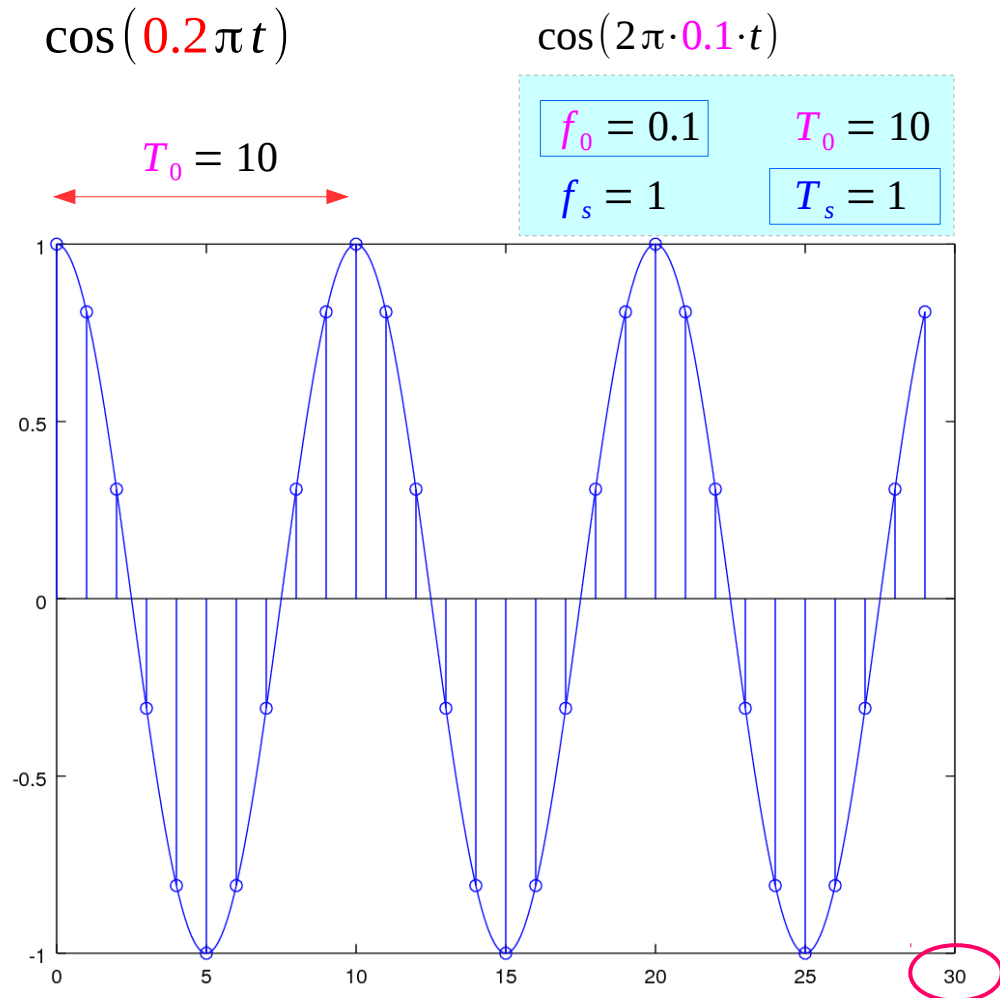
```
x = cos(0.2*pi*n);
```

```
t = [0:29];  
y = cos(0.2*pi*t);  
stem(t, y)  
hold on  
t2 = [0:299]/10;  
y2 = cos(0.2*pi*t2);  
plot(t2, y2)
```

