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Chapter 12

SPECIAL RELATIVITY

Albert Einstein

Eighteen years after the results of the Michelson-Morley experiment were announced, the solution to the puzzle they created came in 1905 from an obscure examiner of the Swiss Patent Office, Albert Einstein, with his "Special Theory of Relativity", which was followed in 1916 by his broader "General Theory of Relativity".

In 1905, at the age of 26, Einstein published four papers, which changed forever our view of the Universe:

- The first paper provided a theoretical explanation of Brownian motion. As previously discussed, it led to wide acceptance of the atomic theory of matter.
- The second paper, in which Einstein proposed a new theory of light, represents a major contribution to quantum theory at its very beginnings. Thus, we find Einstein playing a leading role in both key theories of 20th-century physics.
- The third paper introduced Einstein's special theory of relativity.
- The fourth paper was a follow-up to the third. It established the equivalence of mass and energy in physics' most famous equation: $E = m \times c^2$ (Energy = mass x the square of the speed of light).

There had not been a comparable outburst of scientific creativity since the plague years of 1665-66, when, within a few months, the 23-year old Newton discovered the composite nature of light, analyzed the action of gravity, and invented calculus.

In the 1600's, the shock created by the "Copernican Revolution" had been enormous. Mankind had to adjust to the idea that we live on a planet rushing through space at more than 66,000 miles per hour. In retrospect, however, we must consider this shock rather mild, compared to the violence that Einstein's theory of relativity did to the deeply entrenched notions of time and space. More blows to a common-sense view of reality were to come from the quantum theory, more than even Einstein could accept!

One cannot say about Einstein the usual things that are told about great geniuses - for instance, that the brilliance of his mind was recognized early in his life, or that, in his college days, like Newton, he caught the eye of a professor willing to recommend him for his own job as a man of "unparalleled genius". Like Newton, he was not a precocious child. In fact, he was slow to learn to speak, and his parents feared he might be retarded. In high school, he was told that he would never amount to anything. After he graduated from college, he had difficulty finding a permanent position for lack of recommendations - he later recalled that he had not been in the good graces of any of his former professors (one of them called him a "lazy dog").

He was born in Ulm, Germany, in 1879. The following year, his family moved to Munich, where his father and an uncle started a small electrochemical factory. After his father's business failed, in 1894 his family moved to Milan, Italy. Albert, then 15, was left behind in the care of relatives so that he could complete the requirements for the diploma he would need to enter a university.

Albert, however, paid increasingly less attention to his studies, until he was asked to leave the school. He then rejoined his family in Milan. In 1896, after passing a special examination, he was admitted to the Swiss Polytechnic Institute of Zurich in a 4-year program in physics and mathematics.

Einstein was soon bored by most of his courses at the Polytechnic. His insatiable appetite for reading seldom extended to the books required for his courses. Gifted with charm and wit, he made many friends, and attended cafes much more regularly than lectures. Among his new friends was classmate Marcel Grossman, a bright mathematics student. Having skipped so many classes, Einstein would have flunked his graduation examinations, if he had not been rescued by Grossman, who lent him his very careful notes. Einstein crammed for the examinations, and graduated in 1900.

In 1903, he married a former classmate at the Polytechnic; the following year, the first of their two sons was born. For two years, he got by with tutoring and part-time teaching, until his friend Marcel, through family connections, was able to get him an undemanding job at the Swiss Patent Office in Bern.

During the next three years, in the back room of his small apartment, Einstein conceived the revolutionary ideas that led to his prodigious outpouring of papers in 1905. At work, he quickly mastered his job of examining patent applications, and was able to bootleg time for his own research.

After his papers were published in 1905, the value of his work was soon recognized by some of the most distinguished scientists in Europe, particularly the German physicist Max Planck. This, however, did not catapult him to the international fame he would achieve later on. For a while, Einstein continued to work at the Patent Office, which he later called "that secular cloister where I hatched my most beautiful ideas".

In 1909, he left the Patent Office. Over the next few years, he moved to increasingly more attractive university positions, whose demands, however, interfered with his research. Finally, in 1914, at the insistence of Max Planck, he accepted a professorship at the University of Berlin, where he would be free to devote himself to his research.

