

# Applications of Array Pointers (1A)

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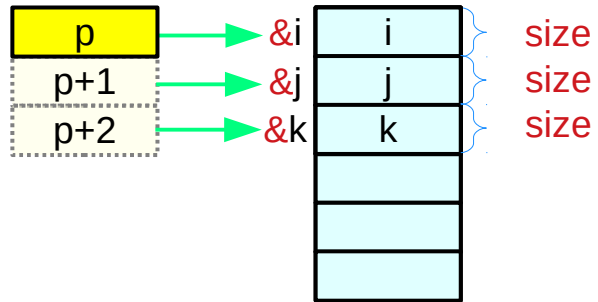
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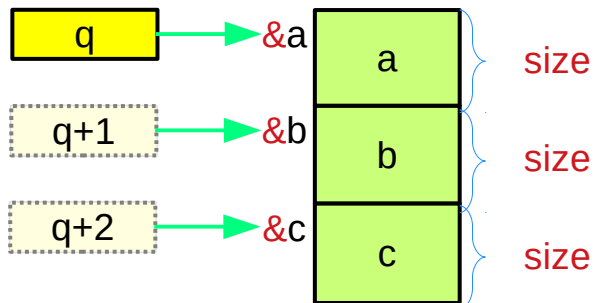
# Virtual Array Pointers in Multi-dimensional Arrays

# Pointers to various data types

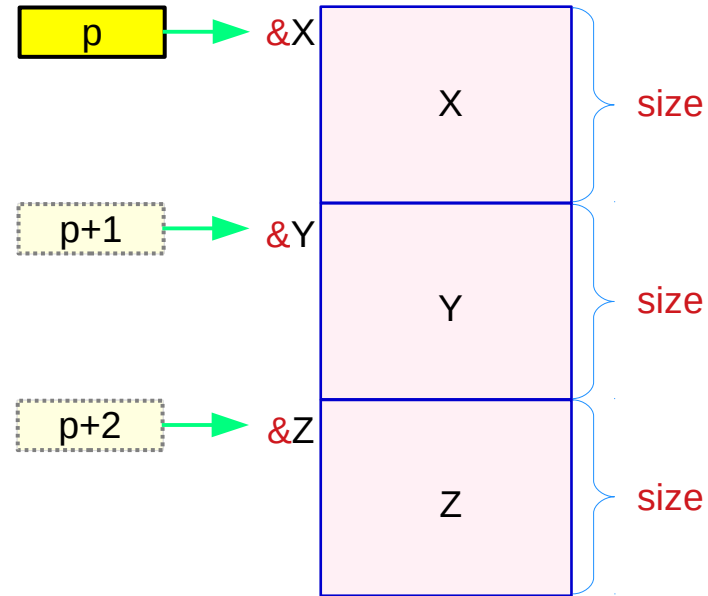
**int \*p;**      **int i, j, k;**



**double \*q;**      **double a, b, c;**



**T \*p;**      **T X, Y, Z;**



pointer

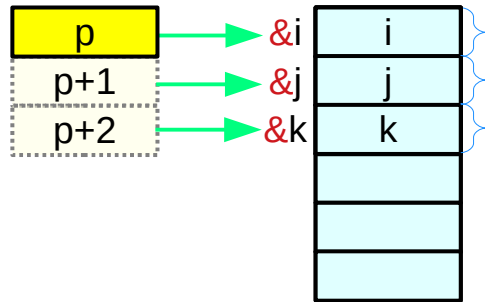
abstract data

# Pointers to primitive data

**int \*p;**

**int i, j, k;**

**sizeof(int) = 4 bytes**



size  
size  
size

$\text{size} = \text{sizeof}(i) = \text{sizeof}(*p)$   
 $\text{size} = \text{sizeof}(j) = \text{sizeof}(*(p+1))$   
 $\text{size} = \text{sizeof}(k) = \text{sizeof}(*(p+2))$

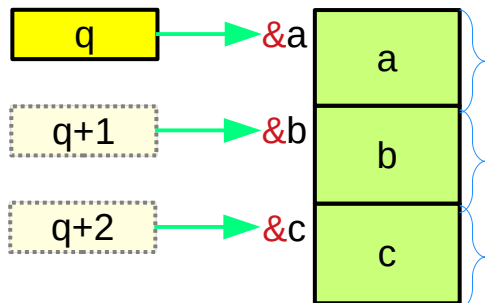
$\neq \text{sizeof}(p)$   
 $\neq \text{sizeof}(p+1)$   
 $\neq \text{sizeof}(p+2)$

pointer size  
4 or 8 bytes

**double \*q;**

**double a, b, c;**

**sizeof(double) = 8 bytes**



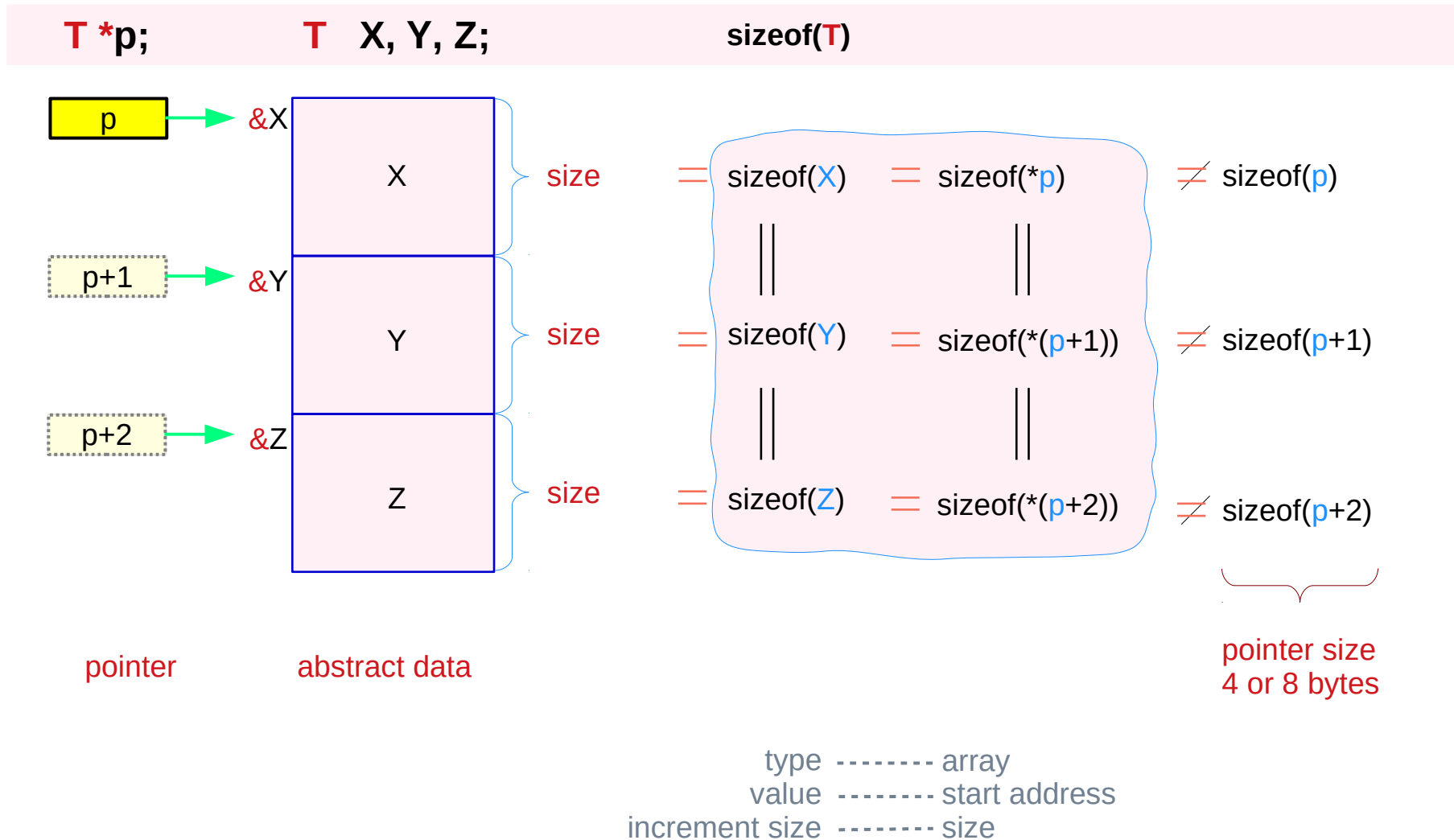
size  
size  
size

$\text{size} = \text{sizeof}(a) = \text{sizeof}(*q)$   
 $\text{size} = \text{sizeof}(b) = \text{sizeof}(*(q+1))$   
 $\text{size} = \text{sizeof}(c) = \text{sizeof}(*(q+2))$

$\neq \text{sizeof}(q)$   
 $\neq \text{sizeof}(q+1)$   
 $\neq \text{sizeof}(q+2)$

pointer size  
4 or 8 bytes

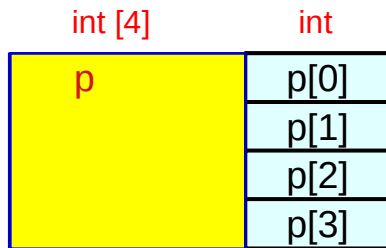
# Pointers to abstract data



# Virtual pointers in an array of integers

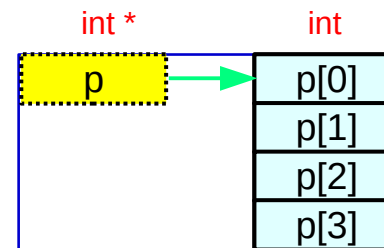
```
int p[3];
```

**p** is an abstract data (array)



- p** is the name of an array
- p** has the size of the whole array
- p** has an array type (abstract data)

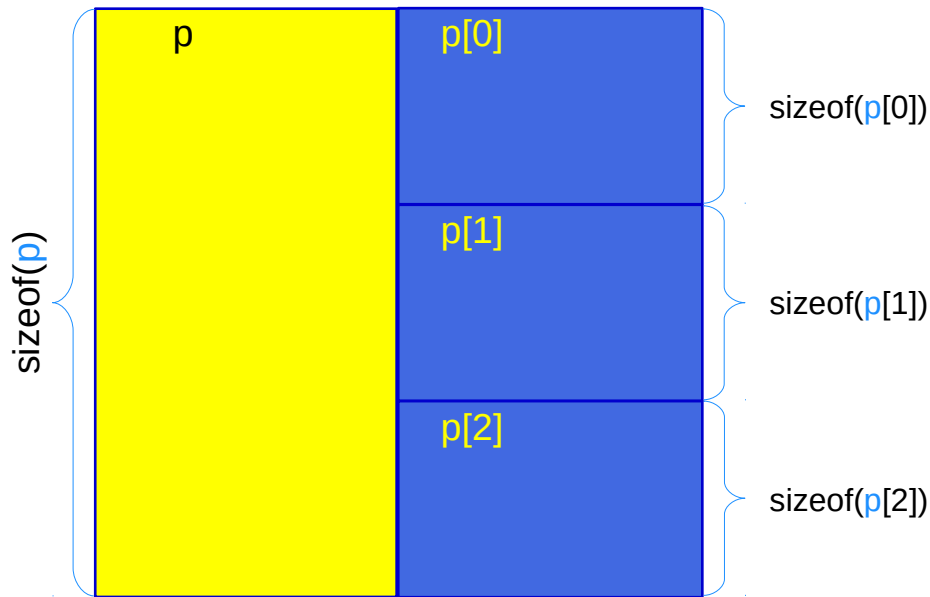
**p** can also be viewed as a pointer



- p** also has pointer characteristics
- p** has the value of the starting address
- p** is a virtual pointer

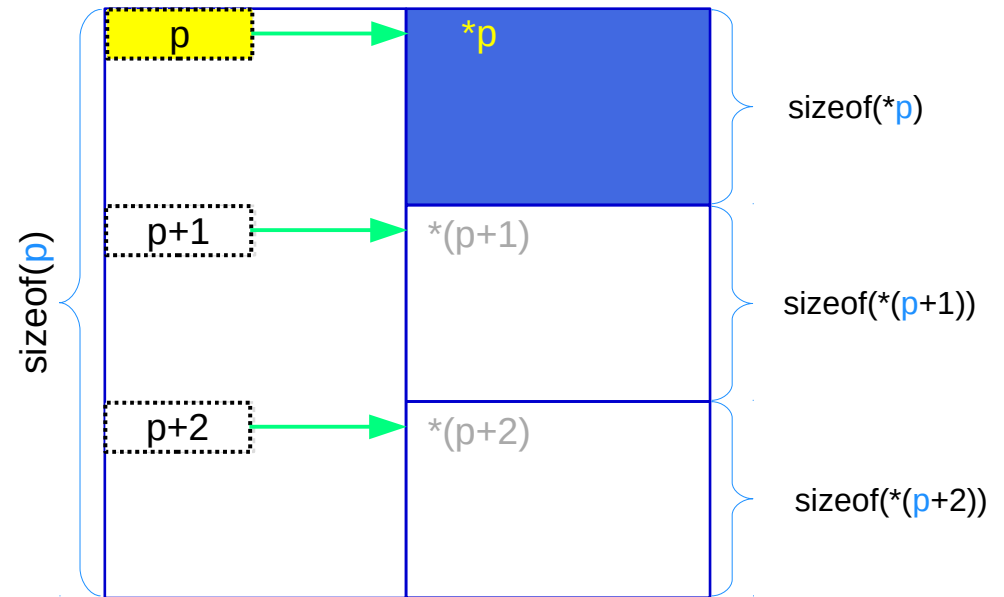
# Virtual pointers in an array of abstract data

## Abstract data array p



- $p$  has an array type (abstract data element)
- $p$  is the name of an array
- $p$  has the size of the whole array

## Virtual pointer p

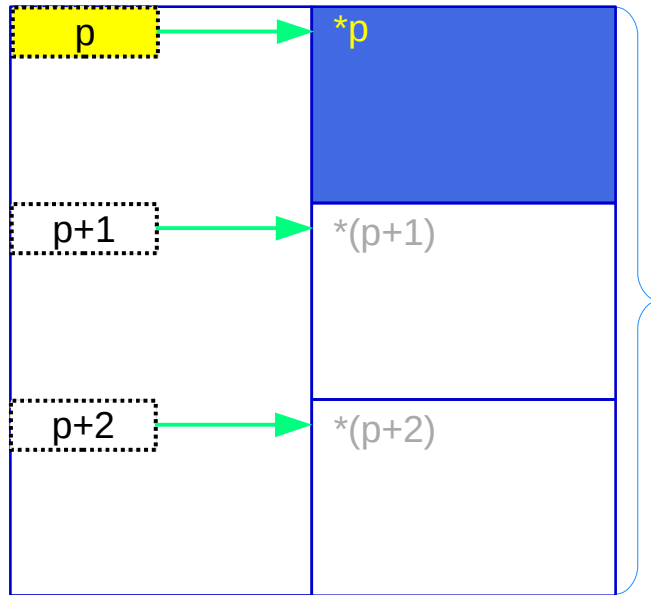


- $p$  also has a pointer type
- $p$  has the value of the starting address
- $p$  is a virtual array pointer

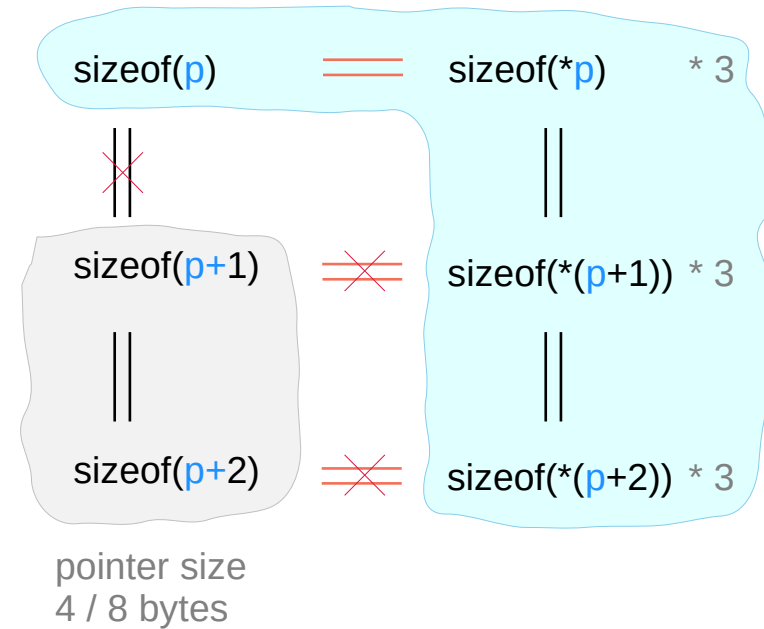


# Virtual pointer to abstract data

virtual pointer p    abstract data \*p

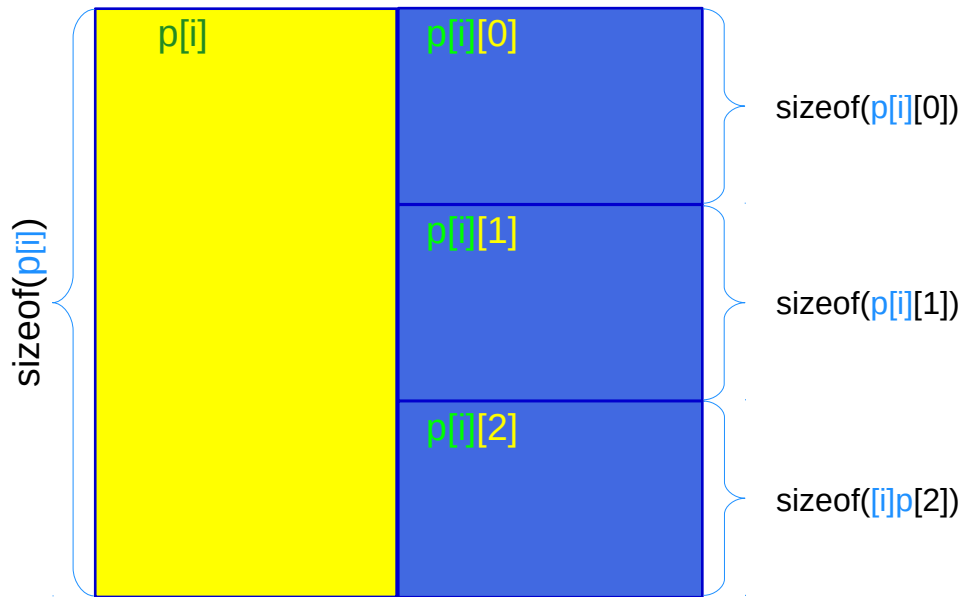


whole array size



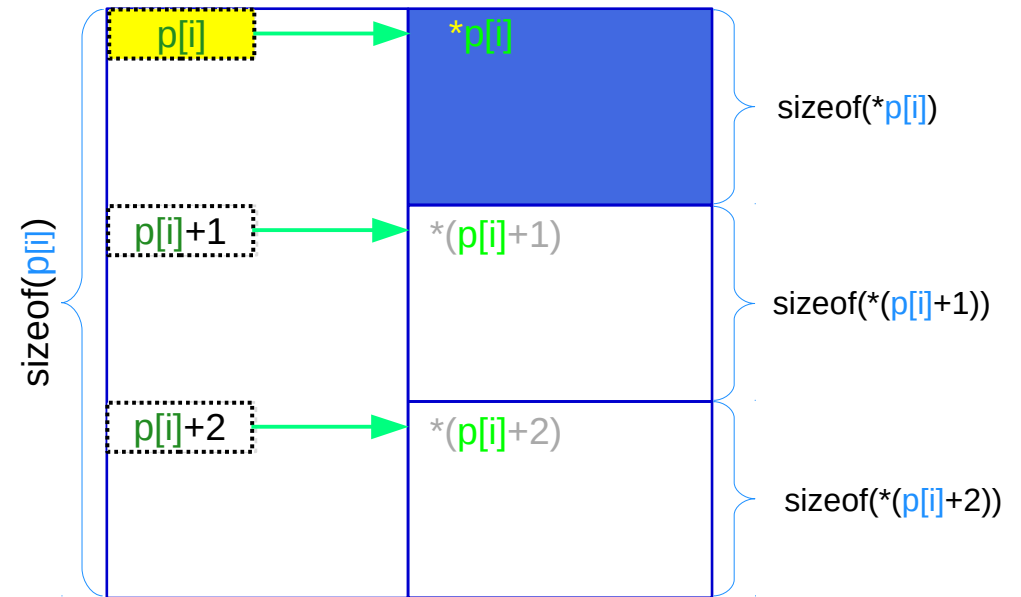
# Virtual array pointers in a multi-dimensional array

## Abstract data (array) $p[i]$



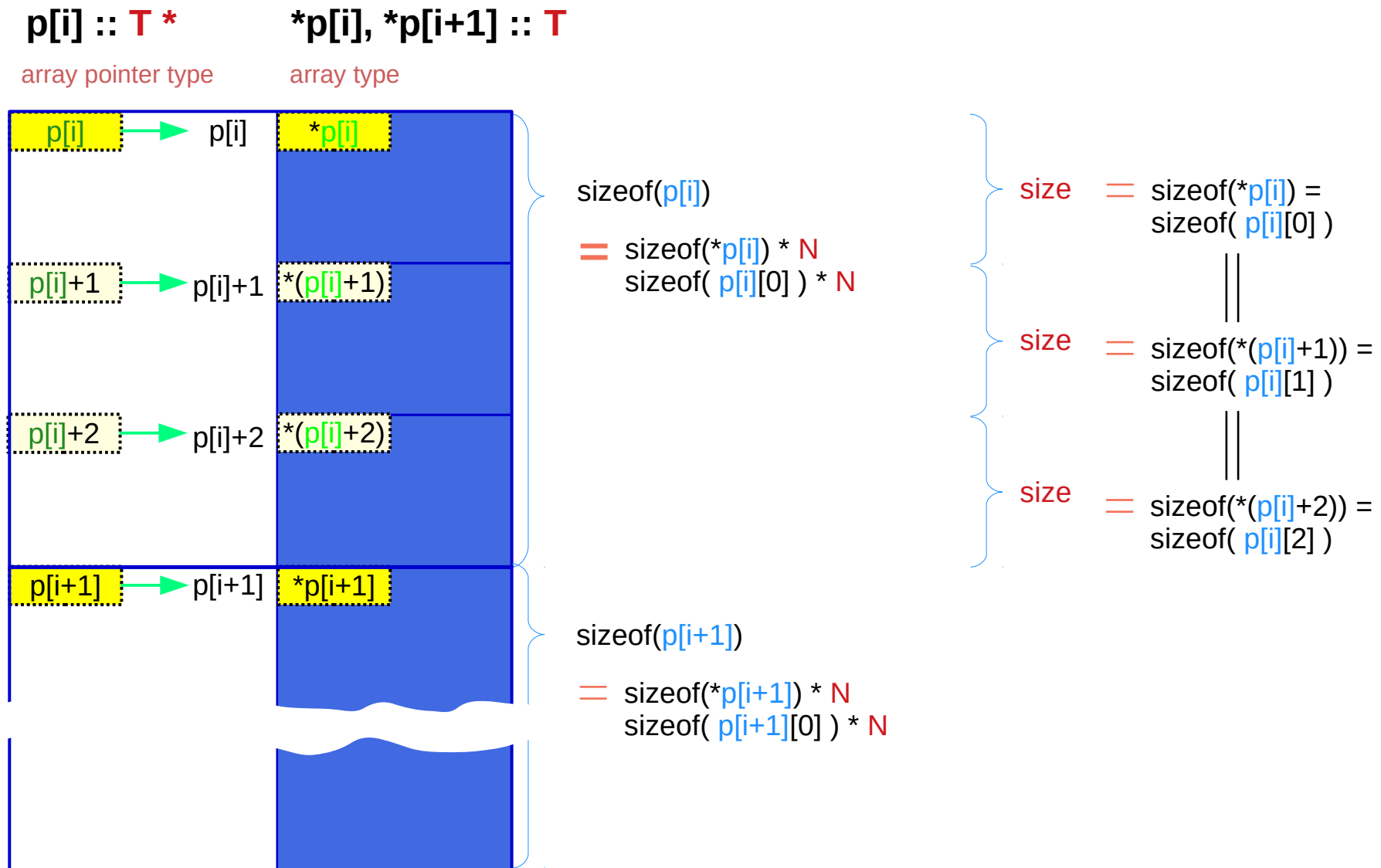
$p[i]$  has an array type (abstract data)  
 $p[i]$  is the name of an array  
 $p[i]$  has the size of the whole array

## Virtual array pointer $p[i]$

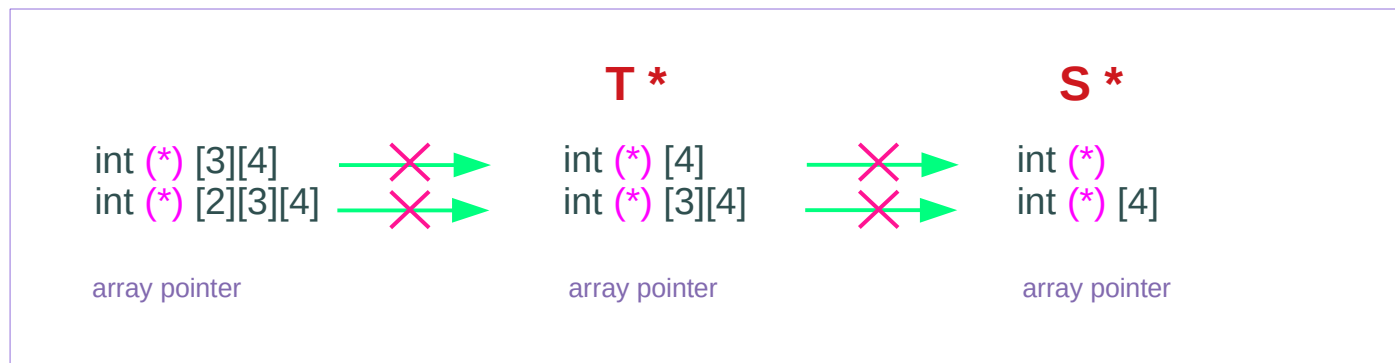
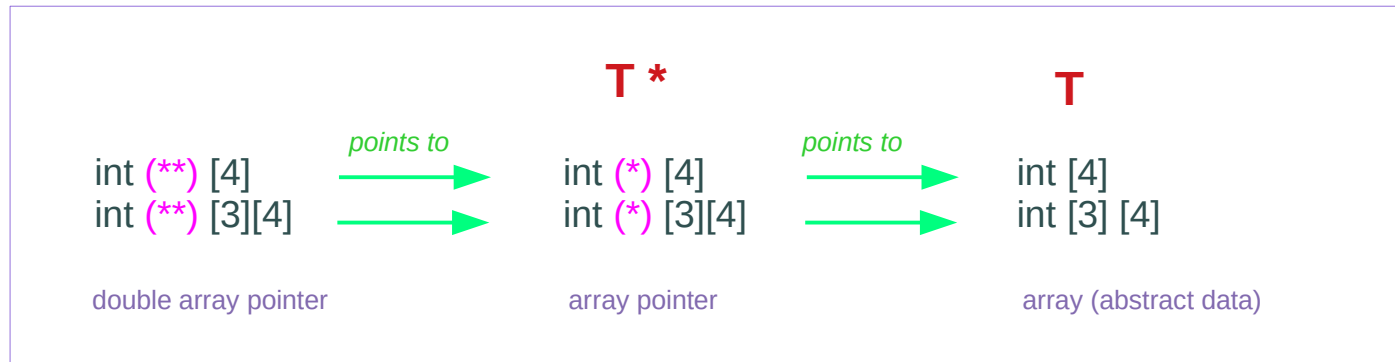


