

# Logic Background (1A)

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<https://courses.csail.mit.edu/6.825/>

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# Proposition : etymology

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From Old French, from Latin *prōpositiō*  
 (“a **proposing**, design, theme, case”).

**The content of an assertion**

that may be taken as being **true** or **false**

and is considered abstractly without reference to  
the linguistic sentence that constitutes the assertion.

# Predicate : etymology

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From Middle French predicate (French prédicat),  
from post-classical Late Latin praedicatum (“**thing said of a subject**”),  
a noun use of the neuter past participle of praedicare (“**proclaim**”)

From Latin predicātus, perfect passive participle of praedicō,  
from prae + dicō (“**declare, proclaim**”), from dicō (“say, tell”).

# Proposition

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In Aristotelian logic a proposition is a particular kind of sentence, one which affirms or denies a **predicate** of a subject.

In formal logic a proposition is considered as objects of a formal language. A formal language begins with different types of **symbols**.

# Predicate

(grammar) The part of the sentence (or clause) which states something about the subject or the object of the sentence.

<p><u>The dog</u> <u>barked very loudly</u> <i>subject</i>    <i>predicate</i></p>
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(logic) A term of a statement, where the statement may be true or false depending on whether the thing referred to by the values of the statement's variables has the property signified by that (predicative) term.

# Propositional Logic

**Propositional logic** includes only

- operators and
  - propositional constants
- as symbols in its language.

The propositions in this language are

- propositional constants  
considered atomic propositions
- composite propositions  
recursive application of operators to propositions

# Predicate Logic

**Predicate logic** include

- variables,
- operators,
- predicate and
- function symbols, and
- quantifiers

as symbols in their languages.



## A Formal Language

**Syntax** – legal expressions

**Semantics** – the meaning of legal expressions

**Proof System** – a way of manipulating syntactic expressions  
to get another syntactic expressions

Multiple Percepts → Conclusions

Current State, Operators → Next State Properties

# Propositional Logic

**Sentences** (WFFs : Well Formed Formulas)

T and F are **sentences**

**Propositional variables** are **sentences** (A, B, C, ...)

If A and B are sentences, the followings are also sentences

(A),  $\neg A$ ,  $A \wedge B$ ,  $A \vee B$ ,  $A \rightarrow B$ ,  $A \leftrightarrow B$

# Precedence of Connectives

$\neg$  highest

$\wedge$

$\vee$

$\Rightarrow$

$\Leftrightarrow$  lowest

$$A \vee B \wedge C = A \vee (B \wedge C)$$

$$A \wedge B \Rightarrow C \vee D = (A \wedge B) \Rightarrow (C \vee D)$$

$$A \Rightarrow B \vee C \Leftrightarrow D = (A \Rightarrow (B \vee C)) \Leftrightarrow D$$

# Semantics

**Meaning** of a sentence : **true** or **false**

**Interpretation** : an assignment of **true** or **false**  
to the **propositional variables**

$\models_i \varphi$  : **Sentence**  $\varphi$  is true in the interpretation **i**

$\not\models_i \varphi$  : **Sentence**  $\varphi$  is true in the interpretation **i**

# Semantic Rules

$\models_i \varphi$  : Sentence  $\varphi$  is true in the interpretation  $\mathbf{i}$

$\models_i \varphi$  : Sentence  $\varphi$  is true in the interpretation  $\mathbf{i}$

$\models_i \mathbf{T}$  for all  $\mathbf{i}$

$\not\models_i \mathbf{F}$  for all  $\mathbf{i}$

$\models_i \mathbf{T}$  for all  $\mathbf{i}$

Since  $\mathbf{i}$  is a mapping from variables to truth values,  
Look  $\mathbf{P}$  up in  $\mathbf{i}$  and return the truth value assigned to  $\mathbf{P}$

$\models_i \neg\varphi$  iff  $\not\models_i \varphi$

$\models_i \varphi \wedge \psi$  iff  $\models_i \varphi$  **and**  $\models_i \psi$

$\models_i \varphi \vee \psi$  iff  $\models_i \varphi$  **or**  $\models_i \psi$

