Multiple Random Variables

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Based on
Probability, Random Variables and Random Signal Principles,
P.Z. Peebles,Jr. and B. Shi

Outline

1. Joint Distribution and its Properties

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1 Joint Distribution and its Properties

Probability Distribution Function

Definition

events :

$A = \{X \leq x\}$

$B = \{Y \leq y\}$

probability distribution functions :

$F_X(x) = P\{X(s) \leq x\}$

$F_Y(y) = P\{Y(s) \leq y\}$

Joint Distribution Function

Definition

events $A = \{X \leq x\}$ $B = \{Y \leq y\}$
joint event $\{X \leq x, Y \leq y\} = (A \cap B)$

joint probability distribution function
$F_{XY}(x,y) = P\{X \leq x, Y \leq y\} = P(A \cap B)$

Joint Distribution Function

for two discrete random variables

Definition

let X have N possible values x_n
let Y have M possible values y_m

$$F_{XY}(x, y) = P\{X \leq x, Y \leq y\}$$

$$= \sum_{n=1}^{N} \sum_{m=1}^{M} P(x_n, y_n) u(x - x_n) u(y - y_m)$$

$P(x_n, y_n)$ the probability of the joint event $\{X = x_n, Y = y_n\}$
$u(.)$ the unit step function

Joint Distribution Function
for N random variables

Definition

let N random variables X_n, $n = 1, 2, \ldots, N$

$$F_{X_1, X_2, \ldots, X_N}(x_1, x_2, \ldots, x_n) = P\{X_1 \le x_1, X_1 \le x_1, \ldots, X_N \le x_N\}$$

Properties of Joint Distribution Function

for 2 random variables

1. $F_{X,Y}(-\infty,-\infty)=0 \quad F_{X,Y}(-\infty,y)=0 \quad F_{X,Y}(x,-\infty)=0$
2. $F_{X,Y}(+\infty,y)=F_Y(y) \quad F_{X,Y}(x,+\infty)=F_X(x)$
3. $F_{X,Y}(+\infty,+\infty)=1$
4. $0\le F_{X,Y}(x,y)\le 1$
5. $F_{X,Y}(x,y)$ is non-decreasing function of both x and y
6. $0\le F_{X,Y}(x_2,y_2)+F_{X,Y}(x_1,y_1)-F_{X,Y}(x_1,y_2)-F_{X,Y}(x_2,y_1)$
$= P\{x_1<x\le x_2, y_1<y\le y_2\}$

Marginal Distribution Function
for 2 random variables

marginal distribution functions

$$F_X(x) = F_{X,Y}(x, +\infty)$$
$$F_Y(y) = F_{X,Y}(+\infty, y)$$

let $A = \{X \le x\}$ and $B = \{Y \le y\}$

$$F_{X,Y}(x, y) = P\{X \le x, Y \le y\} = P(A \cap B)$$

let $S = \{Y \le +\infty\}$

$$F_{X,Y}(x, \infty) = P\{X \le x, Y \le \infty\} = P(A \cap S)$$
$$= P(A) = P\{X \le x\} = F_x(x)$$