$p[i]$  also has an array pointer type  
 $p[i]$  has the value of the starting address  
 $p[i]$  is a virtual array pointer

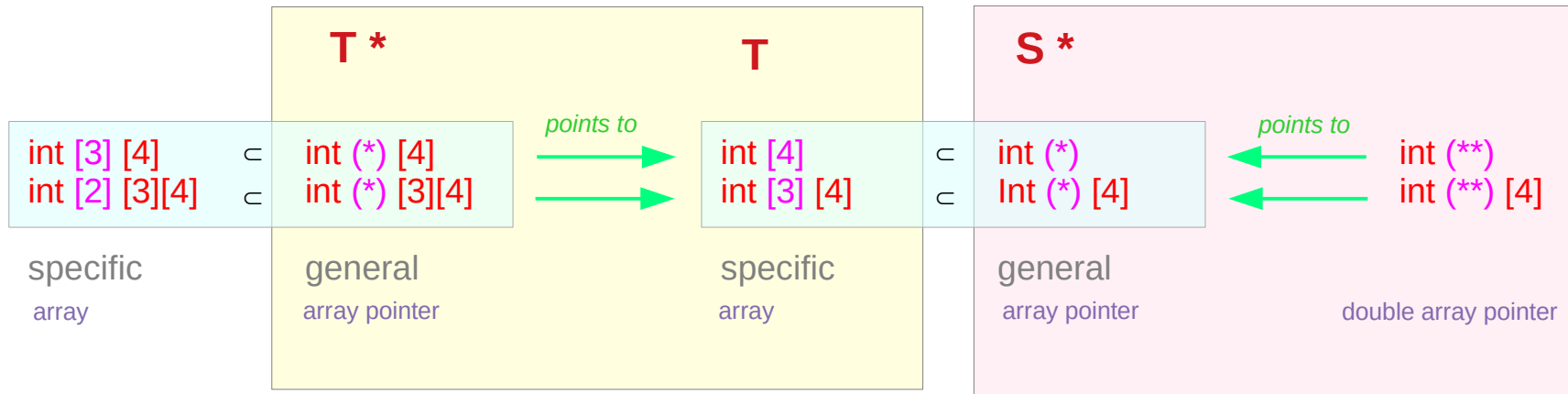
# Virtual pointers to a sub array



# Array pointer types



# Array pointer types point to array types



# Virtual array pointers pointing to arrays

```
typedef int (*T1) [4];  
typedef int (*T1) [3][4];
```

int (\*) [4]  
int (\*) [3][4]  
general

```
typedef int T2[4];  
typedef int T2[3][4];
```

int [4]  
int [3] [4]  
specific

```
T1 a;  
T2 b;
```

T1 is a pointer type  
T2 is an array type  
T1 has one more dimension than T2

```
a = &b;  
*a = b;
```

a references b

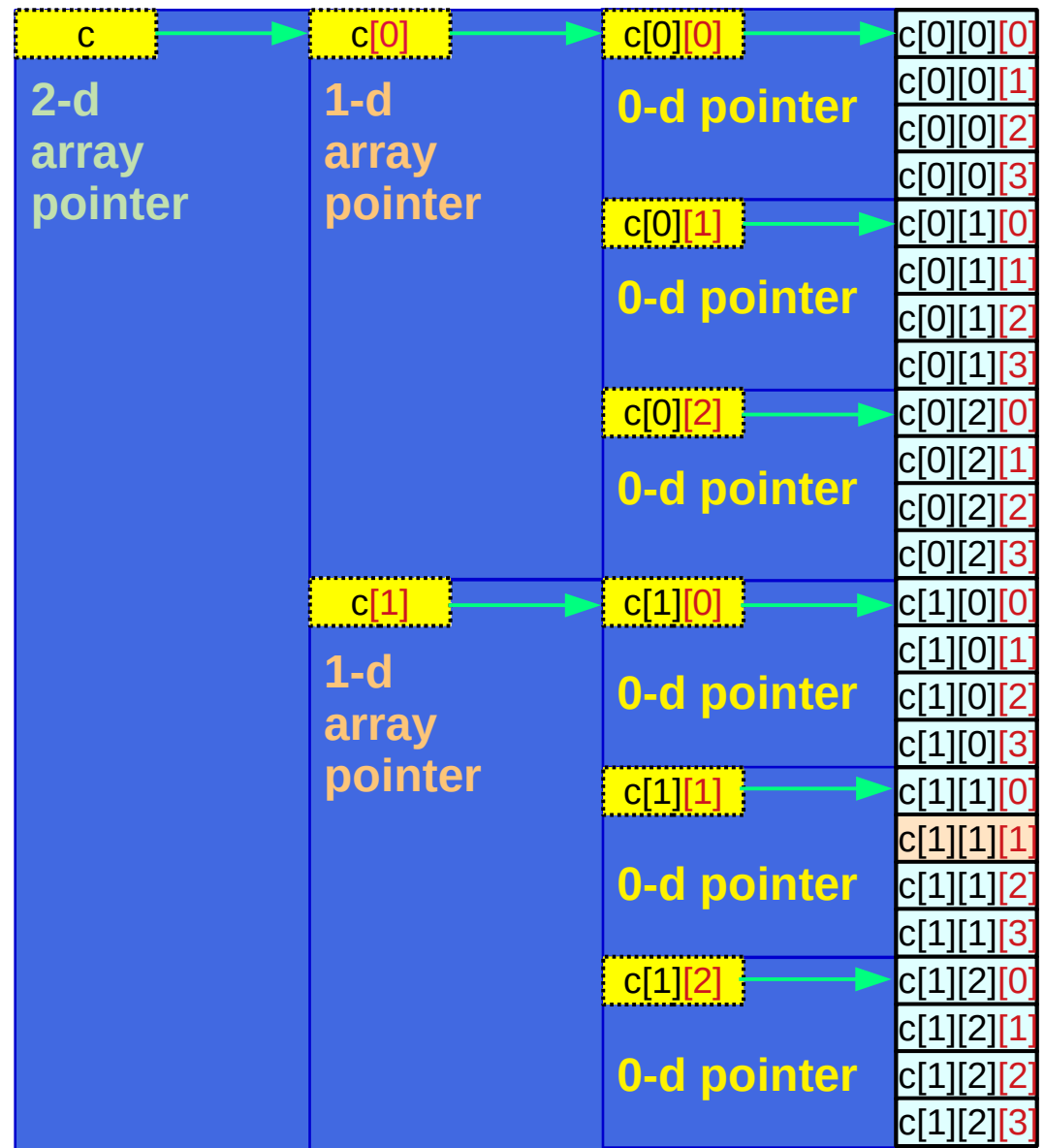
b is the dereference of a

# 3-d array structure – pointer representation

```
int c[2][3][4];
```

```
*(***(c +i) +j) +k)
```

- Hierarchical
- Nested Structure
- Virtual Array Pointers to abstract data (subarrays)
- Contiguous and Linear Data Layout
- Row Major Order



# 3-d array structure – abstract data representation

```
int c[2][3][4];
```

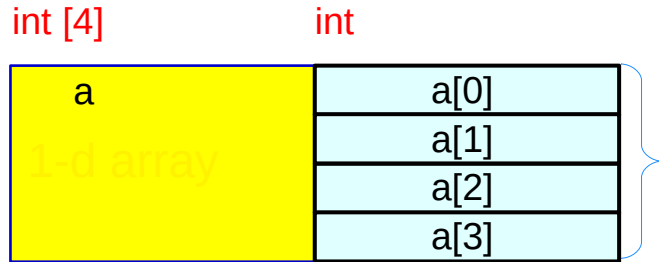
```
((c [i])[j])[k]
```

- Hierarchical
- Nested Structure
- Virtual Array Pointers to abstract data (subarrays)
- Contiguous and Linear Data Layout
- Row Major Order

c 3-d array name	c[0] 2-d array name	c[0][0] 1-d array name	c[0][0][0]
			c[0][0][1]
			c[0][0][2]
		c[0][0][3]	
		c[0][1] 1-d array name	c[0][1][0]
			c[0][1][1]
		c[0][1][2]	
		c[0][1][3]	
		c[0][2] 1-d array name	c[0][2][0]
			c[0][2][1]
			c[0][2][2]
			c[0][2][3]
	c[1] 2-d array name	c[1][0] 1-d array name	c[1][0][0]
			c[1][0][1]
			c[1][0][2]
		c[1][0][3]	
		c[1][1] 1-d array name	c[1][1][0]
			c[1][1][1]
		c[1][1][2]	
		c[1][1][3]	
		c[1][2] 1-d array name	c[1][2][0]
			c[1][2][1]
			c[1][2][2]
			c[1][2][3]

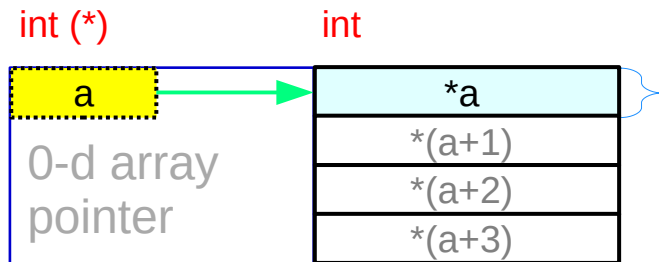


# Array **a** and pointer **a**



**1-d array **a**** specific array type

$\text{sizeof}(a)$



**pointer **a**** general pointer type

$\text{sizeof}(a) = \text{sizeof}(*a) * 4$

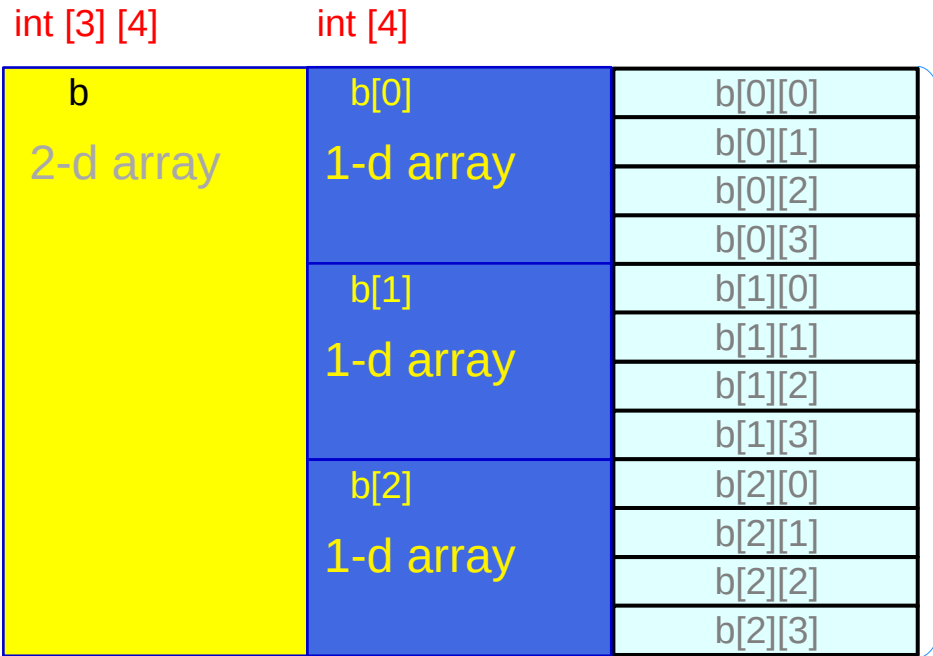
- a** is the name of a 1-d array
- a** also has a pointer type
- a** has the size of the array
- a** has the value of the starting address

**a** is a virtual array pointer

# Array **b** and pointer **b**

**2-d array **b**** specific array type

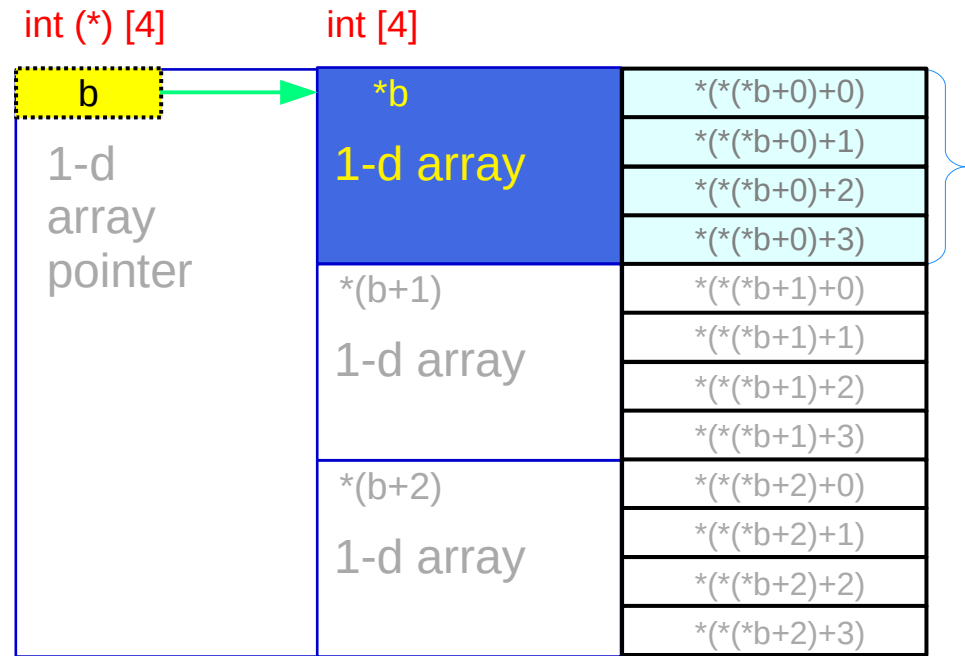
`sizeof(b)`



**b** is the name of a 2-d array  
**b** has the size of the array

**1-d array pointer **b**** general pointer type

`sizeof(b) = sizeof(*b) * 3`



**b** also has a 1-d array pointer type  
**b** has the value of the starting address

**b** is a virtual array pointer

# Array c

## 3-d array c

specific array type

sizeof(c)

c is the name of a 3-d array  
c has the size of the array

int [2][3][4]	int [3][4]	int [4]	
c 3-d array	c[0] 2-d array	c[0][0] 1-d array	c[0][0][0]
			c[0][0][1]
			c[0][0][2]
			c[0][0][3]
		c[0][1] 1-d array	c[0][1][0]
			c[0][1][1]
			c[0][1][2]
			c[0][1][3]
		c[0][2] 1-d array	c[0][2][0]
			c[0][2][1]
			c[0][2][2]
			c[0][2][3]
	c[1] 2-d array	c[1][0] 1-d array	c[1][0][0]
			c[1][0][1]
			c[1][0][2]
		c[1][0][3]	
		c[1][1] 1-d array	c[1][1][0]
			c[1][1][1]
			c[1][1][2]
			c[1][1][3]
		c[1][2] 1-d array	c[1][2][0]
			c[1][2][1]
			c[1][2][2]
			c[1][2][3]

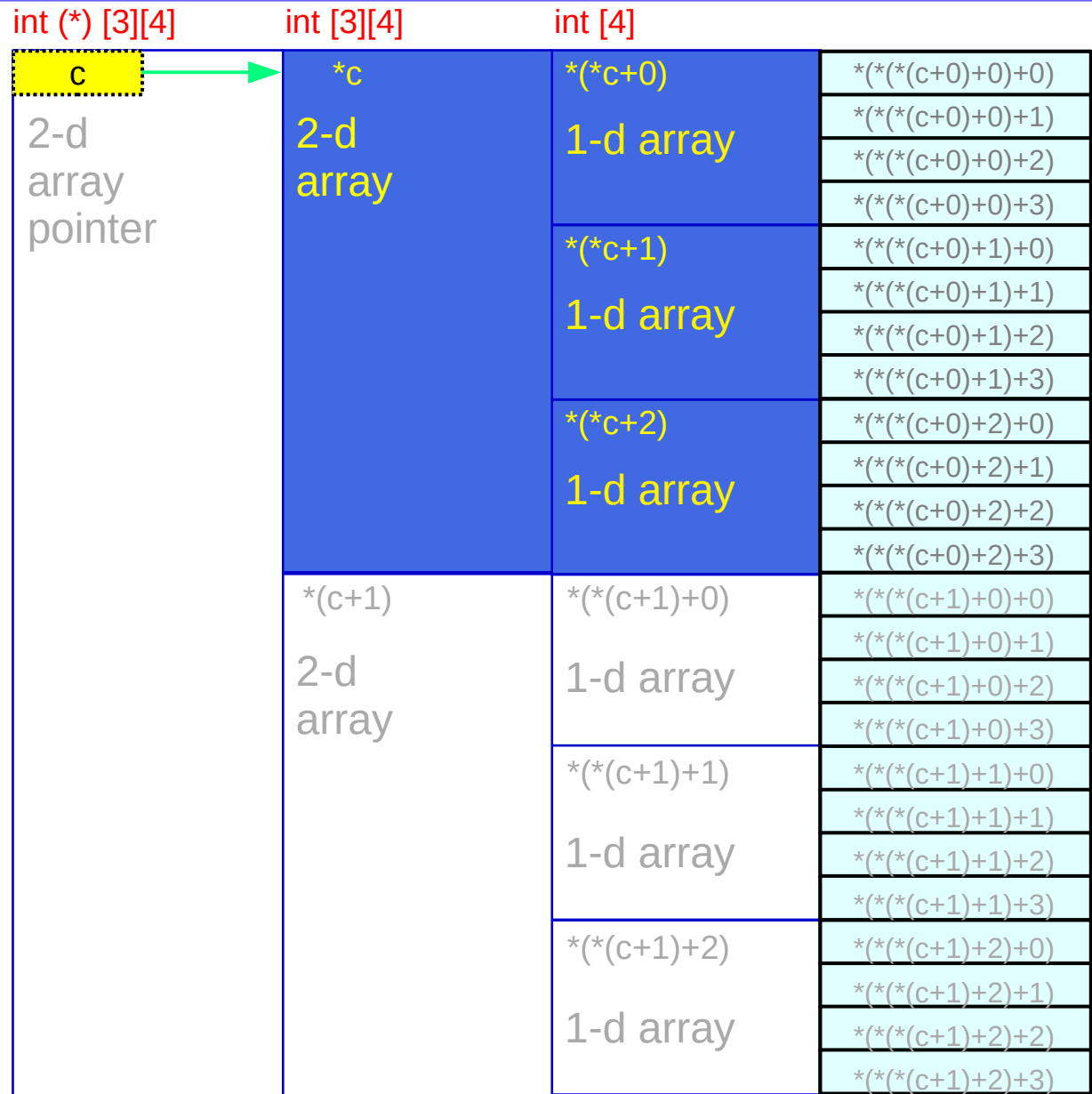
# Pointer c

## 2-d array pointer c

general pointer type

$\text{sizeof}(c) = \text{sizeof}(*c) * 2$

- c also has a 2-d array pointer type
- c has the value of the starting address
- c is a virtual array pointer



# Types of virtual array pointers in a 3-d array

```
int c[2][3][4];
```

`c[i][j][k]`

`c[i][j]`  
[k]

`c[i]`  
[j] [k]

`c`  
[i] [j] [k]

int

int [4]  
[k]

int [3][4]  
[j] [k]

int [2][3][4]  
[i] [j] [k]

array type (name)

int

int (\*)  
[k]

int (\*)[4]  
[j] [k]

int (\*)[3][4]  
[i] [j] [k]

array pointer type

# Sizes of virtual array pointers in a 3-d array

```
int c[2][3][4];
```

$\text{sizeof}(c[i][j][k]) = \text{sizeof}(\text{int})$

$\text{sizeof}(c[i][j])_{[k]} = \text{sizeof}(\text{int}) * 4_{[k]}$

$\text{sizeof}(c[i])_{[j] [k]} = \text{sizeof}(\text{int}) * 3 * 4_{[j] [k]}$

$\text{sizeof}(c)_{[i] [j] [k]} = \text{sizeof}(\text{int}) * 2 * 3 * 4_{[i] [j] [k]}$

# Address values of virtual array pointers in a 3-d array

```
int c[2][3][4];
```

$c[i][j][k] = \&c[i][j][k]$

$c[i][j]+k = \&c[i][j][0] + k * \text{sizeof}(c[i][j][k])$        $\text{sizeof}(*c[i][j]) = \text{sizeof}(c[i][j][0]) = \text{sizeof}(\text{int})$

$c[i]+j = \&c[i][0][0] + j * \text{sizeof}(c[i][j])$        $\text{sizeof}(*c[i]) = \text{sizeof}(c[i][0]) = \text{sizeof}(\text{int}) * 4$   
[k]

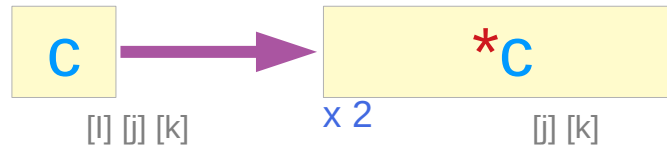
$c+i = \&c[0][0][0] + i * \text{sizeof}(c[i])$        $\text{sizeof}(*c) = \text{sizeof}(c[0]) = \text{sizeof}(\text{int}) * 3 * 4$   
[j] [k]

# Types in a multi-dimensional 3-d array

```
int c [2][3][4];
```

abstract data `int [2] [3][4]`

array pointer `int (*) [3][4]`

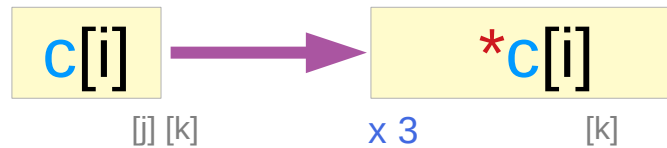


`int [3] [4]` abstract data

`int (*) [4]` array pointer

abstract data `int [3] [4]`

array pointer `int (*) [4]`

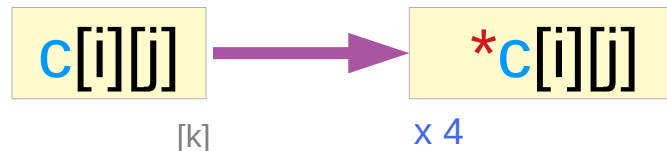


`int [4]` abstract data

`int (*)` array pointer

abstract data `int [4]`

array pointer `int (*)`



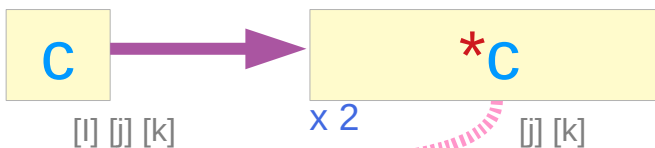
`int` primitive data



# Virtual array pointers and abstract data

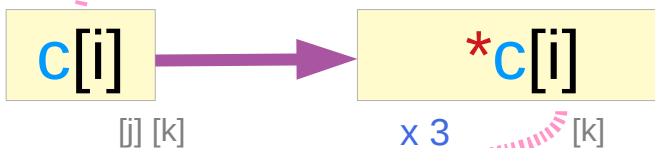
```
int c [2][3][4];
```

2-d array pointer `int (*) [3][4]`



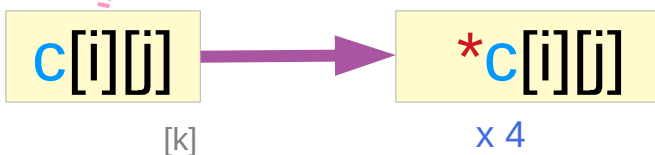
`int [3][4]` 2-d array

1-d array pointer `int (*) [4]`



`int [4]` 1-d array

0-d array pointer `int (*)`



`int` 0-d array

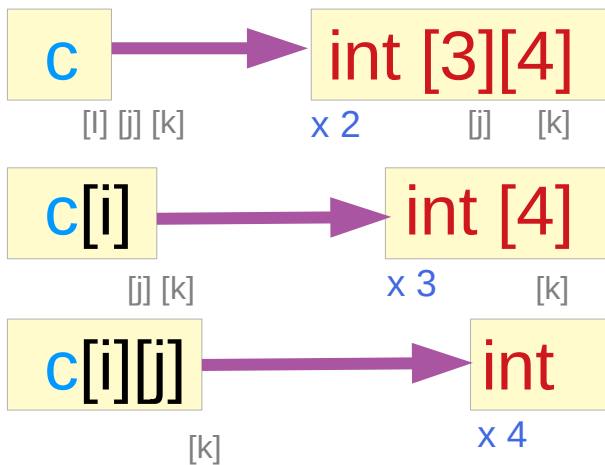
all these pointers are virtual, and take no actual memory locations

exploiting the **contiguity** of allocated memory locations

# Abstract data sizes

```
int c [2][3][4];
```

the size of a pointer type is fixed  
Here, the sizes of virtual pointers are shown  
i.e, the sizes of different abstract data types



```
sizeof( c) = sizeof(int [2][3][4])
sizeof(*c) = sizeof(int [3][4])

sizeof( c[i]) = sizeof(int [3][4])
sizeof(*c[i]) = sizeof(int [4])

sizeof( c[i][j]) = sizeof(int [4])
sizeof(*c[i][j]) = sizeof(int)
```

all are sizes of arrays

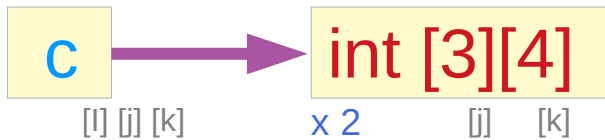
`c`, `c[i]`, `c[i][j]` are virtual array pointers  
and they are also abstract data (arrays)

when sizes are considered,  
view them as abstract data (arrays)

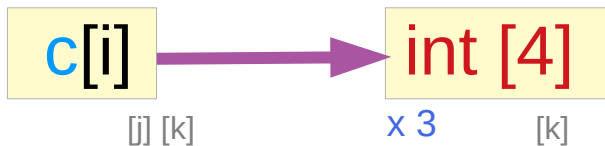
# Virtual array pointer sizes and abstract data sizes

```
int c [2][3][4];
```

$$\text{size of a virtual array pointer} = \text{size of the pointed abstract data type} * \text{the number of such types}$$



$$\text{sizeof}( c ) = \text{sizeof}( *c ) * 2$$



$$\text{sizeof}( c[i] ) = \text{sizeof}( *c[i] ) * 3$$



$$\text{sizeof}( c[i][j] ) = \text{sizeof}( *c[i][j] ) * 4$$

# Sizes of array pointer types

```
int c [2][3][4];
```

`c`  $\rightarrow$  `int [3][4]`  
[i] [j] [k]                    [j] [k]

`c[i]`  $\rightarrow$  `int [4]`  
[j] [k]                    [k]

