

Binary Angle Measurement (3A)

- Look ahead CORDIC
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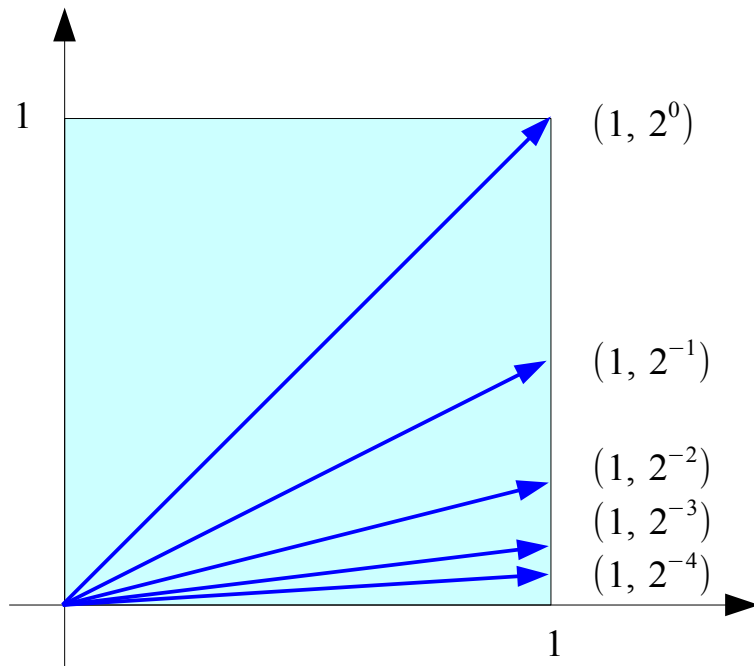
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BAM Background

C. C. Kao, "High-performance CORDIC rotation algorithm based on look-ahead techniques", International Journal of Electronics, 2011

CORDIC Iteration Equations

Choose θ_i such that $\tan \theta_i = \begin{cases} +2^{-i} \\ -2^{-i} \end{cases}$
 $\tan \theta_i = \sigma_i 2^{-i} \quad \sigma_i \in \{+1, -1\}$



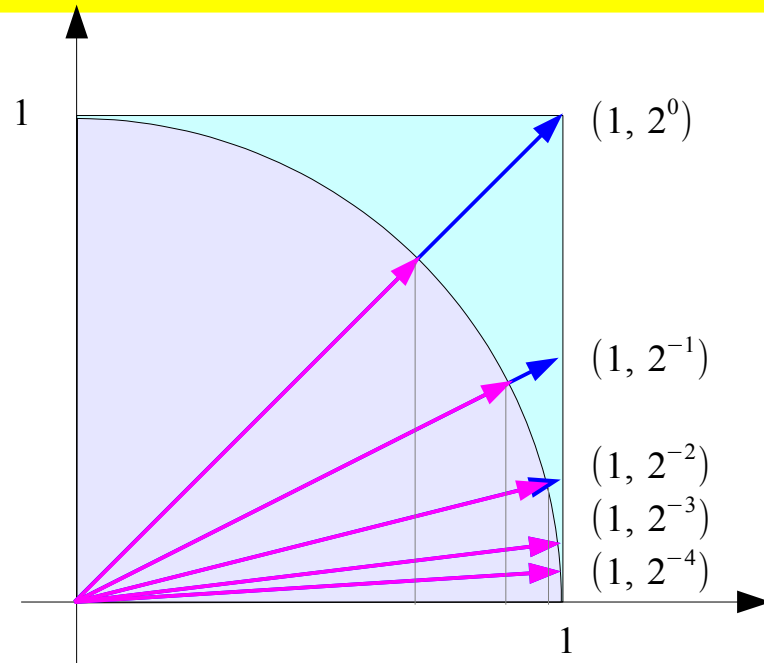
$$\tan \theta_i = \pm 2^{-i} \quad \cos \theta_i = \frac{+1}{\sqrt{1 + 2^{-2i}}}$$

$$\sin \theta_i = \frac{\pm 2^{-i}}{\sqrt{1 + 2^{-2i}}}$$

$$x'_{i+1} = (x'_i - y'_i \sigma_i 2^{-i})$$

$$y'_{i+1} = (x'_i \sigma_i 2^{-i} + y'_i)$$

$$\theta_{i+1} = \theta_i - \tan(\sigma_i 2^{-i})$$



$$\begin{pmatrix} +\cos \theta_i & -\sin \theta_i \\ +\sin \theta_i & +\cos \theta_i \end{pmatrix} = \frac{1}{\sqrt{1 + 2^{-2i}}} \begin{pmatrix} +1 & \mp 2^{-i} \\ \pm 2^{-i} & +1 \end{pmatrix}$$