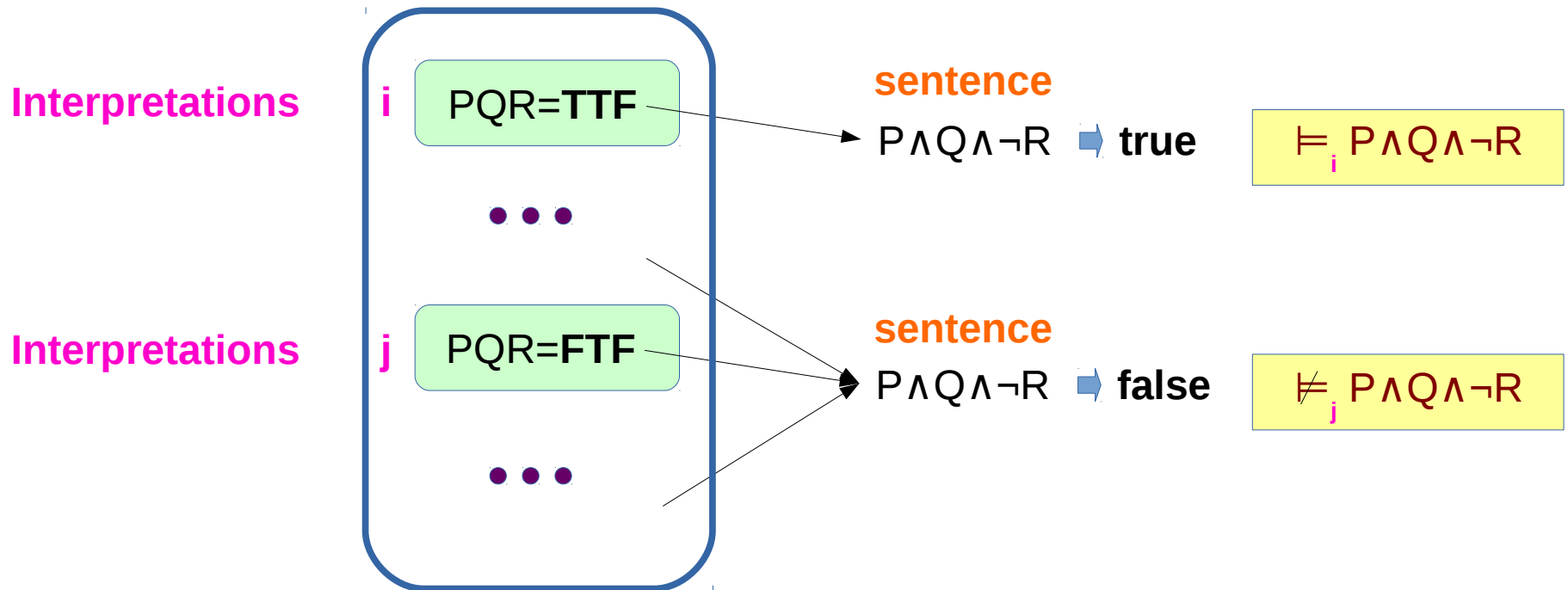
$\models_i \mathbf{P}$  iff  $\mathbf{i}(\mathbf{P}) = \mathbf{T}$

# Semantics

**Meaning** of a sentence : **true** or **false**

**Interpretation** : an assignment of **true** or **false**

to the **propositional variables** (P, Q, R)



# Valid, Satisfiable, Unsatisfiable

A sentence is **valid** iff

Its truth value is **T** in all interpretations

ex)  $\mathbf{T}, \neg\mathbf{F}, P \vee \neg P$  (**tautology:  $\top$** )

A sentence is **satisfiable** iff

Its truth value is **T** in at least one interpretation

ex)  $P, \mathbf{T}, \neg P$

A sentence is **unsatisfiable** iff

Its truth value is **F** in all interpretations

ex)  $\mathbf{F}, \neg\mathbf{T}, P \wedge \neg P$  (**contradiction:  $\perp$** )

# Models

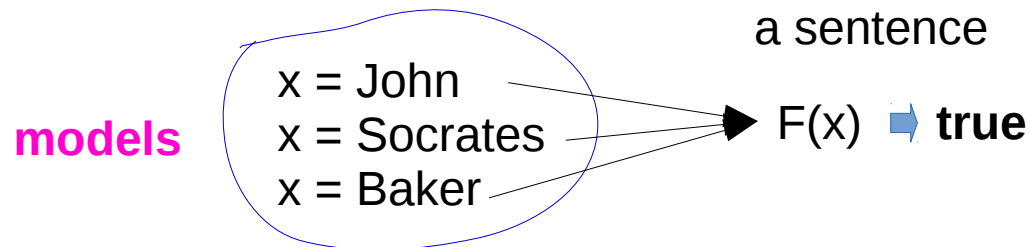
Semantics :

