

Multiple Random Variables

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Based on
Probability, Random Variables and Random Signal Principles,
P.Z. Peebles, Jr. and B. Shi

Outline

1 Joint Distribution and its Properties

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Probability Distribution Function

Definition

events :

$$A = \{X \leq x\}$$

$$B = \{Y \leq y\}$$

probability distribution functions :

$$F_X(x) = P\{X(s) \leq x\}$$

$$F_Y(y) = P\{Y(s) \leq y\}$$

Joint Distribution Function

Definition

events $A = \{X \leq x\}$ $B = \{Y \leq y\}$

joint event $\{X \leq x, Y \leq y\} = (A \cap B)$

joint probability distribution function

$$F_{XY}(x, y) = P\{X \leq x, Y \leq y\} = P(A \cap B)$$

Joint Distribution Function

for two discrete random variables

Definition

let X have N possible values x_n

let Y have M possible values y_m

$$\begin{aligned} F_{XY}(x, y) &= P\{X \leq x, Y \leq y\} \\ &= \sum_{n=1}^N \sum_{m=1}^M P(x_n, y_m) u(x - x_n) u(y - y_m) \end{aligned}$$

$P(x_n, y_m)$ the probability of the joint event $\{X = x_n, Y = y_m\}$

$u(\cdot)$ the unit step function

Joint Distribution Function

for N random variables

Definition

let N random variables $X_n, n = 1, 2, \dots, N$

$$F_{X_1, X_2, \dots, X_N}(x_1, x_2, \dots, x_N) = P\{X_1 \leq x_1, X_2 \leq x_2, \dots, X_N \leq x_N\}$$

Properties of Joint Distribution Function for 2 random variables

$$\textcircled{1} F_{X,Y}(-\infty, -\infty) = 0 \quad F_{X,Y}(-\infty, y) = 0 \quad F_{X,Y}(x, -\infty) = 0$$

$$\textcircled{2} F_{X,Y}(+\infty, y) = F_Y(y) \quad F_{X,Y}(x, +\infty) = F_X(x)$$

$$\textcircled{3} F_{X,Y}(+\infty, +\infty) = 1$$

$$\textcircled{4} 0 \leq F_{X,Y}(x, y) \leq 1$$

$\textcircled{5}$ $F_{X,Y}(x, y)$ is non-decreasing function of both x and y

$$\textcircled{6} 0 \leq F_{X,Y}(x_2, y_2) + F_{X,Y}(x_1, y_1) - F_{X,Y}(x_1, y_2) - F_{X,Y}(x_2, y_1) \\ = P\{x_1 < x \leq x_2, y_1 < y \leq y_2\}$$

Marginal Distribution Function

for 2 random variables

marginal distribution functions

$$F_X(x) = F_{X,Y}(x, +\infty)$$

$$F_Y(y) = F_{X,Y}(+\infty, y)$$

let $A = \{X \leq x\}$ and $B = \{Y \leq y\}$

$$F_{X,Y}(x, y) = P\{X \leq x, Y \leq y\} = P(A \cap B)$$

let $S = \{Y \leq +\infty\}$

$$\begin{aligned} F_{X,Y}(x, \infty) &= P\{X \leq x, Y \leq \infty\} = P(A \cap S) \\ &= P(A) = P\{X \leq x\} = F_X(x) \end{aligned}$$