Laurent Series and z-Transform
- Geometric Series
Double Pole Properties B

20190107 Mon

2 formulas of z

① $$\frac{-1}{(z-1)(z-2)} = \left(\frac{1}{z-1} - \frac{1}{z-2}\right)$$

$$\Big\updownarrow z^{-1}$$

② $$\frac{-0.5z^2}{(z-1)(z-0.5)} = \left(-\frac{z}{(z-1)} + \frac{0.5z}{(z-0.5)}\right)$$

$$f(z) = \begin{cases} f_1(z) \\ f_2(z^{-1}) \end{cases} \qquad g(z) = \begin{cases} g_1(z) \\ g_2(z^{-1}) \end{cases}$$

$$X(z) = \begin{cases} X_1(z) \\ X_2(z^{-1}) \end{cases} \qquad Y(z) = \begin{cases} Y_1(z) \\ Y_2(z^{-1}) \end{cases}$$

① $\dfrac{-1}{(z-1)(z-2)}$ ② $\dfrac{-0.5z^2}{(z-1)(z-0.5)}$

$$+\frac{1}{z-1} - \frac{1}{z-2} \qquad -\frac{z}{(z-1)} + \frac{0.5z}{(z-0.5)}$$

$$-\frac{1}{1-z} + \frac{0.5}{1-0.5z} \quad |z|<1$$

causal $f(z)$

anti-causal $X(z)$

$$-\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}} \quad |z|>1$$

anti-causal $f(z)$

causal $X(z)$

$$+\frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \quad |z|>2$$

anti-causal $f(z)$

causal $X(z)$

$$+\frac{z}{1-z} - \frac{z}{1-2z} \quad |z|<0.5$$

causal $f(z)$

anti-causal $X(z)$

① $\dfrac{-1}{(z-1)(z-2)}$

② $\dfrac{-0.5z^2}{(z-1)(z-0.5)}$

$$+\frac{1}{z-1} - \frac{1}{z-2}$$

$\downarrow \cdot 1$ $\quad$ $\downarrow \cdot \frac{1}{2}$

$$-\frac{1}{1-z} + \frac{0.5}{1-0.5z} \qquad |z|<1$$

$$-\frac{z}{(z-1)} + \frac{0.5z}{(z-0.5)}$$

$\downarrow \cdot \frac{1}{z}$ $\quad$ $\downarrow \cdot \frac{1}{z}$

$$-\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}} \qquad |z|>1$$

$$+\frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \qquad |z|>2$$

$\uparrow \cdot \frac{1}{z}$ $\quad$ $\uparrow \cdot \frac{1}{z}$

$$+\frac{1}{z-1} - \frac{1}{z-2}$$

$$+\frac{z}{1-z} - \frac{z}{1-2z} \qquad |z|<0.5$$

$\uparrow \cdot \frac{1}{1}$ $\quad$ $\uparrow \cdot 2$

$$-\frac{z}{(z-1)} + \frac{0.5z}{(z-0.5)}$$

$$\boxed{① \ \frac{-1}{(z-1)(z-2)} \qquad ② \ \frac{-0.5z^2}{(z-1)(z-0.5)}}$$

$|z|<1 \qquad |0.5z|<1$

$|z|<1 \qquad |z|<2$

$|z^{-1}|<1 \qquad |0.5z^{-1}|<1$

$1<|z| \qquad 0.5<|z|$

$$\boxed{-\frac{1}{1-z}+\frac{0.5}{1-0.5z}} \ \boxed{|z|<1}$$

$$\boxed{-\frac{1}{1-z^{-1}}+\frac{0.5}{1-0.5z^{-1}}} \ \boxed{|z|>1}$$

$$\boxed{+\frac{z^{-1}}{1-z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}} \ \boxed{|z|>2}$$

$$\boxed{+\frac{z}{1-z}-\frac{z}{1-2z}} \ \boxed{|z|<0.5}$$

$|z^{-1}|<1 \qquad |2z^{-1}|<1$

$1<|z| \qquad 2<|z|$

$|z|<1 \qquad |2z|<1$

$|z|<1 \qquad |z|<0.5$

$$-\frac{1}{1-z}+\frac{0.5}{1-0.5z} \qquad |z|<1$$

$$\Big\downarrow \cdot\frac{1}{z} \qquad\qquad \Big\downarrow \cdot\frac{2}{z}$$

$$+\frac{z^{-1}}{1-z^{-1}}-\frac{z^{-1}}{1-2z^{-1}} \qquad |z|>2$$

$$-\frac{1}{1-z^{-1}}+\frac{0.5}{1-0.5z^{-1}} \qquad |z|>1$$

$$\Big\downarrow \cdot z \qquad\qquad \Big\downarrow \cdot 2z$$

$$+\frac{z}{1-z}-\frac{z}{1-2z} \qquad |z|<0.5$$

$$-\frac{1}{1-z}+\frac{0.5}{1-0.5z} \qquad |z|<1$$

$$\Big\uparrow \cdot z \qquad\qquad \Big\uparrow \cdot\frac{z}{2}$$

$$+\frac{z^{-1}}{1-z^{-1}}-\frac{z^{-1}}{1-2z^{-1}} \qquad |z|>2$$

$$-\frac{1}{1-z^{-1}}+\frac{0.5}{1-0.5z^{-1}} \qquad |z|>1$$

$$\Big\uparrow \cdot\frac{1}{z} \qquad\qquad \Big\uparrow \cdot\frac{1}{2z}$$

$$+\frac{z}{1-z}-\frac{z}{1-2z} \qquad |z|<0.5$$

Causal sequence a_n & x_n

$$-\frac{1}{1-z} + \frac{0.5}{1-0.5z} \qquad |z|<1$$

causal $f_1(z) =$

$-[1 + 1^2 z^1 + 1^3 z^2 + \cdots]$ $\qquad -1^{n+1}$

$+[(\frac{1}{2}) + (\frac{1}{2})^2 z^1 + (\frac{1}{2})^3 z^2 + \cdots]$ $\qquad +(\frac{1}{2})^{n+1}$

0 1 2

$$-\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}} \qquad |z|>1$$

causal $Y_1(z) =$

$-[1^1 z^0 + 1^2 z^{-1} + 1^3 z^{-2} + \cdots]$ $\qquad -1^{n+1}$

$+[(\frac{1}{2})^1 z^0 + (\frac{1}{2})^2 z^{-1} + (\frac{1}{2})^3 z^{-2} + \cdots]$ $\qquad +(\frac{1}{2})^{n+1}$

0 1 2

$$+\frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \qquad |z|>2$$

causal $X_2(z)$

$+[(\frac{1}{1})^0 z^{-1} + (\frac{1}{1})^1 z^{-2} + (\frac{1}{1})^2 z^{-3} + \cdots]$ $\qquad +(\frac{1}{1})^{n-1}$

$-[2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots]$ $\qquad -2^{n-1}$

1 2 3

$$+\frac{z}{1-z} - \frac{z}{1-2z} \qquad |z|<0.5$$

causal $g_2(z)$

$+[(\frac{1}{1})^0 z^1 + (\frac{1}{1})^1 z^2 + (\frac{1}{1})^2 z^3 + \cdots]$ $\qquad +(\frac{1}{1})^{n-1}$

$-[2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \cdots]$ $\qquad -2^{n-1}$

1 2 3

Anti-causal sequence a_n & x_n

$$\boxed{-\frac{1}{1-z}+\frac{0.5}{1-0.5z}}\ \boxed{|z|<1}$$

anti-causal $X_1(z)$

$$\boxed{\begin{array}{c} 2=\left(\frac{1}{2}\right)^{-1} \\ \left(\frac{1}{2}\right)=2^{-1} \end{array}}$$

$$-\left[\left(\frac{1}{1}\right)^{-1}+\left(\frac{1}{1}\right)^{-2}z^{1}+\left(\frac{1}{1}\right)^{-3}z^{2}+\cdots\right] \quad -\left(\frac{1}{1}\right)^{n-1}$$

$$+\left[2^{-1}+2^{-2}z^{1}+2^{-3}z^{2}+\cdots\right] \quad +2^{n-1}$$

0   -1   -2

$$\boxed{-\frac{1}{1-z^{-1}}+\frac{0.5}{1-0.5z^{-1}}}\ \boxed{|z|>1}$$

anti-causal $g_1(z)$

$$-\left[\left(\frac{1}{1}\right)^{-1}z^{0}+\left(\frac{1}{1}\right)^{-2}z^{-1}+\left(\frac{1}{1}\right)^{-3}z^{-2}+\cdots\right] \quad -\left(\frac{1}{1}\right)^{n-1}$$

$$+\left[2^{-1}z^{0}+2^{-2}z^{-1}+2^{-3}z^{-2}+\cdots\right] \quad +2^{n-1}$$

0   -1   -2

$$\boxed{+\frac{z^{-1}}{1-z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}}\ \boxed{|z|>2}$$

anti-causal $f_2(z)$

$$\boxed{\begin{array}{c} 2=\left(\frac{1}{2}\right)^{-1} \\ \left(\frac{1}{2}\right)=2^{-1} \end{array}}$$

$$+\left[1^{0}z^{-1}+1^{-1}z^{-2}+1^{-2}z^{-3}+\cdots\right] \quad +1^{n+1}$$

$$-\left[\left(\frac{1}{2}\right)^{0}z^{-1}+\left(\frac{1}{2}\right)^{-1}z^{-2}+\left(\frac{1}{2}\right)^{-2}z^{-3}+\cdots\right] \quad -\left(\frac{1}{2}\right)^{n+1}$$