After the outbreak of World War I, Einstein was an outspoken critic of German militarism. He continued, however, to be primarily absorbed in perfecting his General Theory of Relativity, on which he had been working for nearly ten years. This theory, which was published in 1916, was later described as "the greatest feat of human thinking about nature, the most amazing combination of philosophical penetration, physical intuition and mathematical skill". [2]

One of the predictions of the new theory was that light coming from a star, when passing by the Sun, would bend by some amount that Einstein had computed. This effect could be observed only during a solar eclipse. In spite of the war, news of Einstein's work spread quickly. A copy of his publication was submitted by a Dutch astronomer to the British Royal Astronomical Society, whose secretary was Arthur Eddington, a brilliant astrophysicist. Although there was much hostility among British scientists against German research, Eddington, who was a pacifist, embarked in the study of Einstein's work, in spite of the very difficult mathematics involved.

Eddington, then 34, had declared he would seek deferment from the draft as a conscientious objector. Instead of being sent to a labor camp, however, he was sent on an arduous trip to Africa to observe a full solar eclipse, which might confirm Einstein's theory.

In November 1919, the Royal Society of London announced that its scientific expedition to Africa had completed calculations that verified Einstein's prediction. Overnight, his name became a household word. Worldwide fame brought thousands of invitations to lecture and to write, most of which he turned down. For three years, however, he did travel widely to lecture on relativity, often arriving at his destination by third-class rail, with a violin tucked under his arm.

Although the 1920's were years of wide acclaim, Einstein concentrated on his new quest: to find the mathematical formulation of a "unified field theory" that would encompass both gravitation and electromagnetism, just as Maxwell had succeeded in unifying electricity and magnetism under a single theory. This would have been the first step, he felt, toward discovering the common laws that govern the behavior of everything in the universe, from electrons to planets. It turned out, however, to be a fruitless search, which occupied the rest of his life.

Einstein remained in Berlin until Hitler came to power in 1933. Although greatly admired by some Germans, he was disliked by the Nazis, who resented his Jewish heritage, outspoken pacifism and active support of Zionism. Soon after Hitler became Chancellor of Germany, Einstein renounced his German citizenship and left the country. He later accepted a full-time position at the newly founded Institute for Advanced Studies in Princeton, New Jersey.

He remained in Princeton for more than 20 years until he died. He lived with his second wife in a simple house, and most mornings walked to the Institute. In 1950, he published a new version of his unified field theory but, like the first version published in 1929, it was found untenable by most physicists. In comparison to his eminence a generation earlier, Einstein was now virtually neglected. In 1955, at the age of 76, he died in his sleep at Princeton Hospital.

THE SPECIAL THEORY OF RELATIVITY

Einstein's Postulates

In the years preceding his 1905 publications, Einstein had been largely out of the mainstream of physics. Working in the isolation of his small apartment in Bern and the "intellectual cloister" of his job at the Patent Office, he had not interacted with any professional physicists, which may have kept his imagination free of constraints. What guided his work was an unshakable faith in the unity of the universe and its laws.

Einstein was firmly convinced that the same laws of electromagnetism and optics "will be valid for all frames of reference for which the equations of mechanics hold good." He proposed that this conjecture be adopted as a postulate (something to be accepted on faith). Like Poincare's principle, Einstein's postulate states that the laws of both mechanics and electromagnetism are invariant to (do not change with) straight uniform motion without rotation. It represents a broader restatement of Newton's special principle of relativity, which applied only to the laws of mechanics. Einstein's theory of relativity is as much about the absolute nature of the laws of physics as it is about the relative nature of motion.

Einstein proposed then a second postulate: in empty space, light always propagates in all directions with the same speed c, regardless of whether its source or its observers are at rest or moving.

The two principles of the *invariance of the laws of physics* and the *constancy of the speed of light* could not be reconciled with Newtonian mechanics, which predicted that observers on different inertial frames would measure different values for the speed of light. Very boldly, however, Einstein adopted both principles as his fundamental premises, and then proceeded to show that their incompatibility with Newtonian mechanics could be resolved by drastically reevaluating notions about time and distance that had always been taken to be self-evident.

Gedanken Experiments

Einstein developed his theory with the aid of relatively simple mathematics. To explain his results, he used as examples what he called "gedanken experiments" (thought experiments), which are performed in one's mind, without too much concern for the practical details of implementation. In his examples, he used a train, which was then the fastest available means of locomotion. Today, he probably would have used a space ship.