The relationship between **sentences** and **interpretations**

There are some **set of interpretations** that makes **a sentence true** → **models** of a sentence

An interpretation is a **model** of a sentence  
if the **sentence** is **true** in that **interpretation**

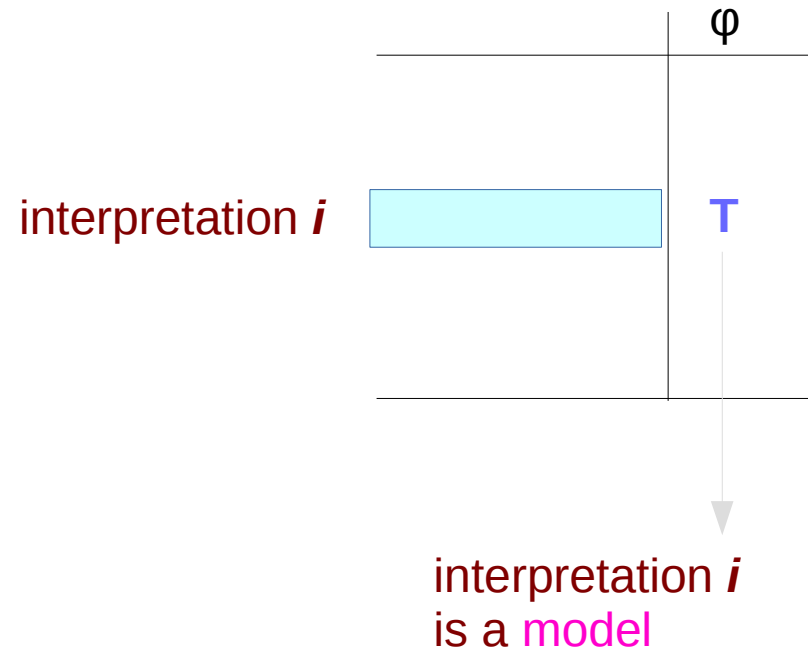
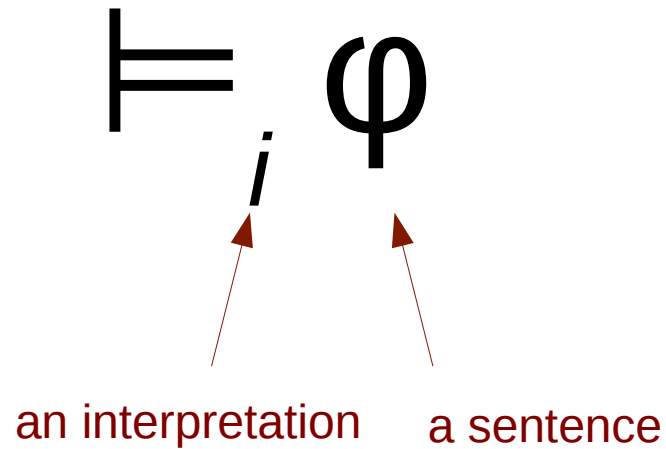
An **interpretation  $i$**  is a **model** of a **sentence  $\varphi$**  **iff**  $\models_i \varphi$





# Models

An interpretation  $i$  is a **model** of a sentence  $\varphi$  **iff**  $\models_i \varphi$



# Models and Interpretations

models

8 interpretations

P	Q	R	$P \wedge (Q \vee R)$
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	F
F	T	T	F
F	T	F	F
F	F	T	F
F	F	F	F

T	T	T
T	T	F

T	T	T
T	F	T

T	F	T
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# Entailment

An interpretation  $i$  is a **model** of a sentence  $\varphi$  **iff**  $\models_i \varphi$