-1   -2   -3

$$\boxed{+\frac{z}{1-z}-\frac{z}{1-2z}}\ \boxed{|z|<0.5}$$

anti-causal $Y_2(z)$

$$+\left[1^{0}z^{1}+1^{-1}z^{2}+1^{-2}z^{3}+\cdots\right] \quad +1^{n+1}$$

$$-\left[\left(\frac{1}{2}\right)^{0}z^{1}+\left(\frac{1}{2}\right)^{-1}z^{2}+\left(\frac{1}{2}\right)^{-2}z^{3}+\cdots\right] \quad -\left(\frac{1}{2}\right)^{n+1}$$

-1   -2   -3

$$\boxed{-\frac{1}{1-z}+\frac{0.5}{1-0.5z}} \quad \boxed{|z|<1}$$

causal $f_1(z) =$

$-[1 + 1^2 z^1 + 1^3 z^2 + \cdots]$ $\quad -1^{n+1}$

$+[(\frac{1}{2}) + (\frac{1}{2})^2 z^1 + (\frac{1}{2})^3 z^2 + \cdots]$ $\quad +(\frac{1}{2})^{n+1}$

0 1 2

anti-causal $X_1(z)$

$-[(\frac{1}{1})^{-1} + (\frac{1}{1})^{-2} z^1 + (\frac{1}{1})^{-3} z^2 + \cdots]$ $\quad -(\frac{1}{1})^{n-1}$

$+[2^{-1} + 2^{-2} z^1 + 2^{-3} z^2 + \cdots]$ $\quad +2^{n-1}$

0 -1 -2

$$\boxed{-\frac{1}{1-z^{-1}}+\frac{0.5}{1-0.5z^{-1}}} \quad \boxed{|z|>1}$$

anti-causal $g_1(z)$

$-[(\frac{1}{1})^{-1} z^0 + (\frac{1}{1})^{-2} z^{-1} + (\frac{1}{1})^{-3} z^{-2} + \cdots]$ $\quad -(\frac{1}{1})^{n-1}$

$+[2^{-1} z^0 + 2^{-2} z^{-1} + 2^{-3} z^{-2} + \cdots]$ $\quad +2^{n-1}$

0 -1 -2

causal $Y_1(z) =$

$-[1^1 z^0 + 1^2 z^{-1} + 1^3 z^{-2} + \cdots]$ $\quad -1^{n+1}$

$+[(\frac{1}{2})^1 z^0 + (\frac{1}{2})^2 z^{-1} + (\frac{1}{2})^3 z^{-2} + \cdots]$ $\quad +(\frac{1}{2})^{n+1}$

0 1 2

$$\boxed{+\frac{z^{-1}}{1-z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}} \quad \boxed{|z|>2}$$

anti-causal $f_2(z)$

$+[1^0 z^{-1} + 1^{-1} z^{-2} + 1^{-2} z^{-3} + \cdots]$ $\quad +1^{n+1}$

$-[(\frac{1}{2})^0 z^{-1} + (\frac{1}{2})^{-1} z^{-2} + (\frac{1}{2})^{-2} z^{-3} + \cdots]$ $\quad -(\frac{1}{2})^{n+1}$

-1 -2 -3

causal $X_2(z)$

$+[(\frac{1}{1})^0 z^{-1} + (\frac{1}{1})^1 z^{-2} + (\frac{1}{1})^2 z^{-3} + \cdots]$ $\quad +(\frac{1}{1})^{n-1}$

$-[2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots]$ $\quad -2^{n-1}$

1 2 3

$$\boxed{+\frac{z}{1-z}-\frac{z}{1-2z}} \quad \boxed{|z|<0.5}$$

causal $g_2(z)$

$+[(\frac{1}{1})^0 z^1 + (\frac{1}{1})^1 z^2 + (\frac{1}{1})^2 z^3 + \cdots]$ $\quad +(\frac{1}{1})^{n-1}$

$-[2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \cdots]$ $\quad -2^{n-1}$

1 2 3

anti-causal $Y_2(z)$

$+[1^0 z^1 + 1^{-1} z^2 + 1^{-2} z^3 + \cdots]$ $\quad +1^{n+1}$

$-[(\frac{1}{2})^0 z^1 + (\frac{1}{2})^{-1} z^2 + (\frac{1}{2})^{-2} z^3 + \cdots]$ $\quad -(\frac{1}{2})^{n+1}$

-1 -2 -3

$$\boxed{-\frac{1}{1-z}+\frac{0.5}{1-0.5z}} \quad \boxed{|z|<1}$$

$$\boxed{-\frac{1}{1-z^{-1}}+\frac{0.5}{1-0.5z^{-1}}} \quad \boxed{|z|>1}$$

$$f(z) = -\left[1 + 1^2 z + 1^3 z^2 + \cdots\right] + \left[\left(\tfrac{1}{2}\right) + \left(\tfrac{1}{2}\right)^2 z + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right]$$

$$\boxed{a_n = -1^{n+1} + \left(\tfrac{1}{2}\right)^{n+1} \quad (n \geq 0)}$$

$$f(z) = -\left[\left(\tfrac{1}{1}\right)^{-1} z^0 + \left(\tfrac{1}{1}\right)^{-2} z^{-1} + \left(\tfrac{1}{1}\right)^{-3} z^{-2} + \cdots\right] + \left[2^{-1} z^0 + 2^{-2} z^{-1} + 2^{-3} z^{-2} + \cdots\right]$$

$$\boxed{a_n = -\left(\tfrac{1}{1}\right)^{n-1} + 2^{n-1} \quad (n < 1)}$$

$$X(z) = -\left[\left(\tfrac{1}{1}\right)^{-1} + \left(\tfrac{1}{1}\right)^{-2} z^1 + \left(\tfrac{1}{1}\right)^{-3} z^2 + \cdots\right] + \left[2^{-1} + 2^{-2} z^1 + 2^{-3} z^2 + \cdots\right]$$

$$\boxed{x_n = -\left(\tfrac{1}{1}\right)^{n-1} + 2^{n-1} \quad (n < 1)}$$

$$X(z) = -\left[1^1 z^0 + 1^2 z^{-1} + 1^3 z^{-2} + \cdots\right] + \left[\left(\tfrac{1}{2}\right)^1 z^0 + \left(\tfrac{1}{2}\right)^2 z^{-1} + \left(\tfrac{1}{2}\right)^3 z^{-2} + \cdots\right]$$

$$\boxed{x_n = -1^{n+1} + \left(\tfrac{1}{2}\right)^{n+1} \quad (n \geq 0)}$$

$$\boxed{+\frac{z^{-1}}{1-z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}} \quad \boxed{|z|>2}$$

$$\boxed{+\frac{z}{1-z}-\frac{z}{1-2z}} \quad \boxed{|z|<0.5}$$

$$f(z) = +\left[1^0 z^{-1} + 1^{-1} z^{-2} + 1^{-2} z^{-3} + \cdots\right] - \left[\left(\tfrac{1}{2}\right)^0 z^{-1} + \left(\tfrac{1}{2}\right)^{-1} z^{-2} + \left(\tfrac{1}{2}\right)^{-2} z^{-3} + \cdots\right]$$

$$\boxed{a_n = +1^{n+1} - \left(\tfrac{1}{2}\right)^{n+1} \quad (n < 0)}$$

$$f(z) = +\left[\left(\tfrac{1}{1}\right)^0 z^1 + \left(\tfrac{1}{1}\right)^1 z^2 + \left(\tfrac{1}{1}\right)^2 z^3 + \cdots\right] - \left[2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \cdots\right]$$