$$f_0 = 0.1$$

$$T_s = 1$$

$$F_0 = f_0 T_s = 0.1$$



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# Sampled Sinusoids

$$g[n] = A \cos(2\pi F_0 n + \theta)$$

$$g[n] = A \cos(2\pi n m / N_0 + \theta)$$

$$g[n] = A \cos(\Omega_0 n + \theta)$$

$$F_0$$
$$m/N_0$$
$$\Omega_0/2\pi$$

$$2\pi F_0$$
$$2\pi m/N_0$$
$$\Omega_0$$

$$N_0 = \frac{m}{F_0}$$

$$N_0 \neq \frac{1}{F_0}$$

$$g[n] = A e^{\beta n}$$

$$g[n] = A z^n \quad z = e^\beta$$

M.J. Roberts, Fundamentals of Signals and Systems

# Sampling Period $T_s$ and Frequency $f_s$

$$g(t) = A \cos(2\pi f_0 t + \theta)$$

$$F_0 \leftarrow f_0 \cdot T_s$$

$$f_0 \leftarrow F_0 \cdot f_s$$

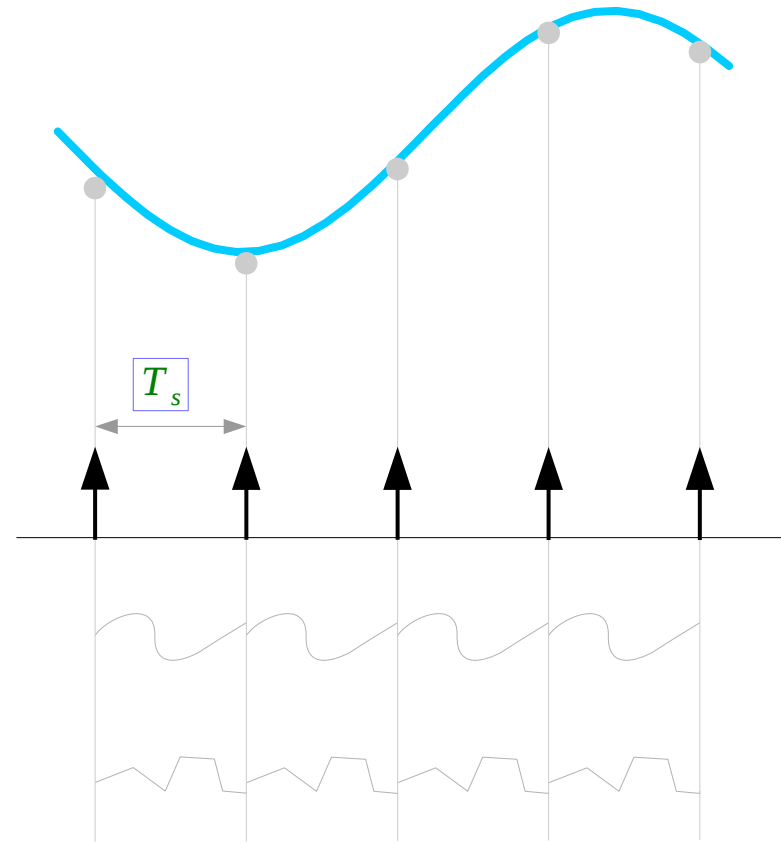
$$g[n] = A \cos(2\pi F_0 n + \theta)$$

$$T_s = \frac{1}{f_s}$$

sampling period

$$\frac{1}{T_s} = f_s$$

sampling frequency  
sampling rate



M.J. Roberts, Fundamentals of Signals and Systems

# $f_0$ and $F_0$

$$g(t) = A \cos(2\pi f_0 t + \theta)$$

$$\begin{aligned} g(t) &= 4 \cos\left(\frac{72\pi t}{19}\right) \\ &= 4 \cos\left(2\pi \cdot \frac{36}{19} \cdot t\right) \end{aligned}$$

$$f_0 = \frac{36}{19}$$



$$g[n] = A \cos(2\pi F_0 n + \theta)$$

$$\begin{aligned} g[n] &= 4 \cos\left(\frac{72\pi n}{19}\right) \\ &= 4 \cos\left(2\pi \cdot \frac{36}{19} \cdot n\right) \end{aligned}$$

there are many  $F_0$

$$F_0 = f_0 T_s = \frac{f_0}{f_s}$$

$$T_s = 1 \rightarrow F_0 = f_0$$

$$F_0 = \frac{36}{19}$$



$$g(t) = A \cos(2\pi f_0 t + \theta)$$

$$\begin{aligned} g(t) &= 4 \cos\left(\frac{72\pi t}{19}\right) \\ &= 4 \cos\left(2\pi \cdot \frac{36}{19} \cdot t\right) \end{aligned}$$

