FPGA Carry Chain Adder (1A)

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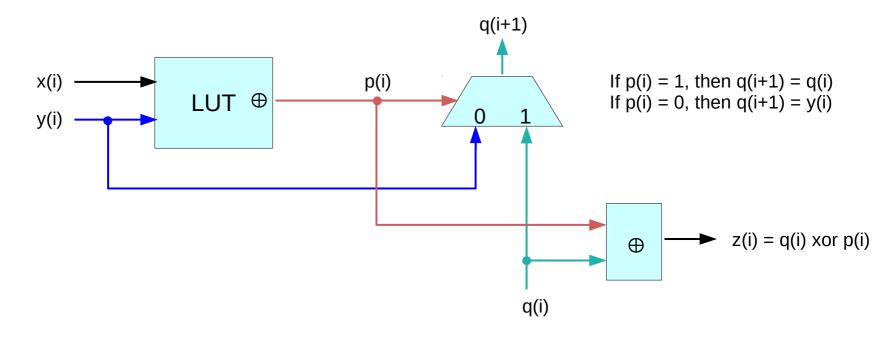
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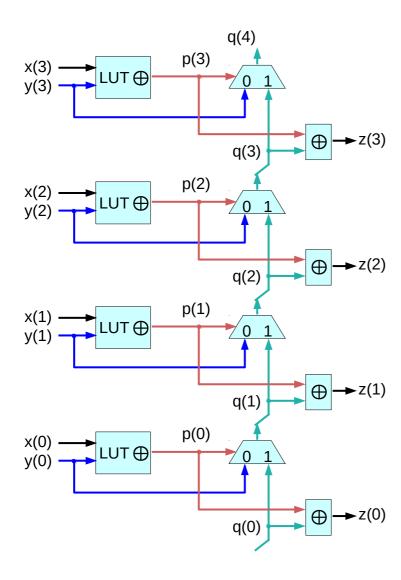
$$\begin{aligned} s_i &= (a_i \oplus b_i) \oplus c_i = p_i \oplus c_i \\ c_{i+1} &= (a_i \cdot b_i) + (a_i \oplus b_i) c_i = \overline{p_i} \cdot g_i + p_i \cdot c_i = \overline{p_i} \cdot a_i + p_i \cdot c_i = \overline{p_i} \cdot b_i + p_i \cdot c_i \end{aligned}$$

when
$$\overline{p}_i = 1$$
, then $a_i = b_i$
when $g_i = 1$, then $a_i = b_i = 1$

p(i)	0	1
0	0	1
1	1	0

g(i)	0	1
0	0	0
1	0	1

Synthesis of Arithmetic Circuits: FPGA, ASIC and Ebedded Systems, J-P Deschamps et al



Synthesis of Arithmetic Circuits: FPGA, ASIC and Ebedded Systems, J-P Deschamps et al

FPGA Carry Chain

FPGAs generally contain dedicated computation resources for generating fast adders

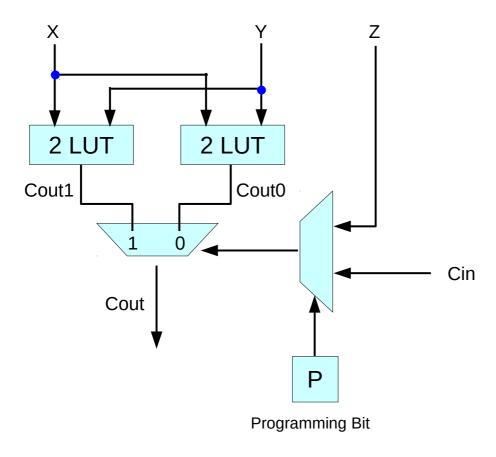
The Virtex family programmable arrays include logic gates (XOR) and multiplexers that along with the general purpose lookup tables allow one to build effective carry-chain adders

The carry chain is made up of multiplexers belonging to adjacent configurable blocks

the lookup table is used for implementing the exclusive or function

$$p(i) = x(i) xor y(i)$$

https://en.wikipedia.org/wiki/Carry-lookahead_adder



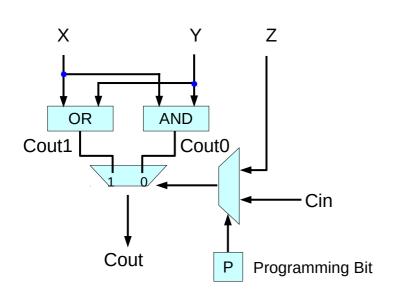
Cout1, Cout2: functions of X, Y, Cin

Cout1 = X+Y when Cin=1 Cout0 = X Y when Cin=0

Cout = $(X + Y) Cin + X Y \overline{Cin}$

Cout = P' Cin + G $\overline{\text{Cin}}$... P' = relaxed P

Cout1	Cout0	Cout	Name
0	0	0	Kill
0	1	Cin	Inverse Propagate
1	0	Cin	Propagate
1	1	1	Generate



		Cin	Cin	
Χ	Υ	Cout1	Cout0	
0	0	0	0	$\overline{X} \overline{Y}$
0	1	1	0	$\overline{X}Y$
1	0	1	0	$X\overline{Y}$
1	1	1	1	ΧY

Cout: functions of X, Y, Cin

Cout(X, Y, 1) = Cout1 = X + YCout(X, Y, 0) = Cout0 = X Y

Cout1 = X + Y when Cin=1 Cout0 = XY when Cin=0

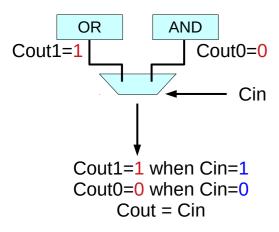
Cout1 = P' $\underline{\text{Cin}}$... P' = relaxed P Cout0 = $\underline{\text{Cin}}$

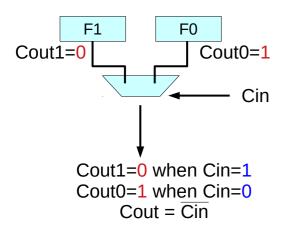
High Performance Carry Chains for FPGAs, S. Hauck, M. M. Hosler, T. W. Fry

If \underline{Cin} , then $\underline{Cout} = (\overline{X} \ Y + X \ \overline{Y} + X \ Y)$ If \underline{Cin} , then $\underline{Cout} = X \ Y$

Cin $(X + Y) + \overline{Cin} X Y$ Cin $(X Y + X \overline{Y} + X Y) + \overline{Cin} X Y$ Cin $(X Y + X \overline{Y}) + (Cin + \overline{Cin}) X Y$ P Cin + G