$$\boxed{a_n = +\left(\tfrac{1}{1}\right)^{n-1} - 2^{n-1} \quad (n \geq 1)}$$

$$X(z) = +\left[\left(\tfrac{1}{1}\right)^0 z^{-1} + \left(\tfrac{1}{1}\right)^1 z^{-2} + \left(\tfrac{1}{1}\right)^2 z^{-3} + \cdots\right] - \left[2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots\right]$$

$$\boxed{x_n = +\left(\tfrac{1}{1}\right)^{n-1} - 2^{n-1} \quad (n \geq 1)}$$

$$X(z) = +\left[1^0 z^1 + 1^{-1} z^2 + 1^{-2} z^3 + \cdots\right] - \left[\left(\tfrac{1}{2}\right)^0 z^1 + \left(\tfrac{1}{2}\right)^{-1} z^2 + \left(\tfrac{1}{2}\right)^{-2} z^3 + \cdots\right]$$

$$\boxed{x_n = +1^{n+1} - \left(\tfrac{1}{2}\right)^{n+1} \quad (n < 0)}$$

1 - A

2 - A

1 - B

2 - B

$f(z)$ $\quad |z| < 0.5 \quad |z| > 2$

causal anticausal

$$\frac{-1}{(z-1)(z-2)} = +\frac{1}{z-1} - \frac{1}{z-2}$$

① – Ⓐ $\boxed{-\frac{1}{1-z} + \frac{0.5}{1-0.5z}}$ $\boxed{|z|<1}$ $\boxed{-1^{n+1} + \left(\frac{1}{2}\right)^{n+1}}$ $\boxed{(n \geq 0)}$

$$-\underset{n=0}{(1} + \underset{n=1}{1^2 z} + \underset{n=2}{1^3 z^2} + \cdots) + \left(\underset{n=0}{\left(\tfrac{1}{2}\right)} + \underset{n=1}{\left(\tfrac{1}{2}\right)^2 z} + \underset{n=2}{\left(\tfrac{1}{2}\right)^3 z^2} + \cdots\right)$$

① – Ⓑ $\boxed{\frac{z^{-1}}{1-z^{-1}} - \frac{1}{1-2z^{-1}}}$ $\boxed{|z|>2}$ $\boxed{+1^{n+1} - \left(\frac{1}{2}\right)^{n+1}}$ $\boxed{(n<0)}$

$$\left(z^{-1} + \left(\tfrac{1}{1}\right)^1 z^{-2} + \left(\tfrac{1}{1}\right)^2 z^{-3} + \cdots\right) - \left(z^{-1} + 2z^{-2} + 2^2 z^{-3} + \cdots\right)$$

$$\left(\underset{n=-1}{1^0 z^{-1}} + \underset{n=-2}{1^{-1} z^{-2}} + \underset{n=-3}{1^{-2} z^{-3}} + \cdots\right) - \left(\underset{n=-1}{\left(\tfrac{1}{2}\right)^0 z^{-1}} + \underset{n=-2}{\left(\tfrac{1}{2}\right)^{-1} z^{-2}} + \underset{n=-3}{\left(\tfrac{1}{2}\right)^{-2} z^{-3}} + \cdots\right)$$

$$\frac{-0.5z^2}{(z-1)(z-0.5)} = -\frac{z}{(z-1)} + \frac{0.5z}{(z-0.5)}$$

② – Ⓐ $\boxed{-\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}}}$ $\boxed{|z|>1}$ $\boxed{-\left(\frac{1}{1}\right)^{n-1} + 2^{n-1}}$ $\boxed{(n<1)}$

$$-\left(1 + 1^2 z^{-1} + 1^3 z^{-2} + \cdots\right) + \left(\left(\tfrac{1}{2}\right) + \left(\tfrac{1}{2}\right)^2 z^{-1} + \left(\tfrac{1}{2}\right)^3 z^{-2} + \cdots\right)$$

$$-\left(\underset{n=0}{\left(\tfrac{1}{1}\right)^{-1}} + \underset{n=-1}{\left(\tfrac{1}{1}\right)^{-2} z^{-1}} + \underset{n=-2}{\left(\tfrac{1}{1}\right)^{-3} z^{-2}} + \cdots\right) + \left(\underset{n=0}{2^{-1}} + \underset{n=-1}{2^{-2} z^{-1}} + \underset{n=-2}{2^{-3} z^{-2}} + \cdots\right)$$

② – Ⓑ $\boxed{+\frac{z}{1-z} - \frac{z}{1-2z}}$ $\boxed{|z|<0.5}$ $\boxed{+1^{n-1} - 2^{n-1}}$ $\boxed{(n \geq 1)}$

$$+\left(\underset{n=1}{z} + \underset{n=2}{1z^2} + \underset{n=3}{1^2 z^3} + \cdots\right) - \left(\underset{n=1}{z} + \underset{n=2}{2z^2} + \underset{n=3}{2^2 z^3} + \cdots\right)$$

$X(z)$ $\quad |z| < 0.5 \quad |z| > 2$

anticausal $\quad$ causal

$$\frac{-1}{(z-1)(z-2)} = +\frac{1}{z-1} - \frac{1}{z-2}$$

① − Ⓐ $\quad \boxed{-\frac{1}{1-z} + \frac{0.5}{1-0.5z}} \quad \boxed{|z| < 1} \quad \boxed{-\left(\frac{1}{1}\right)^{n-1} + 2^{n-1}} \quad \boxed{(n < 1)}$

$$-\left(1^1 z^0 + 1^2 z^1 + 1^3 z^2 + \cdots\right) + \left(\left(\tfrac{1}{2}\right) z^0 + \left(\tfrac{1}{2}\right)^2 z^1 + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right)$$

$$-\left(\left(\tfrac{1}{1}\right)^{-1} z^0 + \left(\tfrac{1}{1}\right)^{-2} z^1 + \left(\tfrac{1}{1}\right)^{-3} z^2 + \cdots\right) + \left(2^{-1} z^0 + 2^{-2} z^1 + 2^{-3} z^2 + \cdots\right)$$

$n=0 \quad n=-1 \quad n=-2 \qquad n=0 \quad n=-1 \quad n=-2$

① − Ⓑ $\quad \boxed{\frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}} \quad \boxed{|z| > 2} \quad \boxed{+\left(\frac{1}{1}\right)^{n-1} - 2^{n-1}} \quad \boxed{(n \geq 1)}$

$$\left(\left(\tfrac{1}{1}\right)^0 z^{-1} + \left(\tfrac{1}{1}\right)^1 z^{-2} + \left(\tfrac{1}{1}\right)^2 z^{-3} + \cdots\right) - \left(2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots\right)$$

$n=1 \quad n=2 \quad n=3 \qquad n=1 \quad n=2 \quad n=3$

$$\frac{-0.5z^2}{(z-1)(z-0.5)} = -\frac{z}{(z-1)} + \frac{0.5z}{(z-0.5)}$$

② − Ⓐ $\quad \boxed{-\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}}} \quad \boxed{|z| > 1} \quad \boxed{-1^{n+1} + \left(\frac{1}{2}\right)^{n+1}} \quad \boxed{(n \geq 0)}$

$$-\left(1 + 1^2 z^{-1} + 1^3 z^{-2} + \cdots\right) + \left(\left(\tfrac{1}{2}\right) + \left(\tfrac{1}{2}\right)^2 z^{-1} + \left(\tfrac{1}{2}\right)^3 z^{-2} + \cdots\right)$$

$n=0 \quad n=1 \quad n=2 \qquad n=0 \quad n=1 \quad n=2$

② − Ⓑ $\quad \boxed{+\frac{z}{1-z} - \frac{z}{1-2z}} \quad \boxed{|z| < 0.5} \quad \boxed{+1^{n+1} - \left(\frac{1}{2}\right)^{n+1}} \quad \boxed{(n < 0)}$

$$+\left(z + \left(\tfrac{1}{1}\right) z^2 + \left(\tfrac{1}{1}\right)^2 z^3 + \cdots\right) - \left(z + 2z^2 + 2^2 z^3 + \cdots\right)$$

$$+\left(1^0 z + 1^{-1} z^2 + 1^{-2} z^3 + \cdots\right) - \left(\left(\tfrac{1}{2}\right)^0 z + \left(\tfrac{1}{2}\right)^{-1} z^2 + \left(\tfrac{1}{2}\right)^{-2} z^3 + \cdots\right)$$

