Multiple Random Variables

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Based on
Probability, Random Variables and Random Signal Principles,
P.Z. Peebles,Jr. and B. Shi

Outline

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Joint Density Function

for continuous 2 random variable X and Y

Definition

the joint probability density function $f_{X,Y}(x,y)$
is defined by the second derivative
of the joint distribution function

$$f_{X,Y}(x,y) = \frac{\partial^2 F_{X,Y}(x,y)}{\partial x \partial y}$$

Joint Density Function

for discrete 2 random variable X and Y

Definition

the joint probability density function $f_{X,Y}(x,y)$
is defined by the second derivative
of the joint distribution function

$$f_{X,Y}(x,y) = \sum_{i=0}^{N}\sum_{j=0}^{M} P(x_n, y_m)\delta(x - x_n)\delta(y - y_m)$$

Joint Density Function

for continuous N random variable $X_1, X_2, \ldots, X_N$

Definition

the joint probability density function

$$f_{X_1,X_2,\ldots,X_N}(x_1, x_2, \ldots, x_N)$$

is defined by the second derivative of the joint distribution function

$$f_{X_1,X_2,\ldots,X_N}(x_1, x_2, \ldots, x_N) = \frac{\partial^N F_{X_1,X_2,\ldots,X_N}(x_1, x_2, \ldots, x_n)}{\partial x_1 \partial x_2 \cdots \partial x_N}$$

$$F_{X_1,X_2,\ldots,X_N}(x_1, x_2, \ldots, x_N) =$$
$$\int_{-\infty}^{x_N} \cdots \int_{-\infty}^{x_2} \int_{-\infty}^{x_1} f_{X_1,X_2,\ldots,X_N}(\xi_1, \xi_2, \ldots, \xi_N) d\xi_1, d\xi_2, \ldots, d\xi_N$$

Joint Density Function
for discrete N random variable $X_1, X_2, \cdots, X_n$

Definition

the joint probability density function $f_{X_1,X_2,\ldots,X_N}(x_1, x_2, \ldots x_N)$
is defined by the second derivative of the joint distribution function

$$f_{X_1,X_2,\ldots,X_N}(x_1, x_2, \ldots x_N) = \sum_{n_1=0}^{N_1} \sum_{n_2=0}^{N_2} \cdots \sum_{n_N=0}^{N_N} P(x_{n_1}, x_{n_2}, \cdots, x_{n_N})$$

$$\delta(x_1 - x_{n_1})\delta(x_2 - x_{n_2}) \cdots \delta(x_N - x_{n_N})$$

Properties of the Joint Density (1)

$$(1) \quad f_{X,Y}(x,y) \geq 0$$

$$(2) \quad \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f_{X,Y}(x,y)dxdy = 1$$

$$(3) \quad F_{X,Y}(x,y) = \int_{-\infty}^{y}\int_{-\infty}^{x} f_{X,Y}(\xi_1,\xi_2)d\xi_1 d\xi_2$$

$$(4) \quad F_X(x) = \int_{-\infty}^{x}\int_{-\infty}^{+\infty} f_{X,Y}(\xi_1,\xi_2)d\xi_2 d\xi_1$$

$$(5) \quad F_Y(y) = \int_{-\infty}^{y}\int_{-\infty}^{+\infty} f_{X,Y}(\xi_1,\xi_2)d\xi_1 d\xi_2$$

Properties of the Joint Density (2)

(6) $P\{x_1 < X \le x_2, y_1 < Y \le y_2\} = \int_{y_1}^{y_2} \int_{x_1}^{x_2} f_{X,Y}(x, y) dxdy$

(7) $f_X(x) = \int_{-\infty}^{+\infty} f_{X,Y}(x, y) dy$

(8) $f_Y(y) = \int_{-\infty}^{+\infty} f_{X,Y}(x, y) dx$

Marginal Density Functions

for continuous 2 random variable X and Y

- $F_X(x) = \int_{-\infty}^{x} \int_{-\infty}^{+\infty} f_{X,Y}(\xi_1, \xi_2) d\xi_2 d\xi_1$
- $F_Y(y) = \int_{-\infty}^{y} \int_{-\infty}^{+\infty} f_{X,Y}(\xi_1, \xi_2) d\xi_1 d\xi_2$
- $f_X(x) = \int_{-\infty}^{+\infty} f_{X,Y}(x, y) dy$
- $f_Y(y) = \int_{-\infty}^{+\infty} f_{X,Y}(x, y) dx$
- $f_X(x) = \dfrac{dF_x(x)}{dx}$
- $f_Y(y) = \dfrac{dF_y(y)}{dy}$

Marginal Density Functions

for continuous N random variable $X_1, X_2, \cdots, X_N$

$$f_{X_1,X_2,\cdots,X_k}(x_1, x_2, \cdots, x_k) =$$

$$\int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} f_{X_1,\cdots,X_k,X_{k+1},\cdots,X_N}(x_1, \cdots, x_k, x_{k+1}, \cdots, x_N)$$

$$dx_{k+1} dx_{k+2} \cdots dx_N$$