$$g(t) = 4 \cos\left(2\pi \cdot \frac{36}{19} \cdot (t + T_0)\right)$$

$$T_0 = \frac{19}{36}$$

Fundamental Period of  $g(t)$

$$g[n] = A \cos(2\pi F_0 n + \theta)$$

$$\begin{aligned} g[n] &= 4 \cos\left(\frac{72\pi n}{19}\right) \\ &= 4 \cos\left(2\pi \cdot \frac{36}{19} \cdot n\right) \end{aligned}$$

$$g[n] = 4 \cos\left(2\pi \cdot \frac{36}{19} \cdot (n + N_0)\right)$$

there is only one  
 $N_0$  for a given  $F_0$

$$N_0 = 19$$

Fundamental Period of  $g[n]$

# Periodic Condition Examples

$$g[n] = 4 \cos\left(2\pi \cdot \frac{36}{19} \cdot (n + N_0)\right)$$

$$\frac{36}{19} \cdot (n + N_0)$$

integer

$$\frac{1}{19} \cdot N_0$$

integer

$$N_0 = 19$$

integer

$N_0 = 19$  Fundamental period of  $g[n]$

$$g(t) = 4 \cos\left(2\pi \cdot \frac{36}{19} \cdot (t + T_0)\right)$$

$$\frac{36}{19} \cdot (t + T_0)$$

integer

$$\frac{36}{19} \cdot T_0$$

integer

$$T_0 = \frac{19}{36}$$

~~integer~~

$N_0 = \frac{19}{36}$  Fundamental period of  $g(t)$

# Periodic Condition Examples

$$F_0 = \frac{q}{N_0}$$

← the number of cycles in  $N_0$  samples  
← the smallest integer : fundamental period

$$F_0 N_0 = q$$

$$2\pi F_0 N_0 = 2\pi q$$

$q$  cycles in  $N_0$  samples

# Periodic Condition Examples

$$g(t) = 4 \cos\left(2\pi \cdot \frac{36}{19} \cdot (t + T_0)\right) \quad T_0 = \frac{19}{36} \quad \text{Fundamental Period of } g(t)$$

$$g[n] = 4 \cos\left(2\pi \cdot \frac{36}{19} \cdot (n + N_0)\right) \quad N_0 = 19 \quad \text{Fundamental Period of } g[n]$$

$$F_0 = \frac{36}{19} = \frac{q}{N_0}$$

← the number of cycles in  $N_0$  samples  
← the smallest integer : fundamental period

$$N_0 \neq \frac{1}{F_0} \quad N_0 = \frac{q}{F_0} \quad \frac{q}{N_0} = F_0$$

M.J. Roberts, Fundamentals of Signals and Systems

# Periodic Condition Examples

$$F_0 = \frac{36}{19} = \frac{q}{N_0}$$

← the number of cycles in  $N_0$  samples  
← the smallest integer : fundamental period

*“When  $F_0$  is not the reciprocal of an integer ( $q=1$ ), a discrete-time sinusoid may not be immediately recognizable from its graph as a sinusoid.”*

$$F'_0 = \frac{1}{19} = \frac{1}{N_0}$$

# Periodic Condition Examples

$$g[n] = 4 \cos\left(2\pi \cdot \frac{1}{19} \cdot n\right)$$

**1** cycles in  $N_0$  samples

$$g[n] = 4 \cos\left(2\pi \cdot \frac{2}{19} \cdot n\right)$$

**2** cycles in  $N_0$  samples

$$g[n] = 4 \cos\left(2\pi \cdot \frac{3}{19} \cdot n\right)$$

**3** cycles in  $N_0$  samples

$$g[n] = 4 \cos\left(2\pi \cdot \frac{36}{19} \cdot n\right)$$

**36** cycles in  $N_0$  samples

```
clf
n = [0:36]; t = [0:3600]/100;
y1 = 4*cos(2*pi*(1/19)*n);
y2 = 4*cos(2*pi*(2/19)*n);
y3 = 4*cos(2*pi*(3/19)*n);
y4 = 4*cos(2*pi*(36/19)*n);
yt1 = 4*cos(2*pi*(1/19)*t);
yt2 = 4*cos(2*pi*(2/19)*t);
yt3 = 4*cos(2*pi*(3/19)*t);
yt4 = 4*cos(2*pi*(36/19)*t);
```

```
subplot(4,1,1);
stem(n, y1); hold on;
plot(t, yt1);
subplot(4,1,2);
stem(n, y2); hold on;
plot(t, yt2);
subplot(4,1,3);
stem(n, y3); hold on;
plot(t, yt3);
subplot(4,1,4);
stem(n, y4); hold on;
plot(t, yt4);
```

