

Example Random Processes

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September 9, 2019

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Based on
Probability, Random Variables and Random Signal Principles,
P.Z. Peebles, Jr. and B. Shi

Outline

1 Gaussian Random Processes

2 Poisson Random Process

Gaussian Random Process

N Gaussian random variables

Definition

$$f_X(x_1, \dots, x_N; t_1, \dots, t_N) = \frac{\exp \left\{ -(1/2) [x - \bar{X}]^t [C_X]^{-1} [x - \bar{X}] \right\}}{\sqrt{(2\pi)^N |[C_X]|}}$$

The Covariance Matrix

N Gaussian random variables

Definition

$$\bar{X}_i = E[X_i] = E[X(t_i)]$$

$$C_{ik} = C_{X_i X_k} = E[(X_i - \bar{X}_i)(X_k - \bar{X}_k)]$$

$$= E[(X_i - E[X(t_i)])(X_k - E[X(t_k)])]$$

$$C_{ik} = C_{X_i X_k} = C_{XX}(t_i t_k)$$

$$= R_{XX}(t_i, t_k) - E[X(t_i)]E[X(t_k)]$$

Stationary Gaussian Process

N Gaussian random variables

Definition

$$\bar{X}_i = E[X_i] = E[X(t_i)] = \bar{X} = \text{const}$$

$$C_{XX}(t_i, t_k) = C_{XX}(t_k - t_i)$$

$$R_{XX}(t_i, t_k) = R_{XX}(t_k - t_i)$$

Jointly Gaussian Process

N Gaussian random variables

Definition

the two random processes $X(t)$ and $Y(t)$ are jointly Gaussian if the random variables $X(t_1), \dots, X(t_N), Y(t'_1), \dots, Y(t'_M)$ at times t_1, \dots, t_N for $X(t)$ and t'_1, \dots, t'_M for $Y(t)$ are jointly gaussian for any $N, t_1, \dots, t_N, M, t'_1, \dots, t'_M$

Stationary Gaussian Markov Process

N Gaussian random variables

Definition

$$C_{XX}(\tau) = \sigma^2 e^{-\beta|\tau|}$$

$$C_{XX}[k] = \sigma^2 a^{-|k|}$$

$$a = e^{\beta T_s}$$

Poisson Random Process

N Gaussian random variables

Definition

$$p \left[X(t = k) = \frac{(\lambda t)^k e^{-\lambda t}}{k!} \right], \quad k = 0, 1, 2, \dots$$

$$f_X(x) = \sum_{k=0}^{\infty} \frac{(\lambda t)^k e^{-\lambda t}}{k!} \delta(x - k)$$

Poisson Random Process - mean and 2nd moment

N Gaussian random variables

Definition

$$\begin{aligned} E[X(t)] &= \int_{-\infty}^{\infty} x f_X(x) dx = \int_{-\infty}^{\infty} x \sum_{k=0}^{\infty} \frac{(\lambda t)^k e^{-\lambda t}}{k!} \delta(x - k) dx \\ &= \sum_{k=0}^{\infty} \frac{k(\lambda t)^k e^{-\lambda t}}{k!} = \lambda t \end{aligned}$$

$$\begin{aligned} E[X^2 t] &= \int_{-\infty}^{\infty} x^2 f_X(x) dx = \int_{-\infty}^{\infty} x^2 \sum_{k=0}^{\infty} \frac{(\lambda t)^k e^{-\lambda t}}{k!} \delta(x - k) dx \\ &= \sum_{k=0}^{\infty} \frac{k^2 (\lambda t)^k e^{-\lambda t}}{k!} = \lambda t(1 + \lambda t) \end{aligned}$$

