Differentiation

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Based on
Introduction to Matrix Algebra, Autar Kaw
https://ma.mathforcollege.com

Outline

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1. Background on Differentiation
   - Tangent and Secant Lines

Secant Lines

- Let P and Q be two points on the curve of $f(x)$
  P $(a, f(a))$ and Q $(a+h, f(a+h))$
- the secant line is the straight line drawn through P and Q .
- the slope of the secant line

$$m_{secant} = \frac{f(a+h) - f(a)}{(a+h) - a} = \frac{f(a+h) - f(a)}{h}$$

Tangent Lines

- as $h \to 0$, Q $\to$ P
  and the secant line $\to$ the tangent line
- the slope of the tangent line

$$m_{tangent} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{(a+h) - a}$$
$$= \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Derivative of a function

the derivative of a function $f(x)$ at $x = a$

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

or

$$f'(a) = \lim_{h \to 0} \frac{f(x) - f(a)}{x - a}$$

Finding equations of a tangent line

One of the numerical methods used
to solve a nonlinear equation
is called the Newton- Raphson method.
based on the knowledge of finding
the tangent line to a curve at a point.

Theorems of differentiations (1)

- If $f(x) = k$, where k is a constant, $f'(x) = 0$.
- The derivative of $f(x) = x^n$, where $n \neq 0$ is $f'(x) = nx^{n-1}$.
- The derivative of $f(x) = kg(x)$,
  where k is a constant is $f'(x) = kg'(x)$.
- The derivative of $f(x) = u(x) \pm v(x)$ is $f'(x) = u'(x) \pm v'(x)$.

Theorems of differentiations (2)

- The derivative of $f(x) = u(x) \cdot v(x)$ is
$f'(x) = \frac{d}{dx} u(x) \cdot v(x) + u(x) \cdot \frac{d}{dx} v(x)$
- The derivative of $f(x) = \frac{u(x)}{v(x)}$ is
$f'(x) = \frac{\frac{d}{dx} u(x) \cdot v(x) - u(x) \cdot \frac{d}{dx} v(x)}{(v(x))^2}$
- The derivative of $f(x) = u(v(x))$ is
$f'(x) = \frac{d}{dx} u(v(x)) \cdot \frac{d}{dx} v(x)$

Implicit differentiation

- Sometimes, the function to be differentiated is not given explicitly as an expression of the independent variable.
- Find $\frac{dy}{dx}$ if $x^2 + y^2 = 2xy$
  $\frac{d}{dx}(x^2 + y^2) = \frac{d}{dx}(2xy)$
  $2x + 2y\frac{dy}{dx} = 2y + 2x\frac{dy}{dx}$
  $(2y - 2x)\frac{dy}{dx} = 2y - 2x$
  $\frac{dy}{dx} = 1$

Finding maximum and minimum of a function (1)

- The knowledge of first derivative and second derivative of a function is used to find the minimum and maximum of a function.
- First, let us define what the maximum and minimum of a function are.
- Let $f(x)$ be a function in domain D, then
  - $f(a)$ is the maximum of the function if $f(a) \geq f(x)$ for all values of x in the domain D.
  - $f(a)$ is the minimum of the function if $f(a) \leq f(x)$ for all values of x in the domain D.
- The minimum and maximum of a function are also the critical values of a function.

Finding maximum and minimum of a function (2)

- An extreme value can occur in the interval $[c, d]$ at end points $x = c$, $x = d$.
- a point in $[c, d]$ where $f'(x) = 0$.
- a point in $[c, d]$ where $f'(x)$ does not exist.
- These critical points can be the local maximas and minimas of the function

Tables of derivatives (1)

$f(x)$	$f'(x)$
$\sin(x)$	$\cos(x)$
$\cos(x)$	$-\sin(x)$
$\tan(x)$	$\sec^2(x)$
$\sin^{-1}(x)$	$\frac{1}{\sqrt{1-x^2}}$
$\cos^{-1}(x)$	$\frac{-1}{\sqrt{1-x^2}}$
$\tan^{-1}(x)$	$\frac{1}{1+x^2}$

$f(x)$	$f'(x)$
$\sinh(x)$	$\cosh(x)$
$\cosh(x)$	$\sinh(x)$
$\tanh(x)$	$1-\tanh^2(x)$
$\sinh^{-1}(x)$	$\frac{1}{\sqrt{1+x^2}}$
$\cosh^{-1}(x)$	$\frac{-1}{\sqrt{x^2-1}}$
$\tanh^{-1}(x)$	$\frac{1}{1-x^2}$

Tables of derivatives (2)

$f(x)$	$f'(x)$
$\csc^{-1}(x)$	$-\frac{\lvert x\rvert}{x^2\sqrt{x^2-1}}$
$\sec^{-1}(x)$	$\frac{\lvert x\rvert}{x^2\sqrt{x^2-1}}$
$\cot^{-1}(x)$	$-\frac{1}{1+x^2}$
a^x	$\ln(x)a^x$
$\ln(x)$	$\frac{1}{x}$
$\log_a(x)$	$\frac{1}{x\ln(a)}$
e^x	e^x