Cin $(X + \underline{Y}) + \overline{Cin} X Y$ Cin P' + $\overline{Cin} G$... P' : relaxed P

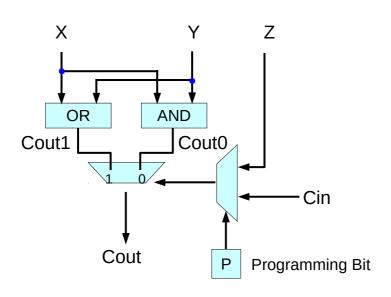




Cout0	Cout1	Cout	Name
0	0	0	Kill
0	1	Cin	Propagate
1	0	Cin	Inverse Propagate
1	1	1	Generate

Cout1	Cout0	Cout	Name
0	0	0	Kill
0	1	Cin	Inverse Propagate
1	0	Cin	Propagate
1	1	1	Generate

Carry Chain



Car	ry Ou	t ,			
Χ	Υ	Cin			
0	0	Cin			
0	1	Cin	Cin		
1	0	Cin	Cin		
1	1	Cin	Cin		

		Cin	Cin	
Χ	Υ	Cout1	Cout0	
0	0	0	0	$\overline{X} \overline{Y}$
0	1	1	0	$\overline{X}Y$
1	0	1	0	$X\overline{Y}$
1	1	1	1	ΧY

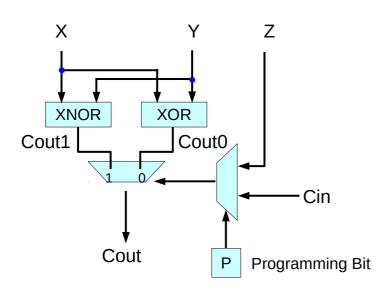
Cout1	Cout0	Cout	Name
0	0	0	Kill
0	1	Cin	Inverse Propagate
1	0	Cin	Propagate
1	1	1	Generate

```
Cout1=1 when Cin=1
Cout0=0 when Cin=0
Cout = Cin propage
Cout1=0 when Cin=1
```

Cout1=0 when Cin=1 Cout0=1 when Cin=0

Cout = $\overline{\text{Cin}}$ inverse propagate

Parity Checker



		Cin	Cin	
X	Υ	Cout1	Cout0	
0	0	1	0	$\overline{X} \overline{Y}$
0	1	0	1	XY
1	0	0	1	$X\overline{Y}$
1	1	1	0	ΧY

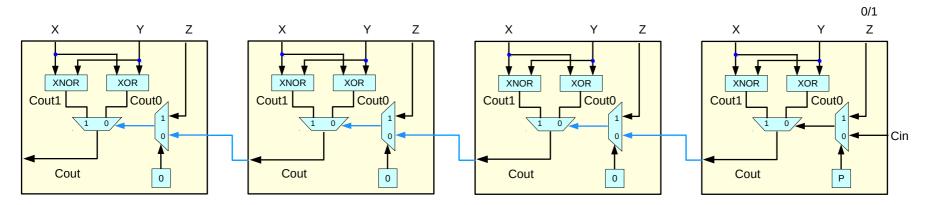
Cout1	Cout0	Cout	Name
0	0	0	Kill
0	1	Cin	Inverse Propagate
1	0	Cin	Propagate
1	1	1	Generate

Computing Par	ity		
X ⊕ Y ⊕ Cin			
0 ⊕ 0 ⊕ Cin	Cin		
0 ⊕ 1 ⊕ Cin	Cin		
1 ⊕ 0 ⊕ Cin	Cin		
1 ⊕ 1 ⊕ Cin	Cin		

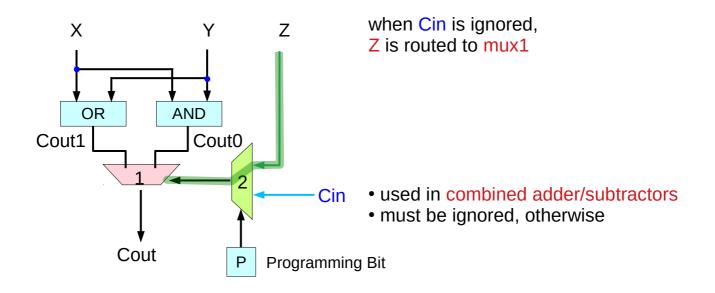
Cout1=1 when Cin=1
Cout0=0 when Cin=0
Cout = Cin propagate

Cout1=0 when Cin=1
Cout0=1 when Cin=0
Cout = Cin inverse propagate

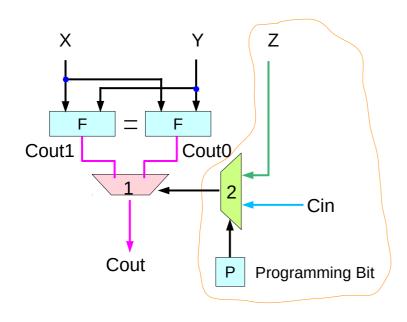
Ripple Carry Chain



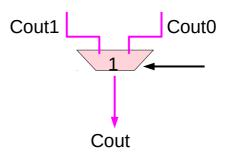
the **first cell** in the chain



the logic cells - resources to compute a function the exact location of logic cells depends on the user. a user can start or end a carry computation at any place in an fpga. But in many carry computations, the first cell has only 2 inputs, and forcing the carry chain to wait for the arrival of an additional, unnecessary input Z will only needlessly slow down the circuit's computation.



when Cin is ignored,
Z can also be ignored
by having the same LUTs



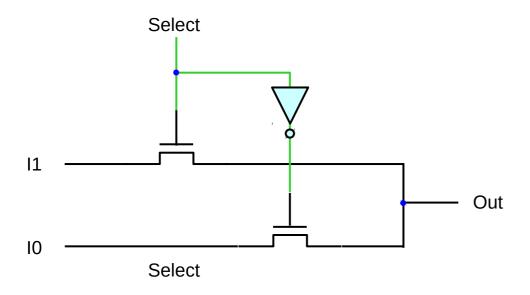
the first cell in the chain

the same LUTs

the <u>same</u> output regardless of Z and Cin

Cout1 = Cout0 = Cout regardless of the select

Ripple Carry Chain



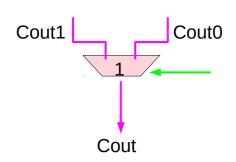


fig1b shows an implementation of a mux that does not obey this requirement

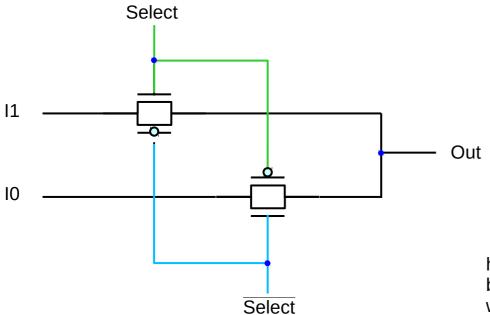
since the carry chain is part of an fpga, the input to this mux could be connected to some unused logic in another row which is generating unknown values.

if that unused logic had multiple transitions which caused the signal to change quicker than the gate could react, then it is possible that the select signal to this mux could be stuck midway between true and false (2.5V for 5V CMOS)

in this case, it will <u>not</u> be able to <u>pass a true value</u> from the input to the output and thus will not function properly for this application.