`c[i][j]`  $\rightarrow$  `int`  
[k]

not real array pointers  
virtual array pointers



`c` `int (*)[3][4]` = sizeof(c)  
sizeof(int (\*) [3][4]) = pointer size  $\neq$  sizeof(c)

`c[i]` `int (*) [4]` = sizeof(c[i])  
sizeof(int (\*) [4]) = pointer size  $\neq$  sizeof(c[i])

`c[i][j]` `int [4]` = sizeof(c[i][j])  
sizeof(int [4]) = pointer size  $\neq$  sizeof(c[i][j])



4 bytes for 32-bit machines  
8 bytes for 64-bit machines

# Virtual array pointer increment size

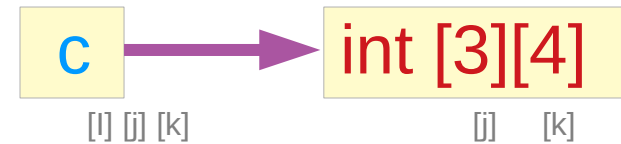
```
int c [2][3][4];
```

c points to a **2-d** array  
increment size: `sizeof(int[2][3][4])`

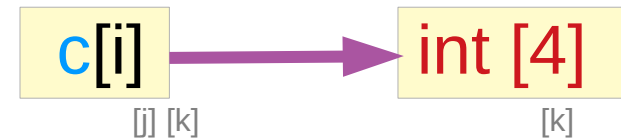
c[i] points to an **1-d** array  
increment size: `sizeof(int[3][4])`

c[i][j] points to an integer  
increment size: `sizeof(int[4])`

int (\*) [3][4]



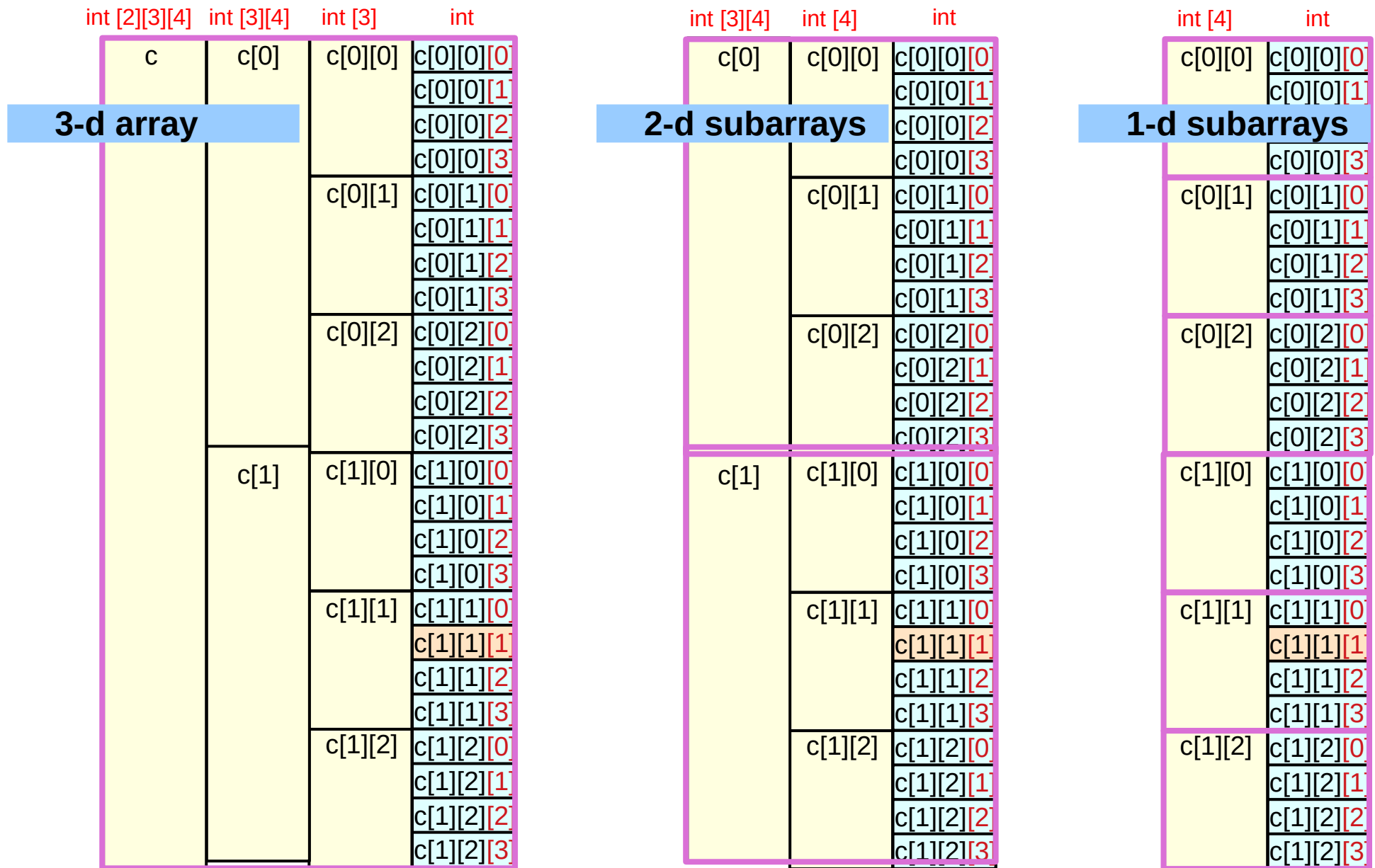
int (\*) [4]



int (\*)

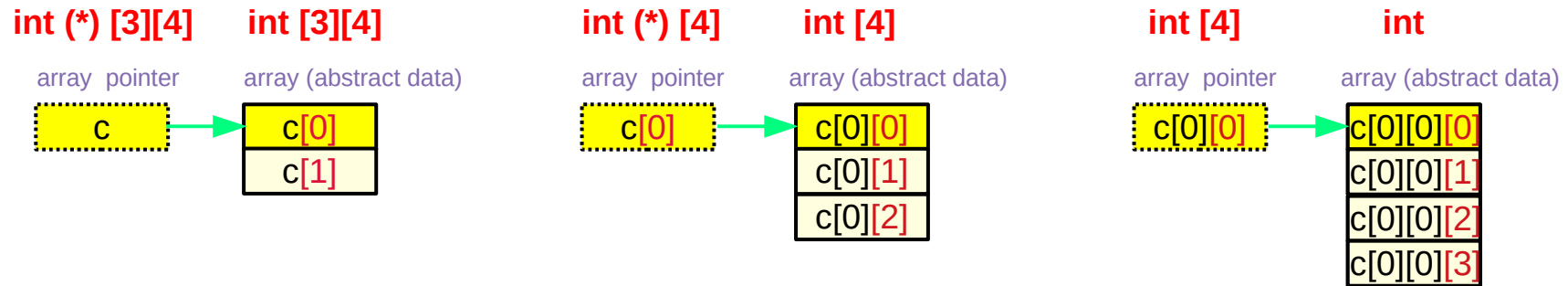


# Subarrays in a 3-d array



# Virtual array pointer c, c[0], c[0][0] – types and sizes

## Types – array pointers



## Sizes – abstract data

`sizeof(c)`  
`sizeof(int [2][3][4])`  
`sizeof(int) * 2 * 3 * 4`

`sizeof(int [2][3][4]) = 96`  
`sizeof(int (*)[3][4]) = 4 / 8`

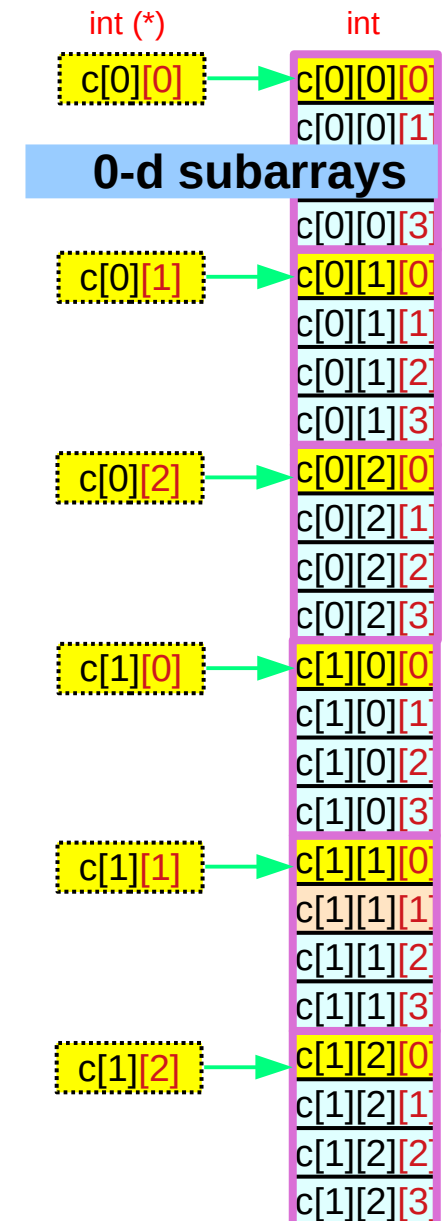
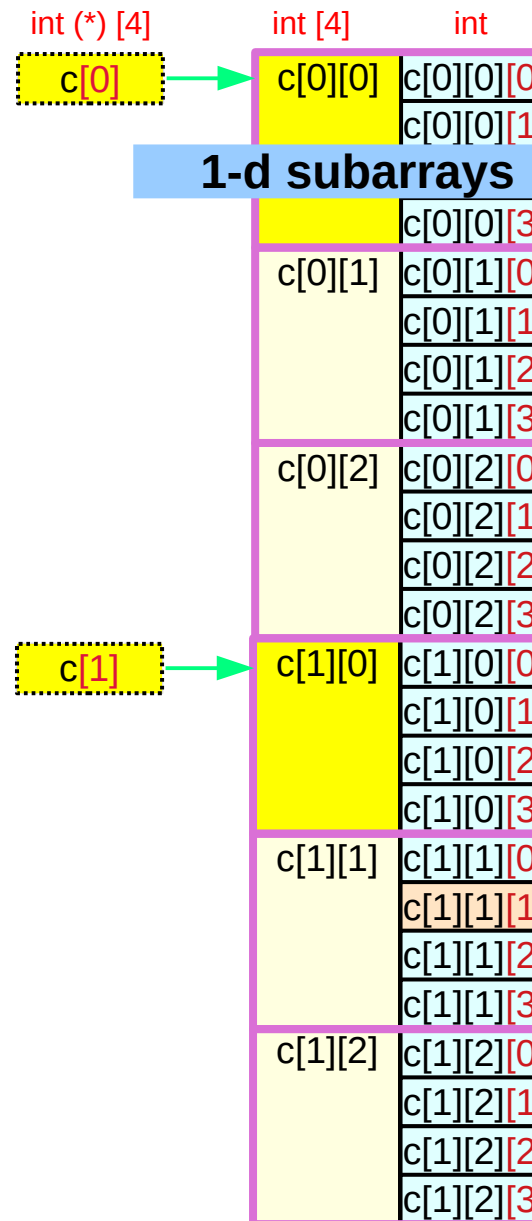
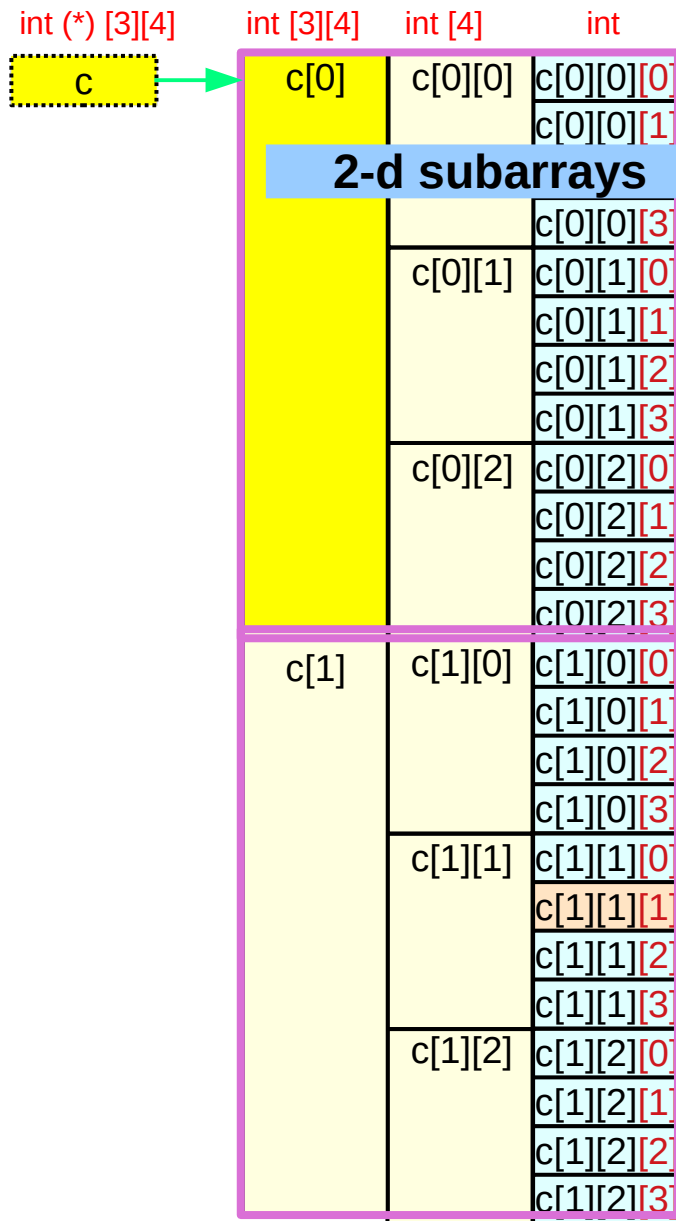
`sizeof(c[0])`  
`sizeof(int [3][4])`  
`sizeof(int) * 3 * 4`

`sizeof(int [3][4]) = 48`  
`sizeof(int (*)[4]) = 4 / 8`

`sizeof(c[0][0])`  
`sizeof(int [4])`  
`sizeof(int) * 4`

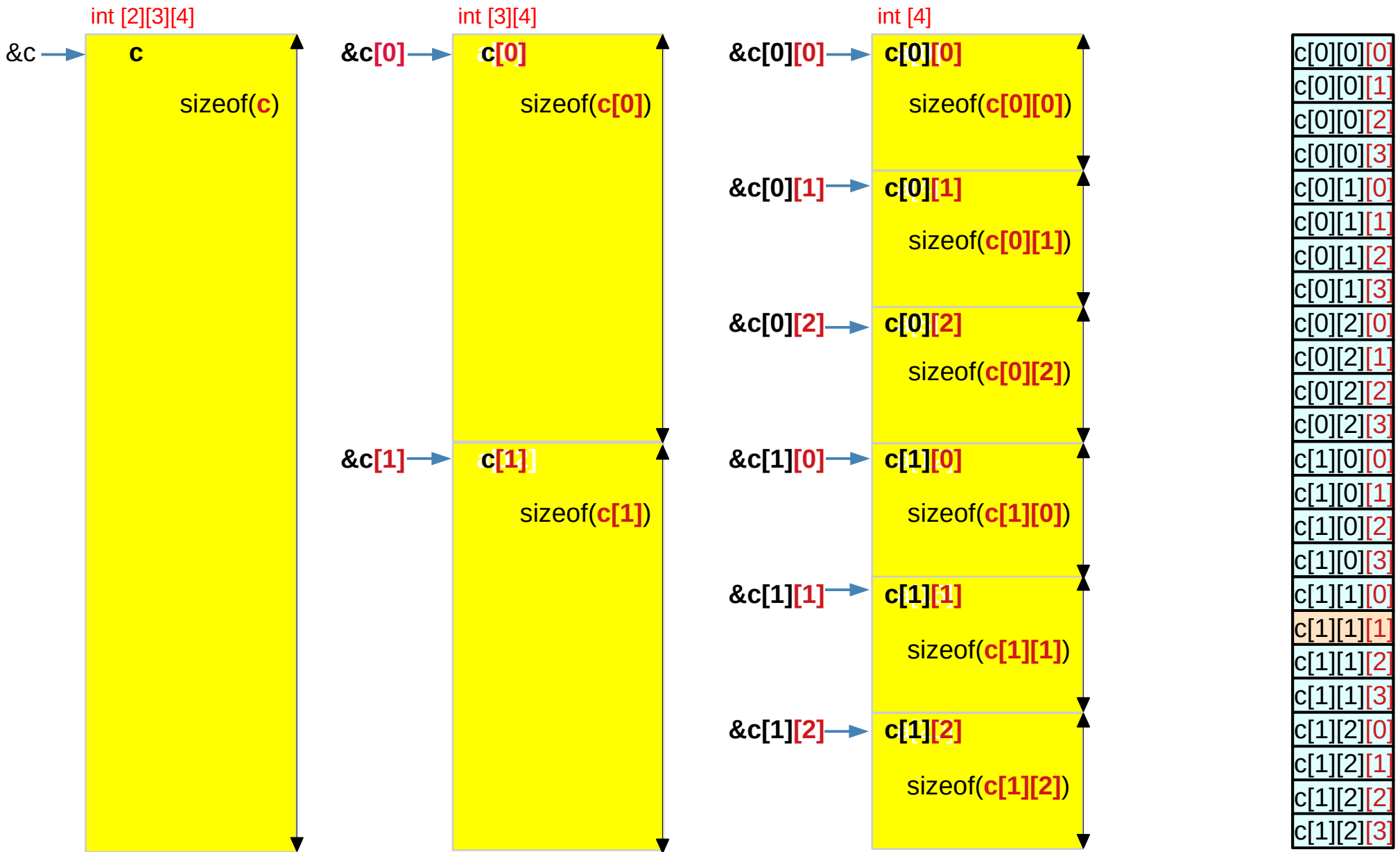
`sizeof(int [4]) = 16`  
`sizeof(int (*) = 4 / 8`

# Pointers to subarrays in a 3-d array





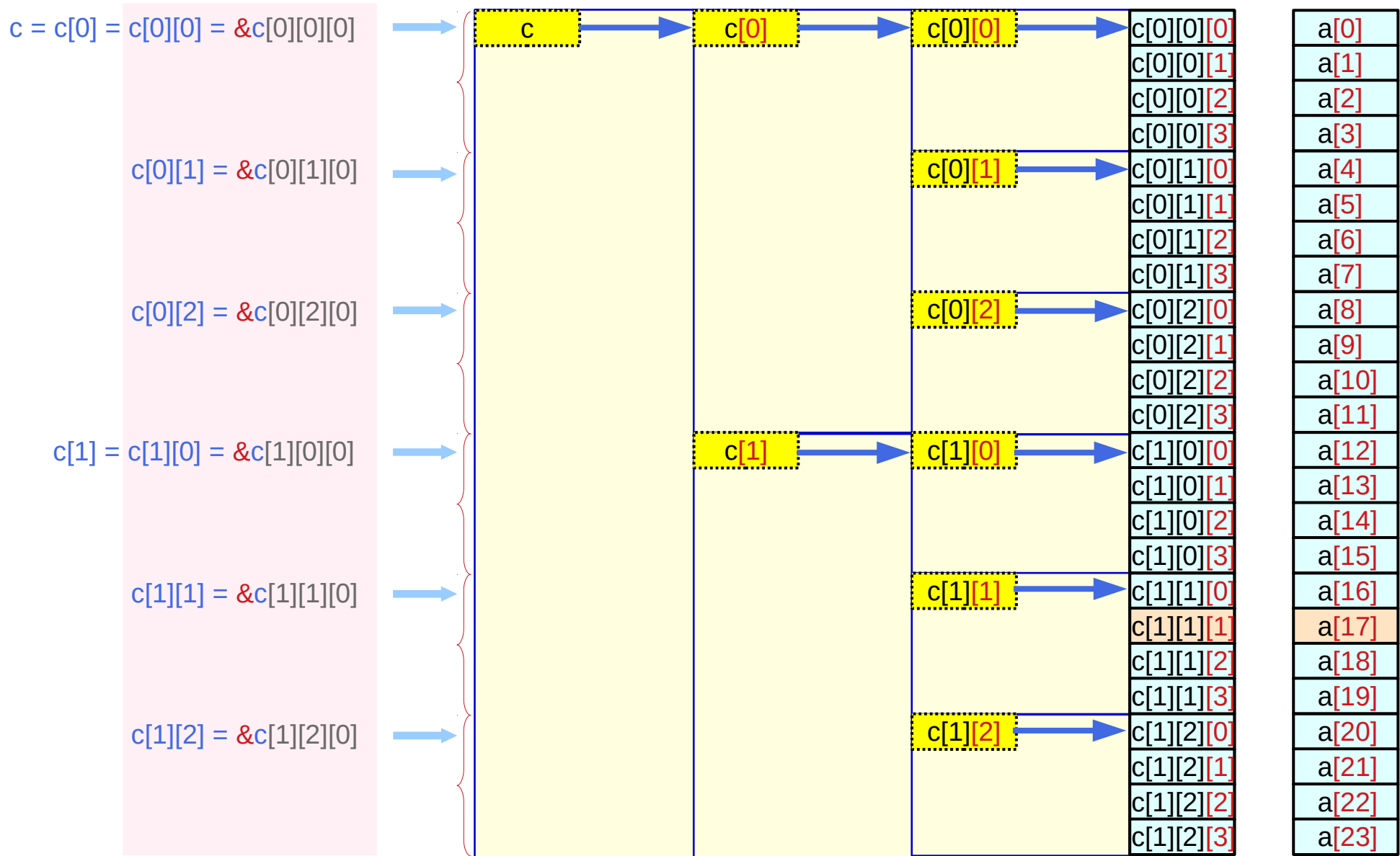
# Abstract Data $c$ , $c[i]$ , $c[i][j]$ – start addresses and sizes



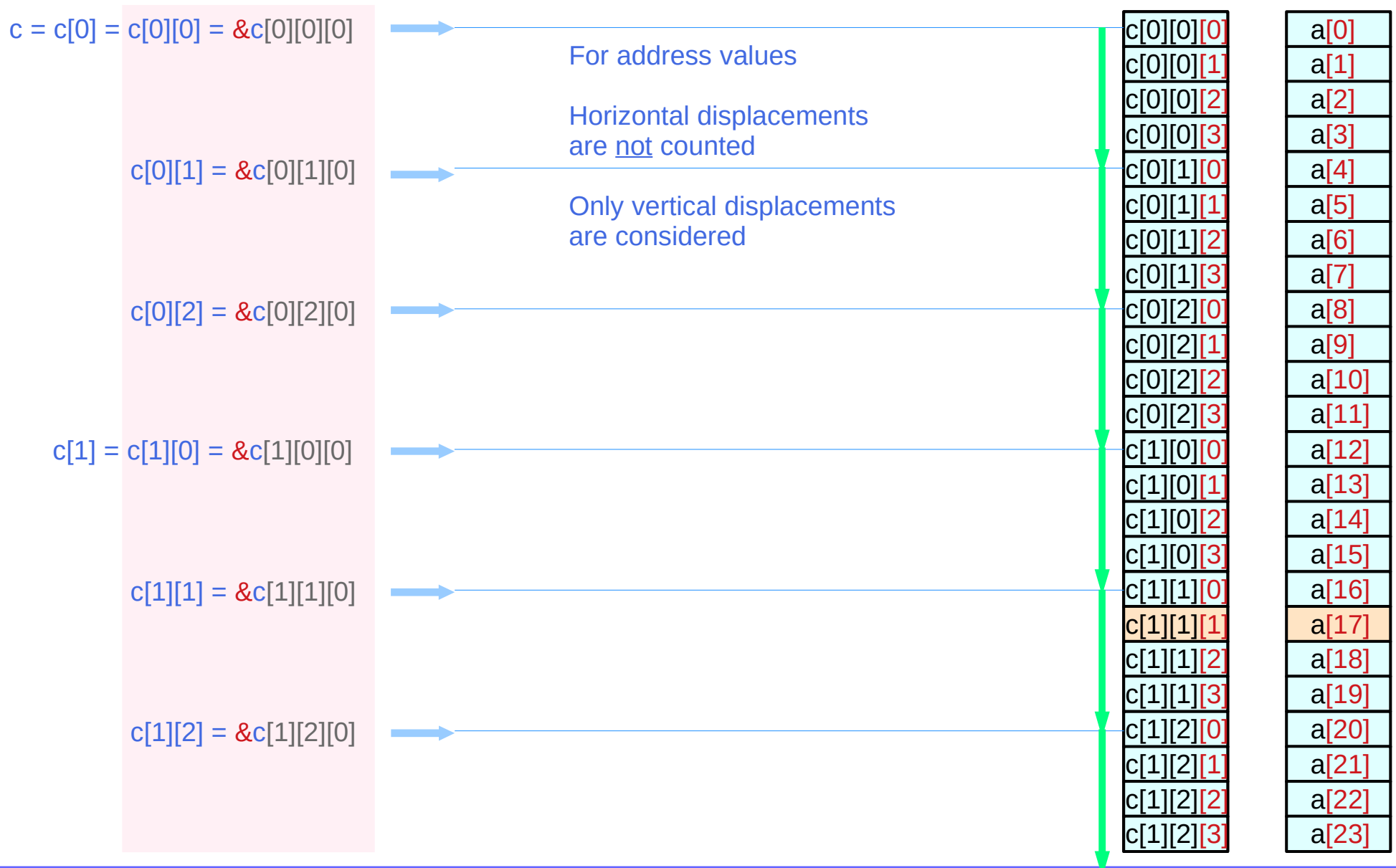
# Virtual array pointers – types, sizes, and values

<b>int c[2][3][4];</b>	<b>c[i][j]</b>	<b>c[i][j][0]</b>	
type	int [4] int (*)	int int	<ul style="list-style-type: none"> <li>abstract data type</li> <li>array pointer type</li> </ul>
size	sizeof(c[i][j]) =	sizeof(c[i][j][0]) * 4	= sizeof(int) * 4
value (address)	c[i][j] =	&c[i][j][0]	
<b>int c[2][3][4];</b>	<b>c[i]</b>	<b>c[i][0]</b>	
type	int [3][4] int (*)[4]	int [4] int (*)	<ul style="list-style-type: none"> <li>abstract data type</li> <li>array pointer type</li> </ul>
size	sizeof(c[i]) =	sizeof(c[i][0]) * 3	= sizeof(int) * 4 * 3
value (address)	c[i] =	&c[i][0][0]	
<b>int c[2][3][4];</b>	<b>c</b>	<b>c[0]</b>	
type	int [2][3][4] int (*)[3][4]	int [3][4] int (*)[4]	<ul style="list-style-type: none"> <li>abstract data type</li> <li>array pointer type</li> </ul>
size	sizeof(c) =	sizeof(c[0]) * 2	= sizeof(int) * 4 * 3 * 2
value (address)	c =	&c[0][0][0]	