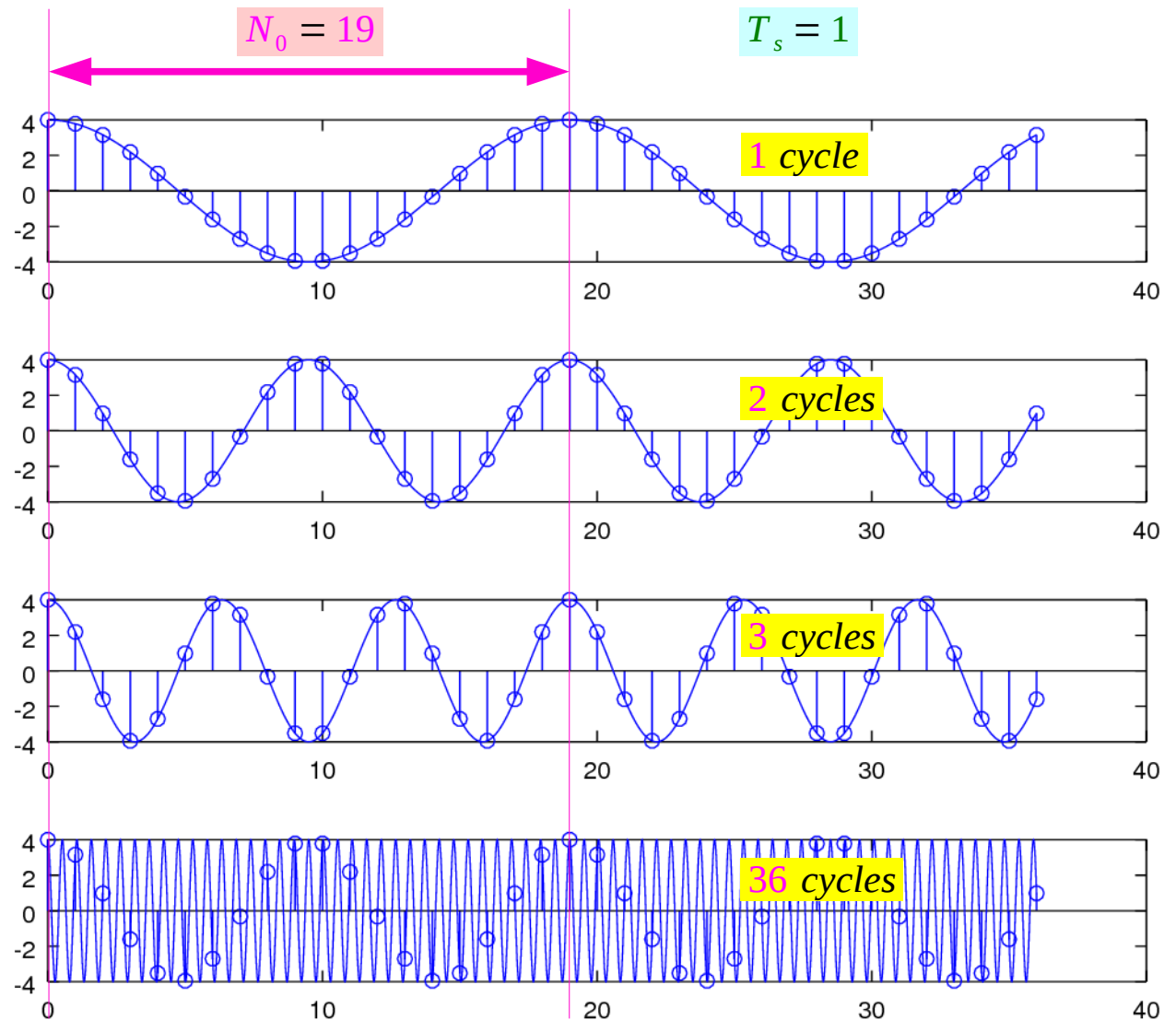
# Periodic Condition Examples

$$g[n] = 4 \cos\left(2\pi \cdot \frac{1}{19} \cdot n\right)$$

$$g[n] = 4 \cos\left(2\pi \cdot \frac{2}{19} \cdot n\right)$$

$$g[n] = 4 \cos\left(2\pi \cdot \frac{3}{19} \cdot n\right)$$

$$g[n] = 4 \cos\left(2\pi \cdot \frac{36}{19} \cdot n\right)$$



# Periodic Condition Examples

$$g(t) = A \cos(2\pi f_0 t + \theta)$$

$$g_1(t) = 4 \cos(2\pi \cdot 1 \cdot t)$$

$$g_2(t) = 4 \cos(2\pi \cdot 2 \cdot t)$$

$$g_3(t) = 4 \cos(2\pi \cdot 3 \cdot t)$$

$$t \leftarrow nT_1$$

$$t \leftarrow nT_2$$

$$t \leftarrow nT_3$$

$$g[n] = A \cos(2\pi F_0 n + \theta)$$

$$g_1[n] = 4 \cos(2\pi n T_{s1})$$

$$g_2[n] = 4 \cos(2\pi n T_{s2})$$

$$g_3[n] = 4 \cos(2\pi n T_{s3})$$

$$t \leftarrow nT_1$$

$$T_1 = \frac{1}{10}$$