Look Ahead CORDIC (1)

$$\begin{aligned}x'_{i+1} &= (x'_i - y'_i \sigma_i 2^{-i}) \\y'_{i+1} &= (x'_i \sigma_i 2^{-i} + y'_i) \\ \theta_{i+1} &= \theta_i - \tan^{-1}(\sigma_i 2^{-i})\end{aligned}$$

$$P_{xi} = f(\sigma_0, \sigma_1, \dots, \sigma_i, i)$$

$$P_{yi} = g(\sigma_0, \sigma_1, \dots, \sigma_i, i)$$

$$x_1 = x_0 - \sigma_0 \cdot 2^{-0} \cdot y_0$$

$$y_1 = y_0 + \sigma_0 \cdot 2^{-0} \cdot x_0$$

$$\theta_1 = \theta_0 - \tan^{-1}(\sigma_0 2^{-0})$$

$$x_1 = P_{x1} x_0 - P_{y1} y_0$$

$$y_1 = P_{x1} y_0 + P_{y1} x_0$$

$$\theta_1 = \theta_0 - \sigma_0 \alpha_0$$

$$x_2 = x_1 - \sigma_1 \cdot 2^{-1} \cdot y_1$$

$$y_2 = y_1 + \sigma_1 \cdot 2^{-1} \cdot x_1$$

$$\theta_2 = \theta_1 - \tan^{-1}(\sigma_1 2^{-1})$$

$$x_2 = P_{x2} x_0 - P_{y2} y_0$$

$$y_2 = P_{x2} y_0 + P_{y2} x_0$$

$$\theta_1 = \theta_0 - \sigma_0 \alpha_0 - \sigma_1 \alpha_1$$

$$x_3 = x_2 - \sigma_2 \cdot 2^{-2} \cdot y_2$$

$$y_3 = y_2 + \sigma_2 \cdot 2^{-2} \cdot x_2$$

$$\theta_3 = \theta_2 - \tan^{-1}(\sigma_2 2^{-2})$$

$$x_3 = P_{x3} x_0 - P_{y3} y_0$$

$$y_3 = P_{x3} y_0 + P_{y3} x_0$$

$$\theta_1 = \theta_0 - \sigma_0 \alpha_0 - \sigma_1 \alpha_1 - \sigma_2 \alpha_2$$

$$x_4 = x_3 - \sigma_3 \cdot 2^{-3} \cdot y_3$$

$$y_4 = y_3 + \sigma_3 \cdot 2^{-3} \cdot x_3$$

$$\theta_4 = \theta_3 - \tan^{-1}(\sigma_3 2^{-3})$$

$$x_4 = P_{x4} x_0 - P_{y4} y_0$$

$$y_4 = P_{x4} y_0 + P_{y4} x_0$$

$$\theta_1 = \theta_0 - \sigma_0 \alpha_0 - \sigma_1 \alpha_1 - \sigma_2 \alpha_2 - \sigma_3 \alpha_3$$

Look Ahead CORDIC (2)

$$\begin{aligned}x'_{i+1} &= (x'_i - y'_i \sigma_i 2^{-i}) \\y'_{i+1} &= (x'_i \sigma_i 2^{-i} + y'_i) \\ \theta_{i+1} &= \theta_i - \tan^{-1}(\sigma_i 2^{-i})\end{aligned}$$

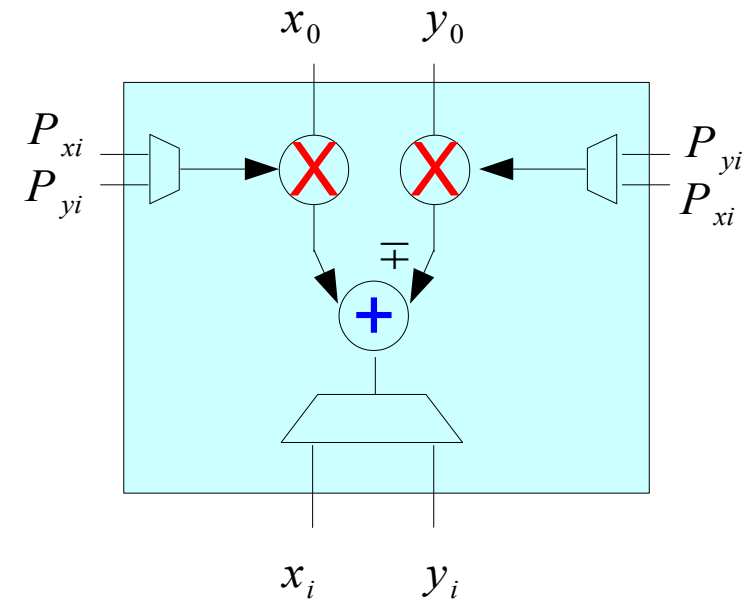


$$\begin{aligned}x_i &= P_{xi} x_0 - P_{yi} y_0 \\y_i &= P_{xi} y_0 + P_{yi} x_0 \\ \theta_1 &= \theta_0 - \sigma_0 \alpha_0 - \sigma_1 \alpha_1 - \dots - \sigma_{i-1} \alpha_{i-1}\end{aligned}$$