# Virtual array pointer $c$ , $c[i]$ , $c[i][j]$ – values (addresses)



# Virtual array pointer $c$ , $c[i]$ , $c[i][j]$ – vertical displacement



# Virtual array pointer c, c[i], c[i][j] – values and types

$c = c[0] = c[0][0] = \&c[0][0][0]$  means

$c[0][1] = \&c[0][1][0]$  means

$c[0][2] = \&c[0][2][0]$  means

$c[1] = c[1][0] = \&c[1][0][0]$  means

$c[1][1] = \&c[1][1][0]$  means

$c[1][2] = \&c[1][2][0]$  means

$\text{value}(c) = \text{value}(c[0]) = \text{value}(c[0][0]) = \text{value}(\&c[0][0][0])$ $\text{type}(c) \neq \text{type}(c[0]) \neq \text{type}(c[0][0]) = \text{type}(\&c[0][0][0])$ $\text{int} (*) [3][4] \quad \text{int} (*) [4] \quad \text{int} * \quad \text{int} *$	$\text{value}(c[0][1]) = \text{value}(\&c[0][1][0])$ $\text{type}(c[0][1]) = \text{type}(\&c[0][1][0])$ $\text{int} * \quad \text{int} *$
$\text{value}(c[0][2]) = \text{value}(\&c[0][2][0])$ $\text{type}(c[0][2]) = \text{type}(\&c[0][2][0])$ $\text{int} * \quad \text{int} *$	$\text{value}(c[1]) = \text{value}(c[1][0]) = \text{value}(\&c[1][0][0])$ $\text{type}(c[1]) \neq \text{type}(c[1][0]) = \text{type}(\&c[1][0][0])$ $\text{int} (*) [4] \quad \text{int} * \quad \text{int} *$
$\text{value}(c[1][1]) = \text{value}(\&c[1][1][0])$ $\text{type}(c[1][1]) = \text{type}(\&c[1][1][0])$ $\text{int} * \quad \text{int} *$	$\text{value}(c[1][2]) = \text{value}(\&c[1][2][0])$ $\text{type}(c[1][2]) = \text{type}(\&c[1][2][0])$ $\text{int} * \quad \text{int} *$

# Summary of virtual array pointers in a 3-d array

$$c[i] \equiv *(c + i)$$

int (\*) [3][4] 2-d array pointer  $c$   
int [2] [3][4] 3-d array name  $c$

address value  $c + i$

$\&c[0][0][0] + i * \text{sizeof}(*c)$   
 $\&c[0][0][0] + i * \text{sizeof}(c[0])$   
 $\&c[0][0][0] + i * 4 * 3 * 4$

leading elements

$c[0][0][0]$

$$c[i][j] \equiv *(c[i] + j)$$

int (\*) [4] 1-d array pointers  $c[i]$   
int [3] [4] 2-d array names  $c[i]$

address value  $c[i] + j$

$\&c[i][0][0] + j * \text{sizeof}(*c[i])$   
 $\&c[i][0][0] + j * \text{sizeof}(c[i][0])$   
 $\&c[i][0][0] + j * 4 * 4$

leading elements

$c[0][0][0]$

$c[1][0][0]$

$$c[i][j][k] \equiv *(c[i][j] + k)$$

int (\*) 0-d array pointers  $c[i][j]$   
int [4] 1-d array names  $c[i][j]$

address value  $c[i][j] + k$

$\&c[i][j][0] + k * \text{sizeof}(*c[i][j])$   
 $\&c[i][j][0] + k * \text{sizeof}(c[i][j][0])$   
 $\&c[i][j][0] + k * 4$

leading elements

$c[0][0][0]$

$c[0][1][0]$

$c[0][2][0]$

$c[1][0][0]$

$c[1][1][0]$

$c[1][2][0]$

# Sub-array properties in multi-dimensional arrays

`int c [2][3][4];` → 3-d access `c [i][j][k]`

2-d array pointer	<code>c</code>	<code>int (*) [3][4]</code>
1-d array pointers	<code>c[i]</code>	<code>int (*) [4]</code>
0-d array pointers	<code>c[i][j]</code>	<code>int (*)</code>

# Hierarchical sub-arrays in a 3-d array

```
int c [L][M][N];
```

```
c [i][j][k]
```

left-to-right associativity

Array Names and Types

Pointers to hierarchical sub-arrays

c	[i]	[j][k]
c[i]	[j]	[k]
c[i][j]	[k]	

c	3-d array names	int (*) [M][N]	2-d array pointer
c[i]	2-d array names	int (*) [N]	1-d array pointer
c[i][j]	1-d array names	int (*)	0-d array pointer



# Associativity and Equivalence Relations

left-to-right associativity

$$((c[i])[j])[k]$$

$$\equiv$$

left-to-right associativity

$$*(*(*(c+i)+j)+k)$$

$$X[n]$$

$$\equiv$$

$$*(X+n)$$

given  $c[i][j]$

$$c[i][j][k]$$

$$\equiv$$

$$*(c[i][j]+k)$$

for all  $k$

given  $c[i]$

$$c[i][j]$$

$$\equiv$$

$$*(c[i]+j)$$

for all  $j$

given  $c$

$$c[i]$$

$$\equiv$$

$$*(c+i)$$

for all  $i$

# Requirements for the expression $c[i][j][k]$

for a given  $c[i][j]$ , for all  $k$

$$c[i][j][k] = *(c[i][j] + k)$$

for a given  $c[i]$ , for all  $j$

$$c[i][j] = *(c[i] + j)$$

for a given  $c$ , for all  $i$

$$c[i] = *(c + i)$$

## 3 contiguity requirements

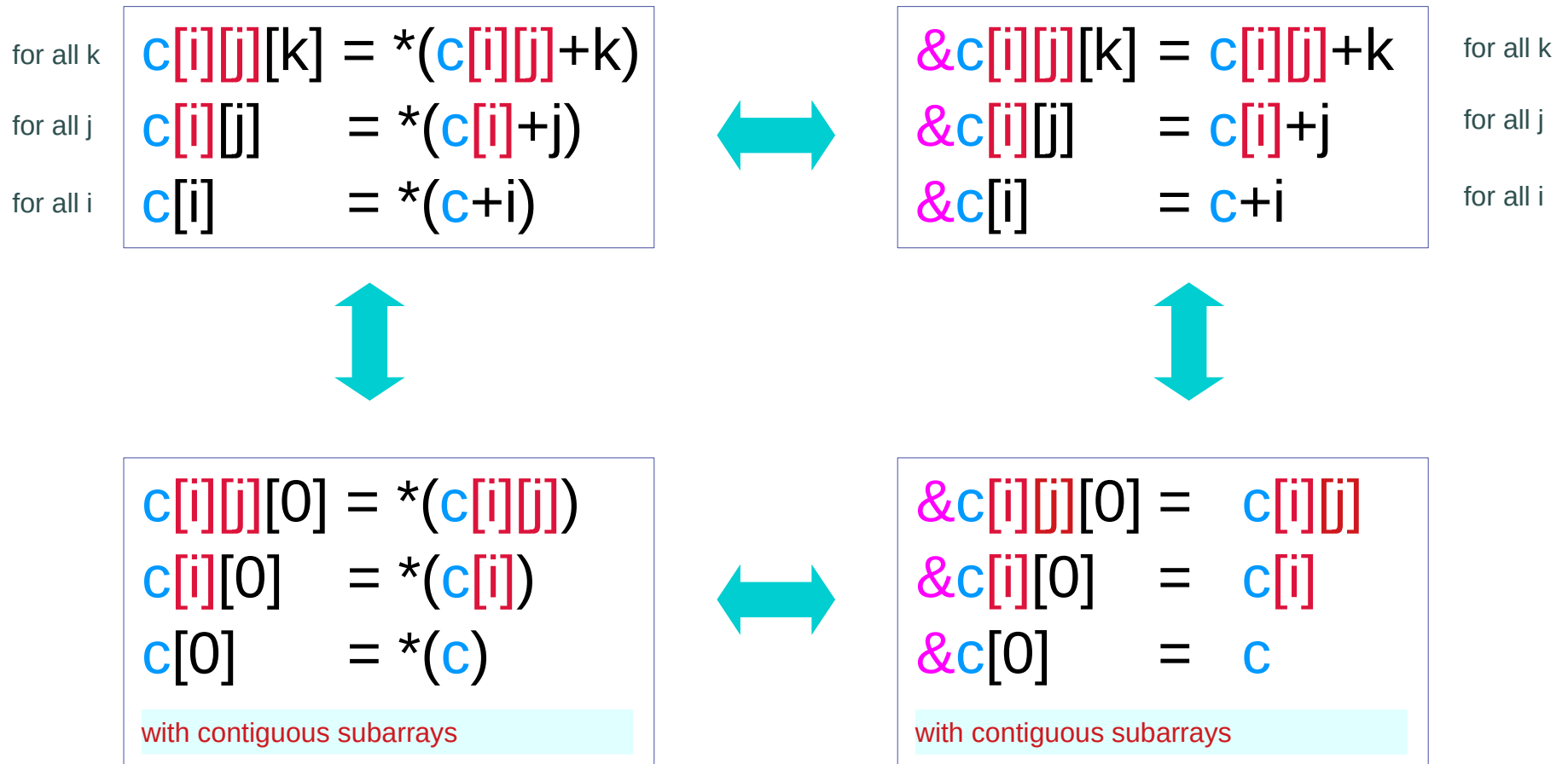
for a given  $c[i][j]$ , contiguous  $c[i][j][k]$ 's

for a given  $c[i]$ , contiguous  $c[i][j]$ 's

for a given  $c$ , contiguous  $c[i]$ 's

for a given subarray pointer  
contiguous subarrays

# Equivalent requirements for the expression $c[i][j][k]$



# 3-d access pattern $c[i][j][k]$

## General requirements

```
c [i][j][k]
```



```
&c[i][j][k] = c[i][j]+k  for all k  
&c[i][j]    = c[i]+j    for all j  
&c[i]       = c+i       for all i
```

## Pointer array approach

```
int** c[2];  
int*  b[2*3];  
int   c[2*3*4];
```

```
c[i][j][k] :: int  
c[i][j]    :: int *  
c[i]       :: int **  
c          :: int ***
```

```
c[i] ← &b[i*3]  
b[j] ← &a[j*4]
```

with contiguous a, b, c

**Explicit**  
Arrays of pointers with  
Multiple Indirection

## N-dim Array approach

```
int c[2][3][4];
```

```
c[i][j][k] :: int  
c[i][j]    :: int [4]  
c[i]       :: int [3][4]  
c          :: int [2][3][4]
```

```
c[i][j] ← &c[i][j][0]  
c[i]    ← &c[i][0][0]  
c       ← &c[0][0][0]
```

with contiguous c

**Implicit**  
Nested  
Virtual Array Pointers

# 3-d access pattern $c[i][j][k]$ – array pointer approach

## General requirements

$c[i][j][k]$



$\&c[i][j][k] = c[i][j] + k$  for all  $k$   
 $\&c[i][j] = c[i] + j$  for all  $j$   
 $\&c[i] = c + i$  for all  $i$



## N-dim array approach

```
int c[2][3][4];
```

```
c[i][j][k] :: int  
c[i][j]    :: int [4]  
c[i]       :: int [3][4]  
c          :: int [2][3][4]
```

```
c[i][j] ← &c[i][j][0]  
c[i]    ← &c[i][0][0]  
c       ← &c[0][0][0]
```

with contiguous  $c$

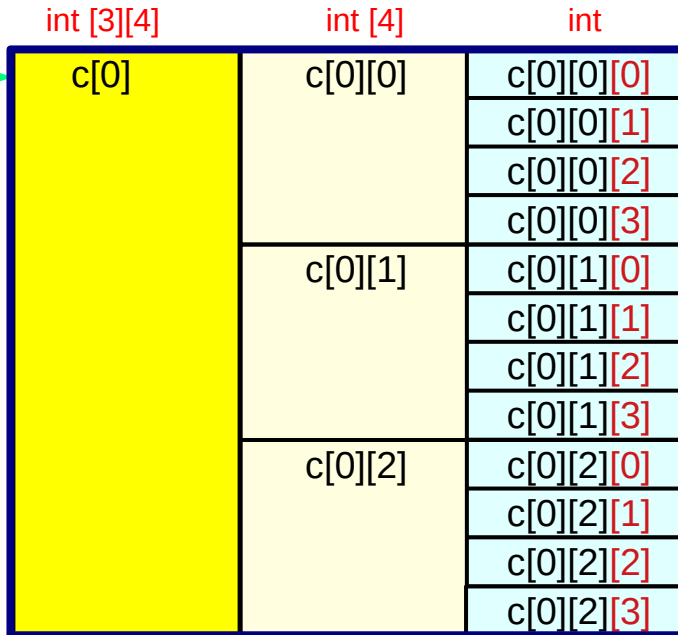
**Implicit  
Nested  
Virtual Array Pointers**

# Virtual pointers to subarrays in a 3-d array

virtual 2-d array pointer

$\text{sizeof}(c) = \text{sizeof}(c[0]) * 2$

$\text{int} (*) [3][4]$   
c



the first 2-d subarray

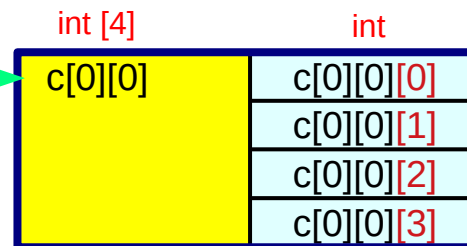
$\text{sizeof}(c[0]) = \text{sizeof}(\text{int} [3][4])$

```
int c [2][3][4];
```

virtual 1-d array pointer

$\text{sizeof}(c[0]) = \text{sizeof}(c[0][0]) * 3$

$\text{int} (*) [4]$   
c[0]



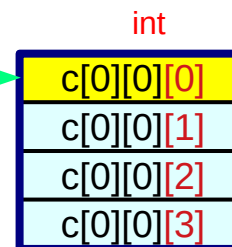
the first 1-d subarray

$\text{sizeof}(c[0][0]) = \text{sizeof}(\text{int} [4])$

virtual 0-d array pointer

$\text{sizeof}(c[0][0]) = \text{sizeof}(c[0][0][0]) * 4$

$\text{int} (*)$   
c[0][0]



the first 0-d subarray

$\text{sizeof}(c[0][0][0]) = \text{sizeof}(\text{int})$

# Virtual assignments

```
int c [2][3][4];
```



```
c [i][j][k];
```

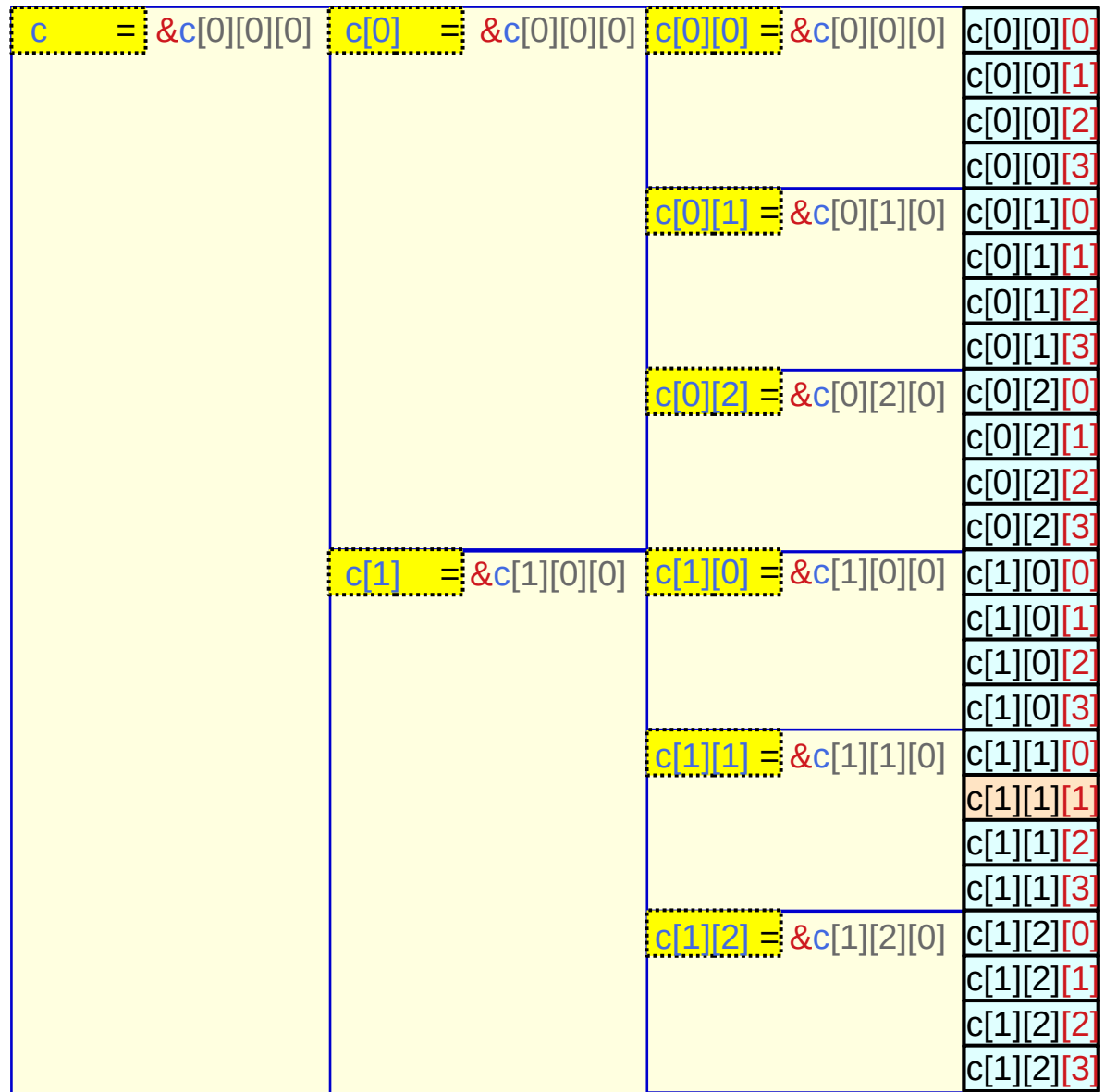
## virtual assignments

```
c ← &c[0][0][0]
```

```
c[i] ← &c[i][0][0]
```

```
c[i][j] ← &c[i][j][0]
```

row major ordering  
contiguous linear layout



# Virtual assignments and type casts

```
int c [2][3][4];
```



```
c [i][j][k];
```

## virtual assignments

```
c ← &c[0][0][0]  
c[i] ← &c[i][0][0]  
c[i][j] ← &c[i][j][0]
```

row major ordering  
contiguous linear layout

```
c ← (int (*)[3][4]) &c[0][0][0]  
c[i] ← (int (*)[4]) &c[i][0][0]  
c[i][j] ← (int *) &c[i][j][0]
```

type casts      address values

if c, c[i], c[i][j] were real pointer variables,  
type casts would be needed



# Dual types of `c`, `c[i]`, `c[i][j]`

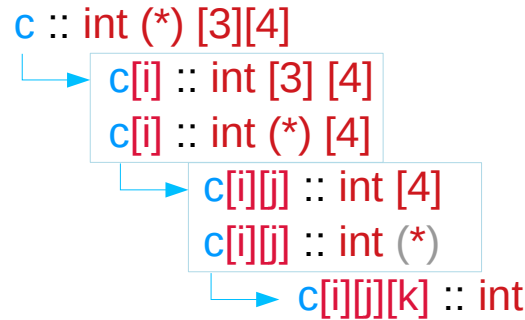
```
int c [2][3][4];
```

```
&c[i][j][0] = c[i][j]
&c[i][0]    = c[i]
&c[0]      = c
```

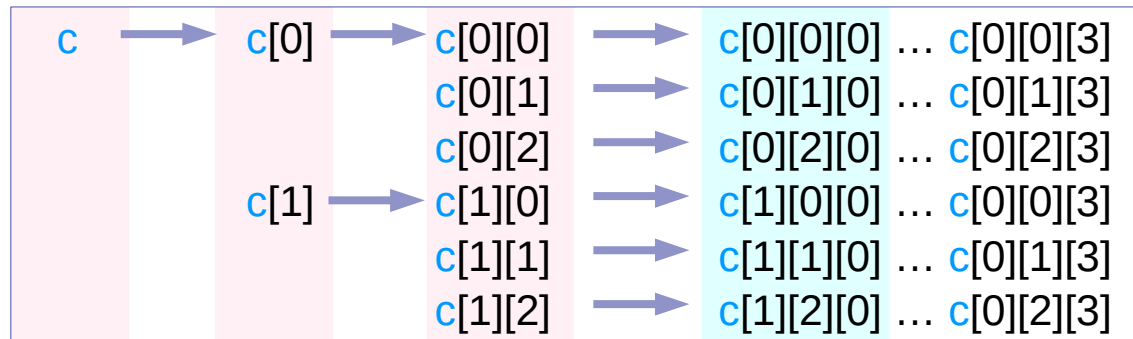
with contiguous subarrays

```
&c[i][j][k] = c[i][j]+k for all k
&c[i][j]    = c[i]+j   for all j
&c[i]       = c+i     for all i
```

```
c [i][j][k]
```



- 2-d array pointers
- 2-d arrays
- 1-d array pointers
- 1-d arrays
- 0-d array pointers
- 0-d arrays (integers)



int [2] [3][4]	int [3] [4]	int [4]	int	...	int
int (*) [3][4]	int (*) [4]	int (*)	int	...	int
pointers to a 2-d array	pointers to a 1-d array	1-d array names	leading element of 4-integer array		

# Values of $c$ , $c[i]$ , $c[i][j]$

```
int c [2][3][4];
```

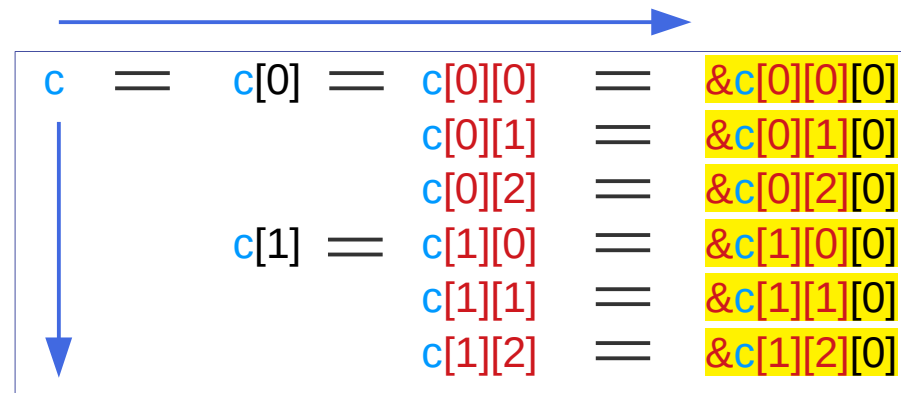
```
&c[i][j][0] = c[i][j]  
&c[i][0] = c[i]  
&c[0] = c
```

with contiguous subarrays

```
&c[i][j][k] = c[i][j]+k for all k  
&c[i][j] = c[i]+j for all j  
&c[i] = c+i for all i
```

```
c [i][j][k]
```

**virtual** array pointers have address values in each row in the following figure have the same address value



**Horizontal displacements** are not counted only **vertical displacements** are considered for address values

**virtual assignments**

```
c[i][j] = &c[i][j][0]  
c[i] = &c[i][0][0]  
c = &c[0][0][0]
```

# Finding address values of $c$ , $c[i]$ , $c[i][j]$

```
int c [2][3][4];
```

```
&c[i][j][0] = c[i][j]
&c[i][0]    = c[i]
&c[0]      = c
```

with contiguous subarrays

```
&c[i][j][k] = c[i][j]+k for all k
&c[i][j]    = c[i]+j   for all j
&c[i]       = c+i     for all i
```

```
c [i][j][k]
```

virtual assignments

```
c[i][j] = &c[i][j][0]
c[i]    = &c[i][0][0]
c       = &c[0][0][0]
```

append  $[0]$  to the right

$c$	$\xrightarrow{+[0]}$	$c[0]$	$\xrightarrow{+[0]}$	$c[0][0]$	$\xrightarrow{+[0]}$	$\&c[0][0][0]$
				$c[0][1]$	$\xrightarrow{+[0]}$	$\&c[0][1][0]$
				$c[0][2]$	$\xrightarrow{+[0]}$	$\&c[0][2][0]$
		$c[1]$	$\xrightarrow{+[0]}$	$c[1][0]$	$\xrightarrow{+[0]}$	$\&c[1][0][0]$
				$c[1][1]$	$\xrightarrow{+[0]}$	$\&c[1][1][0]$
				$c[1][2]$	$\xrightarrow{+[0]}$	$\&c[1][2][0]$

$\text{int (*) [3][4]$     $\text{int (*) [4]$

$\text{int [4]}$

$\text{int}$

$c[0][0][0]$  :  
leading  
elements  
of  $c$

$c[i][0][0]$  :  
leading  
elements  
of  $c[i]$

$c[i][j][0]$  :  
leading  
elements  
of  $c[i][j]$

$\&c[0][0][0]$

$\&c[0][0][0]$

$\&c[1][0][0]$

$\&c[0][0][0]$   
 $\&c[0][1][0]$   
 $\&c[0][2][0]$   
 $\&c[1][0][0]$   
 $\&c[1][1][0]$   
 $\&c[1][2][0]$

# Finding sub-array names with the address $\&c[i][j][0]$

```
int c [2][3][4];
```

```
&c[i][j][0] = c[i][j]
&c[i][0]    = c[i]
&c[0]       = c
```

with contiguous subarrays

```
&c[i][j][k] = c[i][j]+k for all k
&c[i][j]    = c[i]+j   for all j
&c[i]       = c+i     for all i
```

```
c [i][j][k]
```

virtual assignments

```
c[i][j] = &c[i][j][0]
c[i]    = &c[i][0][0]
c       = &c[0][0][0]
```

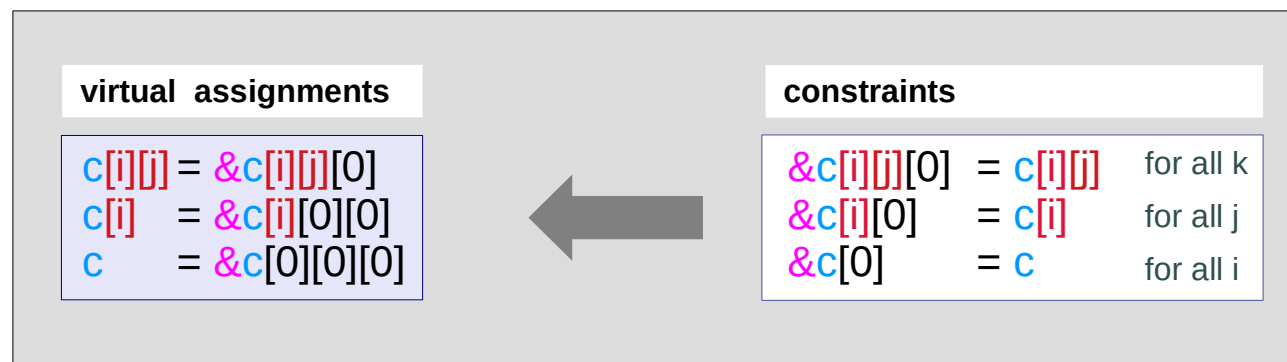
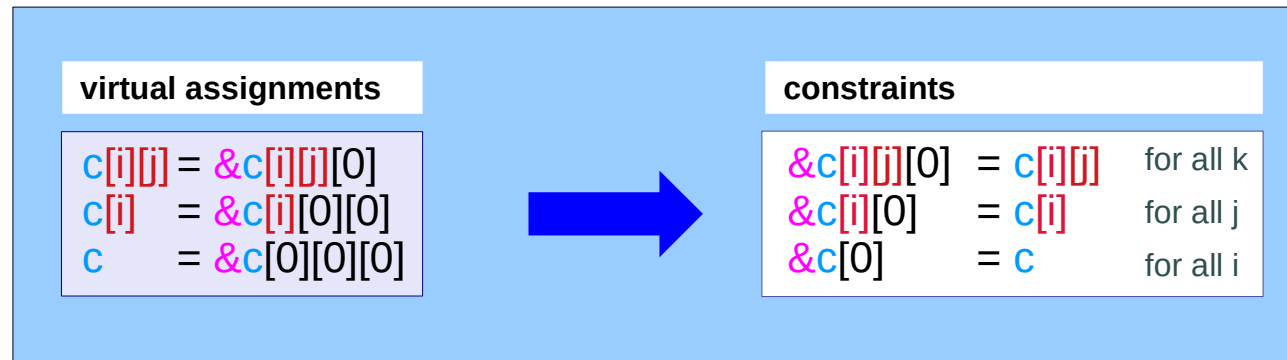
delete [0] from the right