$$n = 0, 1, 2, 3, \dots \rightarrow 1 \cdot t = 0, 0.1, 0.2, 0.3, \dots$$

$$t \leftarrow nT_2$$

$$T_2 = \frac{1}{20}$$

$$n = 0, 1, 2, 3, \dots \rightarrow 2 \cdot t = 0, 0.1, 0.2, 0.3, \dots$$

$$t \leftarrow nT_3$$

$$T_3 = \frac{1}{30}$$

$$n = 0, 1, 2, 3, \dots \rightarrow 3 \cdot t = 0, 0.1, 0.2, 0.3, \dots$$

$$\{g_1[n]\} \equiv \{g_2[n]\} \equiv \{g_3[n]\}$$

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# Periodic Condition Examples

$$g(t) = A \cos(2\pi f_0 t + \theta)$$

$$g[n] = A \cos(2\pi F_0 n + \theta)$$

$$\begin{aligned} g[n] &= 4 \cos\left(\frac{72\pi n}{19}\right) \\ &= 4 \cos\left(2\pi \left(\frac{36}{19}\right) n\right) \\ &= 4 \cos\left(2\pi \left(\frac{36}{19} \cdot (n + N_0)\right)\right) \\ &\quad \text{smallest } N_0 = 19 \end{aligned}$$

