## Integration by Substitutions (4A)

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## Chain Rule

$$
\begin{aligned}
f(g(x)) & \Rightarrow \frac{d}{d x} \Rightarrow f^{\prime}(g(x)) \cdot g^{\prime}(x) \\
\frac{d f}{d x} & =\frac{d f}{d g} \cdot \frac{d g}{d x} \\
f(\square) & \Rightarrow \frac{d}{d x} \Rightarrow f^{\prime}(\square) \cdot \square
\end{aligned}
$$

## Substitution Rule

$$
\begin{aligned}
& f(g(x))+C \backsim \int \cdot d x \backsim f^{\prime}(g(x)) \cdot g^{\prime}(x) \\
& f(g(x))+C=\int f^{\prime}(g(x)) \cdot g^{\prime}(x) d x \\
& f(u)+C \\
& \\
& \begin{array}{rlr} 
& =\int f^{\prime}(u) \cdot d u & \\
\int & \\
\int f^{\prime}(g(x)) \cdot g^{\prime}(x) d x & =\int f^{\prime}(g(x)) \cdot \frac{d g}{d x} d x & \\
& =\int f^{\prime}(u) d u & \\
& =f(u)+C & d u=g(x) \\
d x
\end{array} d x
\end{aligned}
$$

## Chain Rule and Substitution Rule

$$
\begin{aligned}
f(g(x)) & \Rightarrow \frac{d}{d x} \Rightarrow f^{\prime}(g(x)) \cdot g^{\prime}(x) \\
\frac{d f}{d x} & =\frac{d f}{d g} \cdot \frac{d g}{d x}
\end{aligned}
$$

$$
\begin{aligned}
& f(g(x))+C \backsim \int \cdot d x-f^{\prime}(g(x)) \cdot g^{\prime}(x) \\
& f(g(x))+C=\int f^{\prime}(g(x)) \cdot g^{\prime}(x) d x
\end{aligned}
$$

## Substitution Rule

$$
\begin{array}{ll}
f(g(x))+C \curvearrowleft \int \cdot d x \backsim f^{\prime}(g(x)) \cdot g^{\prime}(x) \\
f(g(x))+C=\int f^{\prime}(g(x)) \cdot g^{\prime}(x) d x & f(x)+C=\int f^{\prime}(x) d x \\
F(g(x))+C \curvearrowleft \int \cdot d x \backsim f(g(x)) \cdot g^{\prime}(x) & F(x)+C=\int f(x) d x
\end{array}
$$

## References

[1] http://en.wikipedia.org/
[2] M.L. Boas, "Mathematical Methods in the Physical Sciences"
[3] E. Kreyszig, "Advanced Engineering Mathematics"
[4] D. G. Zill, W. S. Wright, "Advanced Engineering Mathematics"