High Performance Carry Chains for FPGAs, S. Hauck, M. M. Hosler, T. W. Fry

Ripple Carry Chain



however a mux built with both n-transistor and p-transistor pass gates will operate properly for this case

assume this mux implementation will be used

tristate driver based muxes could be used, which restore signal drive and cut series RC chains

Unit Gate Delay Model

All simple gate of two or three inputs that are directly implementable in one logic level in CMOS are considered to have a delay of one.

All other gate must be implemented by such gates, and have the delay of the underlying circuit.

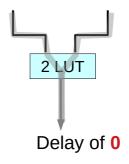
Delay of one

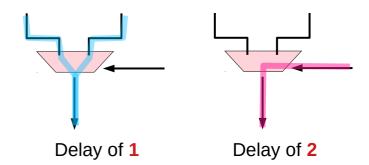
- inverters and
- 2 to 3 input NAND
- 2 to 3 input NOR gates

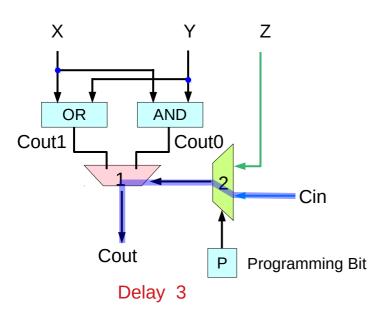
A 2:1 mux has a delay of one from the IO or I1 inputs to the output, But has a delay of two from the select input to the output due to the Inverter delay

Delay of zero (constant delay)

- the delay of the 2-LUTs,
- any routing leading to them,







Significantly slower two muxes on the carry chain in each cell

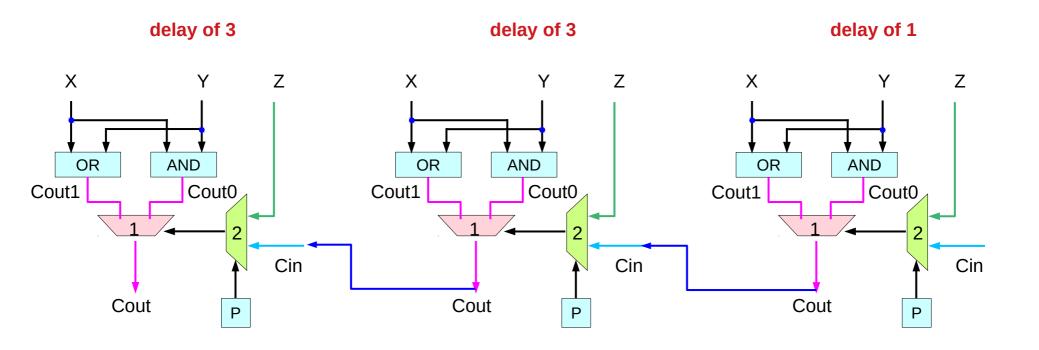
Delay 1 for first cell
Delay 3 for each additional cell in the carry chain
delay 1 for mux2
delays 2 for mux1

Overall 3n-2 for an n-cell carry chain

larger delay

X
Y
Z
Cout1
Cout0
Programming Bit
Delay 1

The critical path comes from the 2-LUTs and not from the input Z since the delay through the 2-LUTs will be larger than through mux 2 in the first cell



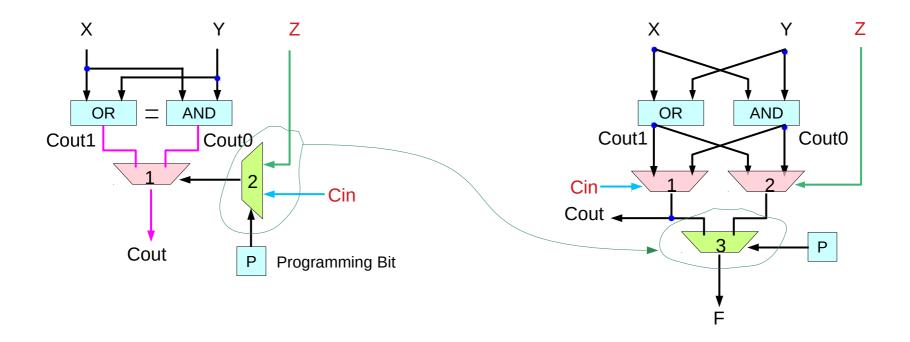
delay of 3n-2 for an n-bit ripple carry chain

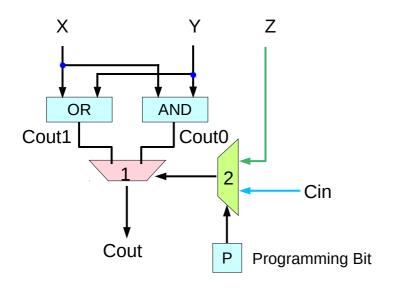
to optimize a ripple carry chain structure for use in FPGAs while this provides some performance gain over the basis ripple carry scheme found in many current FPGAs, it is still much slower than what is done in custom logic There have been tremendous amounts of work done on developing alternative carry chain scheme that overcome the linear delay growth of ripple carry adders Although these techniques have not yet been applied to FPGAs, demonstrate how these advanced adder techniques can be integrated into reconfigurable logic

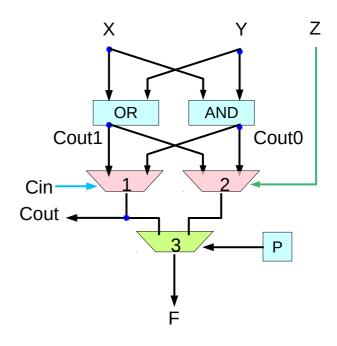
to reduce the delay of the ripple carry chain

- remove mux2 from the carry path.
- no need to choose between Cin and Z for the select line to the output mux1

- two separate muxes, mux1 and mux2, controlled by Cin and Z, respectively.
- the circuit chooses between these outputs with mux3.

