Measurement of Correlation Functions

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Based on
Probability, Random Variables and Random Signal Principles,
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Outline

- in the real world, we can never measure the true correlation functions of two random processes X(t) and Y(t)
- because we never have all sample functions of the ensemble at our disposal
- we may typically have available for measurement only a portion of one sample function from each process
- oure only recourse is to determine time averages based on finite time portion of single sample functions
- because we are able to work only with time functions, we are forced to presume that the given processe satisfy appropriate ergodic theorems

- a possible system for measuring the approximate time cross-correlation function of two cross-correlation-ergodic and stationary random processes $X(t)$ and $Y(t)$
- sample functions $x(t)$ and $y(t)$ are delayed by amounts T and $T - \tau$, respectively
- the product of the delayed waveforms
- integrate to form the output which equals the integral at time $t_1 + 2T_s$, where t_1 is arbitrary and $2T$ is the integration period

Definition

assume $x(t)$ and $y(t)$ exist at least during the interval $t \geq -T$ and that $t_1 \geq 0$ is an arbitrary time instant and that $\tau \leq T$ then the output is as follows

$$R_o(t_1 + 2T) = \frac{1}{2T} \int_{t_1 - T}^{t_{1+} T} x(t) y(t + \tau) dt$$

Definition

if we choose $t_1 = 0$ and assume T is large

$$R_o(t_1 + 2T) = \frac{1}{2T} \int_{t_1 - T}^{t_{1+} T} x(t) y(t+\tau) dt$$

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$$R_o(2T) = \frac{1}{2T} \int_{-T}^{+T} x(t) y(t+\tau) dt = R_{XY}(\tau)$$

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