$n=-1 \quad n=-2 \quad n=-3 \qquad n=-1 \quad n=-2 \quad n=-3$

$f(z) \Longleftrightarrow a_n$

$X(z) \Longleftrightarrow x_n$

① - Ⓐ

$$\boxed{-\frac{1}{1-z} + \frac{0.5}{1-0.5z}} \quad \boxed{|z| < 1}$$

$$a_n = \boxed{-1^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$

$$x_n = \boxed{-1^{n-1} + 2^{n-1}} \quad (n < 1)$$

② - Ⓐ

$$\boxed{-\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}}} \quad \boxed{|z| > 1}$$

$$a_n = \boxed{-1^{n-1} + 2^{n-1}} \quad (n < 1)$$

$$x_n = \boxed{-1^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$

① - Ⓑ

$$\boxed{+\frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}} \quad \boxed{|z| > 2}$$

$$a_n = \boxed{+1^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}} \quad (n < 0)$$

$$x_n = \boxed{+1^{n-1} - 2^{n-1}} \quad (n \geq 1)$$

② - Ⓑ

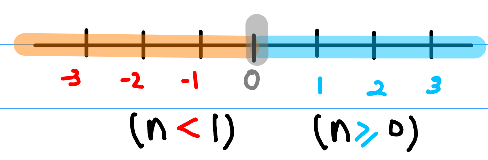
$$\boxed{+\frac{z}{1-z} - \frac{z}{1-2z}} \quad \boxed{|z| < 0.5}$$

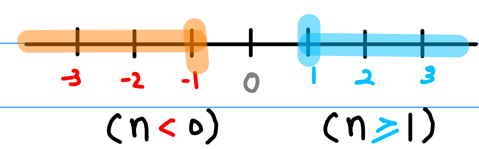
$$a_n = \boxed{+1^{n-1} - 2^{n-1}} \quad (n \geq 1)$$

$$x_n = \boxed{+1^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}} \quad (n < 0)$$

$$x_n = a_{-n}$$

$$a_n = x_{-n}$$





$(n \geq 0) \Longleftrightarrow (n < 1)$

$(n \geq 1) \Longleftrightarrow (n < 0)$

I $(z^{-1}, R^{-1}) \Leftrightarrow (a_{-n}, -N)$

1-A

$$-\frac{1}{1-z} + \frac{0.5}{1-0.5z} \quad |z|<1$$

z^{-1}, R^{-1}

$$a_n = -1^{n+1} + \left(\tfrac{1}{2}\right)^{n+1} \quad (n \geq 0)$$

$a_{-n}, -N$

$$x_n = -1^{n-1} + 2^{n-1} \quad (n < 1)$$

$a_{-n}, -N$

2-A

$$-\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}} \quad |z|>1$$

$$a_n = -1^{n-1} + 2^{n-1} \quad (n<1)$$

$$x_n = -1^{n+1} + \left(\tfrac{1}{2}\right)^{n+1} \quad (n \geq 0)$$

1-B

$$+\frac{z^{-1}}{1-z^{-1}} - \frac{z^{1}}{1-2z^{-1}} \quad |z|>2$$

z^{-1}, R^{-1}

$$a_n = +1^{n+1} - \left(\tfrac{1}{2}\right)^{n+1} \quad (n<0)$$

$a_{-n}, -N$

$$x_n = +1^{n-1} - 2^{n-1} \quad (n \geq 1)$$

$a_{-n}, -N$

2-B

$$+\frac{z}{1-z} - \frac{z}{1-2z} \quad |z|<0.5$$

$$a_n = +1^{n-1} - 2^{n-1} \quad (n \geq 1)$$

$$x_n = +1^{n+1} - \left(\tfrac{1}{2}\right)^{n+1} \quad (n<0)$$

II $\quad (z, R^{-1}) \Leftrightarrow (-a_n, N^c)$

① − Ⓐ

$$-\frac{1}{1-z}+\frac{0.5}{1-0.5z} \qquad |z|<1$$

$\| \qquad \updownarrow R^{-1}$

① − Ⓑ

$$+\frac{z^{-1}}{1-z^{-1}}-\frac{z^{-1}}{1-2z^{-1}} \qquad |z|>2$$

② − Ⓐ

$$-\frac{1}{1-z^{-1}}+\frac{0.5}{1-0.5z^{-1}} \qquad |z|>1$$

$\| \qquad \updownarrow R^{-1}$

② − Ⓑ

$$+\frac{z}{1-z}-\frac{z}{1-2z} \qquad |z|<0.5$$

① − Ⓐ

$a_n = \boxed{-1^{n+1}+\left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$

$\updownarrow -a_n \qquad \updownarrow N^c$

① − Ⓑ

$a_n = \boxed{+1^{n+1}-\left(\tfrac{1}{2}\right)^{n+1}} \quad (n < 0)$

② − Ⓐ

$a_n = \boxed{-1^{n-1}+2^{n-1}} \quad (n<1)$

$\updownarrow -a_n \qquad \updownarrow N^c$

② − Ⓑ

$a_n = \boxed{+1^{n-1}-2^{n-1}} \quad (n \geq 1)$

① − Ⓐ

$x_n = \boxed{-1^{n-1}+2^{n-1}} \quad (n<1)$

$\updownarrow -a_n \qquad \updownarrow N^c$

① − Ⓑ

$x_n = \boxed{+1^{n-1}-2^{n-1}} \quad (n \geq 1)$

② − Ⓐ

$x_n = \boxed{-1^{n+1}+\left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$

$\updownarrow -a_n \qquad \updownarrow N^c$

② − Ⓑ

$x_n = \boxed{+1^{n+1}-\left(\tfrac{1}{2}\right)^{n+1}} \quad (n<0)$

III ①

$$(z^{-1}, R) \Leftrightarrow (-a_{-n}, (-N)^c)$$

$$(z^{-1}, R^{-1}) \rightarrow (z, R^{-1}) \qquad (a_{-n}, -N) \rightarrow (-a_n, N^c)$$

①-Ⓐ

$$\boxed{-\frac{1}{1-z} + \frac{0.5}{1-0.5z}} \; \boxed{|z|<1}$$

$$a_n = \boxed{-1^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$

②-Ⓐ

$$\boxed{-\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}}} \; \boxed{|z|>1}$$

$$a_n = \boxed{-1^{n-1} + 2^{n-1}} \quad (n<1)$$

①-Ⓑ

$$\boxed{+\frac{z^{-1}}{1-z^{-1}} - \frac{z^{1}}{1-2z^{-1}}} \; \boxed{|z|>2}$$

$$a_n = \boxed{+1^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}} \quad (n<0)$$

②-Ⓑ

$$\boxed{+\frac{z}{1-z} - \frac{z}{1-2z}} \; \boxed{|z|<0.5}$$

$$a_n = \boxed{+1^{n-1} - 2^{n-1}} \quad (n \geq 1)$$

①-Ⓐ

$$\boxed{-\frac{1}{1-z} + \frac{0.5}{1-0.5z}} \; \boxed{|z|<1}$$

$$x_n = \boxed{+1^{n-1} - 2^{n-1}} \quad (n \geq 1)$$

②-Ⓐ

$$\boxed{-\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}}} \; \boxed{|z|>1}$$

$$x_n = \boxed{+1^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}} \quad (n<0)$$

①-Ⓑ

$$\boxed{+\frac{z^{-1}}{1-z^{-1}} - \frac{z^{1}}{1-2z^{-1}}} \; \boxed{|z|>2}$$

$$x_n = \boxed{-1^{n-1} + 2^{n-1}} \quad (n<1)$$

②-Ⓑ

$$\boxed{+\frac{z}{1-z} - \frac{z}{1-2z}} \; \boxed{|z|<0.5}$$

$$x_n = \boxed{-1^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$

III ②

$$(z^{-1}, R) \Leftrightarrow (-a_{-n}, (-N)^c)$$

$$(z^{-1}, R^{-1}) \to (z, R^{-1}) \qquad (a_{-n}, -N) \to (-a_n, N^c)$$

① – Ⓐ $\boxed{-\frac{1}{1-z} + \frac{0.5}{1-0.5z}}\ \boxed{|z|<1}$ $\Longleftrightarrow$ ② – Ⓑ $\boxed{+\frac{z}{1-z} - \frac{z}{1-2z}}\ \boxed{|z|<0.5}$

$a_n = \boxed{-1^{n+1} + (\frac{1}{2})^{n+1}}$ $(n \geq 0)$ $\xleftrightarrow{-a_{-n},\, (-N)^c}$ $a_n = \boxed{+1^{n-1} - 2^{n-1}}$ $(n \geq 1)$

① – Ⓑ $\boxed{+\frac{z^{-1}}{1-z^{-1}} - \frac{z^{1}}{1-2z^{-1}}}\ \boxed{|z|>2}$ $\Longleftrightarrow$ ② – Ⓐ $\boxed{-\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}}}\ \boxed{|z|>1}$