In the "experiments" to be described, we will consider a phenomenally fast train moving at uniform speed on rails that are fixed to an inertial "platform" and go on and on, perfectly straight. The speed of this supertrain is some appreciable fraction of the speed of light.

There are two observers. One lives somewhere on the "ground" by the railway track. The second observer lives on the train, which is his “inertial platform." In order to keep these two observers clearly distinct in our minds, the observer on the ground will be called Stan as the one whose platform supposedly stands still. The observer on the train, instead, will be called Moe as the one whose platform is supposedly moving.

Actually, all we can say about the two observers is that their platforms are in relative motion with respect to one another. Stan claims that he and his whole platform are at rest, and that Moe and his train are moving at some speed s in the direction of the track. Moe, on the other hand, claims that he and the train are at rest, and that Stan and his whole platform, track included, are moving at the same speed s, but in the opposite direction. Neither can back up in any way his claim of being at rest.

Let us assume that, initially, everything is at rest. Stan and Moe know that there will be days when they will wake up and find themselves in relative motion, either because the train and Moe are moving, or because the platform, the track and Stan are moving. In anticipation of this, in order to conduct some experiments, they equip themselves with identical yardsticks, and with identical clocks.

They also build a hangar that fits the train exactly, see **Figure 12.1a**. At the two ends of the hangar and the train, they install two sets of special devices. One set will generate a momentary flash of light at the very instant the left ends of both hangar and train are in line; we will call this the L flash. Similarly, the other set will generate a momentary flash of light at the very instant the right ends of both hangar and train are in line; we will call this the R flash.

Length Contraction with Motion

In his work, Einstein stressed the fundamental role that the concept of simultaneity plays in the measurement of a length. Such a measurement must occur all at the same time; for instance, by matching simultaneously both ends of a rod to a ruler.

In Newtonian mechanics, it was considered self-evident that, regardless of their state of rest or motion, any number of observers would always agree on whether two events occurred at the same time, or which occurred first. They would also agree that the length of an object would not be affected by its being at rest or in motion. The following gedanken experiment shows that agreement on either point is not to be taken for granted.

Stan and Moe are now in relative motion with respect to one another. Stan claims the train is moving toward the hangar. Moe, instead, claims the hangar is moving toward the train. In any case, they want to compare the lengths of the train and the hangar in order to determine how length in the direction of motion is affected by motion.

One way they can do this on the fly is to take advantage of the L and R flashes. Moe places himself at the midpoint of his train; Stan, at the midpoint of his hangar (on the roof, safely out of the way). Using mirrors, each can see light flashes coming from either direction. Each is at a midpoint, and each agrees that the time it takes a light flash to reach him from one end of the train (or hangar) is exactly the same as from the other end. If the two flashes occur simultaneously, an observer at rest at exactly the midpoint will see the two flashes at the same time.

Keep in mind that each flash will propagate in all directions with a very high but still finite speed (186,000 miles per second). Neither observer will see a flash the very instant it is generated. When he sees it will depend on whether he is at rest, moving toward it, or away from it, and how far he is.

According to Einstein’s equations, Stan will see the flash from the left (L) before the flash from the right (R), whereas Moe will see R before L. Each is convinced the other is wrong. Each swears that he has rechecked with his yardstick the length of his train or hangar, and found it to be what it always was. Just what is going on? To follow their different points of view, see **Figures 12.1b, c and d**.

The disagreement between Stan and Moe as to which flash occurred first would not arise if they were not in relative motion. The disagreement would not arise either if light propagated instantly (with infinite speed in zero time). Since light propagates at a finite speed, two observers in relative motion will inevitably disagree about the sequence of events, and there is no way of determining which of the two is wrong. Each is right within his own frame of reference.

If two events occur at the same time *and* at the same place, they will be viewed as simultaneous in all reference frames. Disagreement about the simultaneity or sequence of two events can occur only when they are some distance apart. In this case, when large distances are involved, an observer can be present at best at only one of the events, and must obtain his information about the other event by means of some signal. Light is an extremely fast signal, but it too takes time to travel from one place to another.

Stan and Moe - who disagree on their measurements of lengths in the direction of their relative motion - are always in agreement when measuring lengths in a perpendicular direction, for instance, the height or the width of the train or the hangar.