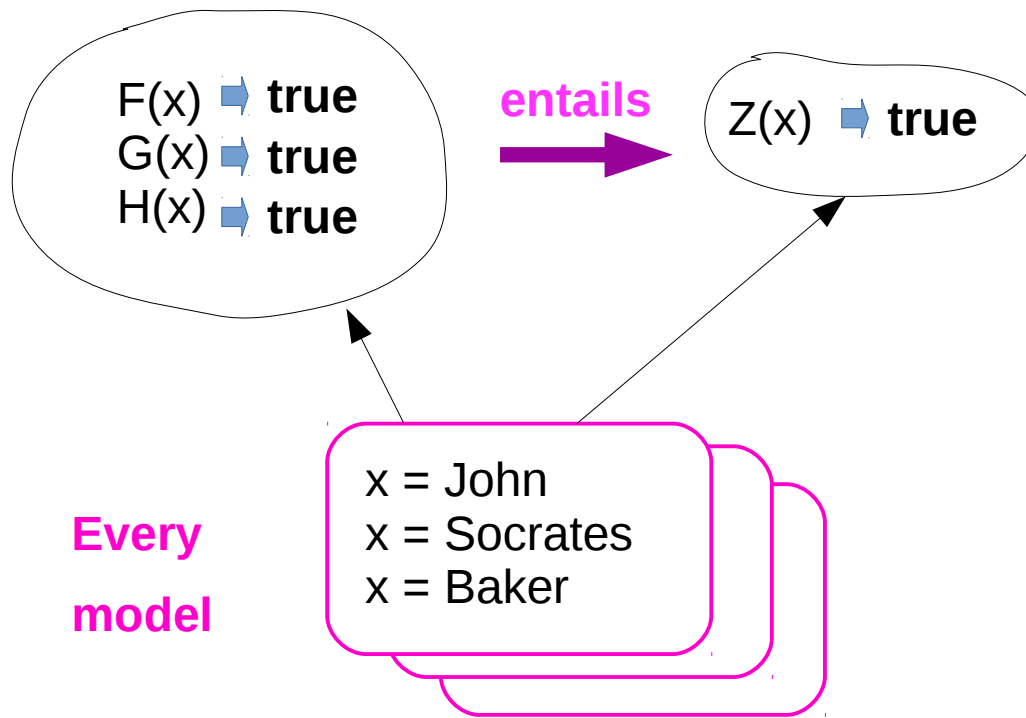
A set of sentences KB **entails** a sentence  $\varphi$   
**iff** **every model** of KB is also a **model** of  $\varphi$