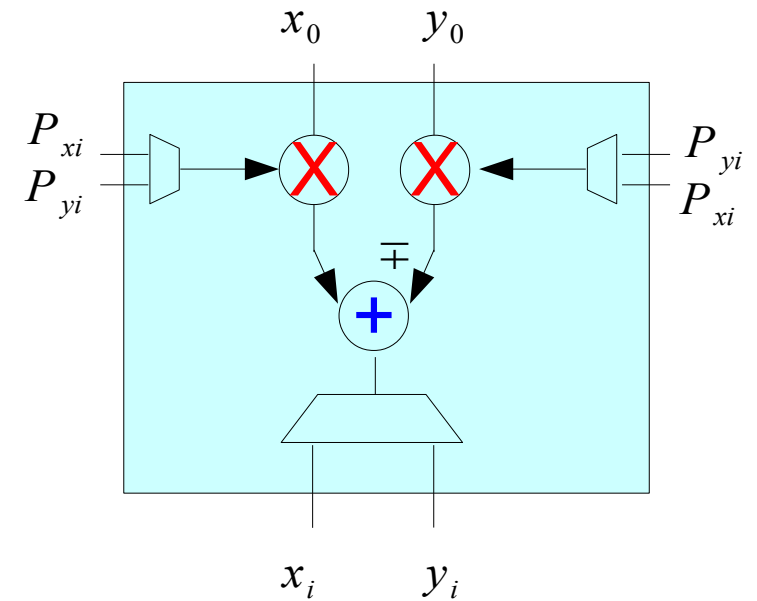
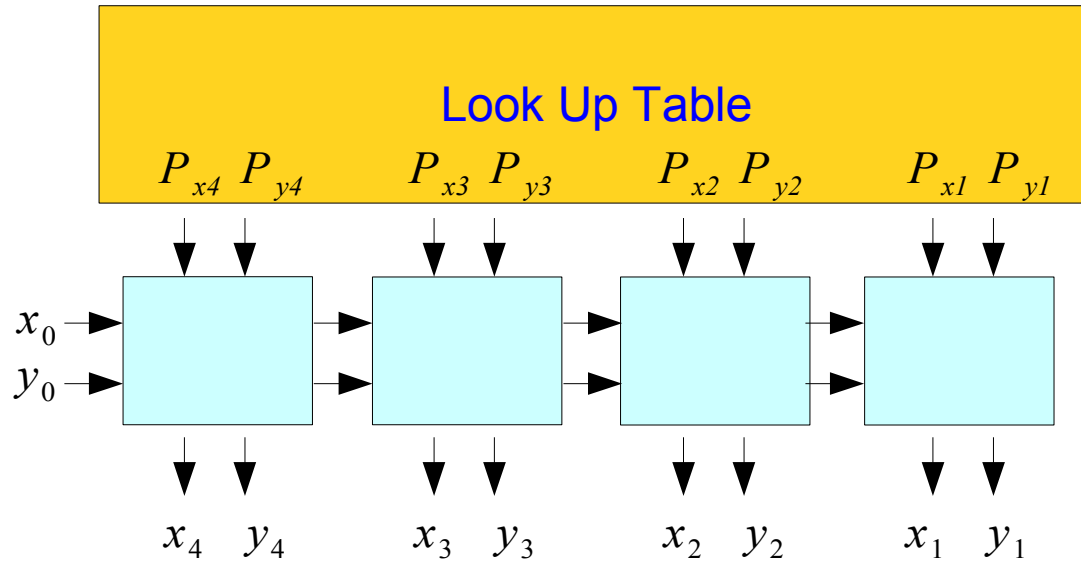
Depend on the initial inputs x_0, y_0
and the precomputing matrixces P_{xi}, P_{yi}

$$P_{xi} = f(\sigma_0, \sigma_1, \dots, \sigma_i, i)$$

$$P_{yi} = g(\sigma_0, \sigma_1, \dots, \sigma_i, i)$$



Look Ahead CORDIC (3)



Vector Rotation (2)

Vector Rotation (3)

Successive Rotations

CORDIC Rotation

$\cos \theta$ in term of $\tan \theta$

References

- [1] <http://en.wikipedia.org/>
- [2] CORDIC FAQ, www.dspguru.com
- [3] R. Andraka, A survey of CORDIC algorithms for FPGA based computers
- [4] J. S. Walther, A Unified Algorithm for Elementary Functions
- [5] J. P. Deschamps, G. A. Bioul, G.D. Sutter, Synthesis of Arithmetic Circuits
- [6] C. C. Kao, "High-performance CORDIC rotation algorithm based on look-ahead techniques", International Journal of Electronics, 2011