$\&c[0][0][0]$	$\underline{\underline{-[0]}}$	$c[0][0]$	$\underline{\underline{-[0]}}$	$c[0]$	$\underline{\underline{-[0]}}$	$c$
$\&c[0][1][0]$	$\underline{\underline{-[0]}}$	$c[0][1]$				
$\&c[0][2][0]$	$\underline{\underline{-[0]}}$	$c[0][2]$				
$\&c[1][0][0]$	$\underline{\underline{-[0]}}$	$c[1][0]$	$\underline{\underline{-[0]}}$	$c[1]$		
$\&c[1][1][0]$	$\underline{\underline{-[0]}}$	$c[1][1]$				
$\&c[1][2][0]$	$\underline{\underline{-[0]}}$	$c[1][2]$				

int                      int [4]                      int (\*) [4]                      int (\*) [3][4]

$c[0][0][0]$  is the leading element of  $c[0][0]$ ,  $c[0]$ ,  $c$   
 $c[0][1][0]$  is the leading element of  $c[0][1]$   
 $c[0][2][0]$  is the leading element of  $c[0][2]$   
 $c[1][0][0]$  is the leading element of  $c[1][0]$ ,  $c[1]$   
 $c[1][1][0]$  is the leading element of  $c[1][1]$   
 $c[1][2][0]$  is the leading element of  $c[1][2]$

## multi-dimensional arrays



# General requirements for `c[i][j][k]`

```
int c [2][3][4];
```

```
&c[i][j][0] = c[i][j]
&c[i][0]    = c[i]
&c[0]      = c
```

with contiguous subarrays

```
&c[i][j][k] = c[i][j]+k for all k
&c[i][j]    = c[i]+j   for all j
&c[i]       = c+i      for all i
```



virtual assignments

```
c[i][j] = &c[i][j][0]
c[i]    = &c[i][0][0]
c       = &c[0][0][0]
```

## Virtual assignments

```
int (*)      c[i][j] = (int *) &c[i][j][0]
int (*) [4]  c[i]    = (int (*) [4]) &c[i][0][0]
int (*) [3][4] c      = (int (*) [3][4]) &c[0][0][0]
```

Pointer  
Types

## Sizes of abstract data types

```
int [4]      c[i][j] size = 4*4
int [3][4]   c[i]    size = 3*4*4
int [2][3][4] c       size = 2*3*4*4
```

Abstract Data  
Types

## Strides of array elements

```
c[i][j][0] stride = 4*4
c[i][0][0] stride = 3*4*4
c[0][0][0] stride = 2*3*4*4
```

```

k=[0:3] c[i][j][k] 4 integers
j=[0:2], k=[0:3] c[i][j][k] 3*4 integers
i=[0:1], j=[0:2], k=[0:3] c[i][j][k] 2*3*4 integers

```

# General requirements for `c[i][j][k]`

```
int c [2][3][4];
```

```
&c[i][j][0] = c[i][j]
&c[i][0]    = c[i]
&c[0]      = c
```

with contiguous subarrays

```
&c[i][j][k] = c[i][j]+k for all k
&c[i][j]    = c[i]+j   for all j
&c[i]       = c+i     for all i
```



virtual assignments

```
c[i][j] = &c[i][j][0]
c[i]    = &c[i][0][0]
c       = &c[0][0][0]
```

```
c[i][j]
c[i]
c
```

Pointer  
Types

has an address of  
has an address of  
has an address of

```
c[i][j][0]
c[i][0][0]
c[0][0][0]
```

```
c[i][j]+1
c[i]+1
c+1
```

Pointer  
Types

has an address of  
has an address of  
has an address of

```
c[i][j][1]
c[i][1][0]
c[1][0][0]
```

1 integer away  
4 integers away  
3\*4 integers away

```
c[i][j]+k
c[i]+j
c+i
```

Pointer  
Types

has an address of  
has an address of  
has an address of

```
c[i][j][k]
c[i][j][0]
c[i][0][0]
```

1\*k integers away  
4\*j integers away  
3\*4\*i integers away

# General requirements for `c[i][j][k]`

```
int c [2][3][4];
```

```
&c[i][j][0] = c[i][j]
&c[i][0]    = c[i]
&c[0]      = c
```

with contiguous subarrays

```
&c[i][j][k] = c[i][j]+k for all k
&c[i][j]    = c[i]+j   for all j
&c[i]       = c+i     for all i
```



virtual assignments

```
c[i][j] = &c[i][j][0]
c[i]    = &c[i][0][0]
c       = &c[0][0][0]
```

```
c[i][j]
c[i]
c
```

Abstract  
Data

starts from  
starts from  
starts from

```
&c[i][j][0]
&c[i][0][0]
&c[0][0][0]
```

```
c[i][j]
c[i]
c
```

Abstract  
Data

has a size of  
has a size of  
has a size of

```
c[i][j][k]
c[i][j][k]
c[i][j][k]
```

4 integers  
3\*4 integers  
2\*3\*4 integers



# General requirements for $c[i][j][k]$

```
int c [2][3][4];
```

```
&c[i][j][0] = c[i][j]
&c[i][0]    = c[i]
&c[0]       = c
```

with contiguous subarrays

```
&c[i][j][k] = c[i][j]+k for all k
&c[i][j]    = c[i]+j    for all j
&c[i]       = c+i       for all i
```



virtual assignments

```
c[i][j] = &c[i][j][0]
c[i]    = &c[i][0][0]
c       = &c[0][0][0]
```

```
c[i][j] = &c[i][j][0]
c[i]    = &c[i][0][0]
c       = &c[0][0][0]
```

```
c[i][j]
c[i]
c
```

starts from  
starts from  
starts from

```
&c[i][j][0]
&c[i][0][0]
&c[0][0][0]
```

Abstract  
Data

```
c[i]
c
```

and  
and

```
c[i][0]
c[0]
```

start from  
start from

```
&c[i][0][0]
&c[0][0][0]
```

Abstract  
Data

Abstract  
Data

```
c[i]
c
```

starts from  
starts from

```
&c[i][0]
&c[0]
```

Abstract  
Data

Abstract  
Data

# General requirements for `c[i][j][k]`

```
int c [2][3][4];
```

```
&c[i][j][0] = c[i][j]
&c[i][0]    = c[i]
&c[0]      = c
```

with contiguous subarrays

```
&c[i][j][k] = c[i][j]+k for all k
&c[i][j]    = c[i]+j   for all j
&c[i]       = c+i      for all i
```



virtual assignments

```
c[i][j] = &c[i][j][0]
c[i]    = &c[i][0][0]
c       = &c[0][0][0]
```

```
c[i]
c
```

and  
and

```
c[i][0]
c[0]
```

start from  
start from

```
&c[i][0][0]
&c[0][0][0]
```

Abstract  
Data

Abstract  
Data

```
c[i]
```

has a size of  
3\*4 integers

```
c[i][0]
```

```
c[i][1]
```

```
c[i][2]
```

has a size of  
4 integers  
has a size of  
4 integers  
has a size of  
4 integers

`c[i]` is the name of an array  
which has 3 elements  
`c[i][0]` `c[i][1]` `c[i][2]`

points to `c[i][0]`

```
&c[i][0] = c[i]
```

# General requirements for $c[i][j][k]$

```
int c [2][3][4];
```

```
&c[i][j][0] = c[i][j]
&c[i][0]    = c[i]
&c[0]      = c
```

with contiguous subarrays

```
&c[i][j][k] = c[i][j]+k for all k
&c[i][j]    = c[i]+j   for all j
&c[i]       = c+i      for all i
```



virtual assignments

```
c[i][j] = &c[i][j][0]
c[i]    = &c[i][0][0]
c       = &c[0][0][0]
```

Each  
Each  
Each

$c[i][j]$   
 $c[i]$   
 $c$

Abstract Data  
Types

sub-array contains  
sub-array contains  
sub-array contains

4 integers  
 $3 \cdot 4$  integers  
 $2 \cdot 3 \cdot 4$  integers

Each  
Each  
Each

$c[i][j]+1$   
 $c[i]+1$   
 $c+1$

Pointer  
Types

starts at  $c[i][j][1]$   
starts at  $c[i][1][0]$   
starts at  $c[1][0][0]$

1 integer away  
4 integers away  
 $3 \cdot 4$  integers away

Each  
Each  
Each

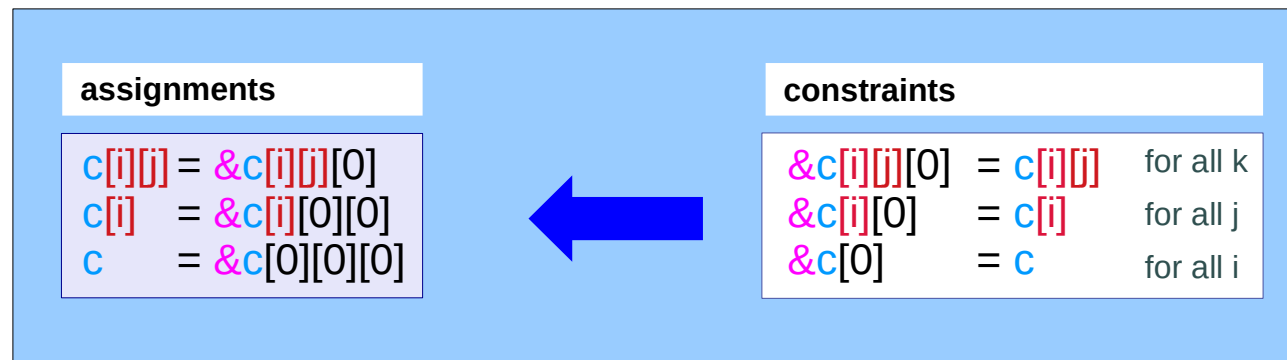
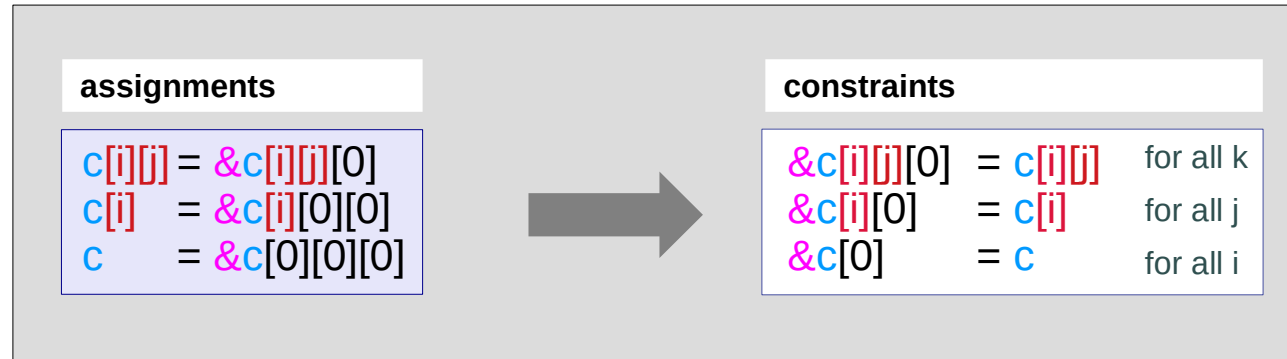
$c[i][j]+k$   
 $c[i]+j$   
 $c+i$

Pointer  
Types

starts at  $c[i][j][k]$   
starts at  $c[i][j][0]$   
starts at  $c[i][0][0]$

$1 \cdot k$  integers away  
 $4 \cdot j$  integers away  
 $3 \cdot 4 \cdot i$  integers away

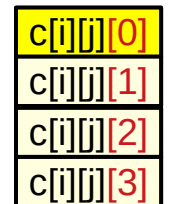
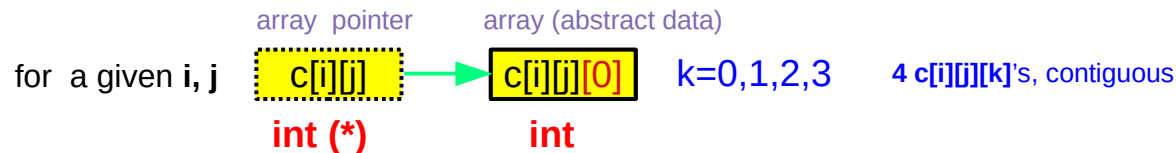
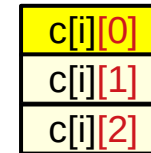
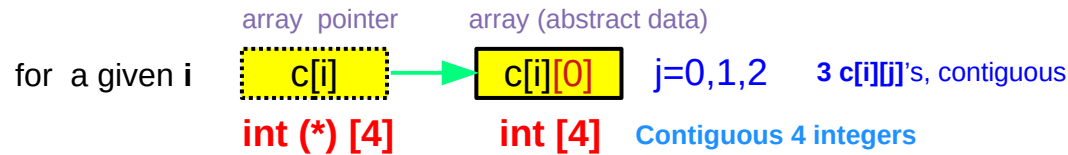
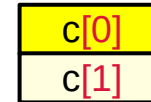
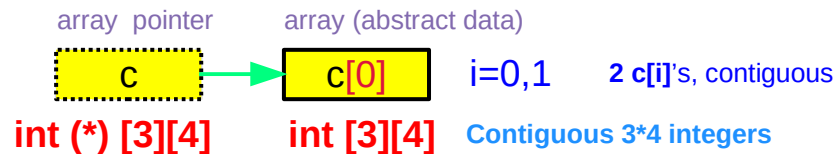
## multi-dimensional arrays



# Contiguous subarrays

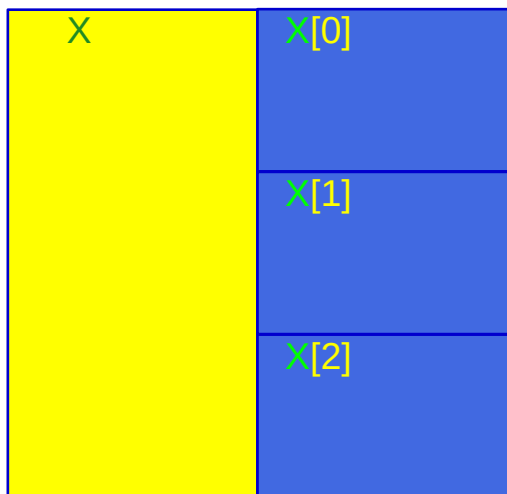


```
int    c [2][3][4];
```



# Dual type – virtual array pointer and abstract array

## Abstract data (array) p[i]

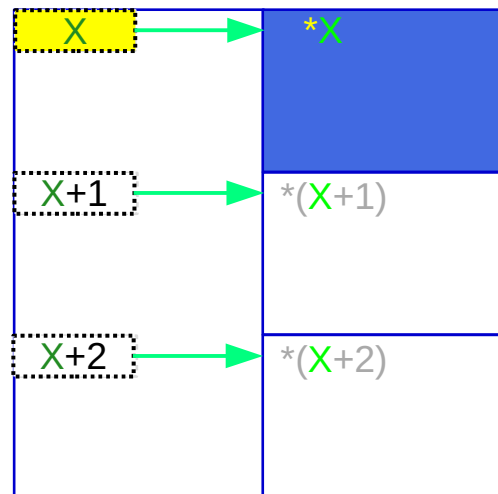


array (abstract data)

$c[i][j]$  starts at  $c[i][j][0]$   
 $c[i]$  starts at  $c[i][0]$   
 $c$  starts at  $c[0]$

$\&c[i][j]$  =  $\&c[i][j][0]$   
 $\&c[i]$  =  $\&c[i][0]$   
 $\&c$  =  $\&c[0]$

## Virtual array pointer p[i]



array pointer

$c[i][j]$  points to  $c[i][j][0]$   
 $c[i]$  points to  $c[i][0]$   
 $c$  points to  $c[0]$

address value

$c[i][j]$  =  $\&c[i][j][0]$   
 $c[i]$  =  $\&c[i][0]$   
 $c$  =  $\&c[0]$

# Virtual array pointer values and addresses

```

&c[i][j][0] = c[i][j]
&c[i][0]     = c[i]
&c[0]       = c
    
```



```

&c[i][j][0] = &c[i][j]
&c[i][0]     = &c[i]
&c[0]       = &c
    
```



```

c[i][j]      = &c[i][j]
c[i]         = &c[i]
c            = &c
    
```

**Virtual  
array  
pointer**

**Abstract  
data  
(array)**

array pointer

c[i][j]	points to	c[i][j][0]
c[i]	points to	c[i][0]
c	points to	c[0]

address value

array (abstract data)

array (abstract data)

c[i][j]	starts at	c[i][j][0]
c[i]	starts at	c[i][0]
c	starts at	c[0]

array (abstract data)

array (abstract data)

c[i][j]	pointer value = pointer address	&c[i][j]
c[i]	pointer value = pointer address	&c[i]
c	pointer value = pointer address	&c

Address of an array pointer





# c[0] = c[0][0] relation

```
int c [2][3][4];
```

```
&c[i][j][0] = c[i][j]
&c[i][0]    = c[i]
&c[0]      = c
```

with contiguous subarrays

```
&c[i][j][k] = c[i][j]+k for all k
&c[i][j]    = c[i]+j   for all j
&c[i]       = c+i     for all i
```



virtual assignments

```
c[i][j] = &c[i][j][0]
c[i]    = &c[i][0][0]
c       = &c[0][0][0]
```

```
c == c[0] == c[0][0] == &c[0][0][0]
```

```
value(c[0]) = &c[0][0][0]
```

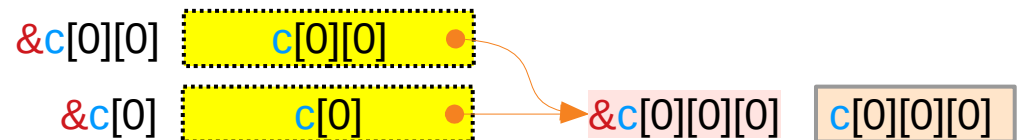
```
value(c[0][0]) = &c[0][0][0]
```

```
type(c[0]) = int (*)[4]
```

```
type(c[0][0]) = int [4]
```

```
c[0] = c[0][0] means
value(c[0]) = value(c[0][0])
```

```
c[0] = c[0][0] does not mean
type(c[0]) = type(c[0][0])
```



# Addresses and Values of `c[0]` and `c[0][0]`

```
int c [2][3][4];
```

```
&c[i][j][0] = c[i][j]
&c[i][0]    = c[i]
&c[0]      = c
```

with contiguous subarrays

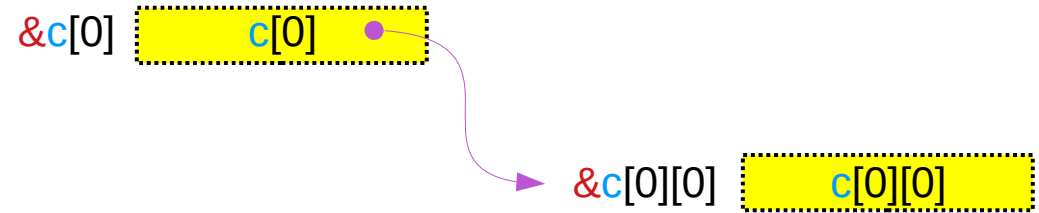
```
&c[i][j][k] = c[i][j] + k for all k
&c[i][j]    = c[i] + j   for all j
&c[i]       = c + i      for all i
```



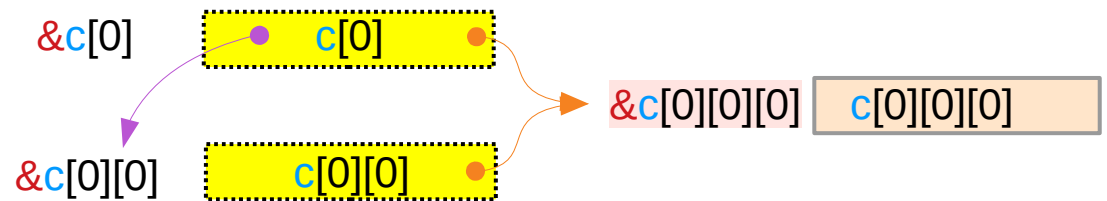
virtual assignments

```
c[i][j] = &c[i][j][0]
c[i]    = &c[i][0][0]
c       = &c[0][0][0]
```

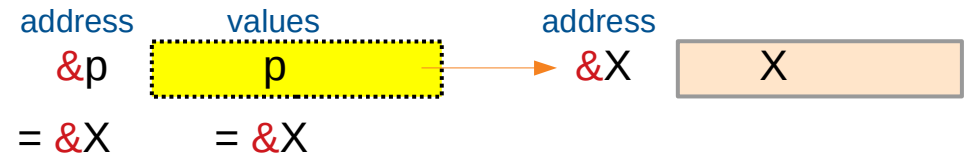
`c[0]` → `c[0][0]`



`c[0]` = `c[0][0]` = `&c[0][0][0]`



A virtual pointer's address and value are the same



# c[i] and c[i][0] point to the same c[i][0][0]

```
int c [2][3][4];
```

```
&c[i][j][0] = c[i][j]
&c[i][0]    = c[i]
&c[0]      = c
```

with contiguous subarrays

```
&c[i][j][k] = c[i][j]+k for all k
&c[i][j]   = c[i]+j   for all j
&c[i]      = c+i      for all i
```

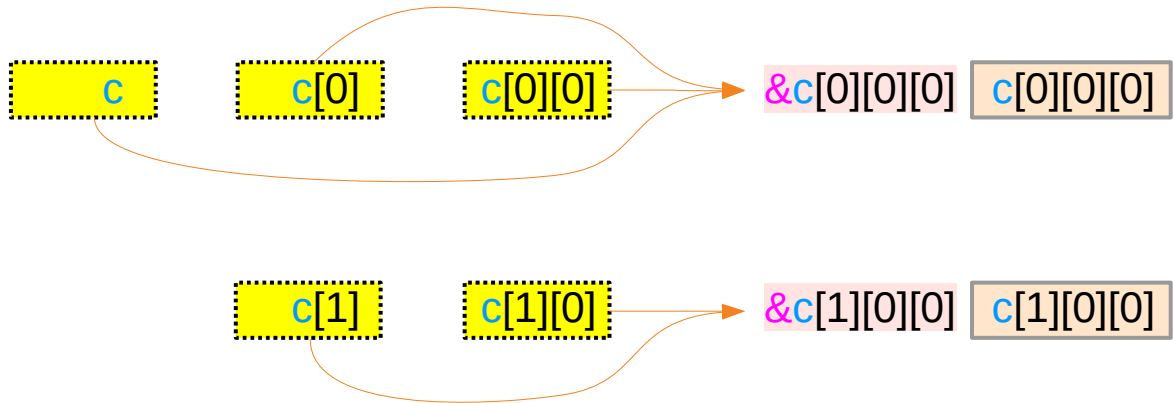


virtual assignments

```
c[i][j] = &c[i][j][0]
c[i]    = &c[i][0][0]
c       = &c[0][0][0]
```

```
c = c[0] = c[0][0] = &c[0][0][0]
int(*)[3][4] int(*)[4] int(*) int
```

```
c[1] = c[1][0] = &c[1][0][0]
int(*)[4] int(*) int
```



These virtual pointers have different types but the same value (address)

# &c[i][0] and &c[i][0][0] – equivalence relations

```
int c [2][3][4];
```

```
&c[i][j][0] = c[i][j]
&c[i][0]    = c[i]
&c[0]      = c
```

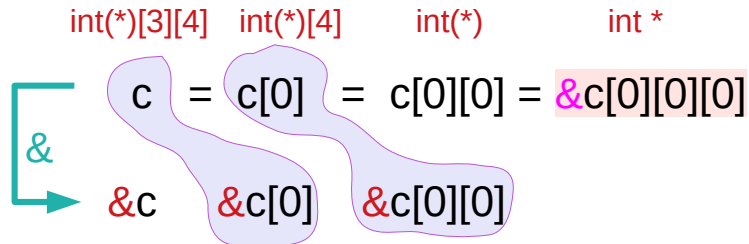
with contiguous subarrays

```
&c[i][j][k] = c[i][j]+k for all k
&c[i][j]    = c[i]+j   for all j
&c[i]       = c+i      for all i
```



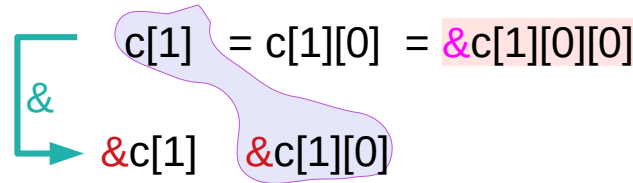
virtual assignments

```
c[i][j] = &c[i][j][0]
c[i]    = &c[i][0][0]
c       = &c[0][0][0]
```



equivalences

```
c ≡ &c[0],
c[0] ≡ &c[0][0]
c[0][0] ≡ &c[0][0][0]
```



equivalences

```
c[1] ≡ &c[1][0]
c[1][0] ≡ &c[1][0][0]
```

Horizontal displacements are not counted  
only vertical displacements are considered  
for address values

equivalences

```
c ≡ &c[0],
c[i] ≡ &c[i][0]
c[i][0] ≡ &c[i][0][0]
```

# $c[i] = \&c[i]$ and $c[i][0] = \&c[i][0]$

```
int c [2][3][4];
```

```
&c[i][j][0] = c[i][j]  
&c[i][0] = c[i]  
&c[0] = c
```

with contiguous subarrays

```
&c[i][j][k] = c[i][j]+k for all k  
&c[i][j] = c[i]+j for all j  
&c[i] = c+i for all i
```

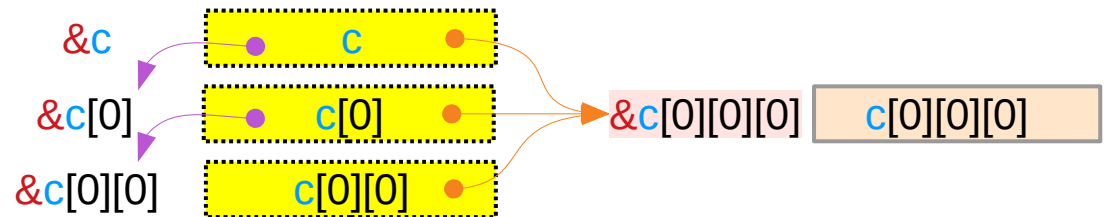


virtual assignments

```
c[i][j] = &c[i][j][0]  
c[i] = &c[i][0][0]  
c = &c[0][0][0]
```

```
c = c[0] = c[0][0] = &c[0][0][0]  
|| || ||  
&c = &c[0] = &c[0][0]
```

```
c[1] = c[1][0] = &c[1][0][0]  
|| ||  
&c[1] = &c[1][0]
```