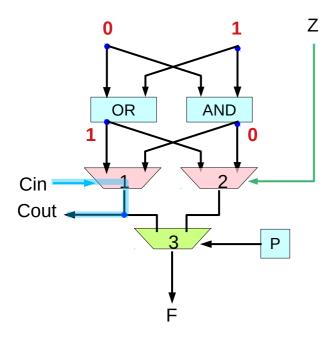






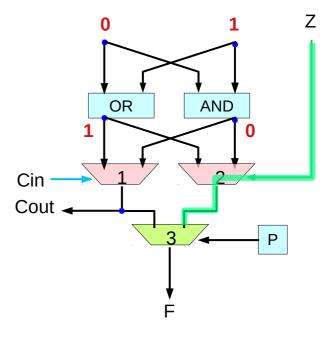
- not logically equivalent
- no longer use the Z input in the <u>first</u> cell since Z is only attached to <u>mux2</u> and <u>mux2</u> does not lead to the carry cells
- no longer use the Cin input in the <u>first</u> cell since Cin is only attached to <u>mux1</u> and <u>mux1</u> does not lead to the carry cells

delay of 2



an additional cell for generating Cin

delay of 2



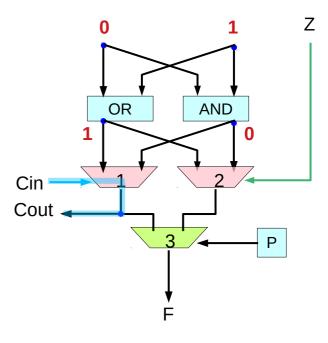
an additional cell for generating Cin

on the other hand, in order to implement a n-bit carry chain with a carry input,

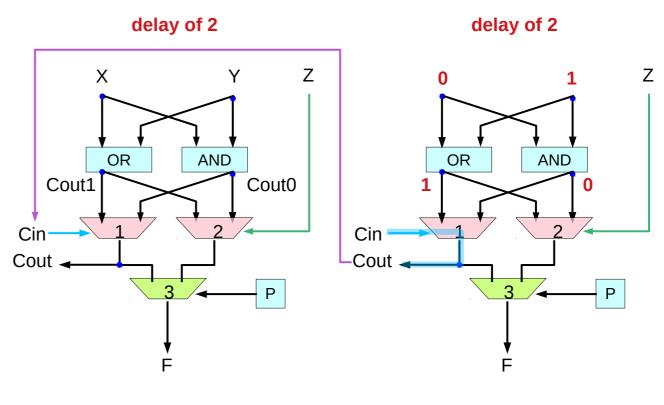
the design of fig 2a requires an additional cell at the beginning of the chain to bring in this input, resulting in a delay of 2(n+1)=2n+2,

which is lower than that of the design in fig2b thus, the design of fig 2b is the preferred ripple carry design among those presented so far

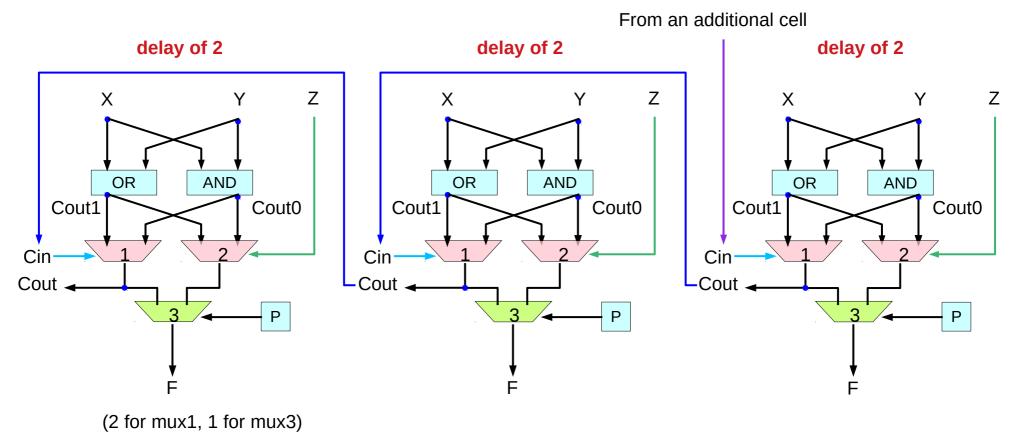
delay of 2



an additional cell for generating Cin



an additional cell for generating Cin

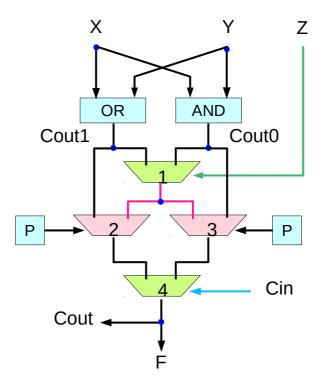


delay of 2(n+1) for an n-bit ripple carry chain

thus, although this design is 1 gate delay slower than that of fig 2a, it provides the ability to have a carry input to the first cell in a carry chain, something that is important in many computations.

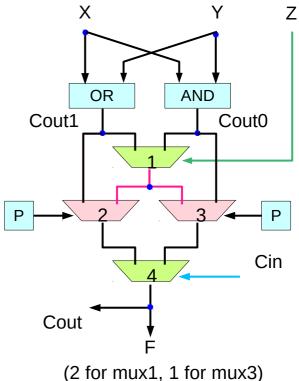
Also, for carry computations that do not need this feature, the first cell in a carry chain built from fig 2b can be configured to bypass mux1, reducing the overall delay to 2n, which is identical to that of fig2a.

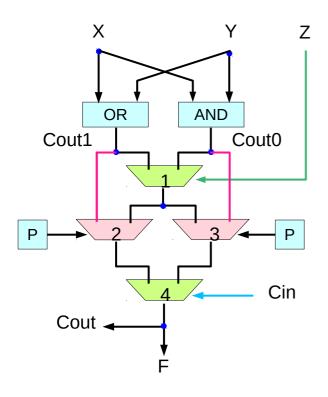
on the other hand, in order to implement a n-bit carry chain with a carry input, the design of fig 2a requires an additional cell at the beginning of the chain to bring in this input, resulting in a delay of 2(n+1)=2n+2, which is lower than that of the design in fig2b thus, the design of fig 2b is the preferred ripple carry design among those presented so far



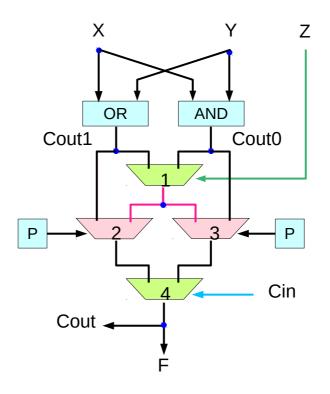
although this design is 1 gate delay slower than that of fig 2a, it provides the ability to have a carry input to the first cell in a carry chain, something that is important in many computations.