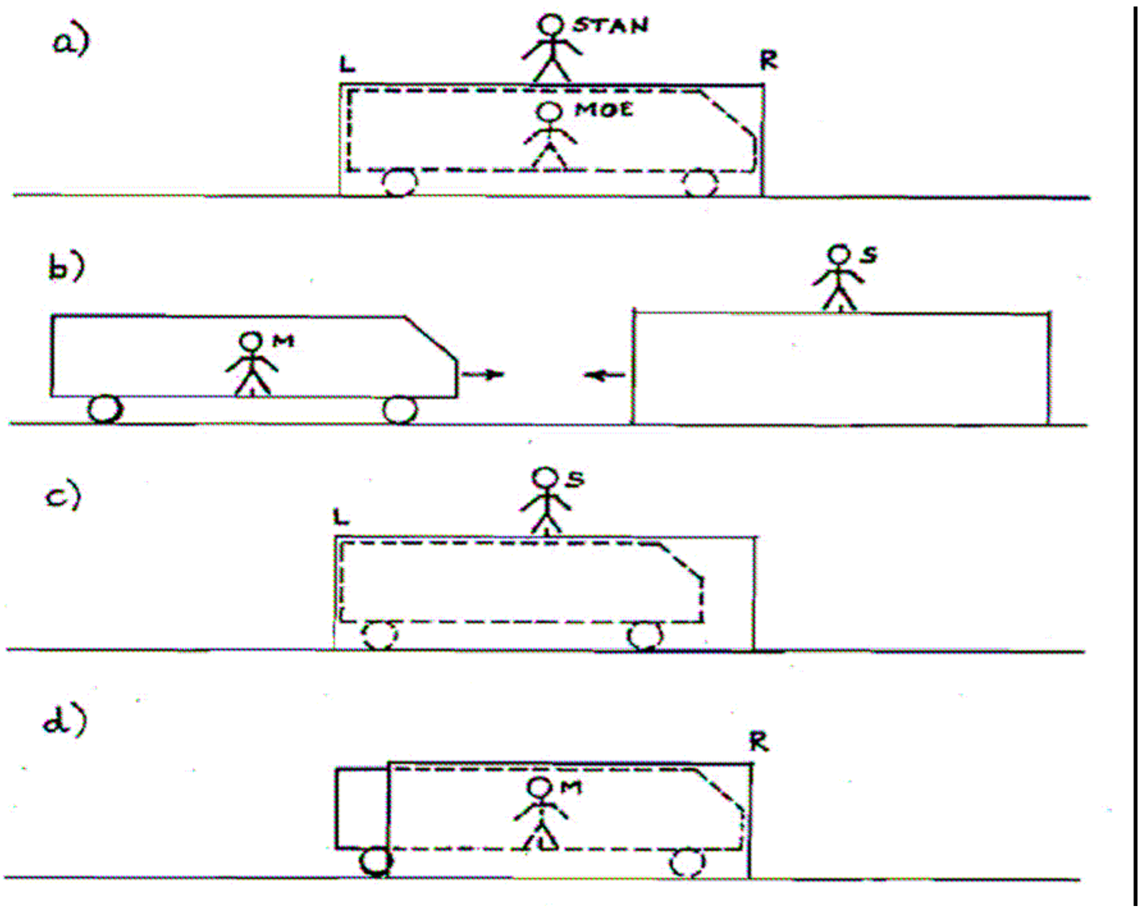

Figure 12.1 - Length contraction with motion

a) Stan and Moe both at rest. b) Stan and Moe in relative motion.

c) <u>Stan's viewpoint</u> - Stan and the hangar have not moved; Moe's train came speeding toward the left end of the hangar. Flash L occurred before flash R, which means that the train has become shorter. Once L occurred, Moe was moving toward R, which occurred second, and away from L, which occurred first. That is why he saw R before L. Since Moe claims he has rechecked the length of his train with his yardstick, it must mean that the moving train and everything on it, including the yardstick, have contracted (shrunken) in the direction of the motion, but Moe is unable to see the change.

d) <u>Moe's viewpoint</u> - Moe and the train have not moved; Stan's hangar came speeding toward the right end of the train. Flash R occurred before flash L, which means that the hangar has become shorter. Once R occurred, Stan was moving toward L, which occurred second, and away from R, which occurred first. That is why he saw L before R. Since Stan claims he has rechecked the length of his hangar with his yardstick, it must mean that everything on Stan's moving platform, including the yardstick and the hangar, has contracted in the direction of motion, but Stan is unable to see the change.