# $c[i] = \&c[i]$ and $c[i][0] = \&c[i][0]$

```
int c [2][3][4];
```

```
&c[i][j][0] = c[i][j]  
&c[i][0] = c[i]  
&c[0] = c
```

with contiguous subarrays

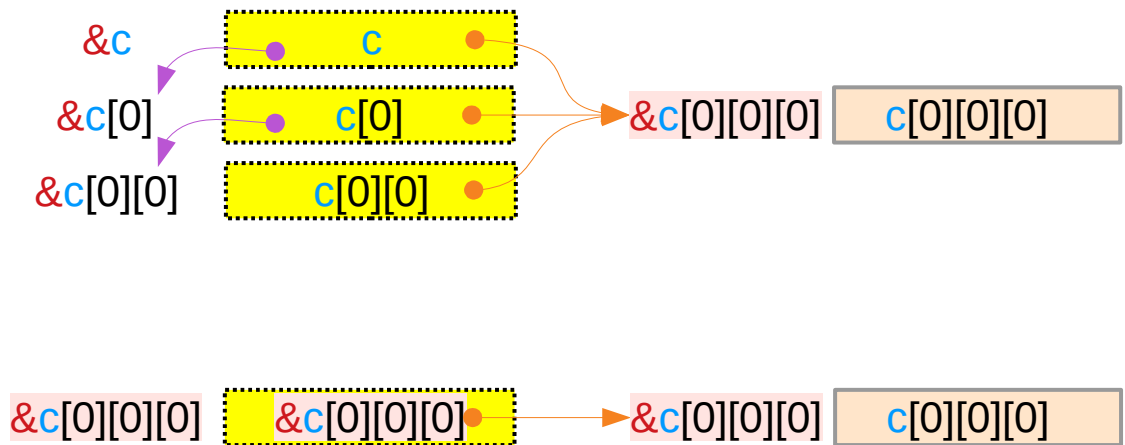
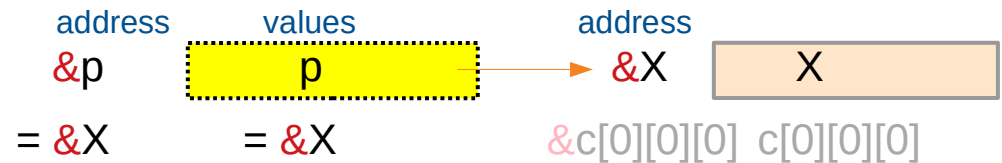
```
&c[i][j][k] = c[i][j]+k for all k  
&c[i][j] = c[i]+j for all j  
&c[i] = c+i for all i
```



virtual assignments

```
c[i][j] = &c[i][j][0]  
c[i] = &c[i][0][0]  
c = &c[0][0][0]
```

A virtual pointer's address and value are the same



**c[i]**

---

# Array Pointers to $c[i][0][0]$

$$\begin{aligned} &\&c[i][j][0] &= c[i][j] \\ &\&c[i][0] &= c[i] \\ &\&c[0] &= c \end{aligned}$$

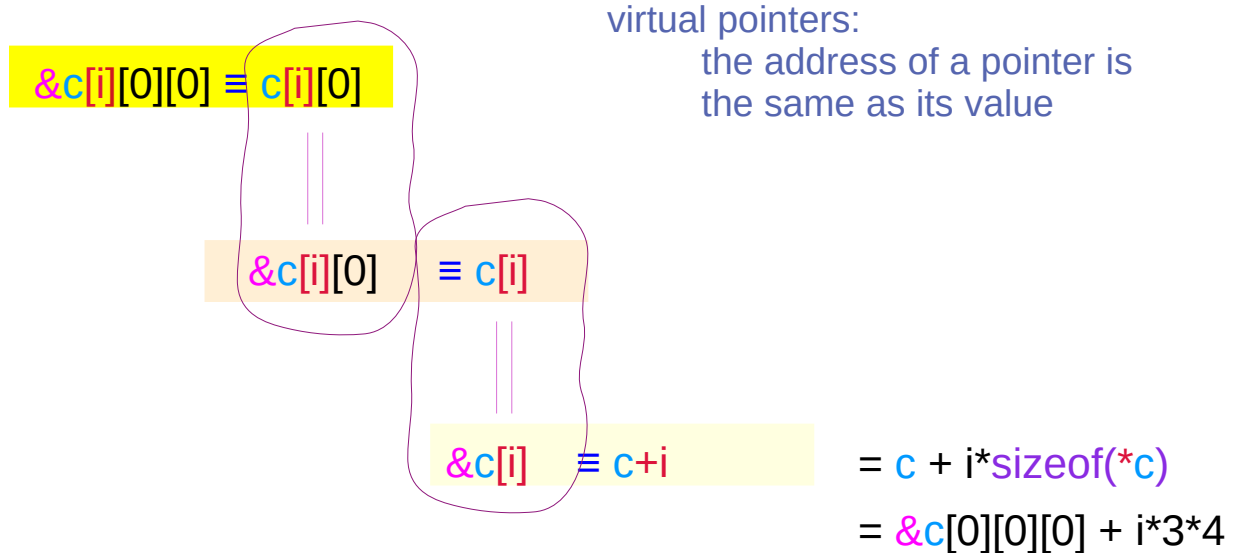
with contiguous subarrays

$$\begin{aligned} &\&c[i][j][k] &= c[i][j] + k && \text{for all } k \\ &\&c[i][j] &= c[i] + j && \text{for all } j \\ &\&c[i] &= c + i && \text{for all } i \end{aligned}$$



assignments

$$\begin{aligned} c[i][j] &= \&c[i][j][0] \\ c[i] &= \&c[i][0][0] \\ c &= \&c[0][0][0] \end{aligned}$$



delete [0] from the right

$$\begin{array}{l} \&c[0][0][0] \quad \underline{\underline{-[0]}} \quad c[0][0] \quad \underline{\underline{-[0]}} \quad c[0] \quad \underline{\underline{-[0]}} \quad c \\ \&c[1][0][0] \quad \underline{\underline{-[0]}} \quad c[1][0] \quad \underline{\underline{-[0]}} \quad c[1] \end{array}$$



# Array Pointers to $c[i][j][0]$

```
&c[i][j][0] = c[i][j]
&c[i][0]    = c[i]
&c[0]      = c
```

with contiguous subarrays

```
&c[i][j][k] = c[i][j]+k for all k
&c[i][j]    = c[i]+j   for all j
&c[i]       = c+i      for all i
```



assignments

```
c[i][j] = &c[i][j][0]
c[i]    = &c[i][0][0]
c       = &c[0][0][0]
```

$\&c[i][j][0] \equiv c[i][j]$

$\&c[i][j] \equiv c[i] + j$

$= c[i] + j * \text{sizeof}(*c[i])$   
 $= c + i * \text{sizeof}(*c) + j * 4$   
 $= \&c[0][0][0] + i * 3 * 4 + j * 4$

delete [0] from the right

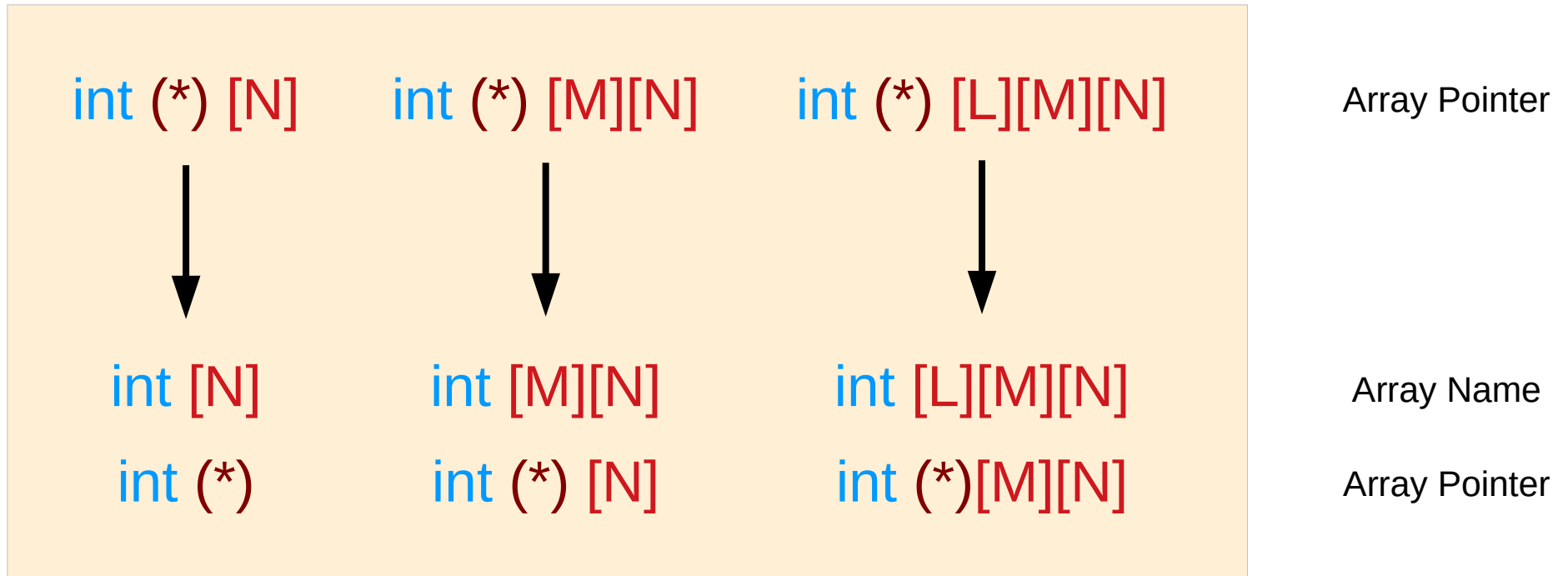
$\&c[0][0][0]$	$\underline{\underline{-[0]}}$	$c[0][0]$	$\underline{\underline{-[0]}}$	$c[0]$	$\underline{\underline{-[0]}}$	$c$
$\&c[0][1][0]$	$\underline{\underline{-[0]}}$	$c[0][1]$				
$\&c[0][2][0]$	$\underline{\underline{-[0]}}$	$c[0][2]$				
$\&c[1][0][0]$	$\underline{\underline{-[0]}}$	$c[1][0]$	$\underline{\underline{-[0]}}$	$c[1]$		
$\&c[1][1][0]$	$\underline{\underline{-[0]}}$	$c[1][1]$				
$\&c[1][2][0]$	$\underline{\underline{-[0]}}$	$c[1][2]$				

# Contiguity Constraints

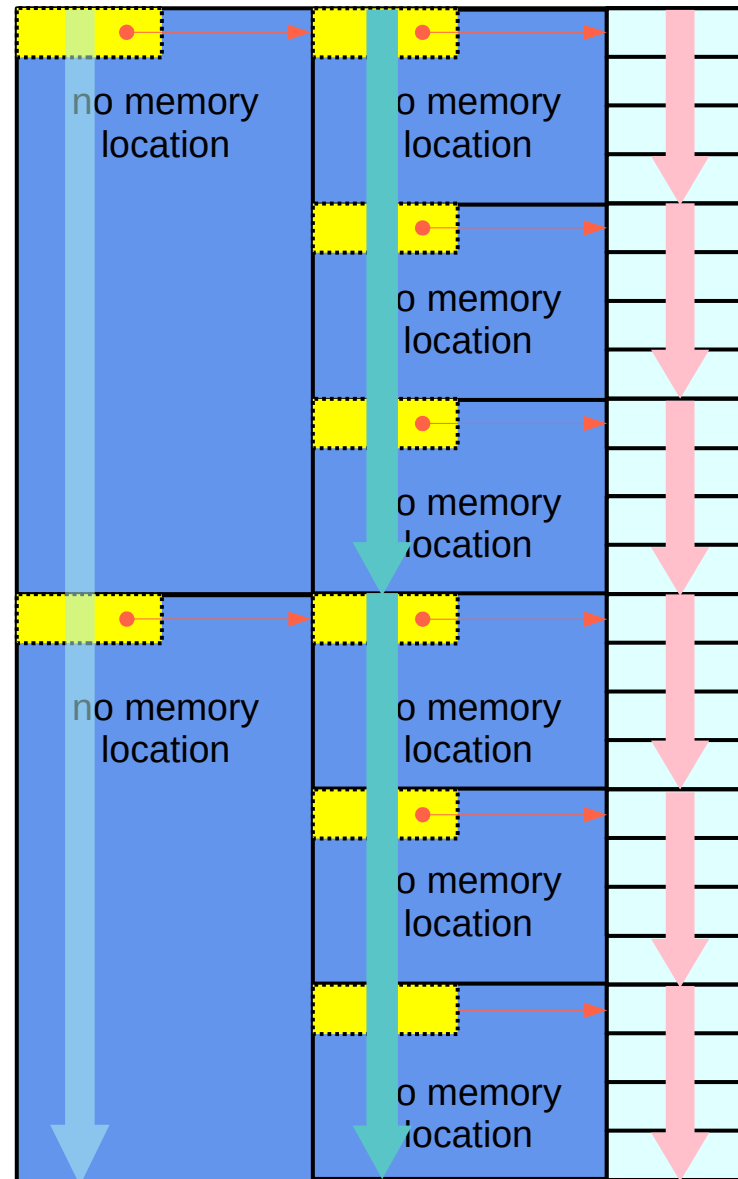
c [i][j][k];

Virtual Array Pointers and Contiguity

# Using array pointers

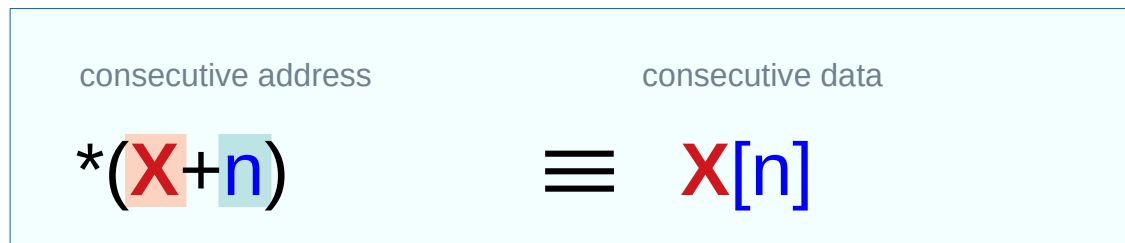


# Array pointer approach – contiguity constraints

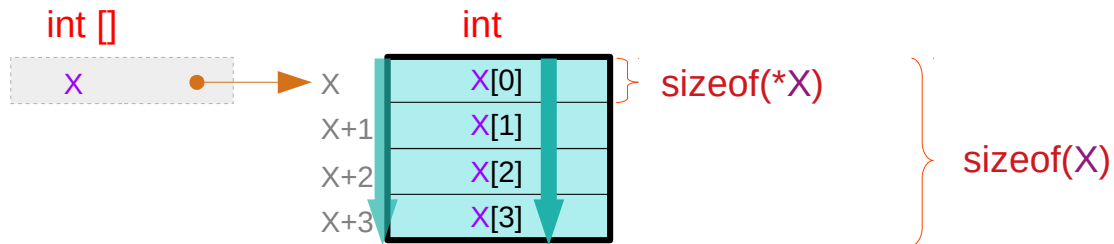


**Array Pointer Approach**  
**(pointer to arrays)**

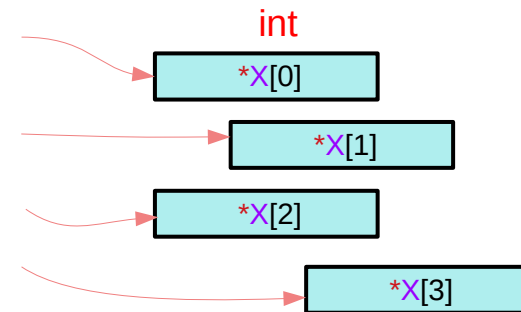
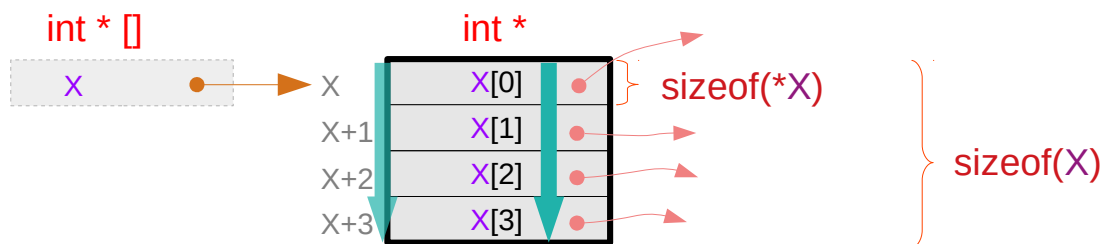
# Equivalence and contiguity (1)



contiguous index : n

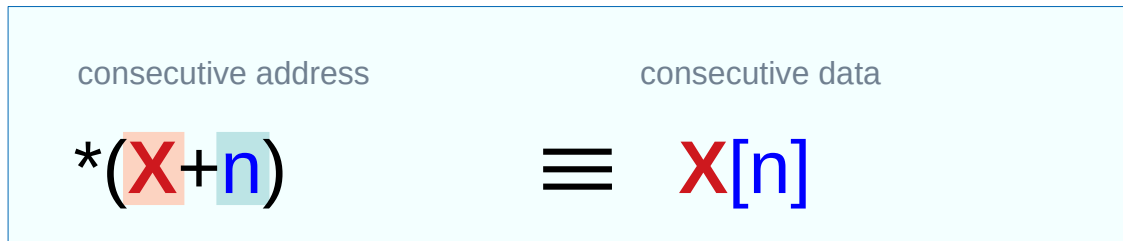


**int X[4];** contiguous X[i] for a given X : **primitive types**



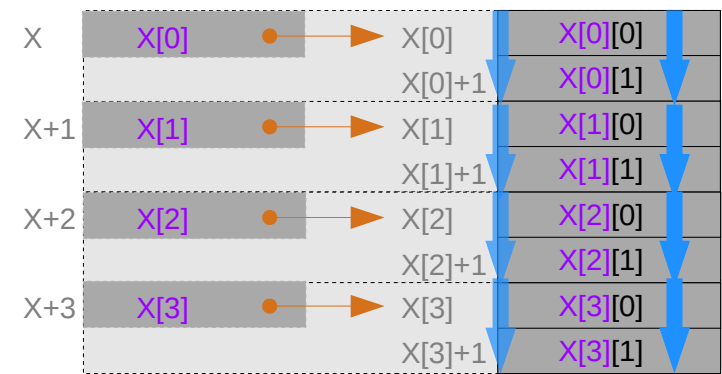
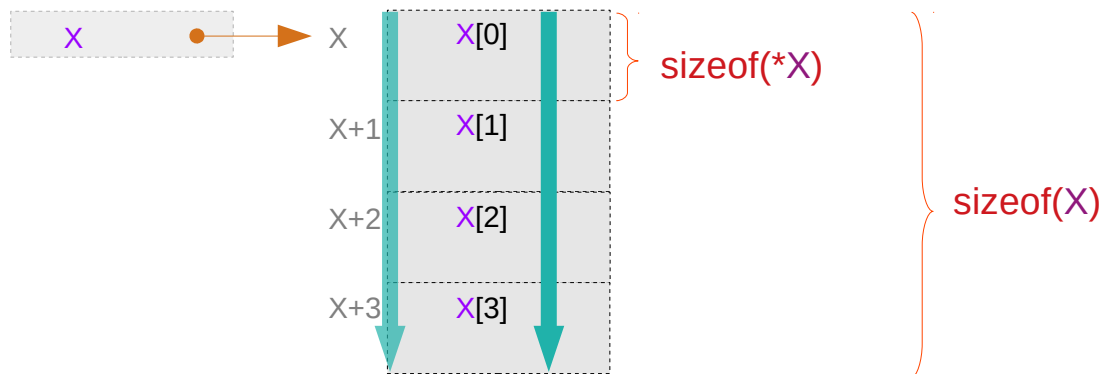
**int \* X[4];** contiguous X[i] for a given X : **pointer types**

# Equivalence and contiguity (2)



contiguous index : n

can be recursively applied



**atype \* X[4]; contiguous X[i] for a given X : abstract data types**

# Recursive applications of equivalences

By definition, contiguous memory locations are assumed

consecutive address

consecutive data

$$*(\mathbf{X} + \mathbf{n}) \equiv \mathbf{X}[\mathbf{n}]$$

contiguous index : n

$$*(\mathbf{p}[\mathbf{m}] + \mathbf{n}) \longleftrightarrow \mathbf{p}[\mathbf{m}][\mathbf{n}]$$

$$\mathbf{X} = \mathbf{p}[\mathbf{m}] \quad \text{contiguous index : n}$$

$$(*(\mathbf{p} + \mathbf{m}))[\mathbf{n}]; \longleftrightarrow \mathbf{p}[\mathbf{m}][\mathbf{n}];$$

$$\mathbf{X} = \mathbf{p} \quad \text{contiguous index : m}$$

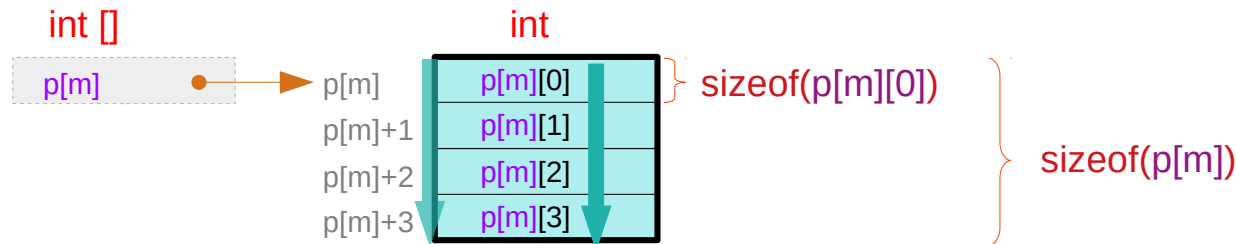
# Equivalence for a given p[m] (1)

$$*(p[m]+n) \iff p[m][n]$$

for a given  $p[m]$  contiguous index :  $n$

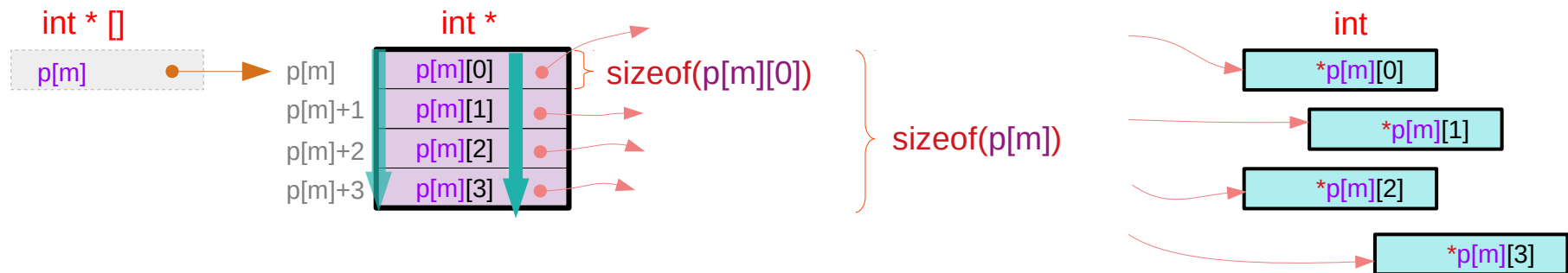
**int p[M][4];** contiguous p[m][n] for a given p[m] : **primitive types**

$m = 0, 1, \dots, M-1$



**int \* p[M][4];** contiguous p[m][n] for a given p[m] : **pointer types**

$m = 0, 1, \dots, M-1$



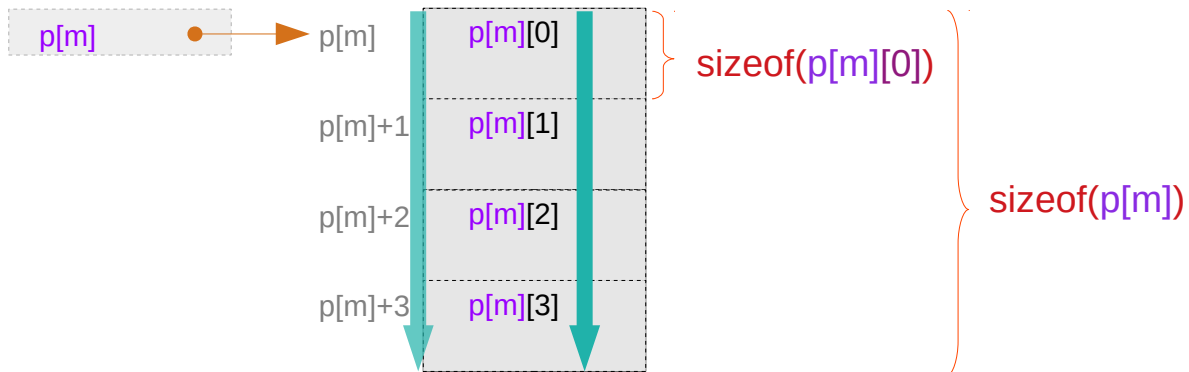


# Equivalence for a given p[m] (2)

$$*(p[m]+n) \iff p[m][n]$$

for a given `p[m]` contiguous index : `n`

`atype * p[M][4];` contiguous `p[m][n]` for a given `p[m]` : **abstract data types**  $m = 0, 1, \dots, M-1$



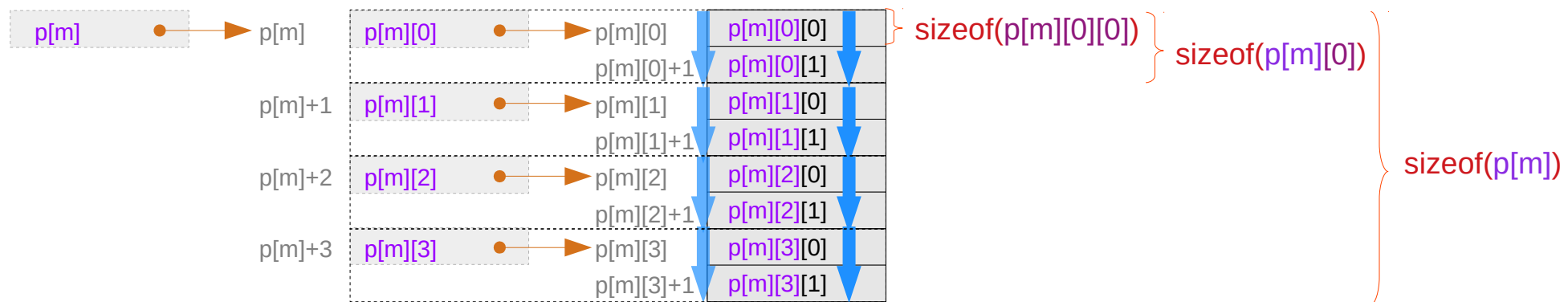
can be recursively applied

# Equivalence for a given p[m][n]

$$*(p[m][n]+k) \iff p[m][n][k]$$

for a given `p[m][n]` contiguous index : `k`

`atype * p[M][4][2];` contiguous `p[m][n][k]` for a given `p[m][n]` : **abstract data types** `m = 0, 1, ..., M-1`