$a_n = \boxed{+1^{n+1} - (\frac{1}{2})^{n+1}}$ $(n<0)$ $\xleftrightarrow{-a_{-n},\, (-N)^c}$ $a_n = \boxed{-1^{n-1} + 2^{n-1}}$ $(n<1)$

① – Ⓐ $\boxed{-\frac{1}{1-z} + \frac{0.5}{1-0.5z}}\ \boxed{|z|<1}$ $\Longleftrightarrow$ ② – Ⓑ $\boxed{+\frac{z}{1-z} - \frac{z}{1-2z}}\ \boxed{|z|<0.5}$

$x_n = \boxed{+1^{n-1} - 2^{n-1}}$ $(n \geq 1)$ $\xleftrightarrow{-a_{-n},\, (-N)^c}$ $x_n = \boxed{-1^{n+1} + (\frac{1}{2})^{n+1}}$ $(n \geq 0)$

① – Ⓑ $\boxed{+\frac{z^{-1}}{1-z^{-1}} - \frac{z^{1}}{1-2z^{-1}}}\ \boxed{|z|>2}$ $\Longleftrightarrow$ ② – Ⓐ $\boxed{-\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}}}\ \boxed{|z|>1}$

$x_n = \boxed{-1^{n-1} + 2^{n-1}}$ $(n<1)$ $\xleftrightarrow{-a_{-n},\, (-N)^c}$ $x_n = \boxed{+1^{n+1} - (\frac{1}{2})^{n+1}}$ $(n<0)$

IV $(a_n, N) \Leftrightarrow (x_{-n}, -N)$

① − Ⓐ

$$-\frac{1}{1-z}+\frac{0.5}{1-0.5z} \qquad |z|<1$$

$$\xleftrightarrow{z^{-1}}$$

② − Ⓐ

$$-\frac{1}{1-z^{-1}}+\frac{0.5}{1-0.5z^{-1}} \qquad |z|>1$$

① − Ⓐ

$$a_n = -1^{n+1} + \left(\tfrac{1}{2}\right)^{n+1} \qquad (n \geq 0)$$

$$\Big\updownarrow a_{-n} \qquad \Big\updownarrow -N$$

$$x_n = -1^{n-1} + 2^{n-1} \qquad (n<1)$$

$$\xleftrightarrow{-n}$$

② − Ⓐ

$$a_n = -1^{n-1} + 2^{n-1} \qquad (n<1)$$

$$\Big\updownarrow a_{-n} \qquad \Big\updownarrow -N$$

$$x_n = -1^{n+1} + \left(\tfrac{1}{2}\right)^{n+1} \qquad (n \geq 0)$$

I

① − Ⓑ

$$+\frac{z^{-1}}{1-z^{-1}}-\frac{z^{-1}}{1-2z^{-1}} \qquad |z|>2$$

$$\xleftrightarrow{z^{-1}}$$

② − Ⓑ

$$+\frac{z}{1-z}-\frac{z}{1-2z} \qquad |z|<0.5$$

① − Ⓑ

$$a_n = +1^{n+1} - \left(\tfrac{1}{2}\right)^{n+1} \qquad (n<0)$$

$$\Big\updownarrow a_{-n} \qquad \Big\updownarrow -N$$

$$x_n = +1^{n-1} - 2^{n-1} \qquad (n \geq 1)$$

$$\xleftrightarrow{-n}$$

② − Ⓑ

$$a_n = +1^{n-1} - 2^{n-1} \qquad (n \geq 1)$$

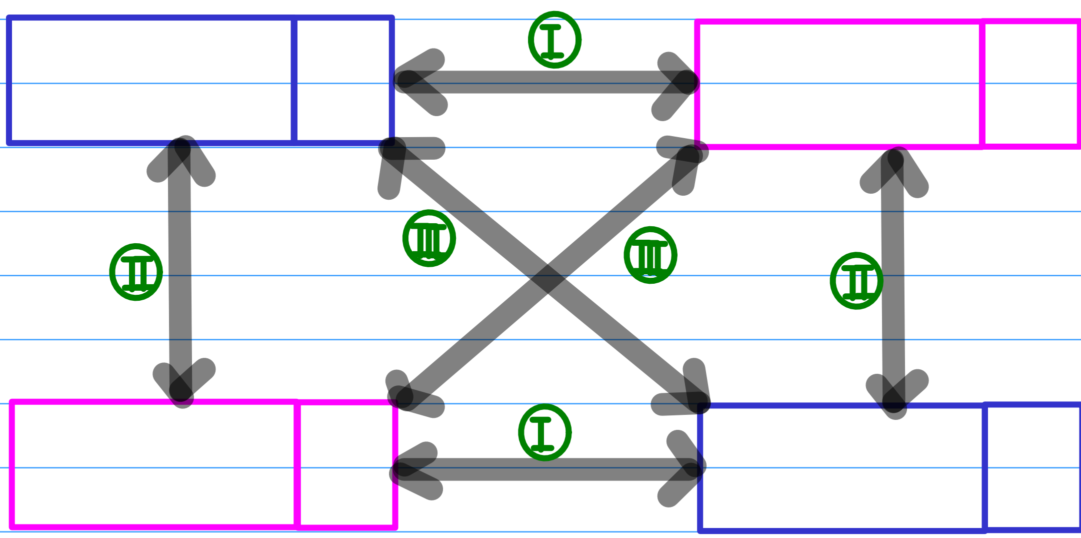
$$\Big\updownarrow a_{-n} \qquad \Big\updownarrow -N$$

$$x_n = +1^{n+1} - \left(\tfrac{1}{2}\right)^{n+1} \qquad (n<0)$$

I $(z^{-1}, R^{-1}) \Leftrightarrow (a_{-n}, -N)$

II $(z, R^{-1}) \Leftrightarrow (-a_n, N^c)$

III $(z^{-1}, R) \Leftrightarrow (-a_{-n}, (-N)^c) = (-a_{-n}, -(N^c))$



$(a_n, N) \Leftrightarrow (x_{-n}, -N)$

ROC's of interests

R1(z) R2(z) R1(z^{-1}) R2(z^{-1})

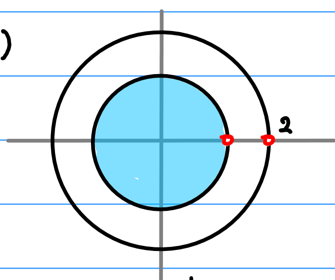
① $\dfrac{-1}{(z-1)(z-2)} \xleftrightarrow{z^{-1}}$ ② $\dfrac{-0.5z^2}{(z-1)(z-0.5)}$

$\boxed{+\dfrac{1}{z-1} - \dfrac{1}{z-2}} \xleftrightarrow{z^{-1}} \boxed{-\dfrac{z}{(z-1)} + \dfrac{0.5z}{(z-0.5)}}$

$p_1 = 1$ $\xleftrightarrow{z^{-1}}$ $p_1^{-1} = 1$

$p_2 = 2$ $\qquad$ $p_2^{-1} = 0.5$

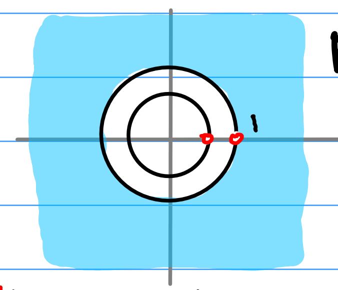
R1(z)



$|z| < 1$

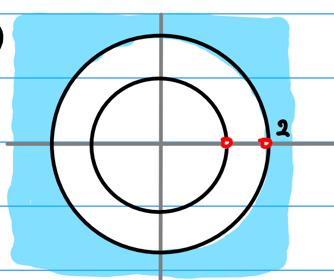
$\xleftrightarrow{z^{-1}}$

R1(z^{-1})



$|z^{-1}| < 1$ $\qquad$ $|z| > 1$

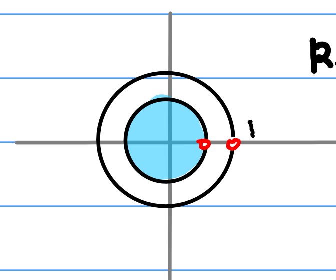
R2(z)



$|z| > 2$

$\xleftrightarrow{z^{-1}}$

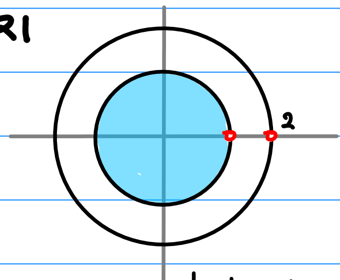
R2(z^{-1})