Time Dilation

Two observers in relative motion will disagree not only on the sequence of events, but also on the very rate at which time flows! This will be shown by another gedanken experiment performed by Stan and Moe using two identical special clocks. See **Figure 12.2.**

Stan makes the startling discovery that Moe's clock, just by virtue of the fact that it is moving, runs slower than his, and that, the greater the speed of the train, the greater the discrepancy between the rhythms of the two clocks. By the same reasoning, however, Moe - who believes that he is at rest, and that Stan is moving - has concluded that Stan's clock runs slower than his.

If, at this point, you feel totally confused, and ready to dismiss the whole thing as nonsensical, you will join a very large crowd of people who have had the same reaction. It is a fact, however, that the theory of special relativity has been verified countless times at speeds close to the speed of light.

You might wonder whether the strange conclusions reached about time have something to do with the peculiar light clocks that have been considered. Keep in mind, however, that one of Einstein's two articles of faith states that the laws of physics are invariant from one inertial frame to another, which requires that it be impossible to detect the motion of one's own inertial frame of reference.

Suppose now that, in addition to the light clocks, Stan and Moe are given two stately grandfather clocks, or whatever you would consider to be "real" reliable clocks. If a discrepancy were found between a light clock and a "real" clock, it would be an indication of motion. But this would violate the very article of faith that was accepted at the beginning. If anything at all changed because of straight uniform motion, one would be able to tell he was moving. If we accept Einstein's postulates, we must agree that the "real" clocks will behave just like the light clocks.

In the words of Nobel Laureate Richard Feynman, "If all moving clocks run slower, if no way of measuring time gives anything but a slower rate, we shall just have to say, in a certain sense, that 'time itself' appears to be slower in our supertrain. All the phenomena there - the man's pulse rate, his thought processes, the time it takes to light a cigar, how long it takes to grow up and get old - all these things must be slowed down in the same proportion. Otherwise, he would be able to tell that he is moving."[1]

Although two observers in relative motion disagree on their measurements of time and distance, they agree on the value they measure for the speed of light. Each, however, claims that the other uses the wrong way to arrive at the correct result.

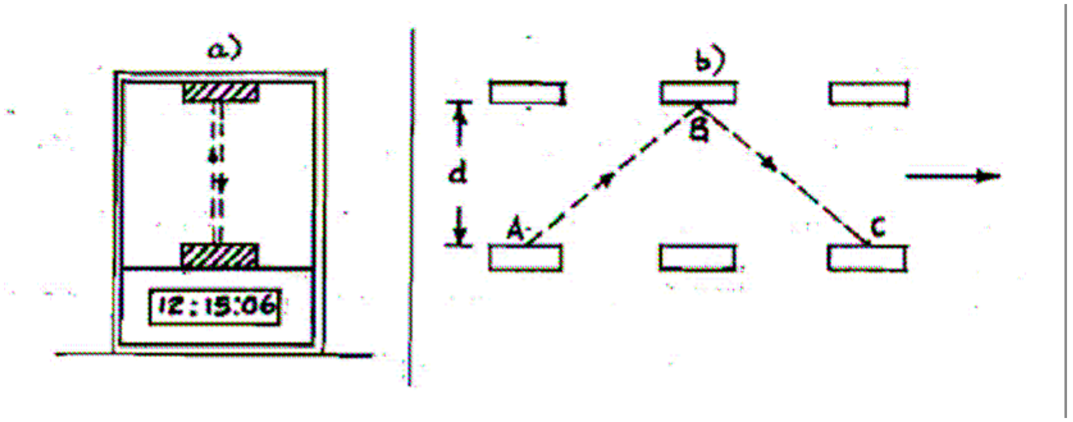

Figure 12.2 - A light clock - Time dilation with motion

The "light clock" shown in a) above consists of an upper and a lower element separated by a fixed distance. The lower element generates a short pulse of light, whose waves propagate in all directions. When the pulse reaches the upper element, which is a plain mirror, it is reflected back toward the lower element. Here, a new pulse is emitted the same instant the old one is received. Also, some kind of a "click" is generated that can be counted to accumulate time into seconds, minutes and so on. Basically, this is no different from an ordinary clock, which counts the time intervals between consecutive swings of a pendulum, or consecutive oscillations of a balance wheel - time intervals that are believed to be always identical.

Stan and Moe have mounted their identical clocks in an upright position, perpendicularly to their relative motion. When they are not in relative motion, they have satisfied themselves that the two clocks keep exactly the same time. When they are in relative motion, each clock looks perfectly normal to its owner.