Also, for carry computations that do not need this feature, without a carry input the first cell in a carry chain built from fig 2b can be configured to bypass mux1, reducing the overall delay to 2n, which is identical to that of fig2a.



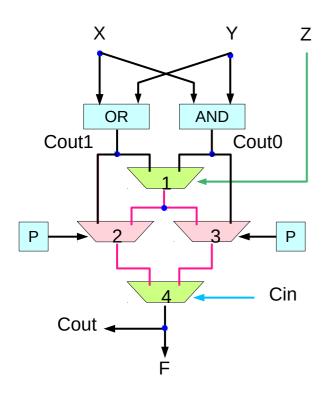


for cells in the middle of a carry chain mux2 passes Cout1 mux3 passes Cout0 mux4 receives Cout1 and Cout0 provides a standard ripple carry path.



For the first cell in a carry chain with a carry input (provided by input Z), mux2 and mux3 both pass the value from mux1

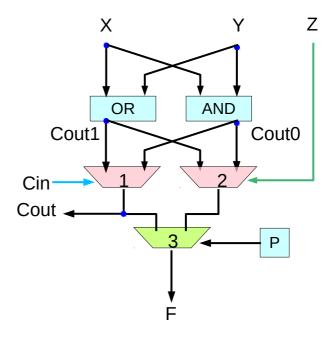
the two main inputs to mux4 are identical the output of mux4 (Cout) will be the same as the output of mux1 (ignoring Cin)

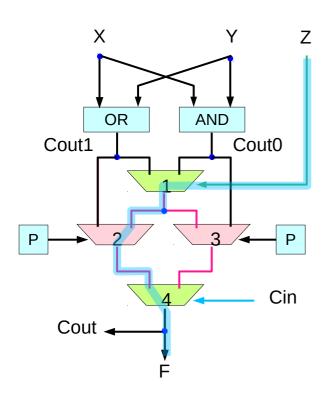


mux1's main inputs are driven by two 2-LUTs (OR, AND) controlled by X and Y mux1 forms a 3-LUT with the other 2-LUTs

When mux2 and mux3 pass the value from mux1 (Cout1 and Cout2 respectively) the circuit is configured to continue the carry chain

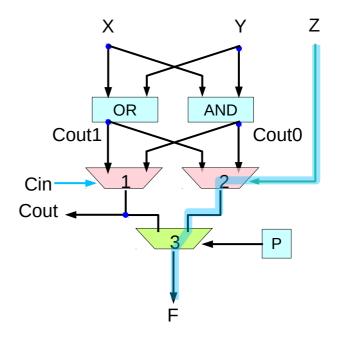
Functionally equivalent

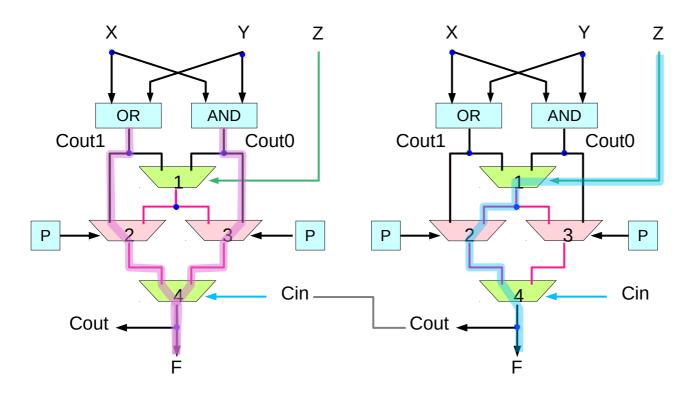




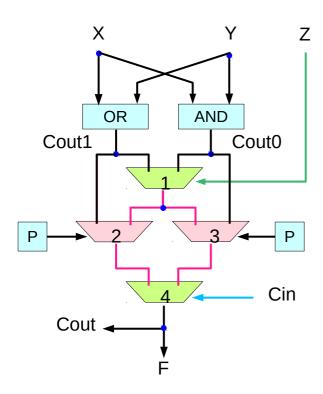
a delay of 3 in the first cell
(1 in mux1, 1 in mux2, 1 in mux4)
2 in all other cells in the carry chain
an total delay of 2n+1 for an n-bit carry chain

1 gate delay slower than that of fig 2a, a carry input to the first cell is enabled





Also, for carry computations that do not need this feature, the first cell in a carry chain built from fig 2b can be configured to bypass mux1, reducing the overall delay to 2n, which is identical to that of fig2a.



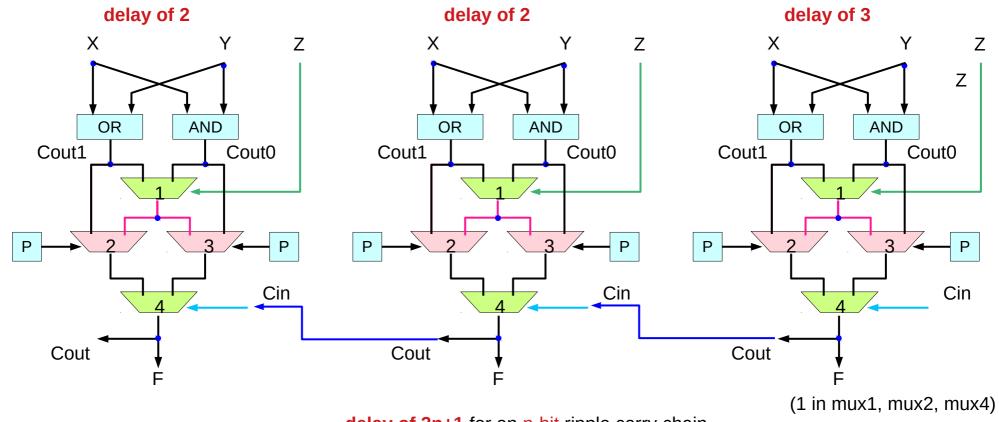
a delay of 3 in the first cell (1 in mux1, 1 in mux2, 1 in mux4) 2 in all other cells in the carry chain an total delay of 2n+1 for an n-bit carry chain

t1 gate delay slower than that of fig 2a, a carry input to the first cell is enabled