# Contiguity constraints in multi-dimensional arrays

$$*(p[m]+n) \iff p[m][n]$$

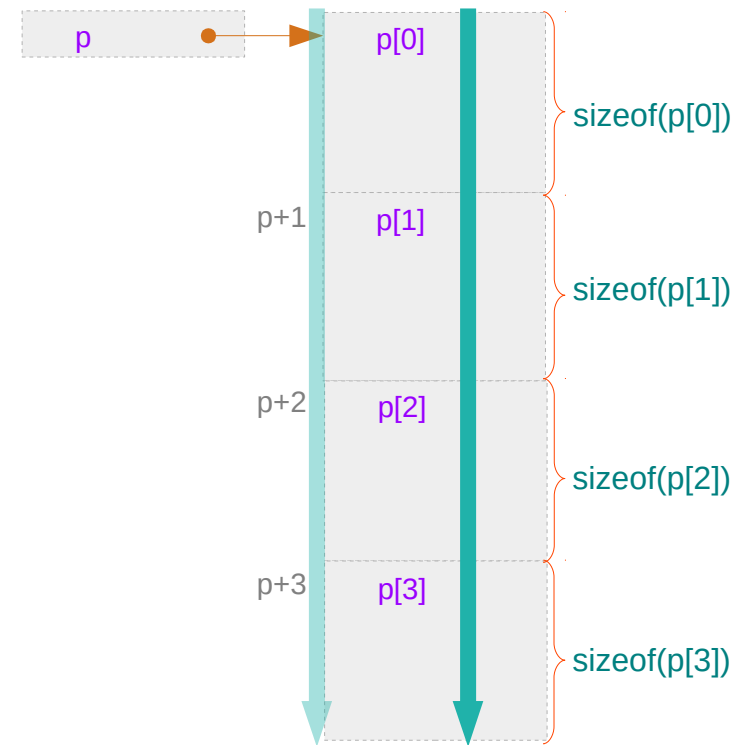
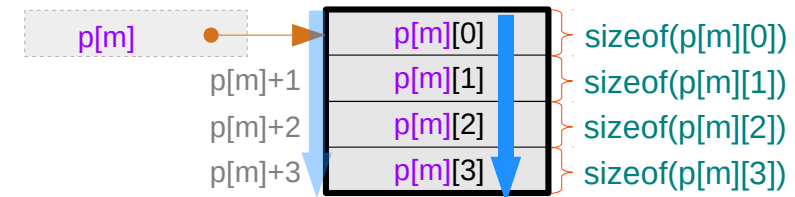
for a given  $p[m]$ , thus for a given  $p$  and  $m$ ,  
 $p[m][n]$ 's must be contiguous for all  $n$ .  
 $p[m][0], p[m][1], \dots, p[m][N-1]$

contiguous index :  $n$

$$*(p+m) \iff p[m]$$

for a given  $p$ ,  
 $p[m]$ 's must be contiguous for all  $m$ .  
 $p[0], p[1], \dots, p[M-1]$

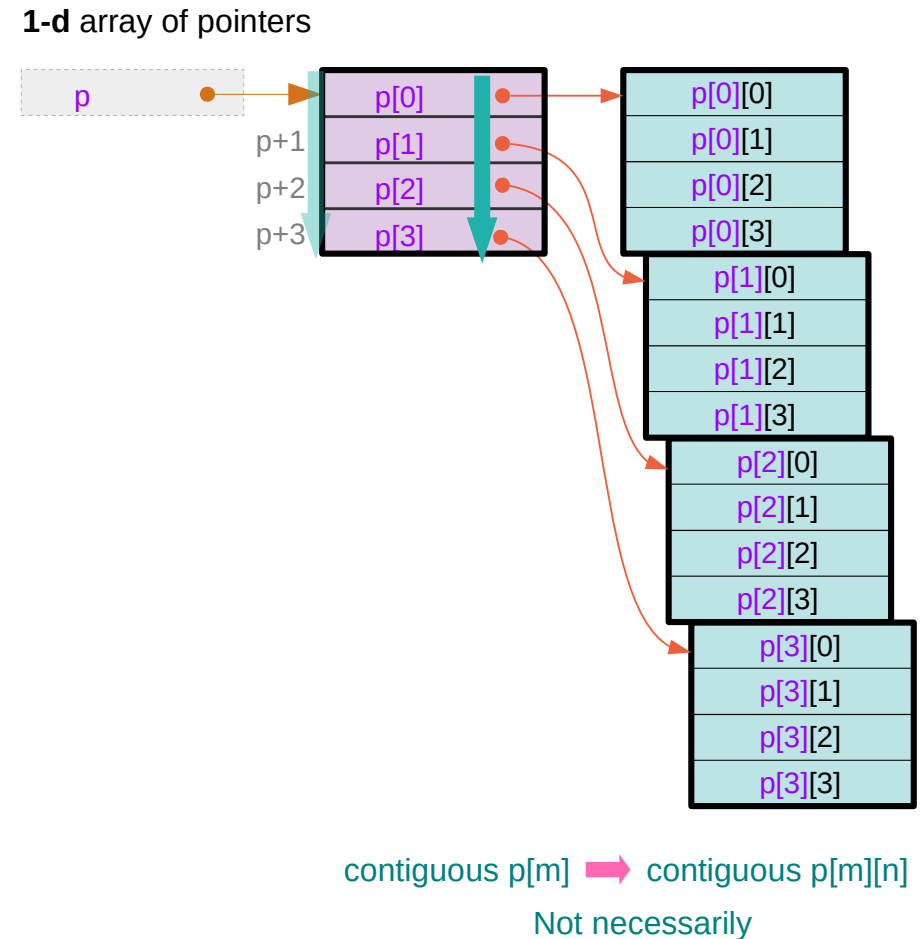
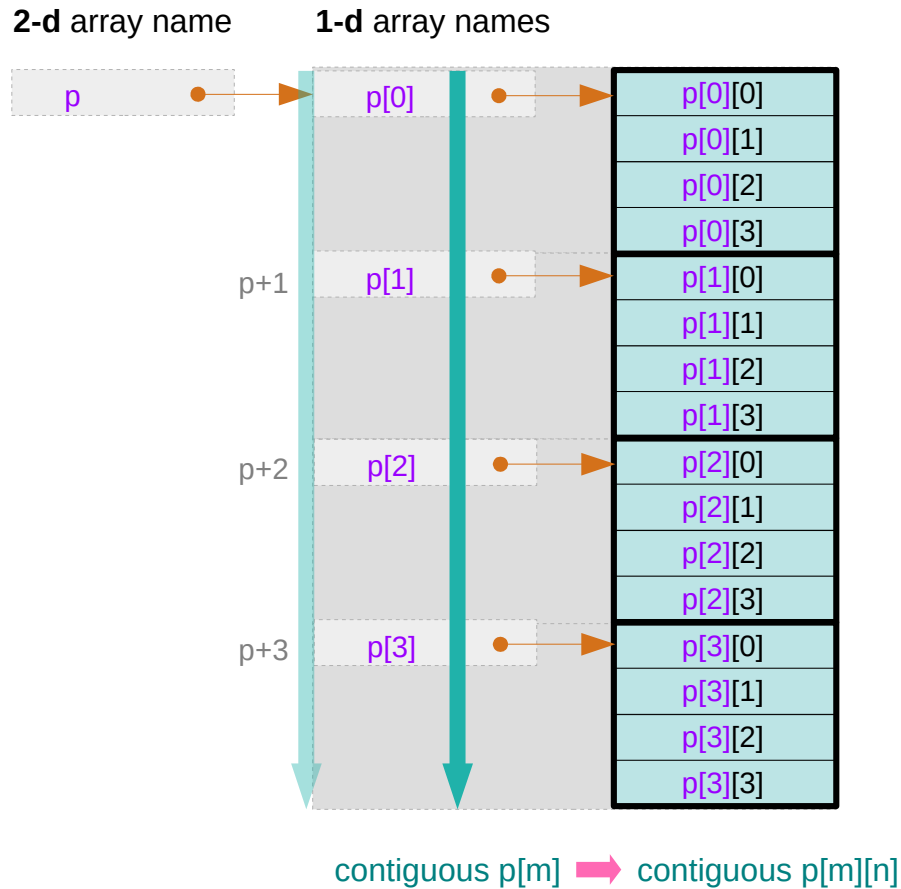
contiguous index :  $m$



# Contiguity constraints for p

$$*(p+m) \iff p[m]$$

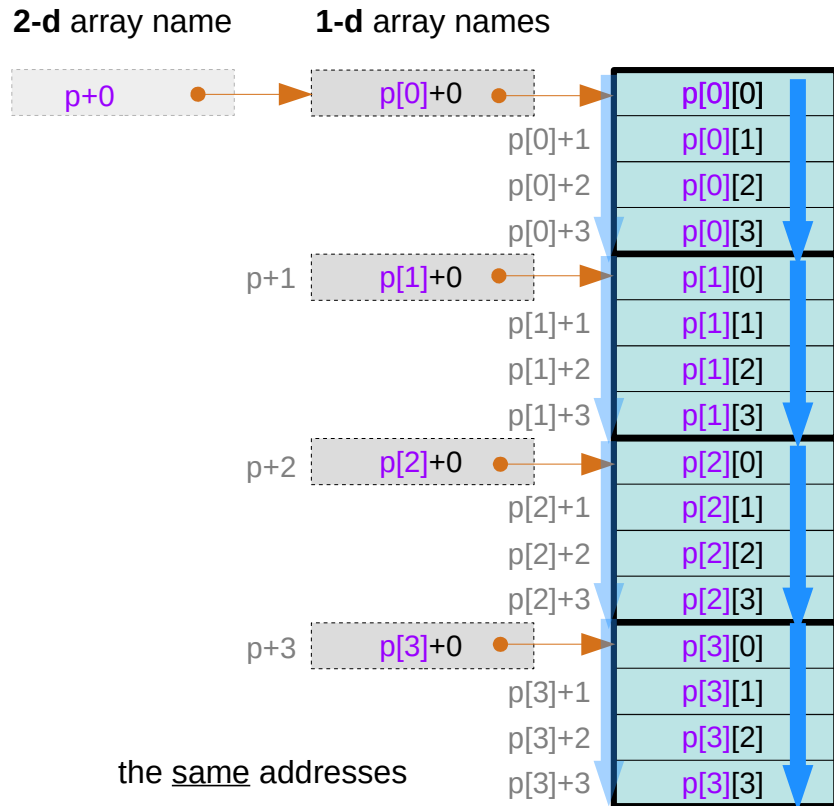
for a given  $p$  contiguous index :  $m$



# Contiguity constraints for p[m] – using array pointers

$$*(p[m]+n) \iff p[m][n]$$

for a given  $p[m]$  contiguous index :  $n$



$$p[0][0] = *(p[0]+0) \xrightarrow{\text{addr}} \underbrace{\&p[0][0] = p[0]}_{\text{addr}} \xrightarrow{\text{addr}} p+0$$

$$p[1][0] = *(p[1]+0) \xrightarrow{\text{addr}} \underbrace{\&p[1][0] = p[1]}_{\text{addr}} \xrightarrow{\text{addr}} p+1$$

$$p[2][0] = *(p[2]+0) \xrightarrow{\text{addr}} \underbrace{\&p[2][0] = p[2]}_{\text{addr}} \xrightarrow{\text{addr}} p+2$$

$$p[3][0] = *(p[3]+0) \xrightarrow{\text{addr}} \underbrace{\&p[3][0] = p[3]}_{\text{addr}} \xrightarrow{\text{addr}} p+3$$

the same addresses

contiguous  $p[m]$   $\rightarrow$  contiguous  $p[m][n]$

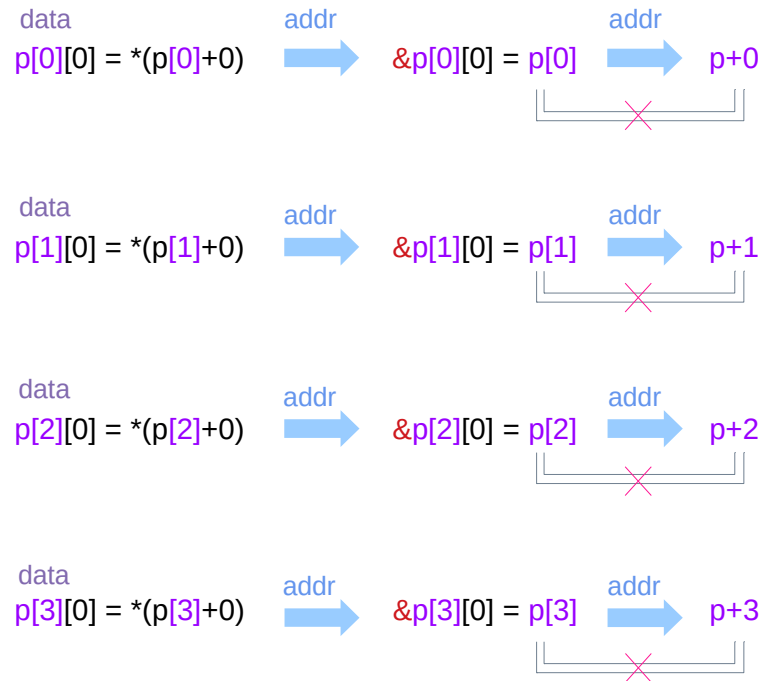
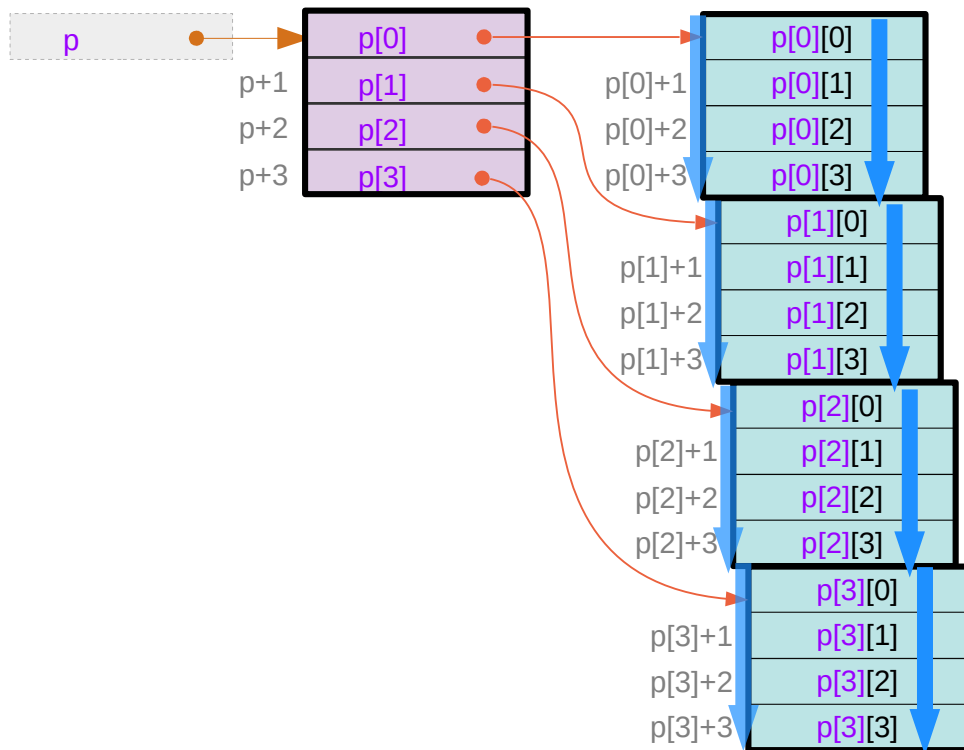
virtual array pointer  $\iff$  no real memory locations

# Contiguity constraints for p[m] – using pointer arrays

$$*(p[m]+n) \iff p[m][n]$$

for a given  $p[m]$  contiguous index :  $n$

1-d array of pointers



the different addresses

contiguous  $p[m]$   $\rightarrow$  contiguous  $p[m][n]$   
Not necessarily

# Contiguity constraints for 2-d arrays

```
int a[M][N] ;
```

$*(a+m) \leftrightarrow a[m]$

$a[0], a[1], \dots, a[M-1]$   
are contiguous

$*(a[m]+n) \leftrightarrow a[m][n]$

$a[m][0], a[m][1], \dots, a[m][N-1]$   
are contiguous

```
int (*b)[N] ;
```

$*(b+m) \leftrightarrow b[m]$

$b[0], b[1], \dots, b[M-1]$   
are contiguous

$*(b[m]+n) \leftrightarrow b[m][n]$

$b[m][0], b[m][1], \dots, b[m][N-1]$   
are contiguous

```
int * c[M] ;
```

$*(c+m) \leftrightarrow c[m]$

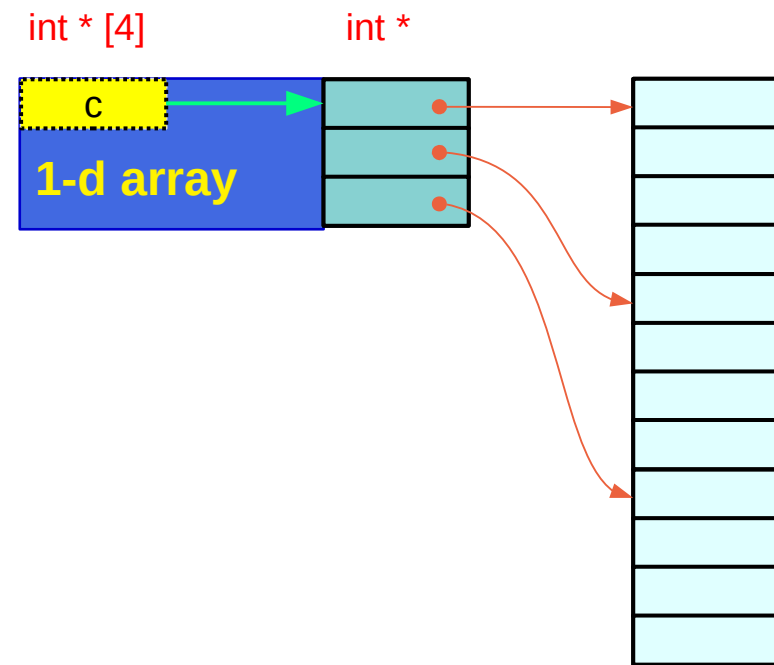
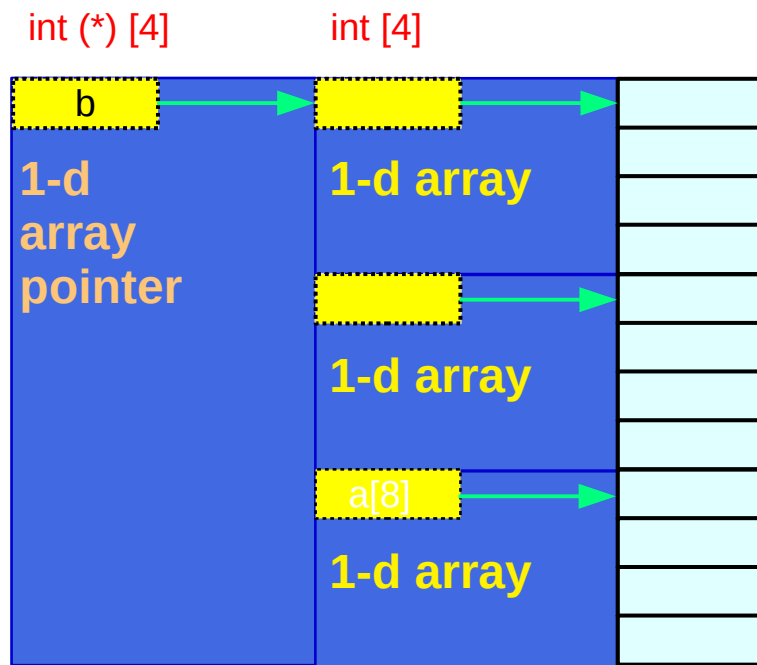
$c[0], c[1], \dots, c[M-1]$   
are contiguous

$*(c[m]+n) \leftrightarrow c[m][n]$

$c[m][0], c[m][1], \dots, c[m][N-1]$   
are contiguous

a set of assignments of pointers  
are necessary for this contiguity

# Pointer Arrays vs Array Pointers



```
int (*b)[N] ;
```

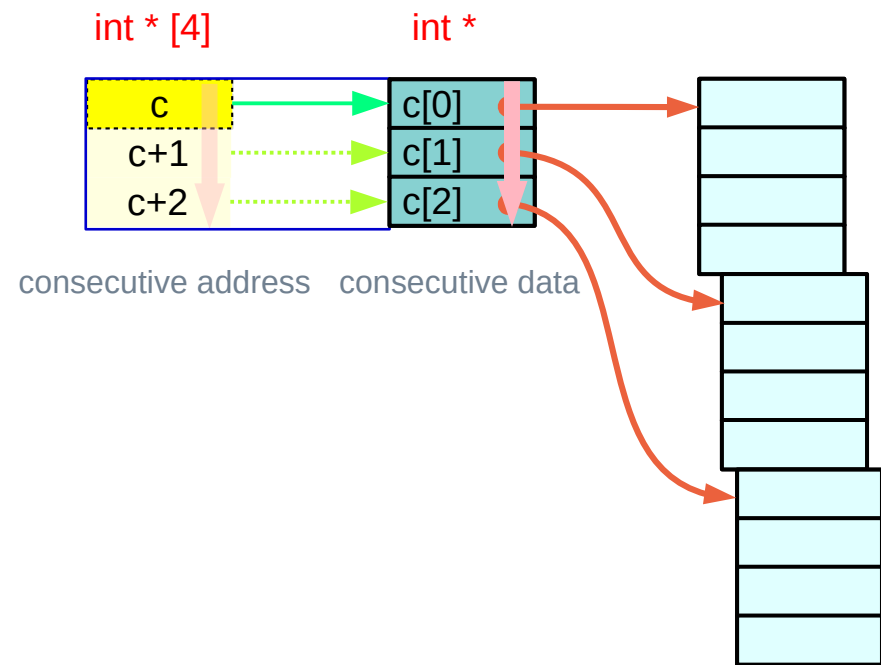
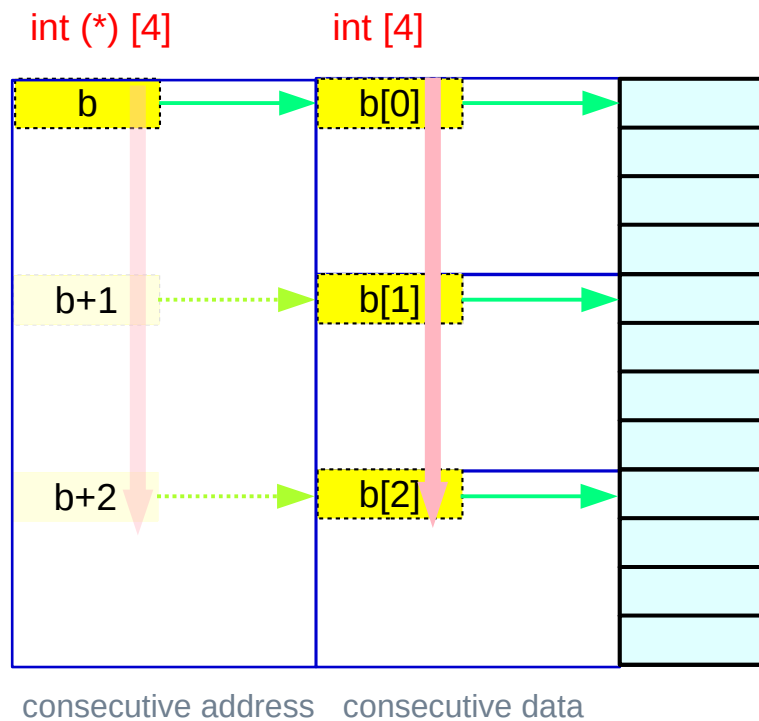
```
int * c[M] ;      with proper assignments
```

```
*(b+m)      ↔      b[m]
*(b[m]+n)   ↔      b[m][n]
```

```
*(c+m)      ↔      c[m] or
*(c[m]+n)   ↔      c[m][n]
```