$$2\pi F_0 n = 2\pi m$$

$$\frac{36}{19} n = m$$

smallest  $n = 19$

$$\frac{36}{19} = \frac{m}{n}$$

$$\frac{36}{19} = \frac{m}{n} = \frac{f_0}{f_s}$$

$$F_0 = \frac{q}{N_0}$$

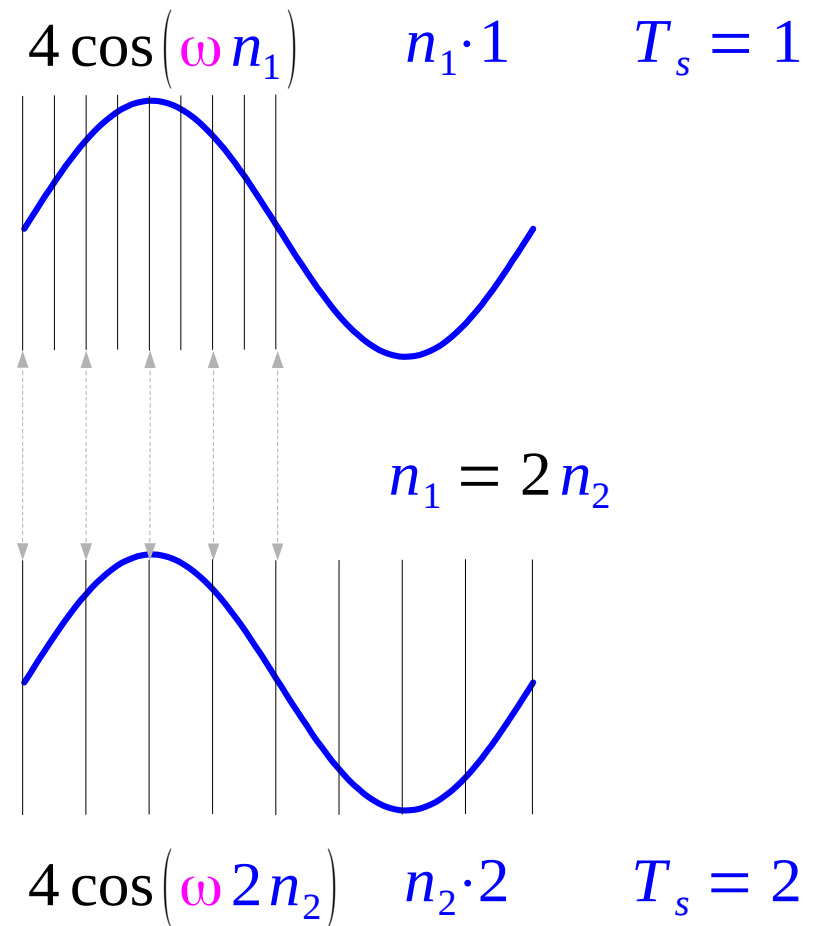
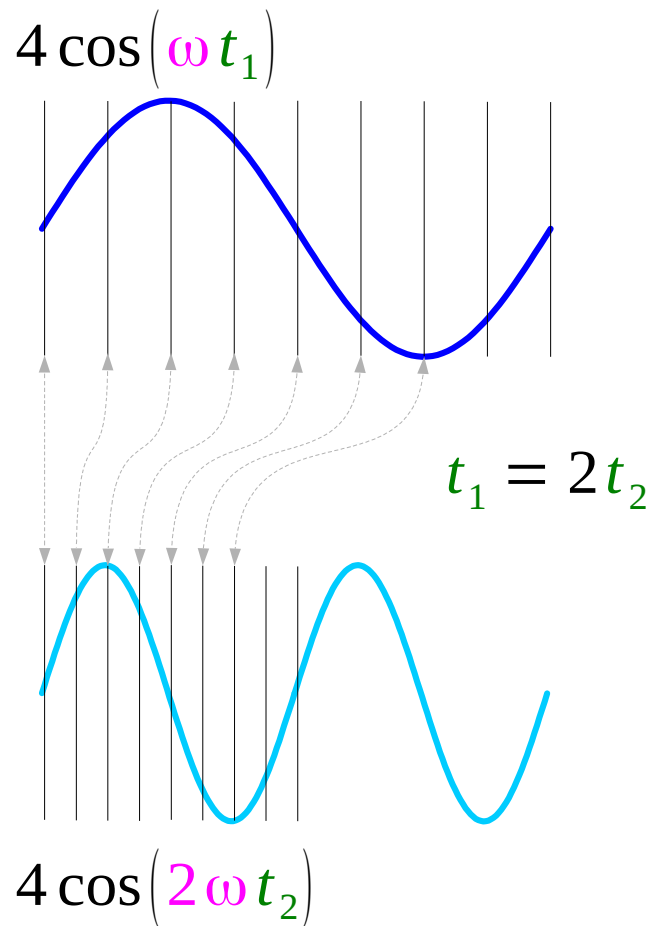
$$1/N_0$$