$|z^{-1}| > 2$ $\qquad$ $|z| < 0.5$

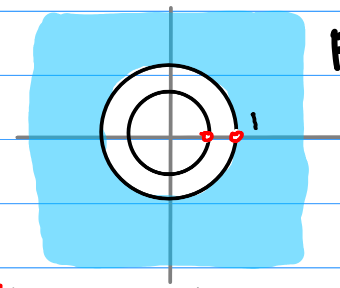
R^{-1}

$R1(z) \triangleq R1$



$|z| < 1$

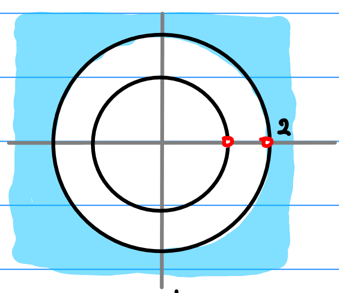
$\overset{z^{-1}}{\longleftrightarrow}$



$R1(z^{-1}) \triangleq$

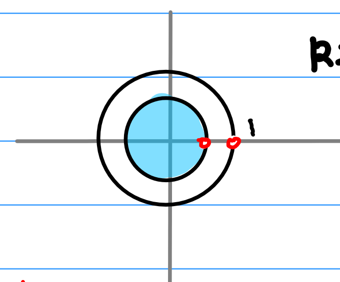
$|z^{-1}| < 1$ $\quad$ $|z| > 1$

$R2(z)$



$|z| > 2$

$\overset{z^{-1}}{\longleftrightarrow}$

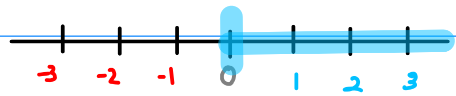


$R2(z^{-1}) \triangleq$

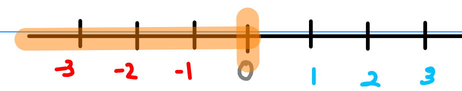
$|z^{-1}| > 2$ $\quad$ $|z| < 0.5$

$-N \quad N^c$

$-N$

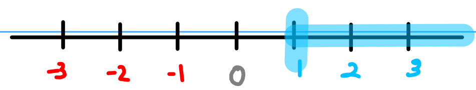


$(n \geq 0)$

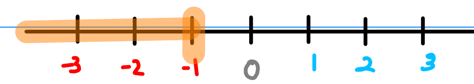


$(n < 1)$

$-N$

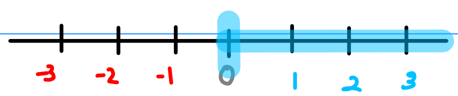


$(n \geq 1)$

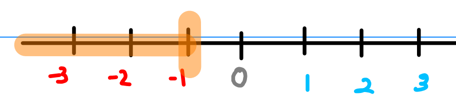


$(n < 0)$

N^c

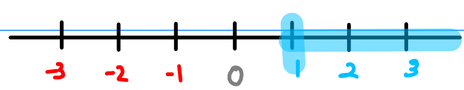


$(n \geq 0)$

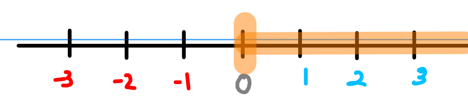


$(n < 0)$

N^c



$(n \geq 1)$



$(n < 1)$

$$(z, R) \Leftrightarrow (a_n, N)$$

$f(z)$	$ROC(z)$	$\Leftrightarrow$	a_n	$RNG(n)$
	$\lvert z\rvert < p$			$n \geq 0$

Ⅰ $$(z^{-1}, R^{-1}) \Leftrightarrow (a_{-n}, -N)$$

$f(z^{-1})$	$ROC(z^{-1})$	$\Leftrightarrow$	a_{-n}	$RNG(-n)$
	$\lvert z\rvert > \frac{1}{p}$			$n < 1$

Ⅱ $$(z, R^{-1}) \Leftrightarrow (-a_n, N^c)$$

$f(z)$	$ROC(z^{-1})$	$\Leftrightarrow$	$-a_n$	$\overline{RNG}(n)$
	$\lvert z\rvert > \frac{1}{p}$			$n < 0$

Ⅲ $$(z^{-1}, R) \Leftrightarrow (-a_{-n}, (-N)^c) = (-a_{-n}, -(N^c))$$

$f(z^{-1})$	$ROC(z)$	$\Leftrightarrow$	$-a_{-n}$	$\ll RNG(n) \gg$
	$\lvert z\rvert < p$			$n \geq 1$

Ⅰ + Ⅱ

Ⅳ $$(a_n, N) \Leftrightarrow (x_{-n}, -N)$$

$X(z)$	$ROC(z)$	$\Leftrightarrow$	a_{-n}	$RNG(-n)$
	$\lvert z\rvert < p$			$n < 1$

$$\text{III} = \text{I} + \text{II}$$

III: $f(z^{-1})$ ROC(z) $|z| < p$ $\Longleftrightarrow$ $-a_{-n}$ ≪RNG(n)≫ $n \geq 1$ I + II

$f(z)$ ROC(z) $|z| < p$ $\Longleftrightarrow$ a_n RNG(n) $n \geq 0$

I: $f(z^{-1})$ ROC(z^{-1}) $|z| > \frac{1}{p}$ $\Longleftrightarrow$ a_{-n} RNG$(-n)$ $n < 1$

II: $f(z)$ ROC(z^{-1}) $|z| > \frac{1}{p}$ $\Longleftrightarrow$ $-a_n$ $\overline{\text{RNG}(n)}$ $n < 0$

$f(z^{-1})$ ROC(z) $|z| < p$ $\Longleftrightarrow$ $-a_{-n}$ $\overline{\text{RNG}(-n)}$ $n \geq 1$

$$(z^{-1}, R^{-1}) \Leftrightarrow (a_{-n}, -N)$$

$$(z, R^{-1}) \Leftrightarrow (-a_n, N^c)$$

$$(z^{-1}, R) \Leftrightarrow (-a_{-n}, (-N)^c) = (-a_{-n}, -(N^c))$$

Compare Ⓘ with Ⓘⓥ

ROC(z)	f(z)	$\Longleftrightarrow$	a_n	RNG(n)
$\lvert z\rvert < p$				$n \geq 0$

Ⓘ $\left(z^{-1},\ R^{-1}\right) \Leftrightarrow \left(a_{-n},\ -N\right)$

f(z^{-1})	ROC(z^{-1})	$\Longleftrightarrow$	a_{-n}	RNG($-n$)
	$\lvert z\rvert > \frac{1}{p}$			$n < 1$

Ⓘⓥ $\left(a_n,\ N\right) \Leftrightarrow \left(x_{-n},\ -N\right)$

$\left(x_n,\ N\right) \Leftrightarrow \left(a_{-n},\ -N\right)$

X(z)	ROC(z)	$\Longleftrightarrow$	a_{-n}	RNG($-n$)
	$\lvert z\rvert < p$			$n < 1$

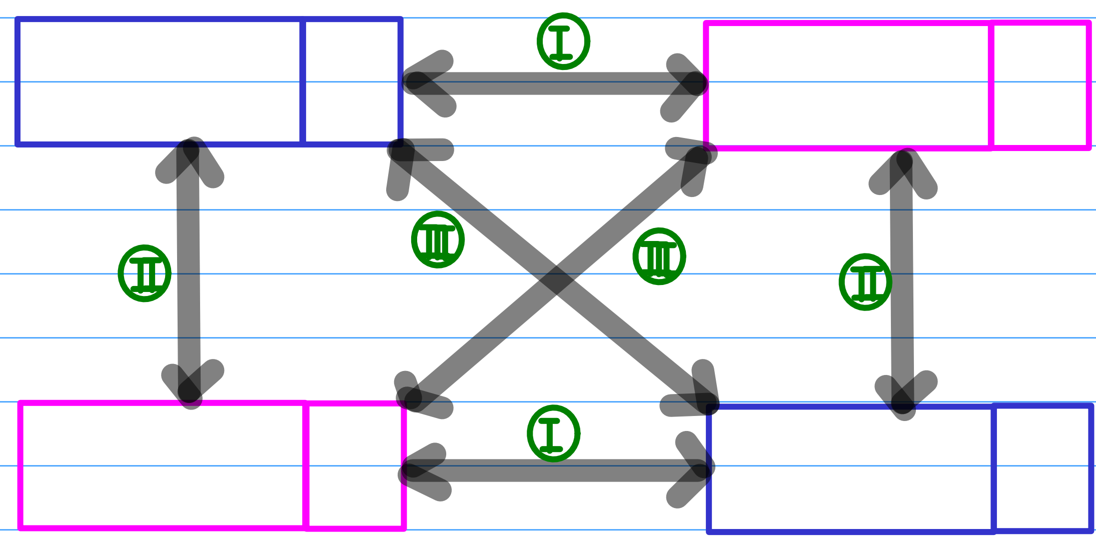
$-n$ $-n$

symmetrical

I $\quad (z^{-1}, R^{-1}) \Leftrightarrow (a_{-n}, -N)$

II $\quad (z, R^{-1}) \Leftrightarrow (-a_n, N^c)$

III $\quad (z^{-1}, R) \Leftrightarrow (-a_{-n}, (-N)^c) = (-a_{-n}, -(N^c))$



$(a_n, N) \Leftrightarrow (x_{-n}, -N)$

$f(z)$ $\quad |z| < 0.5$ $\quad |z| > 2$

causal $\quad$ anticausal

① − Ⓐ $\quad \dfrac{-1}{(z-1)(z-2)} = \left(\dfrac{1}{z-1} - \dfrac{1}{z-2}\right)$