Stan, who believes to be at rest, notices something peculiar about Moe's clock, see b) above. Since the train is moving, by the time a light pulse reaches the upper element of the clock, the latter has moved somewhat. The lower element has moved even further by the time the reflected pulse gets back to it. Instead of traveling the straight up and straight down distance 2 x d, the pulse has traveled the longer distance ABC. The time interval between "clicks" has dilated (become longer), which means that Moe's clock is running slow. Its readings (say, in seconds) are smaller than they should be. For Moe, however, it is Stan's clock that runs slower!

The "Gamma" Correction Factor

In a general way, we have seen how, according to Einstein, Stan and Moe differ in their assessments of time and distance. Each sees the other make measurements of time that are too small, and measurements of distance (in the direction of relative motion) that are too large. Let us see what the quantitative extent of the discrepancies is.

Time Measurements

If Moe says that some interval of time is, say, 100 seconds according to his clock, Stan "knows" that he must use some correction factor, say 2: the "correct" number is 200 seconds (according to Stan's clock). We will call this correction factor "gamma". Einstein concluded that "gamma" depends on the ratio of the speed of relative motion s divided by the speed of light c, or s/c.

If s is much smaller than c, "gamma" is only very slightly larger than 1. For instance, if our supertrain moved at the speed of the Earth around the Sun (about 19 miles/sec.), s would be only one tenthousandth (or 1% of 1%) of c, and gamma would be 1.000000001. It would exceed 1 by only one part in 100 million. There would be essentially no disagreement between Stan and Moe. That is why, at ordinary speeds, we do not notice any relativistic effects.

Only when s is an appreciable percentage of c does the correction factor become significant. As s gets closer and closer to c, "gamma" gets larger and larger, growing faster and faster. For instance, "gamma" is 1.25 when the ratio s/c is 60%; 2.29 when the ratio is 90%; 7.09 when the ratio is 99%. It jumps to 5000 when the ratio s/c is 99.99%; it becomes infinite when s is equal to c.

Length Measurements

Using his yardstick, Moe measures some object on his train to be, say, 20 inches long in the direction of relative motion. Stan, however, "knows" that both the object and the yardstick have shrunken, and that to get the correct number of "real" inches, he must *decrease* Moe's number by some correction factor. It turns out that this correction factor is the same "gamma" just discussed, except that now we must divide by it. For our example, Stan claims that the "correct" length is 20/2=10 inches (according to Stan's yardstick).

Paradoxically, Moe makes exactly the same kind of corrections, using the same gamma factor. Moe believes that he is at rest and Stan is moving. Stan's clock is the moving clock that runs slow, and Stan's yardstick is the moving yardstick that has shrunken.

Relativistic Mass

Einstein's faith in his two postulates led him to revise drastically the concepts of time and distance. Since these concepts are the very foundation of how we describe motion, the whole science of mechanics would appear now to rest on very shaky ground. And yet, for more than two hundred years, Newton's equations had yielded results in remarkable agreement with the observed motions of the planets.

How could this be? The answer is that Newton and his successors had dealt only with a range of phenomena in which the speeds involved were much smaller than the speed of light.

The changes that need to be made in Newtonian mechanics center on the concept of mass. In the Principia, Newton defined mass as the "quantity of matter" contained in a body. He assumed it was always constant, whether the body was at rest or in motion.

Einstein tells us, instead, that the mass of a body is not constant but increases with speed. If a given body is at rest with respect to some observer, the latter will measure a certain mass, which is called the "rest mass" of the body. This is the value of mass that Newton would have used.

If the body, however, is moving at some speed s with respect to the same observer, the latter will measure a larger mass equal to the "rest mass" times the now familiar correction factor "gamma".

You will recall that, as long as the speed s is much smaller than the speed of light c, "gamma" is for all practical purposes equal to 1. Mass in motion and rest mass are then essentially identical. Under these conditions, we can disregard the correction factor introduced by relativity, and use Newton's equations with very satisfactory results. As the speed s becomes a sizeable fraction of c, however, the gamma factor increases more and more: the mass of the moving body offers an ever increasing inertia.

The Cosmic Speed Limit

In Newton's mechanics, since mass is constant, if we want a given body to increase its speed, say, by one foot per second in one second, it does not matter whether the body is starting from 100 or 100,000 feet per second. For a given mass, the force to be applied depends only on the acceleration, the change in speed per unit time.