Also, for carry computations that do not need this feature, the first cell in a carry chain built from fig 2b can be configured to bypass mux1, reducing the overall delay to 2n, which is identical to that of fig2a.

in order to implement a n-bit carry chain with a carry input, the design of fig 2a requires an additional cell at the beginning of the chain to bring in this input, resulting in a delay of 2(n+1)=2n+2, which is lower than that of the design in fig2b

thus, the design of fig 2b is the preferrred ripple carry design among those presented so far



delay of 2n+1 for an n-bit ripple carry chain

50% faster circuit that the original design

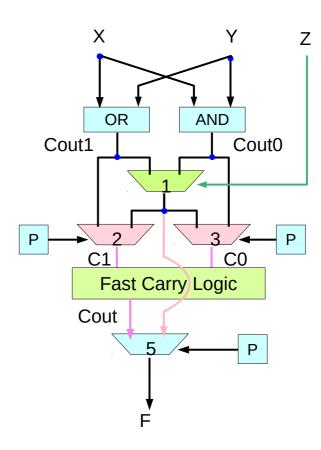
Design C

various high performance carry chains can be developed based on the carry cell of fig 2c

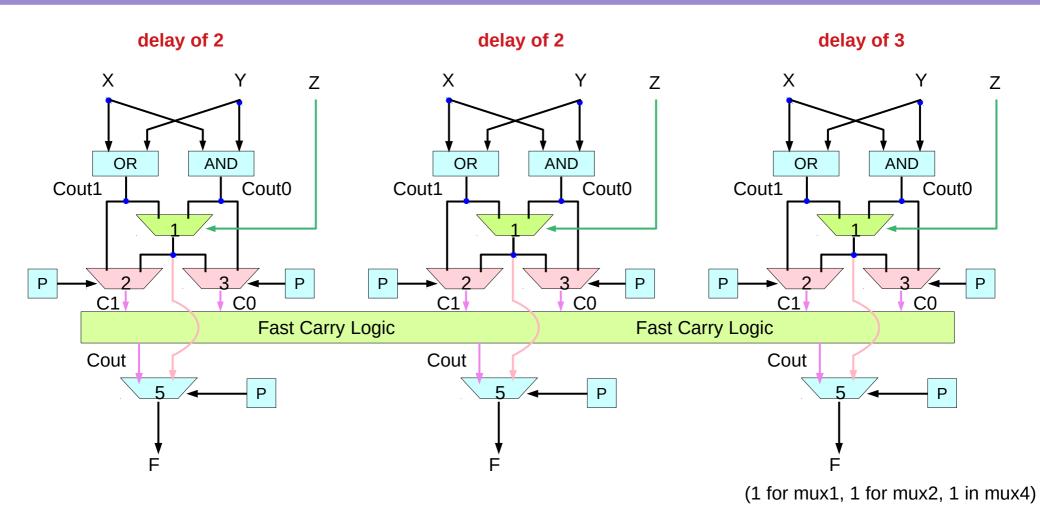
this cell is very similar to that of fig 2b, except that the actual carry chain (mux4) has been replaced by an abstract fast carry logic unit and mux5 has been added

this extra mux5 is present because although some of our faster carry chains will have much faster carry propagation for long carry chains, they incur significant delay for non-carry computations

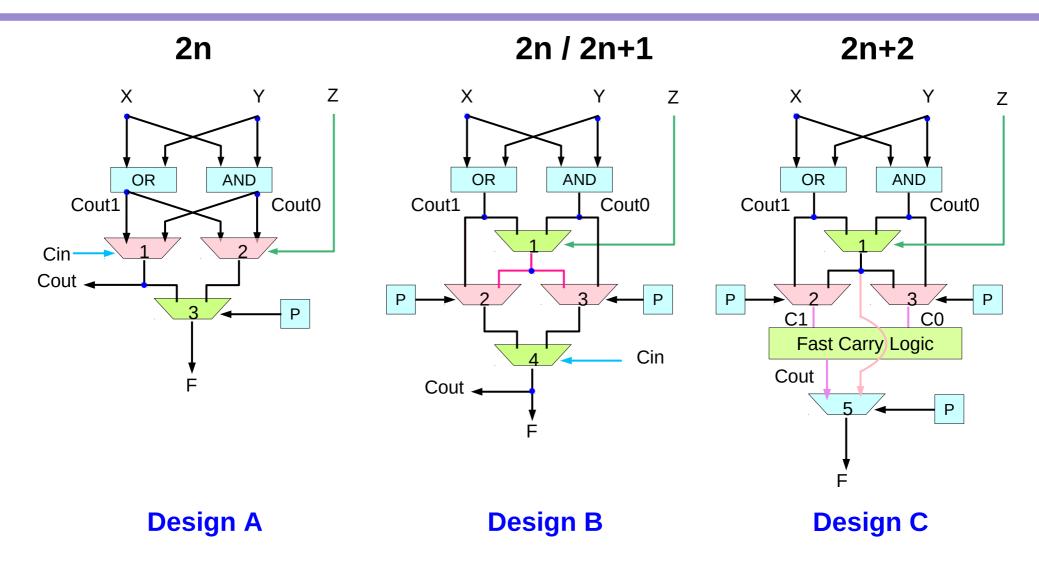
thus, when the cell is used as a simple normal 3LUT, using inputs X, Y, and Z mux5 allows us to bypass the carry chain by selecting the output of mux1

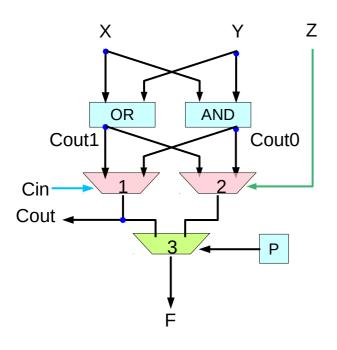


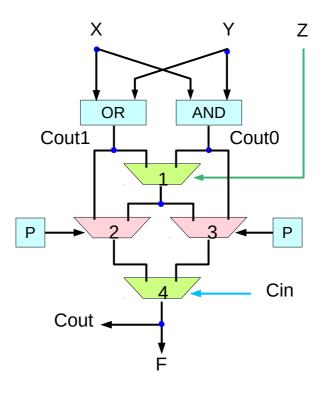
Design C

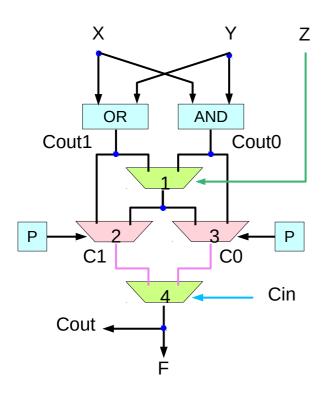


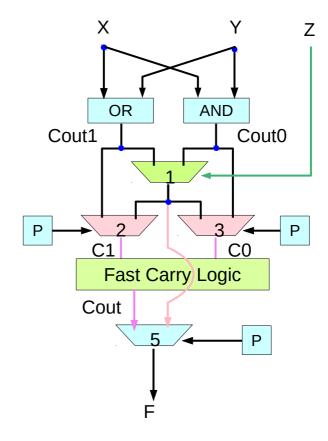
delay of 2n+2 for an n-bit ripple carry chain