$|z| < 1 \quad f(z) = -\dfrac{1}{1-z} + \dfrac{0.5}{1-0.5z} \quad \boxed{-1^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$

$$-\left(\underset{n=0}{1^1} + \underset{n=1}{1^2 z} + \underset{n=2}{1^3 z^2} + \cdots\right) + \left(\underset{n=0}{\left(\tfrac{1}{2}\right)} + \underset{n=1}{\left(\tfrac{1}{2}\right)^2 z} + \underset{n=2}{\left(\tfrac{1}{2}\right)^3 z^2} + \cdots\right)$$

$|z| > 2 \quad f(z) = \dfrac{z^{-1}}{1 - z^{-1}} - \dfrac{z^{-1}}{1 - 2z^{-1}} \quad \boxed{+1^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}} \quad (n < 0)$

$$\left(1^0 z^{-1} + 1^1 z^{-2} + 1^2 z^{-3} + \cdots\right) - \left(2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots\right)$$

$$\left(\underset{n=-1}{1^0 z^{-1}} + \underset{n=-2}{1^{-1} z^{-2}} + \underset{n=-3}{1^{-2} z^{-3}} + \cdots\right) - \left(\underset{n=-1}{\left(\tfrac{1}{2}\right)^0 z^{-1}} + \underset{n=-2}{\left(\tfrac{1}{2}\right)^{-1} z^{-2}} + \underset{n=-3}{\left(\tfrac{1}{2}\right)^{-2} z^{-3}} + \cdots\right)$$

② − Ⓐ $\quad \dfrac{-0.5z^2}{(z-1)(z-0.5)} = \left(-\dfrac{z}{(z-1)} + \dfrac{0.5z}{(z-0.5)}\right)$

$|z| < 0.5 \quad f(z) = +\dfrac{z}{1-z} - \dfrac{z}{1-2z} \quad \boxed{1^{n-1} \; - 2^{n-1}} \quad (n \geq 1)$

$$+\left(\underset{n=1}{1^0 z^1} + \underset{n=2}{1^1 z^2} + \underset{n=3}{1^2 z^3} + \cdots\right) - \left(\underset{n=1}{2^0 z} + \underset{n=2}{2^1 z^2} + \underset{n=3}{2^2 z^3} + \cdots\right)$$

$|z| > 1 \quad f(z) = -\dfrac{1}{1 - z^{-1}} + \dfrac{0.5}{1 - 0.5z^{-1}} \quad \boxed{-1^{n-1} + 2^{n-1}} \quad (n < 1)$

$$-\left(1^1 z^0 + 1^2 z^{-1} + 1^3 z^{-2} + \cdots\right) + \left(\left(\tfrac{1}{2}\right)^1 z^0 + \left(\tfrac{1}{2}\right)^2 z^{-1} + \left(\tfrac{1}{2}\right)^3 z^{-2} + \cdots\right)$$

$$-\left(\underset{n=0}{1^{-1} z^0} + \underset{n=-1}{1^{-2} z^{-1}} + \underset{n=-2}{1^{-3} z^{-2}} + \cdots\right) + \left(\underset{n=0}{2^{-1} z^0} + \underset{n=-1}{2^{-2} z^{-1}} + \underset{n=-2}{2^{-3} z^{-2}} + \cdots\right)$$

$X(z)$ $\quad |z| < 0.5 \quad |z| > 2$

anticausal causal

① - Ⓑ $\dfrac{-1}{(z-1)(z-2)} = \left(\dfrac{1}{z-1} - \dfrac{1}{z-2}\right)$

$|z| < 1$ $\quad X(z) = -\dfrac{1}{1-z} + \dfrac{0.5}{1-0.5z}$ $\quad \boxed{-1^{n-1} + 2^{n-1}}$ $\quad (n<1)$

$-\left(1^1 z^0 + 1^2 z^1 + 1^3 z^2 + \cdots\right) + \left((\tfrac{1}{2})^1 z^0 + (\tfrac{1}{2})^2 z^1 + (\tfrac{1}{2})^3 z^2 + \cdots\right)$

$-\left(1^{-1} z^0 + 1^{-2} z^1 + 1^{-3} z^2 + \cdots\right) + \left(2^{-1} z^0 + 2^{-2} z^1 + 2^{-3} z^2 + \cdots\right)$

$n=0 \quad n=-1 \quad n=-2 \qquad n=0 \quad n=-1 \quad n=-2$

$|z| > 2$ $\quad X(z) = \dfrac{z^{-1}}{1-z^{-1}} - \dfrac{z^{-1}}{1-2z^{-1}}$ $\quad \boxed{+1^{n-1} - 2^{n-1}}$ $\quad (n \geq 1)$

$+\left(1^0 z^{-1} + 1^1 z^{-2} + 1^2 z^{-3} + \cdots\right) - \left(2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots\right)$

$n=1 \quad n=2 \quad n=3 \qquad n=1 \quad n=2 \quad n=3$

② - Ⓑ $\dfrac{-0.5z^2}{(z-1)(z-0.5)} = \left(-\dfrac{z}{(z-1)} + \dfrac{0.5z}{(z-0.5)}\right)$

$|z| < 0.5$ $\quad X(z) = +\dfrac{z}{1-z} - \dfrac{z}{1-2z}$ $\quad \boxed{+1^{n+1} - (\tfrac{1}{2})^{n+1}}$ $\quad (n<0)$

$+\left(1^0 z^1 + 1^1 z^2 + 1^2 z^3 + \cdots\right) - \left(2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \cdots\right)$

$+\left(1^0 z + 1^{-1} z^2 + 1^{-2} z^3 + \cdots\right) - \left(2^0 z + 2^{-1} z^2 + 2^{-2} z^3 + \cdots\right)$

$n=-1 \quad n=-2 \quad n=-3 \qquad n=-1 \quad n=-2 \quad n=-3$

$|z| > 1$ $\quad X(z) = -\dfrac{1}{1-z^{-1}} + \dfrac{0.5}{1-0.5z^{-1}}$ $\quad \boxed{-1^{n+1} + (\tfrac{1}{2})^{n+1}}$ $\quad (n \geq 0)$

$-\left(1^1 z^0 + 1^2 z^{-1} + 1^3 z^{-2} + \cdots\right) + \left((\tfrac{1}{2})^1 z^0 + (\tfrac{1}{2})^2 z^{-1} + (\tfrac{1}{2})^3 z^{-2} + \cdots\right)$

$n=0 \quad n=1 \quad n=2 \qquad n=0 \quad n=1 \quad n=2$

$$\boxed{\begin{array}{llll} \text{ROC}(z^{-1}) & f(z) \Longleftrightarrow -a_n & & \text{RNG}(n) \\ |z| > \frac{1}{p} & & & n < 0 \end{array}}$$ (I)

$$\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$|z| < 0.5$ $X(z)$ $a_n = \boxed{-(\tfrac{1}{2})^{n-1} + 2^{n-1}}$ $(n<1)$

$|z| > 2$ $X(z)$ $b_n = \boxed{+(\tfrac{1}{2})^{n-1} - 2^{n-1}}$ $(n \geq 1)$

$\{|z| < 0.5\} \cap \{|z| > 2\} = \phi \Rightarrow a_n + b_n = 0$

$a_n = -b_n$

ROC

$|z| < a$ $X(z) = \dfrac{a}{1-az} = \displaystyle\sum_{n=0}^{\infty} a^{n+1} z^n$ a^{n+1} $n \geq 0$ $n \geq 1$ $n < 0$ $n < 1$