KB: Knowledge Base  
a set of **sentences**

every model of KB

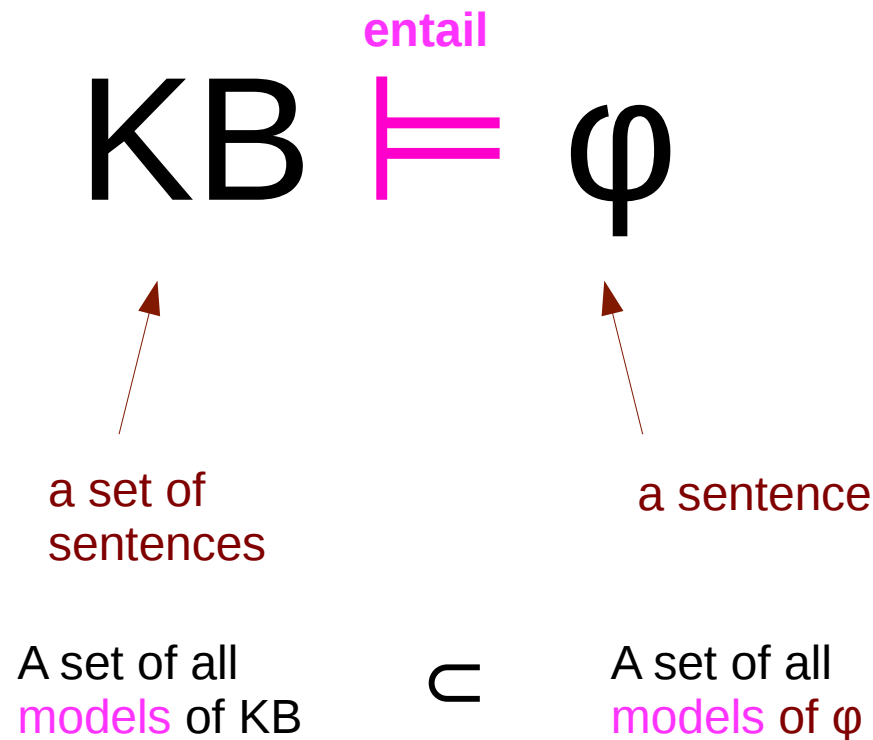
every model of  
each sentence in KB

the model of a sentence  $\varphi$

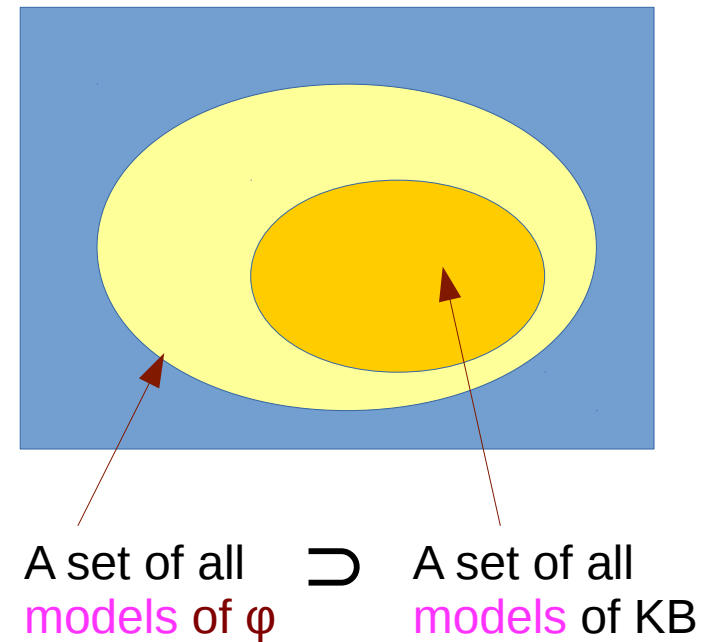


# Models and Entailment

A set of sentences KB **entails** a sentence  $\varphi$   
**iff** every model of KB is also a **model** of  $\varphi$



U: A set of all possible interpretation

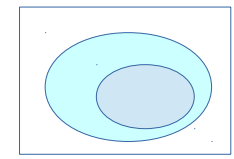
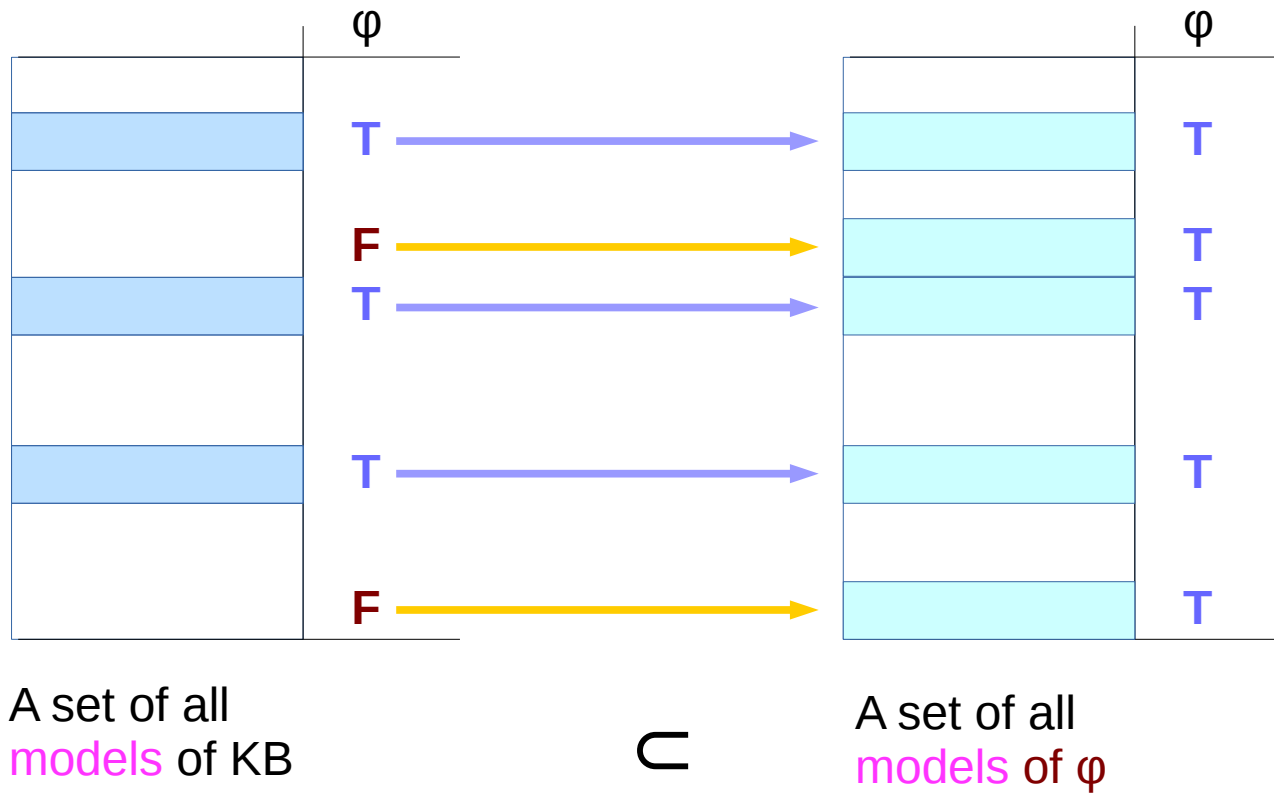