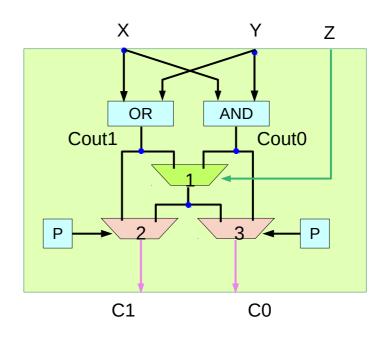








$$Cout_i = (Cout_{i-1} \cdot C1_i) + (\overline{Cout_{i-1}} \cdot C0_i)$$

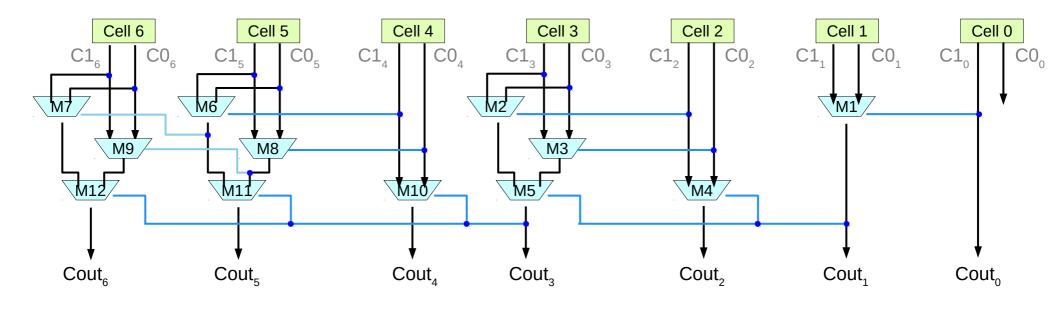


$$Cout_i = (Cout_{i-1} \cdot C1_i) + (\overline{Cout_{i-1}} \cdot C0_i)$$

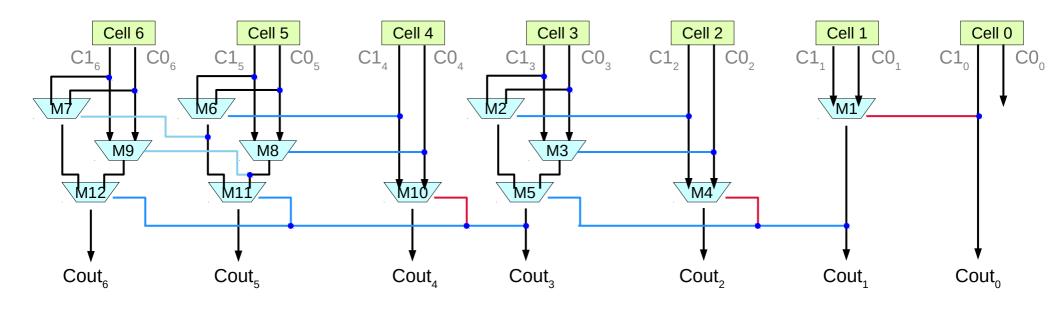
Fast Carry Logc

Carry Select Adder
Carry Lookahead Adder
Brent-Kung
Variable Block
Ripple Carry Adder

https://en.wikipedia.org/wiki/Carry-lookahead_adder



$$Cout_i = (Cout_{i-1} \cdot C1_i) + (\overline{Cout_{i-1}} \cdot C0_i)$$



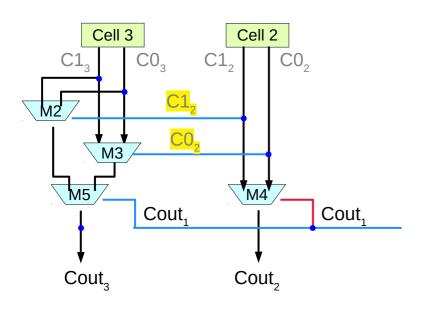
$$Cout_i = (Cout_{i-1} \cdot C 1_i) + (\overline{Cout_{i-1}} \cdot C 0_i)$$

$$Cout_1 = (Cout_0 \cdot C1_1) + (\overline{Cout_0} \cdot C0_1)$$

$$Cout_1 = (C 1_0 \cdot C 1_1) + (\overline{C} 1_0 \cdot C 0_1)$$

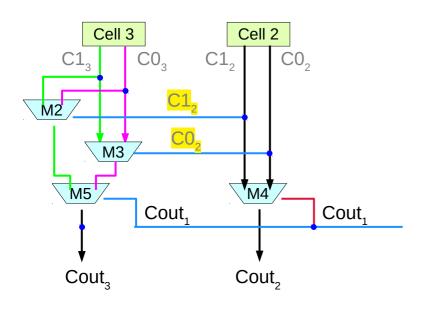
$$Cout_{i+1} = (Cout_i \cdot C 1_{i+1}) + (\overline{Cout_i} \cdot C 0_{i+1})$$

$$Cout_{i+1} = \left(\left[\left(Cout_{i-1} \cdot C \, \mathbf{1}_i\right) + \left(\overline{Cout_{i-1}} \cdot C \, \mathbf{0}_i\right)\right] \cdot C \, \mathbf{1}_{i+1}\right) + \left(\overline{\left[\left(Cout_{i-1} \cdot C \, \mathbf{1}_i\right) + \left(\overline{Cout_{i-1}} \cdot C \, \mathbf{0}_i\right)\right]} \cdot C \, \mathbf{0}_{i+1}\right)$$