$-1\cdot$

ROC'

$|z| > a^{-1}$ $X(z) = -\dfrac{z^{-1}}{1-a^{-1}z^{-1}} = -\displaystyle\sum_{n=0}^{\infty} a^{-n} z^{-n-1}$ $-a^{n+1}$ $n < 0$ $n < 1$ $n \geq 0$ $n \geq 1$

$$= -\sum_{k=-1}^{-\infty} a^{k+1} z^k$$

$$\boxed{\frac{a}{1-az}} = \sum_{n=0}^{\infty} a^{n+1} z^n \qquad \boxed{\frac{z}{1-az}} = \sum_{n=1}^{\infty} a^{n-1} z^n$$

$$\boxed{-\frac{z^{-1}}{1-a^{-1}z^{-1}}} = -\sum_{n=0}^{\infty} a^{-n} z^{-n-1} \qquad \boxed{-\frac{a^{-1}}{1-a^{-1}z^{-1}}} = -\sum_{n=0}^{\infty} a^{-n-1} z^{-n}$$

$$= -\sum_{k=-1}^{-\infty} a^{k+1} z^k \qquad = -\sum_{k=0}^{-\infty} a^{k-1} z^k$$

$$\frac{a}{1-az} \Rightarrow \sum_{n=0}^{\infty} a^{n+1} z^{n} \qquad\qquad \frac{z}{1-az} \Rightarrow \sum_{n=1}^{\infty} a^{n-1} z^{n}$$

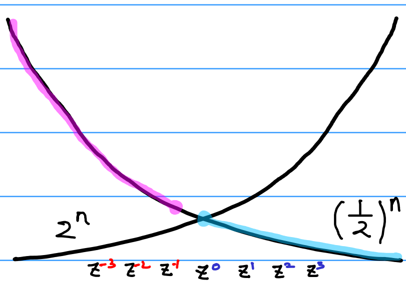
$$\| \qquad\qquad \| \qquad\qquad\qquad \| \qquad\qquad \|$$

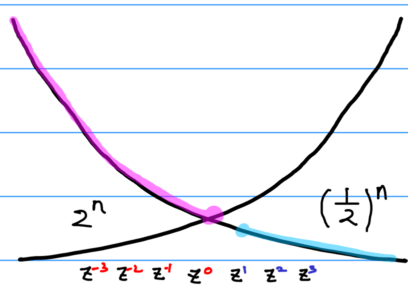
$$-\frac{z^{-1}}{1-a^{-1}z^{-1}} \Rightarrow -\sum_{n=0}^{\infty} a^{-n} z^{-n-1} \qquad\qquad -\frac{a^{-1}}{1-a^{-1}z^{-1}} \Rightarrow -\sum_{n=0}^{\infty} a^{-n-1} z^{-n}$$

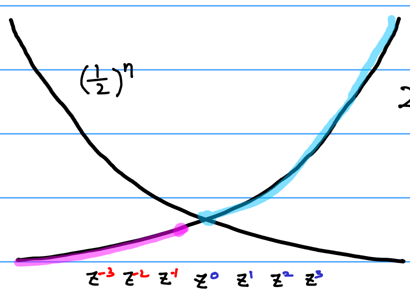
$$= -\sum_{k=-1}^{-\infty} a^{k+1} z^{k} \qquad\qquad = -\sum_{k=0}^{-\infty} a^{k-1} z^{k}$$

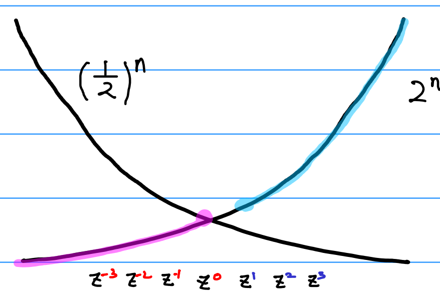
$a + a^2 z^1 + a^3 z^2 + a^4 z^3 + \cdots$ $\qquad$ $z + a z^2 + a^2 z^3 + a^3 z^4 + \cdots$

$z^{-1} + a^{-1} z^{-2} + a^{-2} z^{-3} + a^{-3} z^{-4} + \cdots$ $\qquad$ $a^{-1} + a^{-2} z^{-1} + a^{-3} z^{-2} + a^{-4} z^{-3} + \cdots$









Ⅳ

$$\boxed{\begin{array}{cccc} ROC(z) & X(z) \longleftrightarrow & a_{-n} & RNG(-n) \\ |z| < p & & & n \le 0 \end{array}}$$

$$\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$|z| < 0.5$

$$f(z) = -\frac{2}{1-2z} + \frac{0.5}{1-0.5z} \qquad \boxed{-2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}} \quad (n \ge 0)$$

$$-\left(2z^0 + 2^2 z^1 + 2^3 z^2 + \cdots\right) + \left(\left(\tfrac{1}{2}\right)z^0 + \left(\tfrac{1}{2}\right)^2 z^1 + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right)$$

n=0 n=1 n=2 n=0 n=1 n=2

$|z| < 0.5$

$$X(z) = -\frac{2}{1-2z} + \frac{0.5}{1-0.5z} \qquad \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}} \quad (n \le 0)$$

$$-\left(2^1 z^0 + 2^2 z^1 + 2^3 z^2 + \cdots\right) + \left(\left(\tfrac{1}{2}\right)z^0 + \left(\tfrac{1}{2}\right)^2 z^1 + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right)$$

$$-\left(\left(\tfrac{1}{2}\right)^{-1} z^0 + \left(\tfrac{1}{2}\right)^{-2} z^1 + \left(\tfrac{1}{2}\right)^{-3} z^2 + \cdots\right) + \left(2^{-1} z^0 + 2^{-2} z^1 + 2^{-3} z^2 + \cdots\right)$$

n=0 n=-1 n=-2 n=0 n=-1 n=-2

ROC $\quad f(z) = \sum_{n=0}^{\infty} a^{n+1} z^n \qquad a^{n+1} \qquad n \ge 0 \quad n \ge 1 \quad n < 0 \quad n < 1$

$$\sum_{n=0}^{\infty} \left(\frac{1}{a}\right)^{-n-1} z^n \qquad \downarrow -n$$

ROC $\quad X(z) = \sum_{k=0}^{-\infty} \left(\frac{1}{a}\right)^{k-1} z^{-k} \qquad a^{-n+1} = \left(\frac{1}{a}\right)^{n-1} \qquad n \le 0 \quad n \le 1 \quad n > 0 \quad n > 1$

$$\boxed{\begin{array}{llll} ROC(z^{-1}) & f(z^{-1}) & a_{-n} & RNG(-n) \\ |z| > \frac{1}{p} & & & n < 1 \end{array}}$$

(II)

$|z| < 1$

$$-\frac{1}{1 - z} + \frac{0.5}{1 - 0.5z}$$

$$f(z) = -[1^1 + 1^2 z^1 + 1^3 z^2 + \cdots] + [(\tfrac{1}{2}) + (\tfrac{1}{2})^2 z^1 + (\tfrac{1}{2})^3 z^2 + \cdots]$$

$$\boxed{a_n = -1^{n+1} + (\tfrac{1}{2})^{n+1} \quad (n \geq 0)}$$

$|z| > 1$

$$-\frac{1}{1 - z^{-1}} + \frac{0.5}{1 - 0.5z^{-1}}$$

$$f(z) = -[(\tfrac{1}{1})^{-1} z^0 + (\tfrac{1}{1})^{-2} z^{-1} + (\tfrac{1}{1})^{-3} z^{-2} + \cdots] + [2^{-1} z^0 + 2^{-2} z^{-1} + 2^{-3} z^{-2} + \cdots]$$

$$\boxed{a_n = -1^{n-1} + 2^{n-1} \quad (n < 1)}$$

ROC

$|z| < a$

$$f(z) = \frac{a}{1 - az} = \sum_{n=0}^{\infty} a^{n+1} z^n$$

a^{n+1}

$n \geq 0$ $\quad n \geq 1$ $\quad n < 0$ $\quad n < 1$

z^{-1} $\quad -n$

ROC'

$|z| > a^{-1}$

$$f(z^{-1}) = \frac{a}{1 - az^{-1}} = \sum_{n=0}^{\infty} a^{n+1} z^{-n} = \sum_{k=0}^{-\infty} a^{-k+1} z^k$$

$$a^{-n+1} = (\tfrac{1}{a})^{n-1}$$

$n < 1$ $\quad n < 0$ $\quad n \geq 0$ $\quad n \geq 0$