This is not so, however, in Einstein's mechanics. The nearer a (relative) speed is to that of light, the more difficult it becomes to increase it further. A body resists a change more strongly not only when its rest mass is greater, but also when its speed is greater.

If a force is applied to a body, both the mass and the speed of the body will increase, but not at the same rate. At slower speeds, mass increases more slowly than speed; at higher speeds, mass increases much faster than speed. The speed of the body never exceeds the speed of light c, but gets ever closer to it.

When the speed s is equal to c, the mass becomes infinite: no further acceleration is possible. Thus, the speed of light c represents a cosmic speed limit, whose enforcement is automatically assured by the way mass increases with speed. No object can be accelerated beyond the speed of light.

Mass-Energy Equivalence

According to Newtonian mechanics, if we apply a constant force to a particle, the particle keeps picking up speed at a constant rate, increasing its kinetic energy. This kinetic energy is "paid" for by the "work" we do by applying the force. This is an adequate description of things as long as the speed of the particle is much smaller than the speed of light.

If we apply a force to a particle moving at very high speed, the speed increases at a slower and slower rate. Since we are still doing "work", the particle's energy must be increasing, but less and less of it is going into kinetic energy. Where is the rest of the energy we are "paying" for with our "work"? Since the mass of the particle is also increasing at the same time, we might suspect that the mass itself represents energy. This indeed turns out to be the case. Einstein showed that the total energy of the particle is

$$\text{Energy} = \text{mass x the square of the speed of light} = m \times c^2$$

The theory of relativity leads us to conclude that the mass of a body is a measure of its energy content. Energy, whatever its form, has mass. Mass can be converted into energy, and vice versa. Note that, when multiplied by the square of the speed of light (a very large number), even a small mass can yield an enormous amount of energy.

Since the atomic bombing of Hiroshima, Einstein's famous equation has been popularly associated with nuclear energy. Actually, Einstein had no direct involvement in the development of nuclear energy for either war or peace. His equation applies to all forms of energy. In nuclear reactions, in which mass is partially converted into energy, we can have mass changes of about one part in 1000. On the other hand, heating water from its freezing point to its boiling point increases its mass by merely one part in 100 billions.

Before relativity, physics recognized two fundamental laws of conservation, which appeared to be independent of one another: the conservation of energy and the conservation of mass. The theory of relativity united them into a single law: the conservation of mass-energy.

SPECIAL RELATIVITY IN A NUTSHELL

In his theory of special relativity, Einstein reached the following major conclusions:

- Maxwell's equations, which were at first under suspicion, actually required no change.
- It was Newton's laws of motion that needed to be changed because of a

new definition of mass, no longer viewed as a constant, but as increasing with speed.

- Maxwell's original equations and Newton's modified equations are both invariant from one inertial platform to another.
- Measurements of time and distance are relative to the observer's platform. To somebody "at rest" looking on somebody else's "moving" platform, time there appears to flow more slowly, and distances appear to shrink in the direction of relative motion. Events that are viewed on one platform as simultaneous, may not appear so in another platform. An event that is seen on one platform to occur *before* some other event, may be seen to occur *after* on another platform.
- No object can be accelerated past the speed of light.
- Mass and energy are equivalent.

We considered before the question: If a train is moving away from you at 20 miles/hour and a jogger on the train is running at 5 miles/hour, also away from you, what is the speed of the jogger relative to you? Newtonian mechanics told us: "Obviously, 25 miles/hour, the sum of the two speeds."

This simple way of combining speeds is no longer valid in general. It is still essentially valid for speeds that are small relative to the speed of light, but not for higher speeds. When two speeds are combined by Einstein's rules, even if either or both are equal to c, we still get no more than c! The Michelson-Morley experiment failed to show a winner because it was based on the wrong assumption that speeds could be combined by addition or subtraction.

It should be noted again that all aspects of special relativity have been repeatedly confirmed. Physicists studying subatomic particles work daily with objects traveling at speeds close to the speed of light; their experiments confirm Einstein's predictions. At ordinary speeds, on the other hand, the effects of relativity are totally negligible.

Since special relativity, space and time are viewed not as two independent entities, but as a single 4-dimensional entity, "space-time", something that our three-dimensionally oriented brain finds most difficult, if not impossible, to visualize.