# Truth Tables and Entailment

A set of sentences KB **entails** a sentence  $\varphi$   
**iff** every model of KB is also a **model** of  $\varphi$

$$KB \models \varphi$$

entail

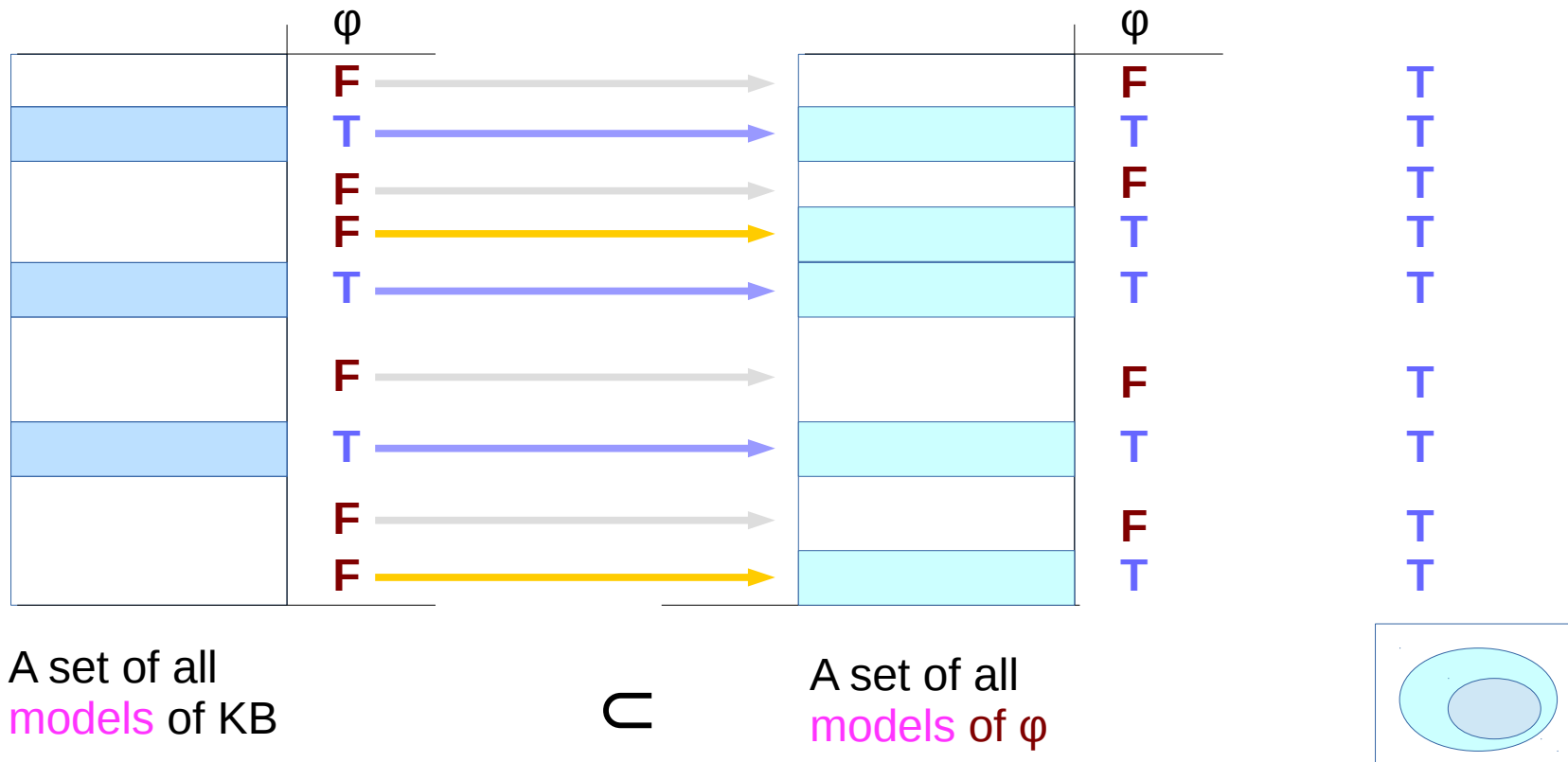


# Truth Tables and Entailment

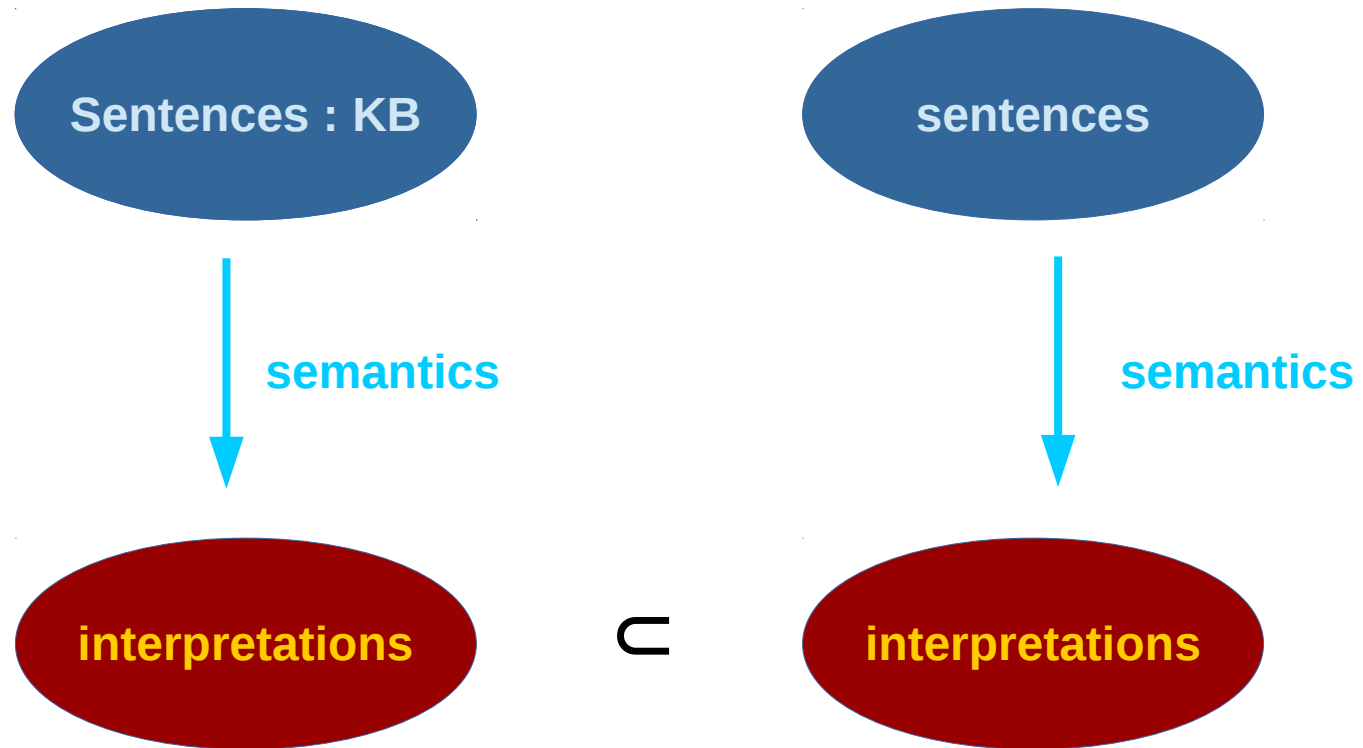
A set of sentences KB **entails** a sentence  $\varphi$   
**iff** every model of KB is also a **model** of  $\varphi$

$$KB \models \varphi$$

entail



# Entailment



## References

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