$$= F_0$$

$$= \Omega_0/2\pi$$

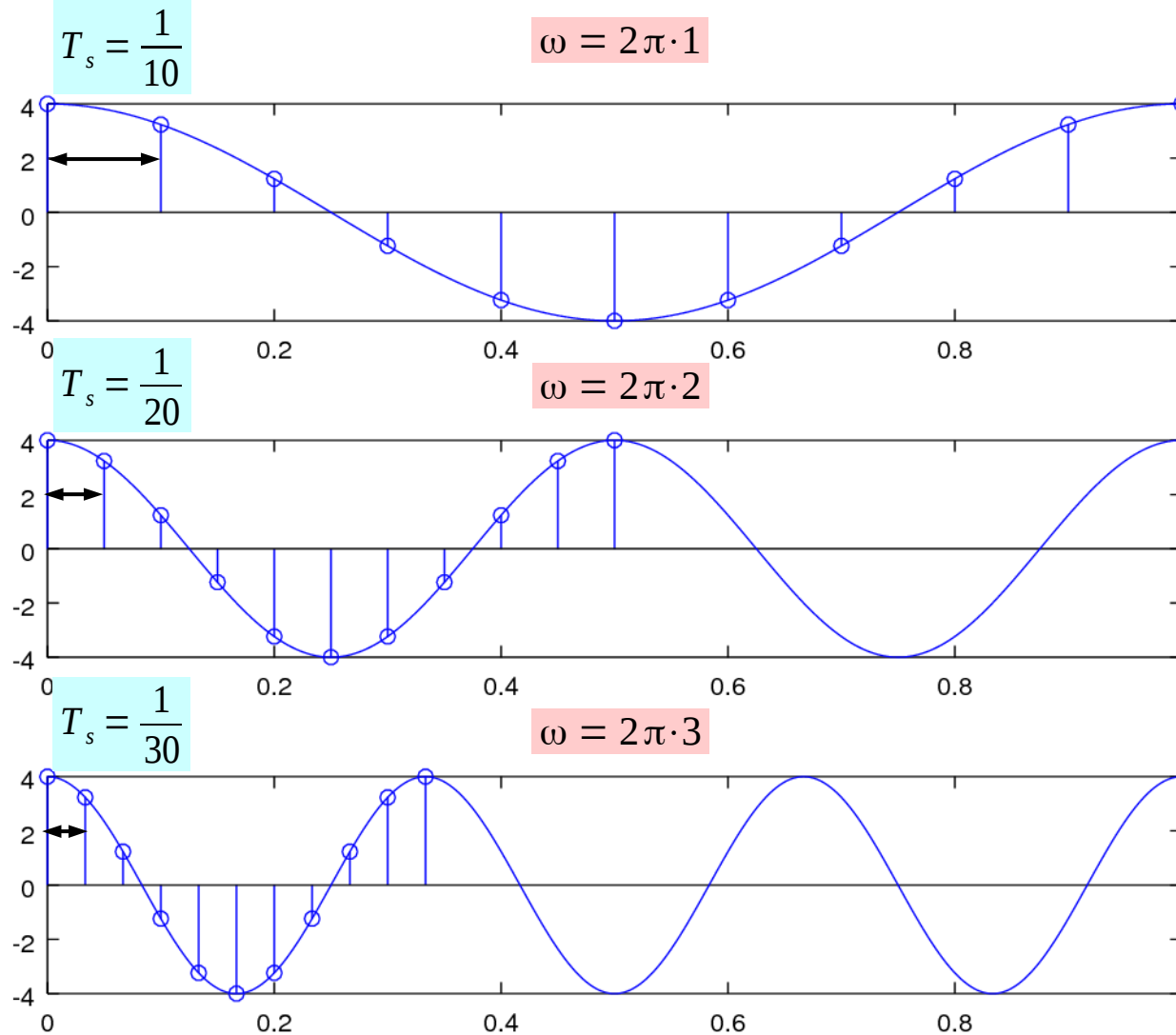
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# Periodic Condition Examples



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# Periodic Condition Examples



```
clf
n = [0:10]; t = [0:1000]/1000;
y1 = 4*cos(2*pi*1*n/10);
y2 = 4*cos(2*pi*2*n/20);
y3 = 4*cos(2*pi*3*n/30);
yt1 = 4*cos(2*pi*t);
yt2 = 4*cos(2*pi*2*t);
yt3 = 4*cos(2*pi*3*t);
```

```
subplot(3,1,1);
stem(n, y1); hold on;
plot(t, yt1);
subplot(3,1,2);
stem(n/20, y2); hold on;
plot(t, yt2);
subplot(3,1,3);
stem(n/30, y3); hold on;
plot(t, yt3);
```

M.J. Roberts, Fundamentals of Signals and Systems

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## References

- [1] <http://en.wikipedia.org/>
- [2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
- [3] M.J. Roberts, Fundamentals of Signals and Systems
- [4] S.J. Orfanidis, Introduction to Signal Processing
- [5] K. Shin, et al., Fundamentals of Signal Processing for Sound and Vibration Engineerings
  
- [6] A “graphical interpretation” of the DFT and FFT, by Steve Mann