# Pointer Arrays vs Array Pointers



```
int (*b)[N] ;
```

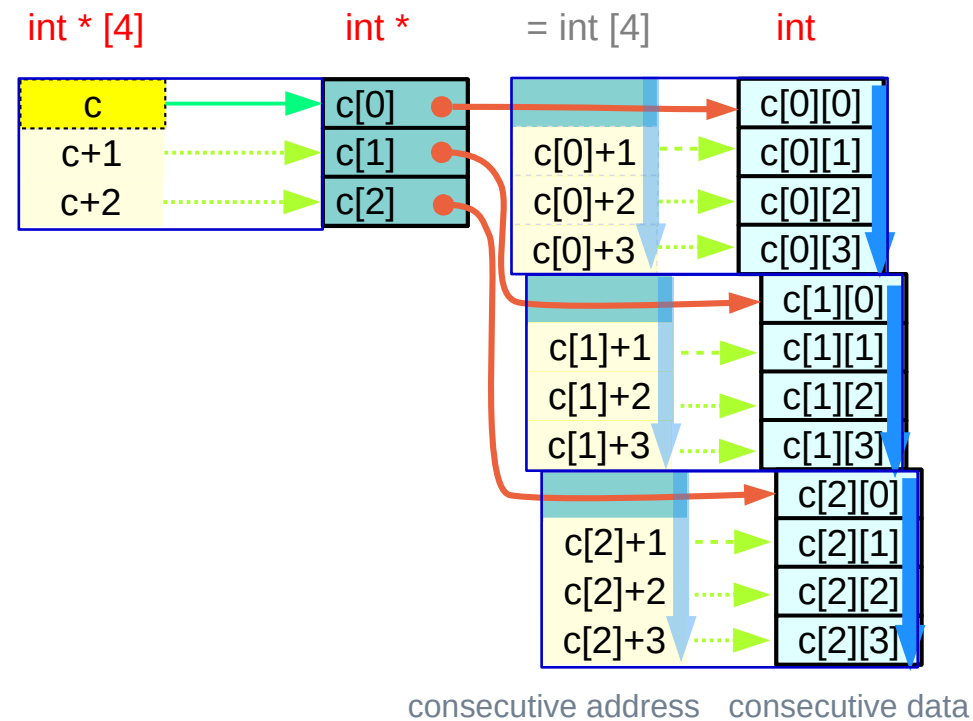
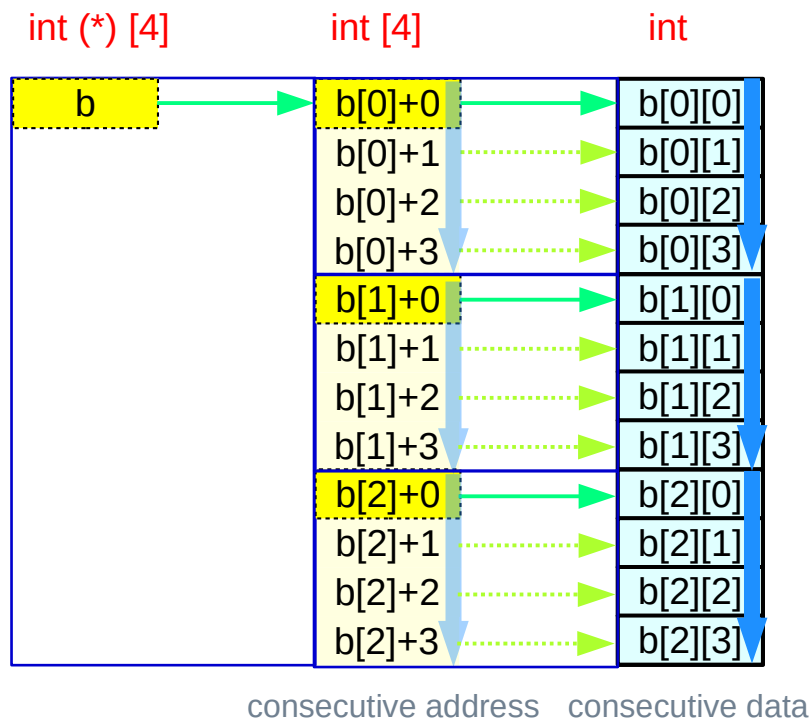
```
*(b+m)        ↔    b[m]
*(b[m]+n)    ↔    b[m][n]
```

```
int * c[M] ;
```

with proper assignments

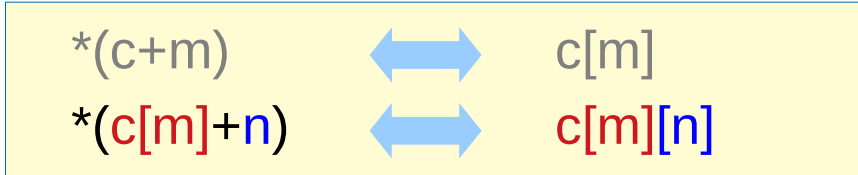
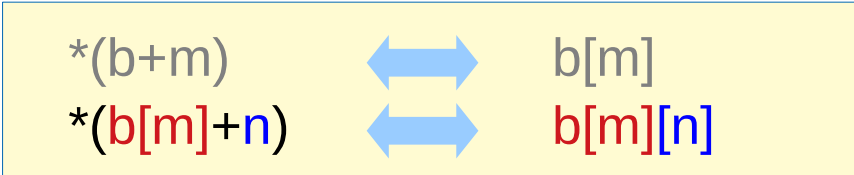
```
*(c+m)        ↔    c[m] or
*(c[m]+n)    ↔    c[m][n]
```

# Pointer Arrays vs Array Pointers



```
int (*b)[N] ;
```

```
int * c[M] ;      with proper assignments
```



# Three contiguity constraints for 3-d arrays

## Pointer Array Approach (array of pointers)

$c[i][j][k]$        $\rightarrow$      $*(c[i][j] + k)$   
 $*(c[i][j] + k)$      $\rightarrow$      $*(*(c[i] + j) + k)$   
 $*(*(c[i] + j) + k)$   $\rightarrow$   $*(**(*c + i) + j) + k)$

contiguous **int**                                    **int**  
contiguous pointers to **int**                    **int \***  
contiguous double pointers to **int**           **int \*\***

the contiguity constraints are satisfied by allocating arrays of pointers

## Array Pointer Approach (pointer to arrays)

$c[i][j][k]$        $\rightarrow$      $*(c[i][j] + k)$   
 $*(c[i][j] + k)$      $\rightarrow$      $*(*(c[i] + j) + k)$   
 $*(*(c[i] + j) + k)$   $\rightarrow$   $*(**(*c + i) + j) + k)$

contiguous **0-d** arrays    **int**                                    **int**  
contiguous **1-d** arrays    **int [4]**                                    **int \***  
contiguous **2-d** arrays    **int [3][4]**                                **int (\*) [4]**

The contiguity constraints are satisfied by row major ordered linear data layout

# Contiguous array pointers $c[i][j][k] \equiv *(c[i][j] + k)$

```

c[0][0][0] = *(c[0][0] + 0)
c[0][0][1] = *(c[0][0] + 1)
c[0][0][2] = *(c[0][0] + 2)
c[0][0][3] = *(c[0][0] + 3)
c[0][1][0] = *(c[0][1] + 0)
c[0][1][1] = *(c[0][1] + 1)
c[0][1][2] = *(c[0][1] + 2)
c[0][1][3] = *(c[0][1] + 3)

```

• •  
• •

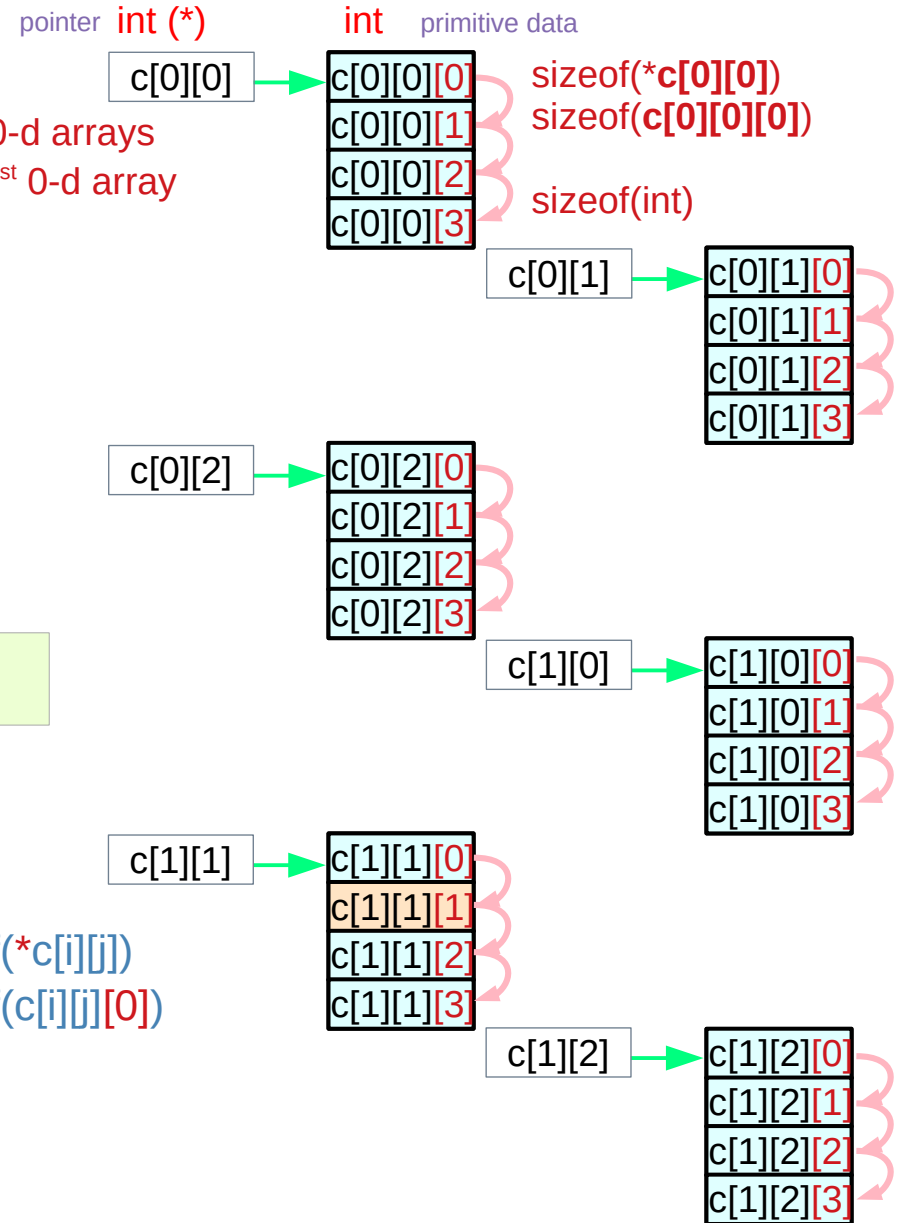
contiguous 1-d  
array elements

$c[i][j]$   
**int [4]** 4 contiguous 0-d arrays  
**int \*** points to the 1<sup>st</sup> 0-d array  
**int** 0-d array

$\text{sizeof}(c[i][j])$  [k]  
 $\text{sizeof}(c[i][j][k]) * 4$   
 $\text{sizeof}(\text{int}) * 4$

```
int c[2][3][4];
```

Address Value  
 $c[i][j] + k$   
 $\&c[i][j][0] + k * \text{sizeof}(*c[i][j])$   
 $\&c[i][j][0] + k * \text{sizeof}(c[i][j][0])$   
 $\&c[i][j][0] + k * 4$



# Contiguous array pointers $c[i][j] \equiv *(c[i] + j)$

```

c[0][0] = *(c[0] + 0)
c[0][1] = *(c[0] + 1)
c[0][2] = *(c[0] + 2)
c[1][0] = *(c[1] + 0)
c[1][1] = *(c[1] + 1)
c[1][2] = *(c[1] + 2)
    
```

**c[i]**  
**int [3][4]** 3 contiguous 1-d arrays  
**int (\*) [4]** points to the 1<sup>st</sup> 1-d array  
**int [4]** 1-d array

sizeof(c[i]) [j] [k]  
 sizeof(c[i][j][k]) \* 3 \* 4  
 sizeof(int) \* 3 \* 4

```
int c[2][3][4];
```

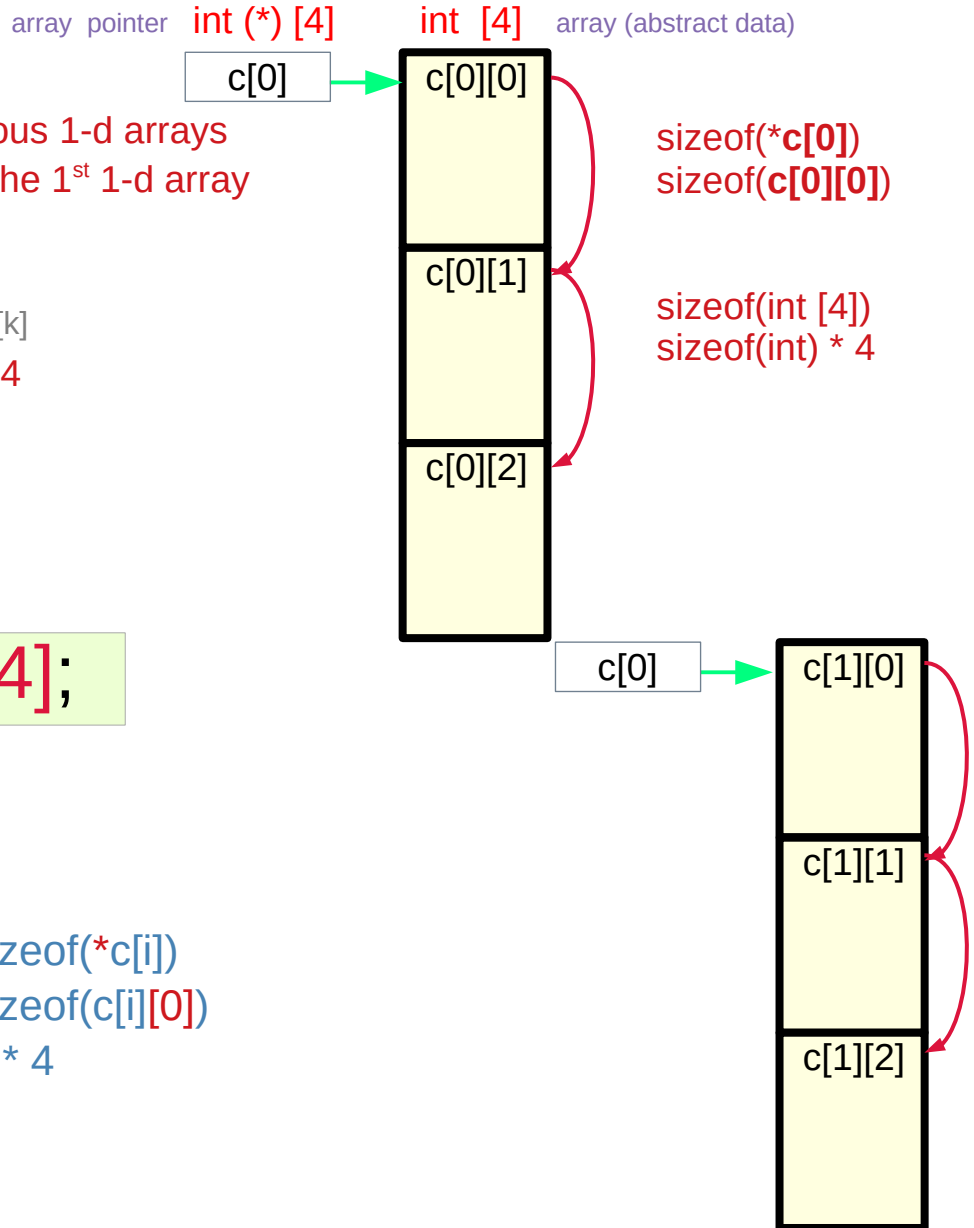
Address Value

$c[i] + j$

$\&c[i][0][0] + j * \text{sizeof}(*c[i])$

$\&c[i][0][0] + j * \text{sizeof}(c[i][0])$

$\&c[i][0][0] + j * 4 * 4$



# Contiguous array pointers $c[i] \equiv *(c + i)$

```
c[0] = *(c + 0)
c[1] = *(c + 1)
```

```
c
int [2][3][4]
int (*) [3][4]
int [3][4]
```

2 contiguous 2-d arrays  
points to the 1<sup>st</sup> 2-d array  
2-d array

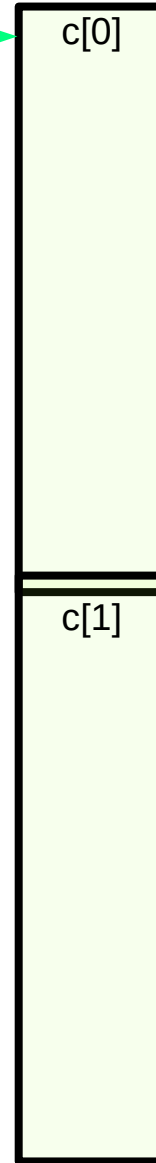
```
sizeof(c)           [i] [j] [k]
sizeof(c[i][j][k]) * 2 * 3 * 4
sizeof(int) * 2 * 3 * 4
```

```
int c[2][3][4];
```

Address Value

```
c + i
&c[0][0][0] + i * sizeof(*c)
&c[0][0][0] + i * sizeof(c[0])
&c[0][0][0] + i * 4 * 3 * 4
```

array pointer  $int (*) [3][4]$   $int [3][4]$  array (abstract data)



sizeof(\*c)  
sizeof(c[0])

sizeof(int [3][4])  
sizeof(int) \* 3 \* 4

# Contiguous linear layout

```
int c [L][M][N];
```

```
C [i][j][k];
```

L	M	N
i	j	k
$i * M * N$	$j * N$	k

Base Index = 0

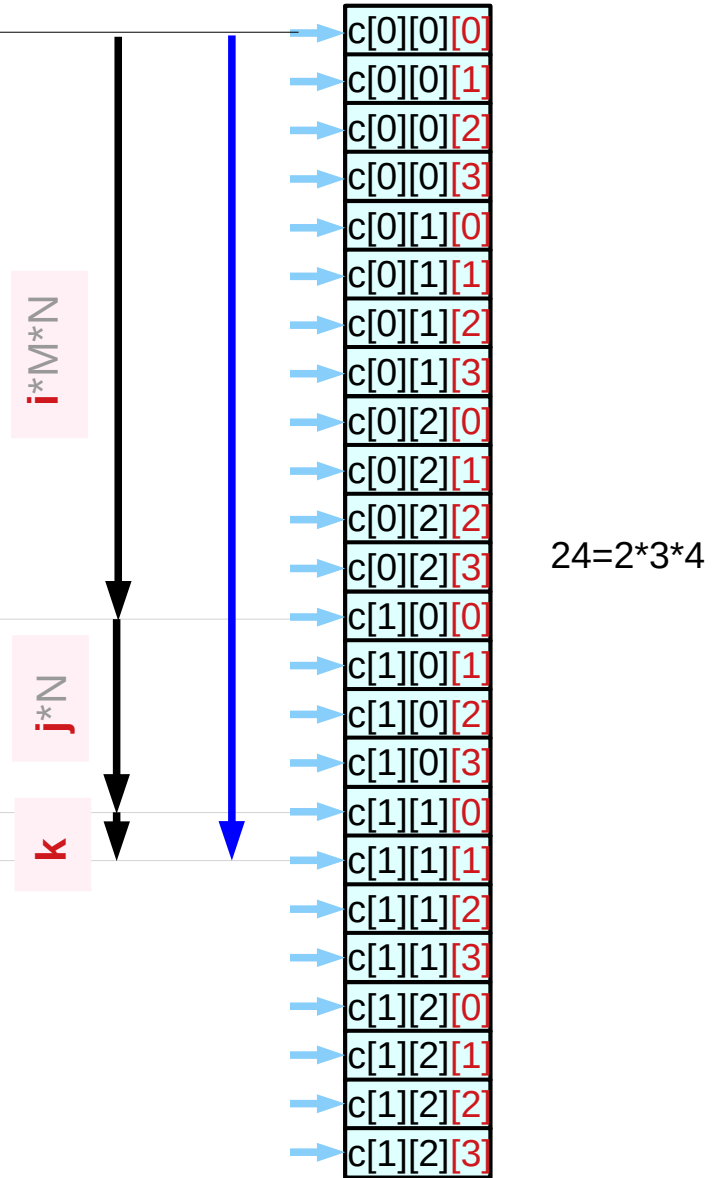
Offset Index 1 (i=1)

Offset Index 2 (j=1)

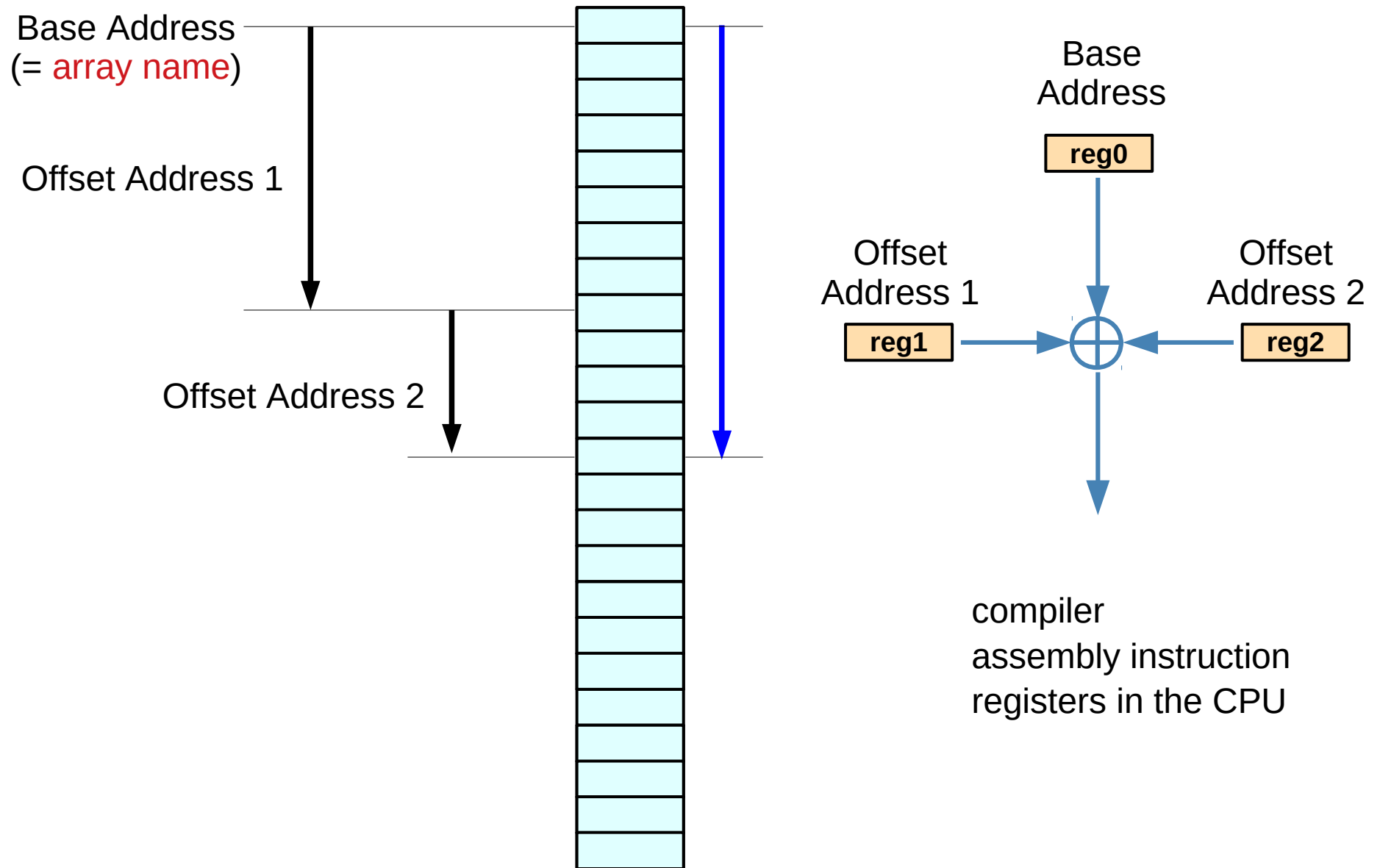
Offset Index 3 (k=1)

$$(i * M * N + j * N + k)$$

$$((i * M + j) * N + k)$$

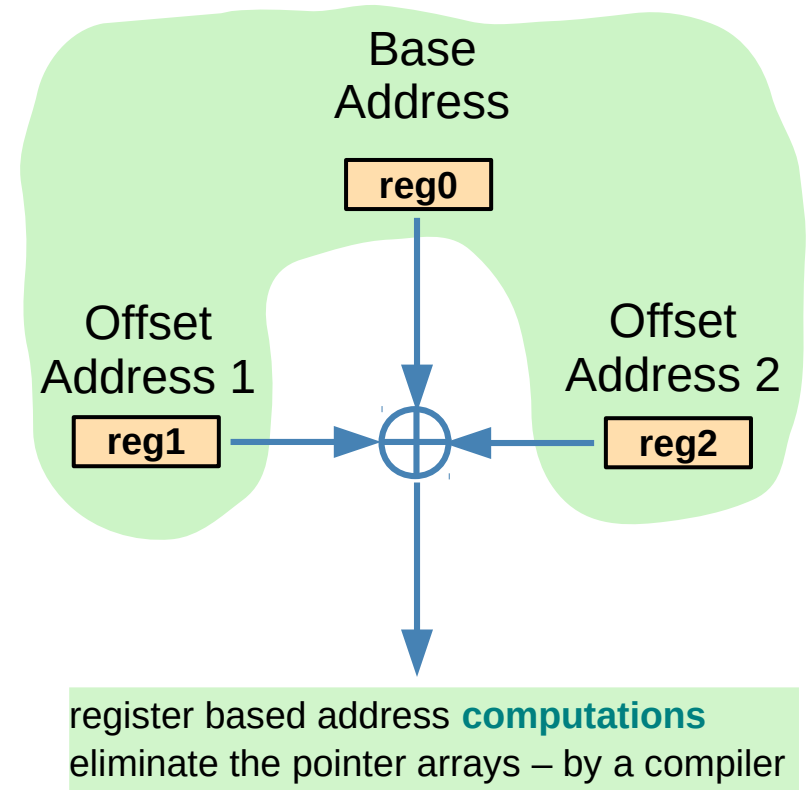
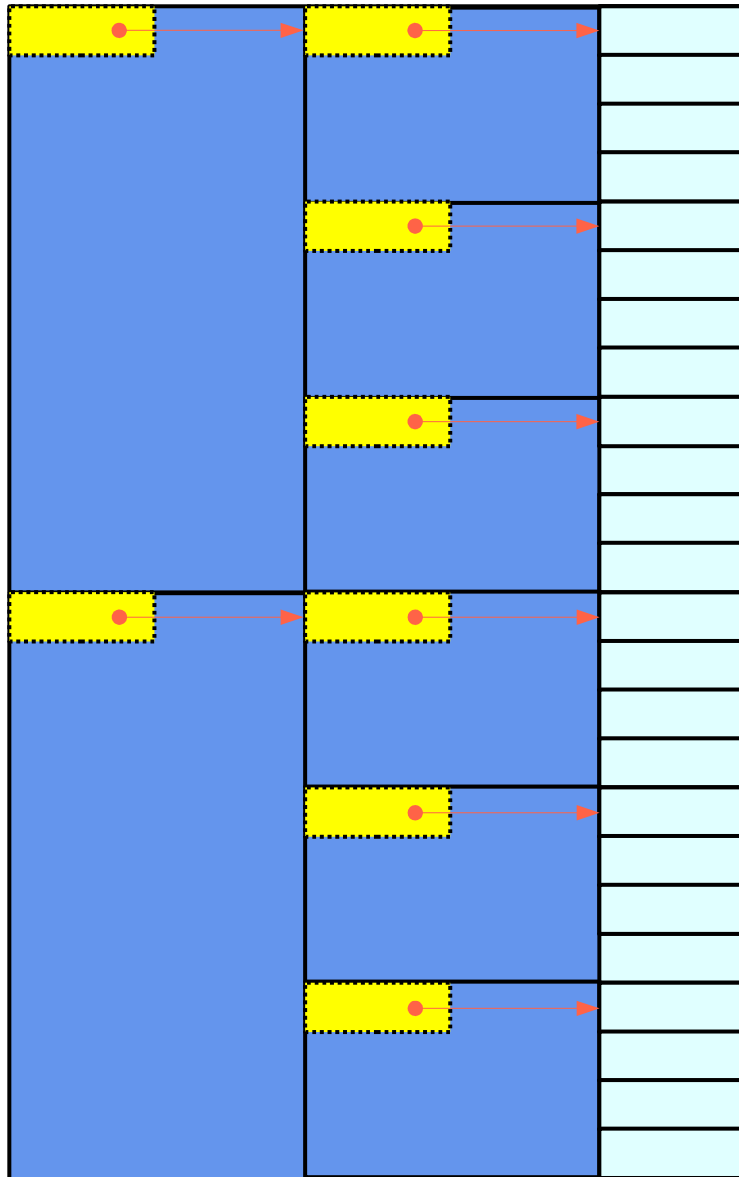


# Base and Offset Addressing





# Array Pointer Approach



**Array Pointer Approach**  
**(pointer to arrays)**

## References

- [1] Essential C, Nick Parlante
- [2] Efficient C Programming, Mark A. Weiss
- [3] C A Reference Manual, Samuel P. Harbison & Guy L. Steele Jr.
- [4] C Language Express, I. K. Chun