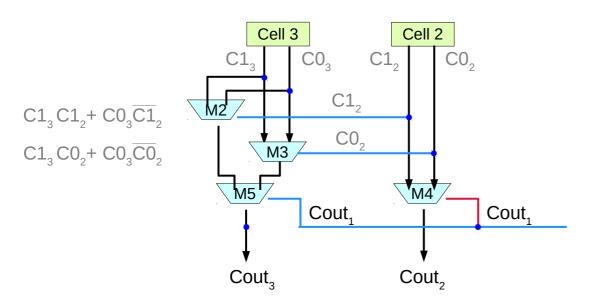
$$(C1_3C1_2 + C0_3\overline{C1}_2)Cout_1 + (C1_3C0_2 + C0_3\overline{C0}_2)\overline{Cout}_1$$

$$\begin{aligned} &Cout_{i} = \left(Cout_{i-1} \cdot C \cdot \mathbf{1}_{i}\right) + \left(\overline{Cout_{i-1}} \cdot C \cdot \mathbf{0}_{i}\right) \\ &Cout_{i+1} = \left(Cout_{i} \cdot C \cdot \mathbf{1}_{i+1}\right) + \left(\overline{Cout_{i}} \cdot C \cdot \mathbf{0}_{i+1}\right) \\ &Cout_{2} = \left(Cout_{1} \cdot C \cdot \mathbf{1}_{2}\right) + \left(\overline{Cout_{1}} \cdot C \cdot \mathbf{0}_{2}\right) \\ &Cout_{3} = \left(Cout_{2} \cdot C \cdot \mathbf{1}_{3}\right) + \left(\overline{Cout_{2}} \cdot C \cdot \mathbf{0}_{3}\right) \\ &= \left(\left(\left(Cout_{1} \cdot C \cdot \mathbf{1}_{2}\right) + \left(\overline{Cout_{1}} \cdot C \cdot \mathbf{0}_{2}\right)\right) \cdot C \cdot \mathbf{1}_{3}\right) \\ &+ \left(\overline{\left(\left(Cout_{1} \cdot C \cdot \mathbf{1}_{2}\right) + \left(\overline{Cout_{1}} \cdot C \cdot \mathbf{0}_{2}\right)\right) \cdot C \cdot \mathbf{0}_{3}\right) \\ &= \left(C \cdot \mathbf{1}_{3} C \cdot \mathbf{1}_{2} Cout_{1} + C \cdot \mathbf{1}_{3} C \cdot \mathbf{0}_{2} \overline{Cout_{1}}\right) \\ &\left(\overline{\left(\left(Cout_{1} \cdot C \cdot \mathbf{1}_{2}\right) \cdot \left(\overline{Cout_{1}} \cdot C \cdot \mathbf{0}_{2}\right)\right) \cdot C \cdot \mathbf{0}_{3}\right) \\ &= \left(\overline{\left(\left(Cout_{1} + \overline{C} \cdot \mathbf{1}_{2}\right) \cdot \left(Cout_{1} + \overline{C} \cdot \mathbf{0}_{2}\right)\right) \cdot C \cdot \mathbf{0}_{3}\right) \\ &= \left(\overline{\left(Cout_{1}} \cdot Cout_{1} + \overline{C} \cdot \mathbf{1}_{2} Cout_{1} + \overline{Cout_{1}} \overline{C} \cdot \mathbf{0}_{2} + \overline{C} \cdot \mathbf{1}_{2} \overline{C} \cdot \mathbf{0}_{2}\right) \cdot C \cdot \mathbf{0}_{3} \\ &= \left(\overline{C} \cdot \mathbf{1}_{2} Cout_{1} + \overline{C} \cdot \mathbf{1}_{2} \overline{Cout_{1}}\right) \cdot C \cdot \mathbf{0}_{3} \\ &= \left(\overline{C} \cdot \mathbf{1}_{2} Cout_{1} + \overline{C} \cdot \mathbf{0}_{2} \overline{Cout_{1}}\right) \cdot C \cdot \mathbf{0}_{3} \\ &= \left(\overline{C} \cdot \mathbf{1}_{2} Cout_{1} + \overline{C} \cdot \mathbf{0}_{2} \overline{Cout_{1}}\right) \cdot C \cdot \mathbf{0}_{3} \end{aligned}$$



$$(C1_3C1_2 + C0_3\overline{C1}_2)Cout_1 + (C1_3C0_2 + C0_3\overline{C0}_2)\overline{Cout}_1$$

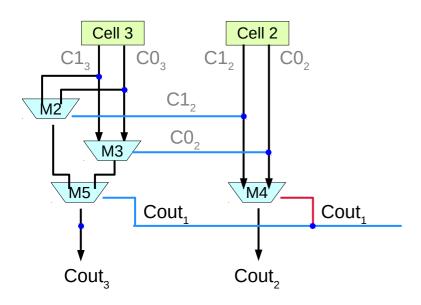
$$\begin{split} &= (\overline{Cout_1}\overline{Cout_1} + \overline{C1_2}Cout_1 + \overline{Cout_1}\overline{C0_2} + \overline{C1_2}\overline{C0_2}) \cdot C0_3 \\ &= (\overline{C1_2}Cout_1 + \overline{C0_2}\overline{Cout_1}) \cdot C0_3 \\ &= (C0_3\overline{C1_2}Cout_1 + C0_3\overline{C0_2}\overline{Cout_1}) \end{split}$$



$$(C1_3C1_2 + C0_3\overline{C1}_2)Cout_1 + (C1_3C0_2 + C0_3\overline{C0}_2)\overline{Cout}_1$$

$$= C1_3 \cdot (C1_2 Cout_1 + C0_2 \overline{Cout_1})$$

+ $C0_3 \cdot (\overline{C1_2} Cout_1 + \overline{C0_2} \overline{Cout_1})$



$$\begin{split} Cout_{i} &= \left(Cout_{i-1} \cdot C \, \mathbf{1}_{i}\right) + \left(\overline{Cout_{i-1}} \cdot C \, \mathbf{0}_{i}\right) \\ Cout_{i+1} &= \left(Cout_{i} \cdot C \, \mathbf{1}_{i+1}\right) + \left(\overline{Cout_{i}} \cdot C \, \mathbf{0}_{i+1}\right) \\ Cout_{i+1} &= \left(\left[\left(Cout_{i-1} \cdot C \, \mathbf{1}_{i}\right) + \left(\overline{Cout_{i-1}} \cdot C \, \mathbf{0}_{i}\right)\right] \cdot C \, \mathbf{1}_{i+1}\right) \\ &+ \left(\overline{\left[\left(Cout_{i-1} \cdot C \, \mathbf{1}_{i}\right) + \left(\overline{Cout_{i-1}} \cdot C \, \mathbf{0}_{i}\right)\right]} \cdot C \, \mathbf{0}_{i+1}\right) \end{split}$$

References

- [1] http://en.wikipedia.org/
- [2] J-P Deschamps, et. al., "Sunthesis of Arithmetic Circuits", 2006